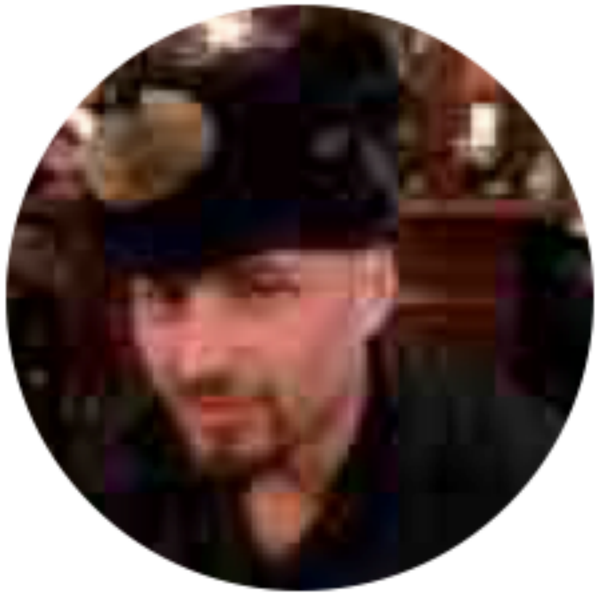
Noether's Second Theorem and Strange Metals

Thanks to: NSF

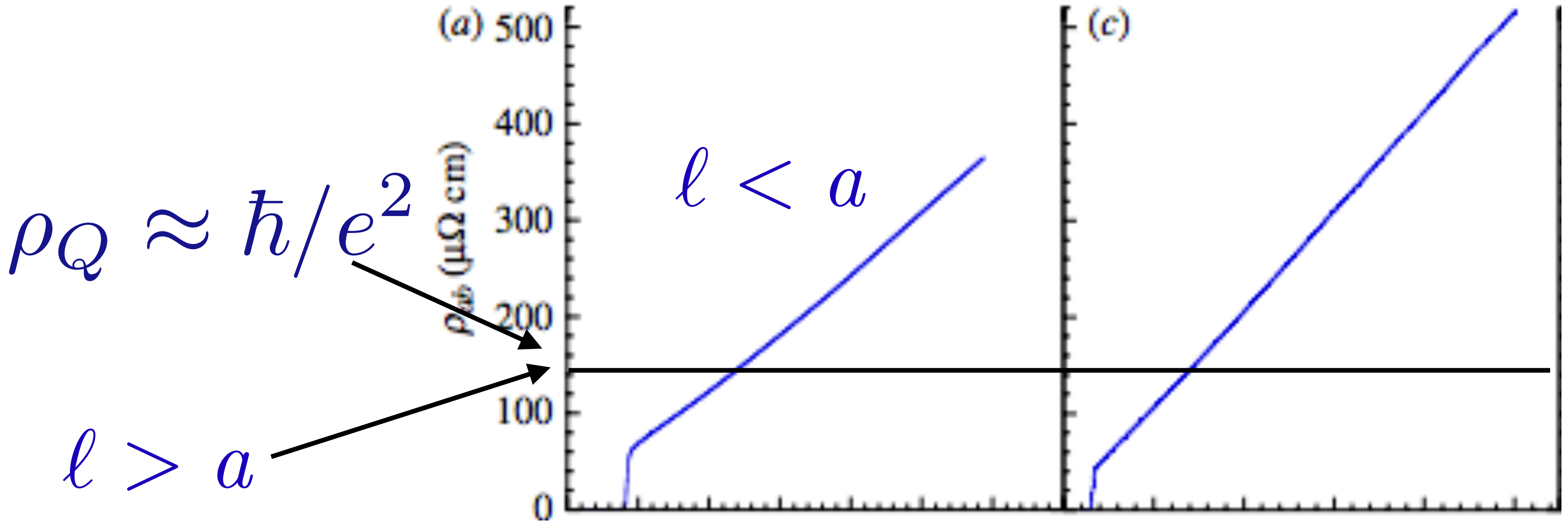
Gabriele La Nave



Rev. Mod. Phys. 2019 (arXiv:1904.01023)
CIMP 2019
Adv. Th. Math. Phys. 2019

Kridsangaphong Limtragool

T-linear resistivity

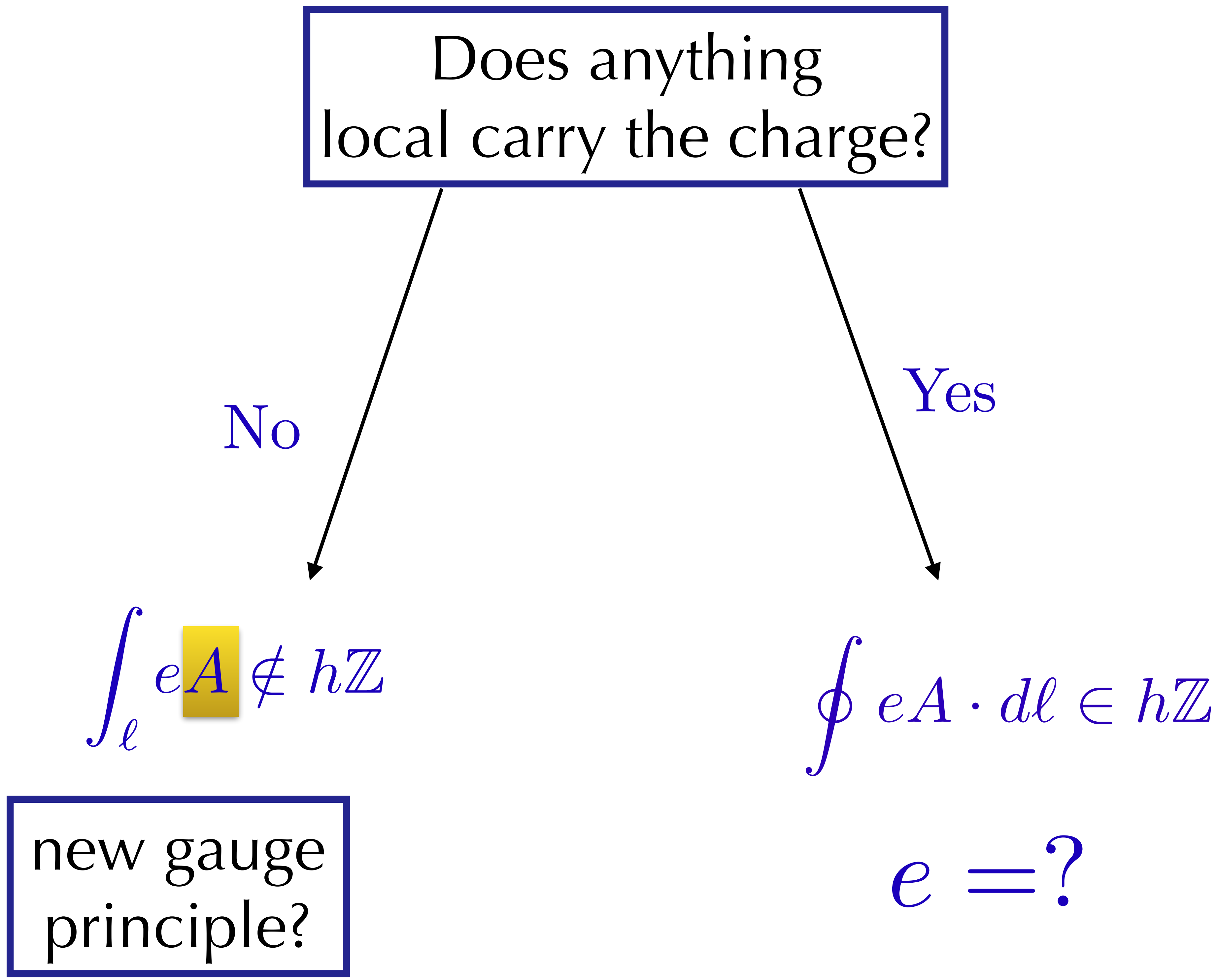


$\rho > \rho_Q$

Violates MIR limit

electrons not charge carriers

?


Does anything local carry the charge?
No
Yes
$\int_\ell eA \notin h\mathbb{Z}$
$\oint eA \cdot d\ell \in h\mathbb{Z}$
new gauge principle?
$e = ?$

Can you mess with A?

Noether's First Theorem

$U(1) \longleftrightarrow qA \to qA - q\partial_\mu \Lambda$

$\psi' = e^{iq\Lambda}\psi$

$[q\Lambda] = 0$

$[qA] = 1$

$[A] = 1$

$$S = \int d^d x (J_\mu A^\mu + \cdots)$$

$$S \to S + \int d^d x J_\mu \partial \Lambda$$

fixes dimension of current

$[d^d x J A] = 0$

$\partial_\mu J^\mu = 0$

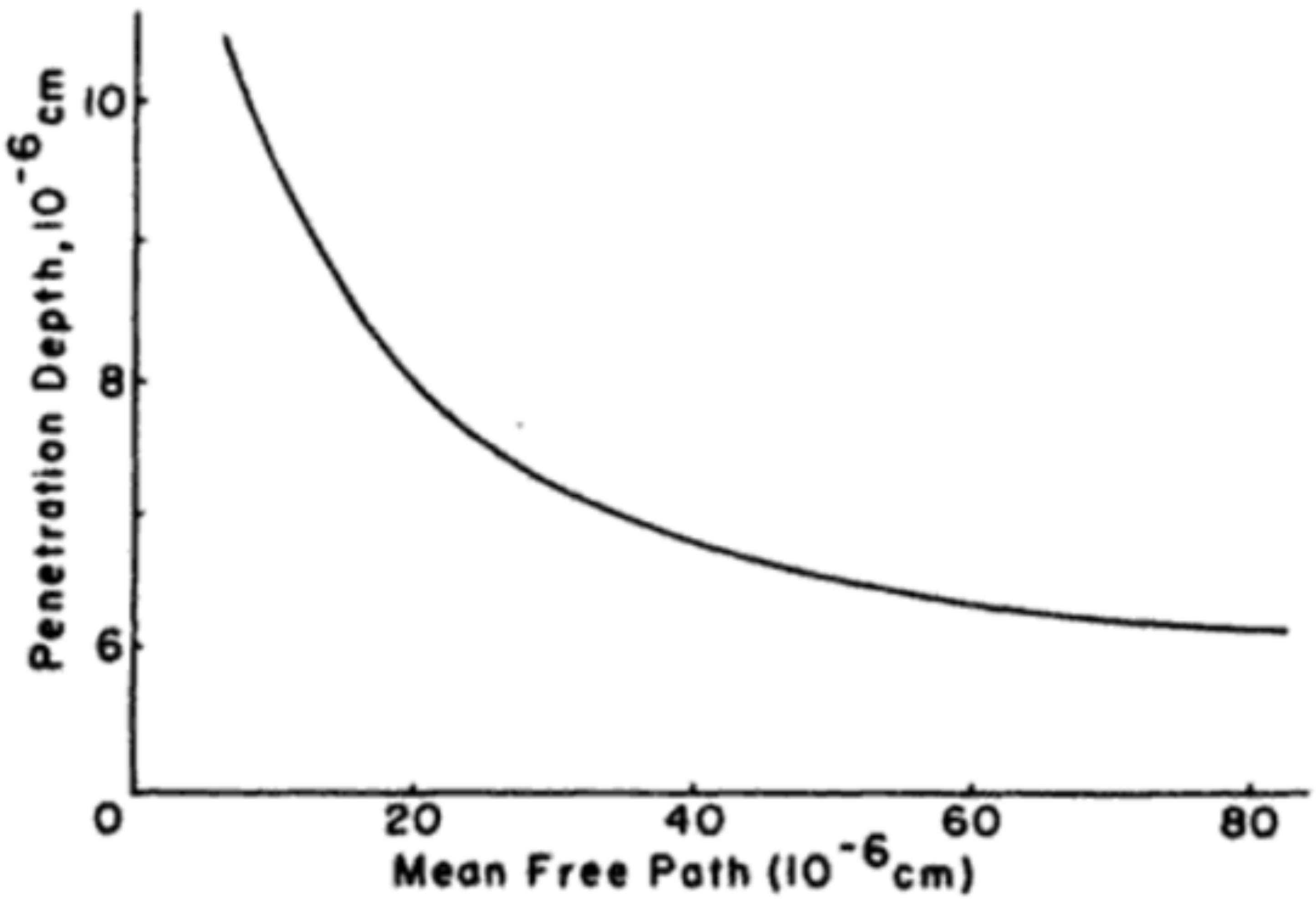
$[J] = d - 1$

Noether's Thm. I

current conservation

Are there exceptions?

Pippard's problem



$$J_s \neq \frac{-c}{4\pi\lambda^2} A$$

London Eq.

failure of local London relations

Superconductivity ala Weinberg

$$U(1) \to \mathbb{Z}_2$$

$$A_\mu - \partial_\mu \phi = 0$$

$U(1)/\mathbb{Z}_2$

$$\nabla \phi - A = 0$$

stable equilibrium

around minimum

$$L_m = L_{m0} - \frac{1}{2} \int C^{\mu\nu}(\mathbf{x}, \mathbf{x}')(\mathbf{A}_\mu(\mathbf{x}) - \partial_\mu \phi(\mathbf{x}))$$

$$\times (A_\nu(\mathbf{x}') - \partial_\nu \phi(\mathbf{x}'))\mathbf{d^3x'd^3x} + \cdots$$

Pippard Current

$$J_\mu(\mathbf{x}) = \frac{\delta \mathbf{L_m}}{\delta \mathbf{A}_\mu} = -\int \mathbf{C}^{\mu\nu}(\mathbf{x}, \mathbf{x}')(\mathbf{A}_\nu(\mathbf{x}') - \partial_\nu \phi(\mathbf{x}'))\mathbf{d^3x}'$$

Pippard kernel

$$J_s = -\frac{3}{4\pi c \xi_0 \lambda} \int \frac{(\vec{r} - \vec{r}')((\vec{r} - \vec{r}') \cdot \vec{A}(\vec{r}'))e^{-(\vec{r}-\vec{r}')/\xi(\ell)}}{(\vec{r} - \vec{r}')^4} d^3\vec{r}'$$

non-local

Units of Current

$$J_\mu(\mathbf{x}) = \frac{\delta \mathbf{L_m}}{\delta \mathbf{A}_\mu} = -\int \mathbf{C}^{\mu\nu}(\mathbf{x},\mathbf{x}')(\mathbf{A}_\nu(\mathbf{x}') - \partial_\nu \phi(\mathbf{x}'))\mathbf{d^3x}'$$

$$[J] = d - d_C - d_A$$

anomalous dimension

Standard Result

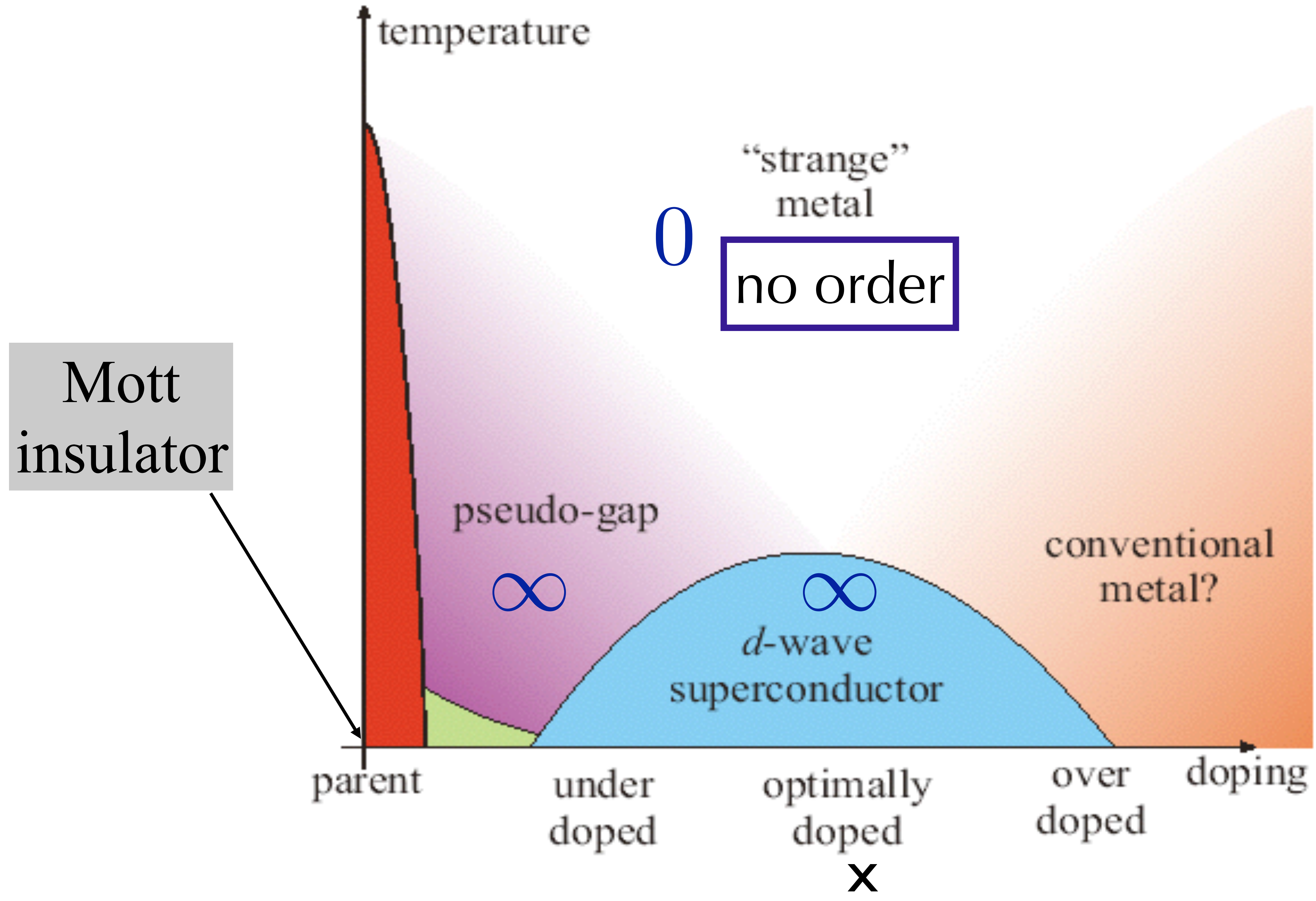
$$\delta(x_0 - y_0)[J_\mu(x), \phi(y)] = \delta^d(x-y)\delta\phi(y)$$

$$[J] = d - 1$$

Are there other examples of currents with anomalous dimensions?

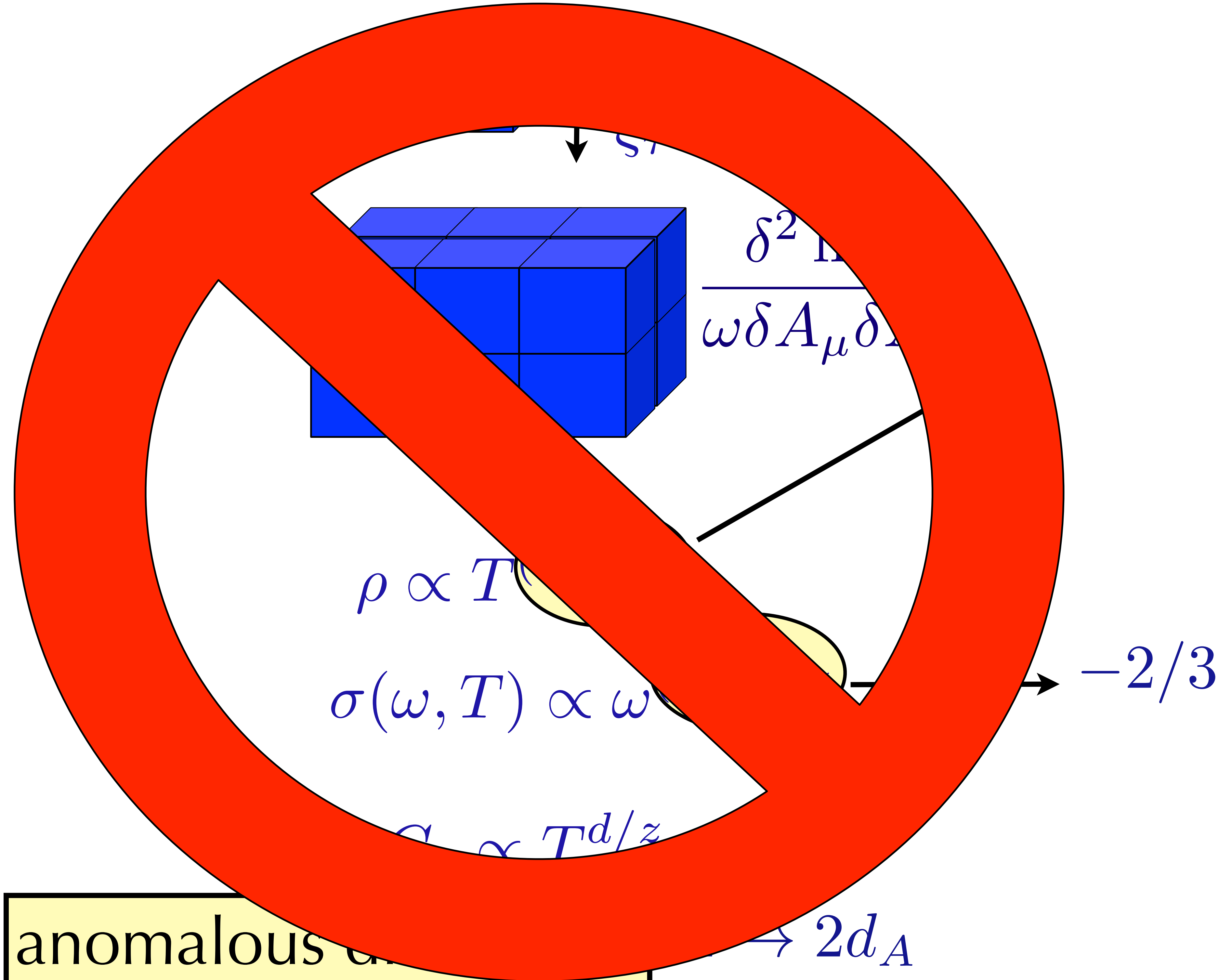
underlying electricity and magnetism?

is symmetry breaking necessary?


temperature
"strange" metal
0
no order
Mott insulator
pseudo-gap
∞
∞
d-wave superconductor
conventional metal?
parent
under doped
optimally doped
over doped
doping
x

why is the problem hard?

single-parameter scaling

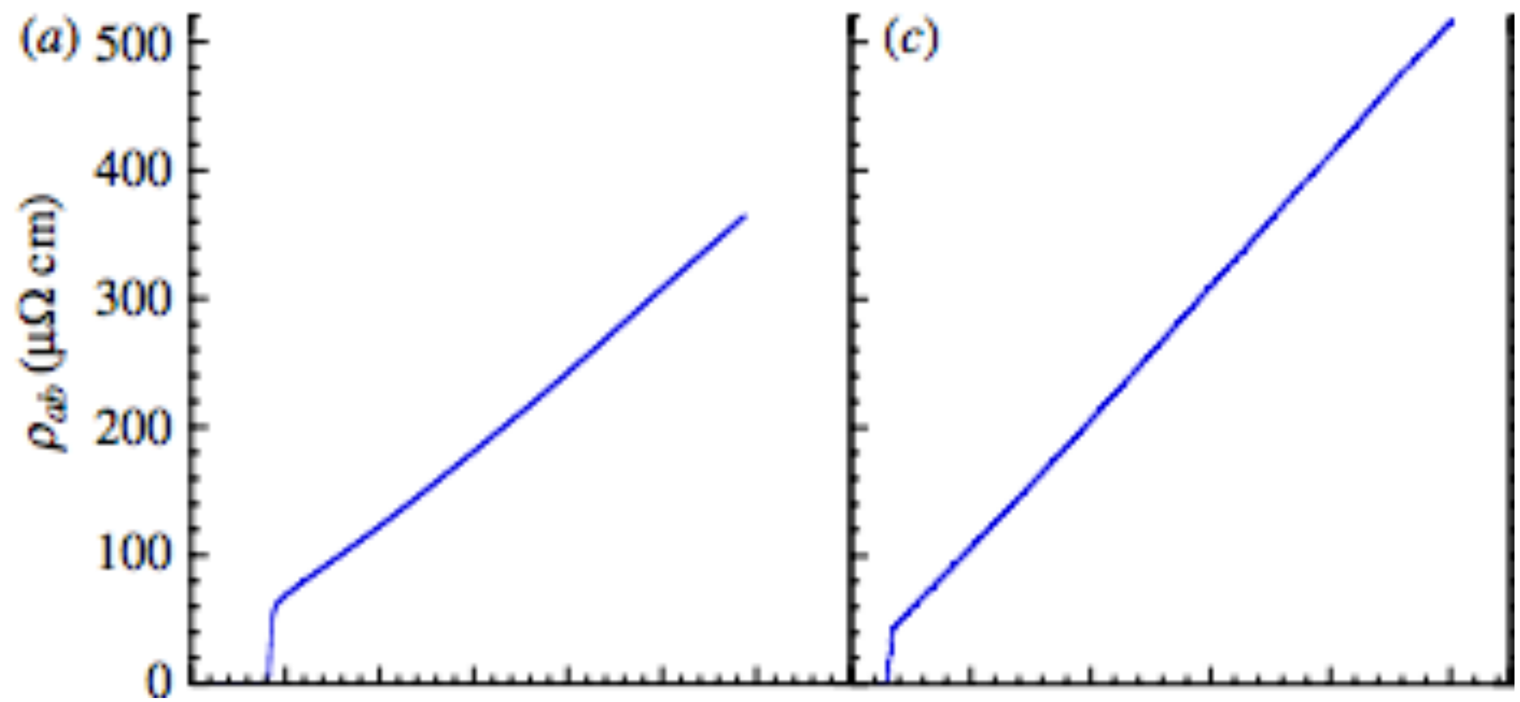


strange metal explained!

Hall Angle

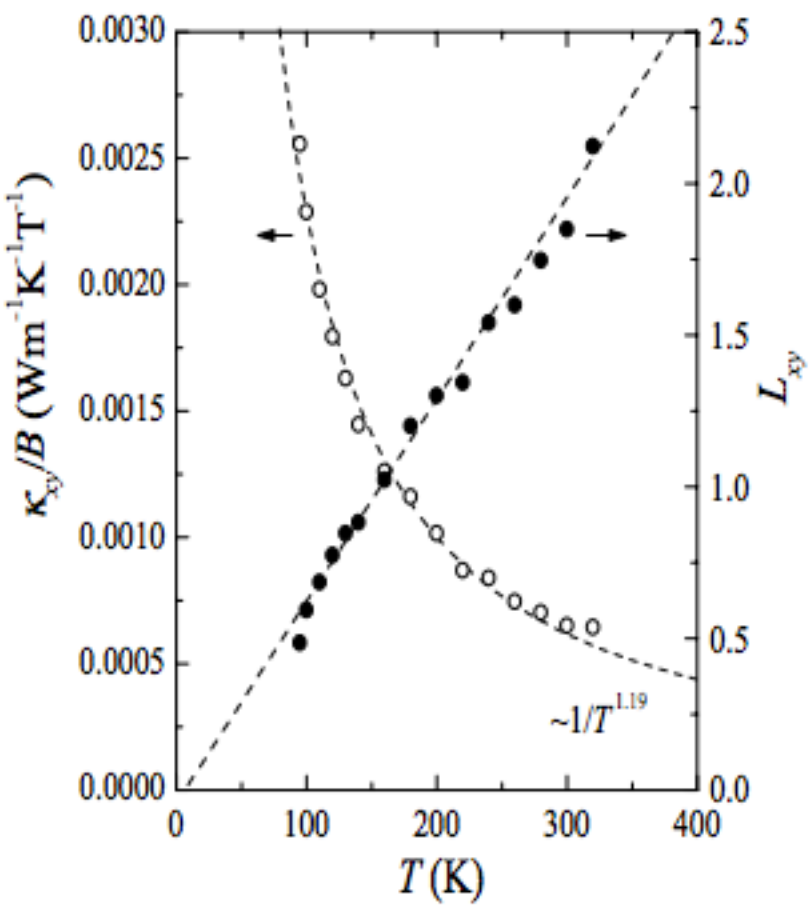
$$\cot\theta_H \equiv \frac{\sigma_{xx}}{\sigma_{xy}} \approx T^2$$

T-linear resistivity



Hall Lorenz ratio

$$L_{xy} = \kappa_{xy}/T\sigma_{xy} \neq \# \propto T$$



all explained if

$$[J_\mu] = d - \theta + \Phi + z - 1$$

$$[A_\mu] = 1 - \Phi$$

$$\Phi = -2/3$$

Hartnoll/Karch

strange metal

$$[J_\mu] = d - \theta + \Phi + z - 1$$

$$[A_\mu] = 1 - \Phi$$

$$\Phi = -2/3$$

$$[E] = 1 + z - \Phi$$

$$[B] = 2 - \Phi$$

note $\pi r^2 B \neq$ flux

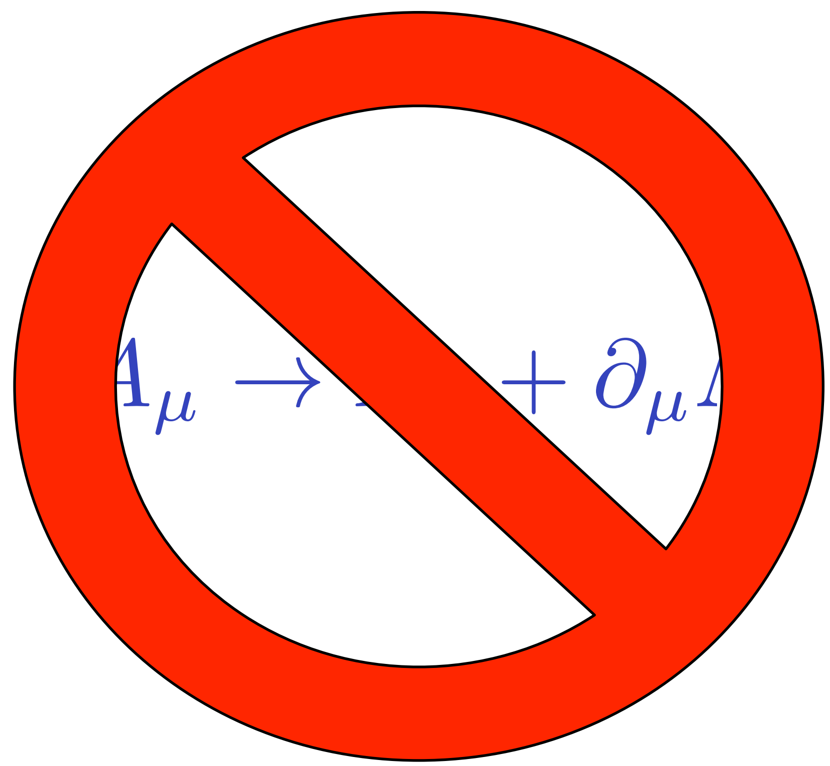
$$\oint A \cdot d\ell \notin h\mathbb{Z}$$

How is this possible - - if at all?

what is the new gauge principle?

if

$$[A_\mu] \neq 1$$



Noether's Second Theorem: precursor

hint

$$\partial_\mu J^\mu = 0$$

current conservation

what if

$$[\partial_\mu, \hat{Y}] = 0$$

new current

$$[\tilde{J}] = d - 1 - D_Y$$

gauge symmetry

$$\partial_\mu \hat{Y} J^\mu = \partial_\mu \tilde{J}^\mu = 0$$

possible gauge transformations

$$S = -\frac{1}{4}\int d^d x F^2$$

$$S = \frac{1}{2}\int \frac{d^d k}{2\pi^d} A_\mu(k)[\underbrace{k^2\eta^{\mu\nu} - k^\mu k^\nu}]A_\nu(k)$$

$$M^{\mu\nu}k_\nu = 0$$

zero eigenvector

$$ik_\mu \to \partial_\nu$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

family of zero eigenvalues

$$M_{\mu\nu} \underbrace{f k^{\nu}} = 0$$

generator of gauge symmetry

1.) rotational invariance

2.) A is still a 1-form

3.) $[f, k_{\mu}] = 0$

only choice

$$f \equiv f(k^2)$$

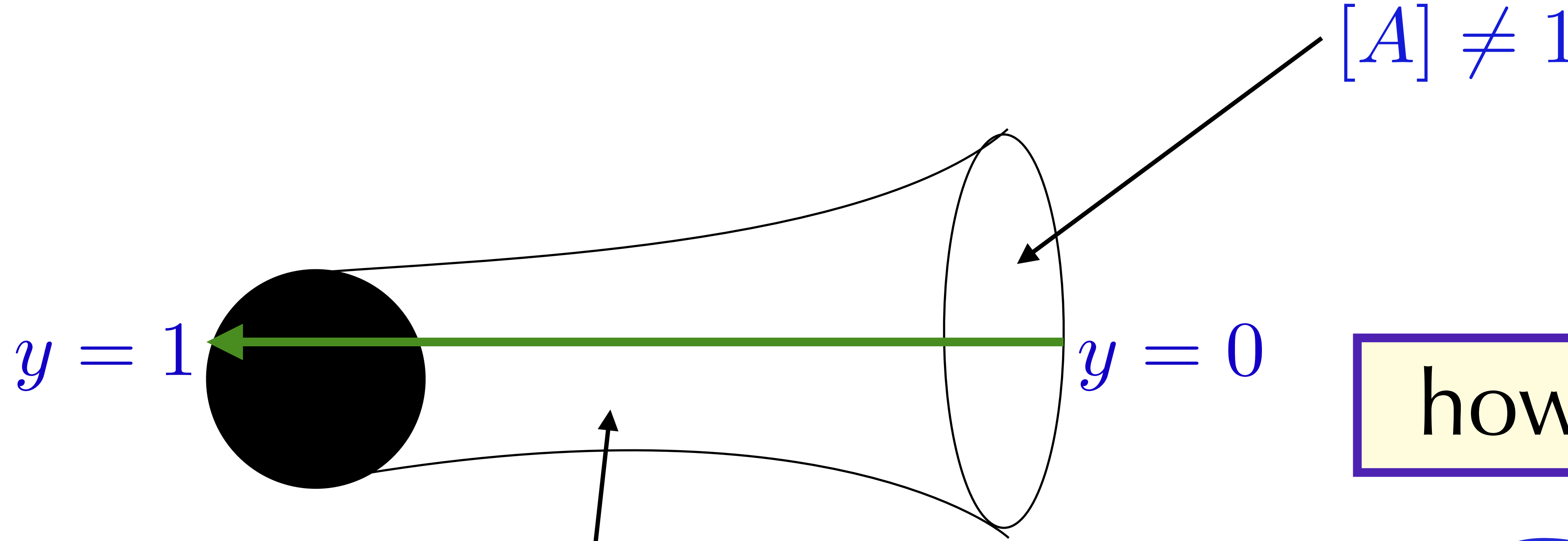
$$(-\Delta)^{\gamma}$$

$$A_\mu \to A_\mu + (-\Delta)^{\frac{\gamma-1}{2}} \partial_\mu \Lambda$$

$$[A_\mu] = \gamma$$

what kind of E&M has such gauge transformations?

claim: extra dimension



how?

?

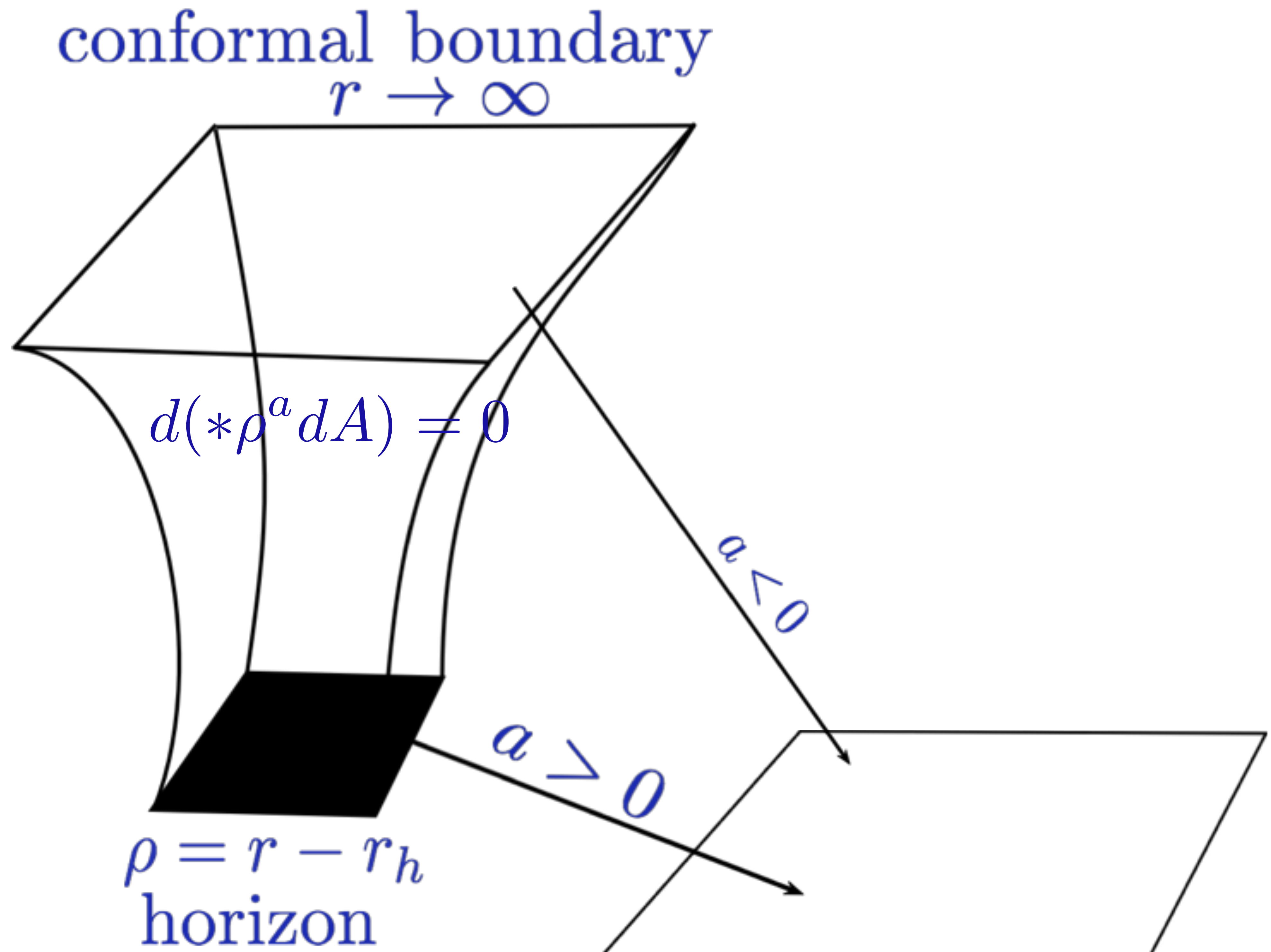
$$S = \int dV_d dy \left(y^a F^2 + \cdots \right)$$

eom $d(y^a \star dA) = 0$

Karch:1405.2926
Gouteraux: 1308.2084

if holography is RG then how can it lead to an anomalous dimension?

membrane paradigm



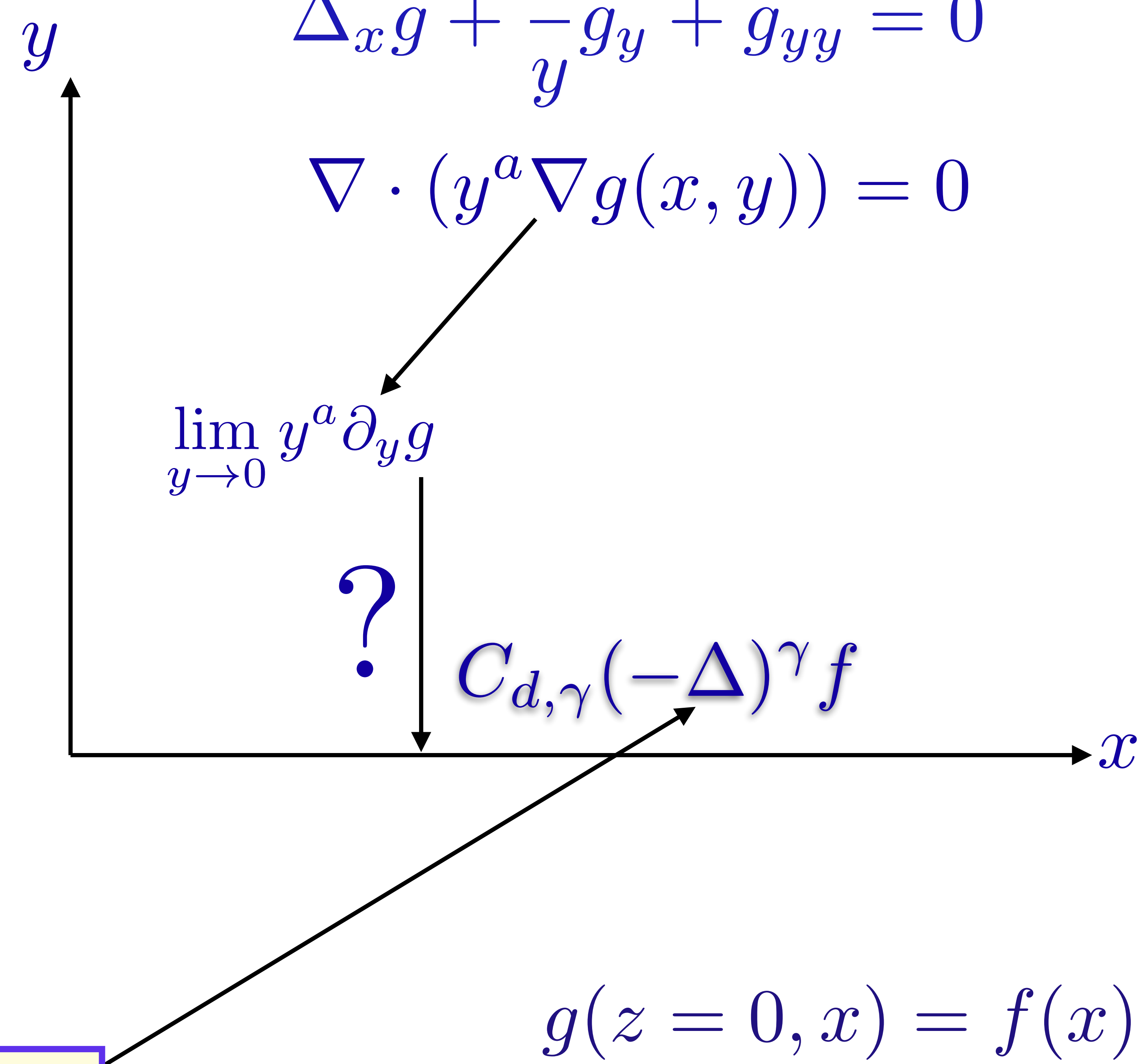
construct `boundary'
theory explicitly

Caffarelli-Silvestre extension theorem (2006)

$$g(x, y = 0) = f(x)$$

$$\Delta_x g + \frac{a}{y} g_y + g_{yy} = 0$$

$$\nabla \cdot (y^a \nabla g(x, y)) = 0$$



fractional Laplacian

$$g(z = 0, x) = f(x)$$

$$\gamma = \frac{1 - a}{2}$$

Caffarelli-Silvestre extension theorem (2006)

Dirichlet

$\mathbb{R}^{n+1}$ $g(x,y)$

$\downarrow (-\Delta)^{\gamma}$

$\mathbb{R}^{n}$ $f(x)$

Neumann

closer look

$$\nabla \cdot (y^a \nabla u) = 0$$

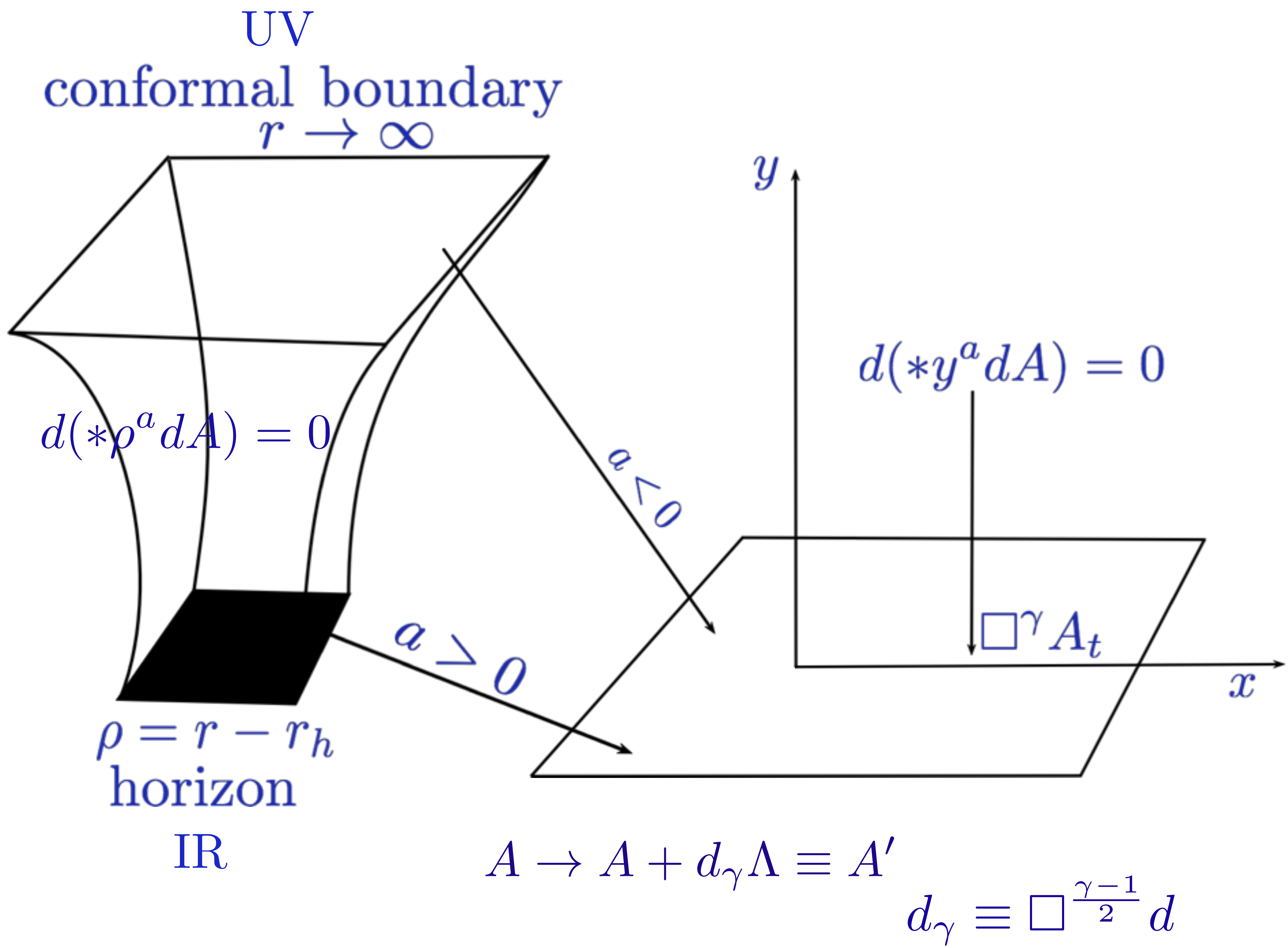
scalar field
(use CS theorem)

$$d(y^a \star dA) = 0$$

holography

similar equations

generalize CS theorem to
p-forms
GL,PP:1708.00863
(CIMP, 366, 199 (2019)))


UV
conformal boundary
$r \to \infty$
$d(*\rho^a dA) = 0$
$\rho = r - r_h$
horizon
IR
$a > 0$
$a < 0$
y
x
$d(*y^a dA) = 0$
$\Box^\gamma A_t$
$A \to A + d_\gamma \Lambda \equiv A'$
$d_\gamma \equiv \Box^{\frac{\gamma-1}{2}} d$

boundary action: fractional Maxwell equations

$$\Box^{\gamma} A_{\perp} = J$$

boundary action has `anomalous dimension' (non-locality)

$$F \to d_{\gamma} A = \partial_{\mu} \Box^{(\gamma-1)/2} A_{\nu} - \partial_{\nu} \Box^{(\gamma-1)/2} A_{\mu}$$

if holography is RG then how can it lead to an anomalous dimension?

$$S = \int dV_d dy \left(y^a F^2 + \cdots \right)$$

$$[A] = 1 - a/2$$

dimension of A is fixed by the bulk theory: not really anomalous dimension

new gauge transformation

$$A \to A + d_\gamma \Lambda \equiv A'$$

$$d_\gamma \equiv \Box^{\frac{\gamma-1}{2}} d$$

$$[A] = \gamma$$

action of gauge group

$$D_{\gamma,A}(e^\Lambda \odot \phi) = e^{i\Box^{(1-\gamma)/2}\Lambda} D_{\gamma,A'}\phi$$

$$D_{\gamma,A}\phi = \left(\partial_\mu + ie\Box^{(1-\gamma)/2} A^\mu\right)\Box^{(1-\gamma)/2}\phi$$

$$L = D_{\gamma,A}\phi(D_{\gamma,A}\phi)^* - m^2\phi^*\phi - F^{\mu\nu}_\gamma F_{\mu\nu\,\gamma}$$

from the bulk

use CS theorem

$$\lim_{y\to 0}[y^a\partial\phi(x,y), y^a\partial\phi(x',y)] = \Delta^\gamma[\phi(x,y,\phi(x',y)]$$

$$= \Delta^\gamma\delta(x-x') = 0$$

no problem
with causality

Ward identities

$$C^{ij}(k) \propto (k^2)^\gamma \left(\eta^{ij} - \frac{k^i k^j}{k^2} \right).$$

standard Ward identity

$$k_i C^{ij}(k) = 0 \quad \rightarrow \quad \partial_i C^{ij}(k) = 0$$

but

$$k^{\gamma-1} k_\mu C^{\mu\nu} = 0 \quad \rightarrow \quad \partial_\mu (-\Delta)^{\frac{\gamma-1}{2}} C^{\mu\nu} = 0$$

inherent ambiguity in E&M

Noether's Second Theorem

$$
\begin{aligned}
\sum \psi_i \, \delta u_i = \delta f &- \frac{d}{dx}\left\{ \sum \left[\binom{1}{1} \frac{\partial f}{\partial u_i^{(1)}} \delta u_i + \binom{2}{1} \frac{\partial f}{\partial u_i^{(2)}} \delta u_i^{(1)} + \cdots + \binom{\kappa}{1} \frac{\partial f}{\partial u_i^{(\kappa)}} \delta u_i^{(\kappa-1)} \right] \right\} + \\
&+ \frac{d^2}{dx^2}\left\{ \sum \left[\binom{2}{2} \frac{\partial f}{\partial u_i^{(2)}} \delta u_i + \binom{3}{2} \frac{\partial f}{\partial u_i^{(3)}} \delta u_i^{(1)} + \cdots + \binom{\kappa}{2} \frac{\partial f}{\partial u_i^{(\kappa)}} \delta u_i^{(\kappa-2)} \right] \right\} + \\
&\;\;\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
&+ (-1)^{\kappa} \frac{d^{\kappa}}{dx^{\kappa}}\left\{ \sum \left[\binom{\kappa}{\kappa} \frac{\partial f}{\partial u_i^{(\kappa)}} \delta u_i \right] \right\} \qquad (6)
\end{aligned}
$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda + \partial_\mu \partial_\nu G^\nu + \cdots ,$$

$$A \to A + d_\gamma \Lambda \equiv A'$$

$$d_\gamma \equiv (-\Delta)^{\frac{\gamma-1}{2}} d$$

$$\Delta \to \Box$$

Noether's Second Theorem and Ward Identities for Gauge Symmetries

Steven G. Avery[a], Burkhard U. W. Schwab[b]

For simplicity, we focus on the case when the transformation may be written in the form[6]

$$\delta_\lambda \phi = f(\phi)\,\lambda + f^\mu(\phi)\partial_\mu\lambda, \tag{10}$$

but it is straightforward to consider transformations, as Noether did, involving arbitrarily high derivatives of λ. (Although, the authors know of no physically interesting examples.) Let us start with

arxiv:1510.07038

family of zero eigenvalues

$$M_{\mu\nu} f k^{\nu} = 0$$

most fundamental conservation law

$$\partial^{\mu} \underbrace{(-\nabla^2)^{(\gamma-1)/2} J_{\mu}}_{J'_{\mu}} = 0$$

is boundary non-locality a problem?

entanglement?

Li/Takayanagi (PRL 106, 141301 (2011)

$$I_B(\phi) = \int d^d x \, \phi (-\Delta)^\gamma \phi$$

$$I_C(\phi) = \int d^d x \, \phi e^{(-\Delta)^\gamma} \phi$$

?

$$S_B = \kappa_{d-2} \left(\frac{1}{\epsilon} \right)^{d-2}$$

$$S_C \sim \kappa_{d-2} \left(\frac{1}{\epsilon} \right)^{d-2+2\gamma}$$

Area

>Area

RESOLUTION

$$I = \int d^dx \phi \mathcal{O}^\gamma \phi + J\phi$$

$$\phi \to \mathcal{O}^{(1-\gamma)/2}\phi$$

$$J \to \mathcal{O}^{(1-\gamma)/2} J$$

Rule

$$\hat{\mathcal{O}(\gamma)} = \hat{\mathcal{O}}^\gamma$$

$$I = \int d^dx \phi \mathcal{O} \phi + J\phi$$

Area entanglement

simplest exception

standard AdS

$$I = \int d^dx \phi \mathcal{O}^\gamma \phi + m^2|\phi|^2$$

$$m^2 > 0$$

$$S = \kappa_{d-2} \left(\frac{1}{\epsilon}\right)^{d-2\gamma}$$

Volume entanglement $\gamma = 1/2$

is there a consistent algebra for fractional currents?

Yes

Virasoro algebra

$$L_n := -z^{n+1}\frac{\partial}{\partial z}$$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}$$

Witt algebra

central extension

conformal transformations on unit disk

$$\mathcal{V} \to \mathcal{W} \to 1$$

Fractional Virasoro algebra

generators

$$L_n^a = -z^{a(n+1)} \left(\frac{\partial}{\partial z} \right)^a \qquad \bar{L}_n^a := -\bar{z}^{a(n+1)} \left(\frac{\partial}{\partial \bar{z}} \right)^a$$

$$[L_n, L_m](z^{ak}) = \left(\frac{\Gamma(a(k+n)+1)}{\Gamma(a(k-1+n)+1)} - \frac{\Gamma(a(k+m)+1)}{\Gamma(a(k-1+m)+1)} \right) L_{n+m}(z^{ak})$$

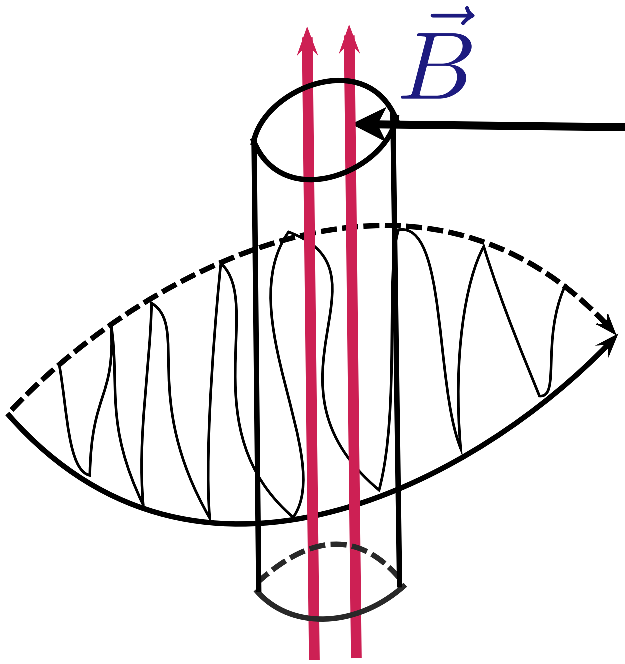
$$= (A_{n,m}^a(k) \otimes L_{n+m})(z^{ak})$$

$$[L_m^a, L_n^a] = A_{m,n} L_{m+n}^a + \delta_{m,n} h(n) c Z^a$$

algebra for conformal non-local actions

$$Z_\star^2(\mathcal{W}_a, \mathcal{H}) / B_\star^2(\mathcal{W}_a, \mathcal{H})$$

experiments?



magnetic flux

$\pi r^2 B$

should be dimensionless

$$[B] = 2 - \Phi = 2 + 2/3 \neq 2$$

what's the resolution?

correct dimensionless quantity

$$a_i \equiv [\partial_i, I_i^{\alpha}{}_{\alpha} A_i] = \partial_i I_i^{\alpha}{}_{\alpha} A_i$$

$\rightarrow \Delta^{-\alpha}$

what's the relationship?

$$\oint_{\partial\Sigma} a \qquad\qquad \oint_{\partial\Sigma} A$$

$$\mathrm{Norm} \oint_{\partial\Sigma} a = \frac{1}{\Gamma(3/2-\gamma)} \oint_{\partial\Sigma} A$$

not an integer

obstruction theorem to charge quantization (NST)

$$A_\mu \to A_\mu + \partial_\mu \Lambda + \partial_\mu \partial_\nu G^\nu + \cdots ,$$

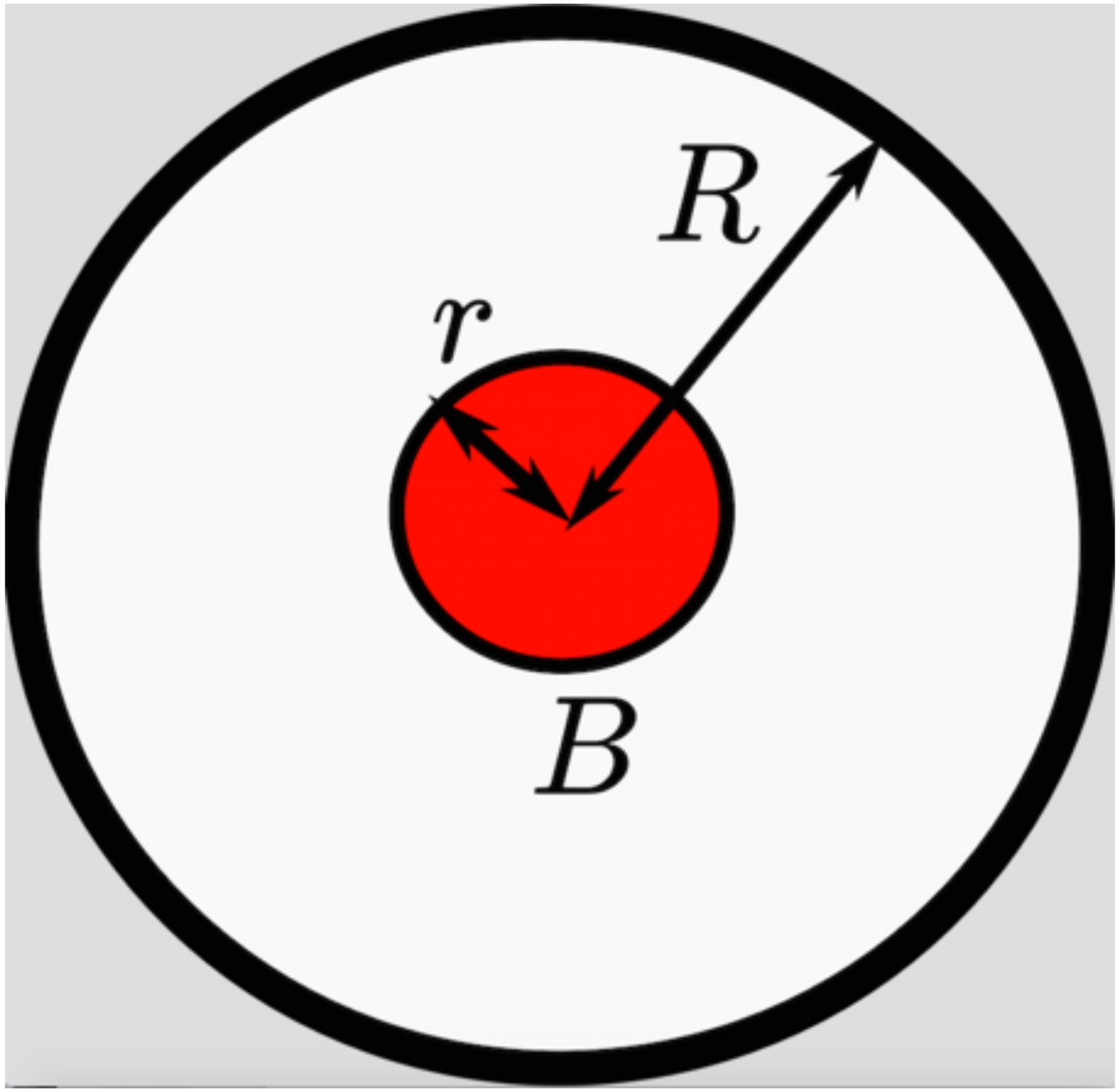
$$A \to A + d_\gamma \Lambda \equiv A'$$

$$d_\gamma \equiv (-\Delta)^{\frac{\gamma-1}{2}} d$$

$\downarrow$ J'_μ

charge ill-defined (new landscape problem)

New Aharonov-Bohm Effect



$$\Delta\phi_{\rm D} = \frac{e}{\hbar}\pi r^2 B R^{2\alpha-2}\left(\frac{\sqrt{\pi}2^{1-\alpha}\Gamma(2-\alpha)\Gamma(1-\frac{\alpha}{2})}{\Gamma(\alpha)\Gamma(\frac{3}{2}-\frac{\alpha}{2})}\sin^2\frac{\pi\alpha}{2}\,{}_2F_1(1-\alpha,2-\alpha;2;\frac{r^2}{R^2})\right)$$

is the correction large?

$\alpha = 1 + 2/3 = 5/3$

$$\Delta\Phi_R = \frac{eB\ell^2}{\hbar} L^{-5/3}/(0.43)^2$$

yes!

if in the strange metal

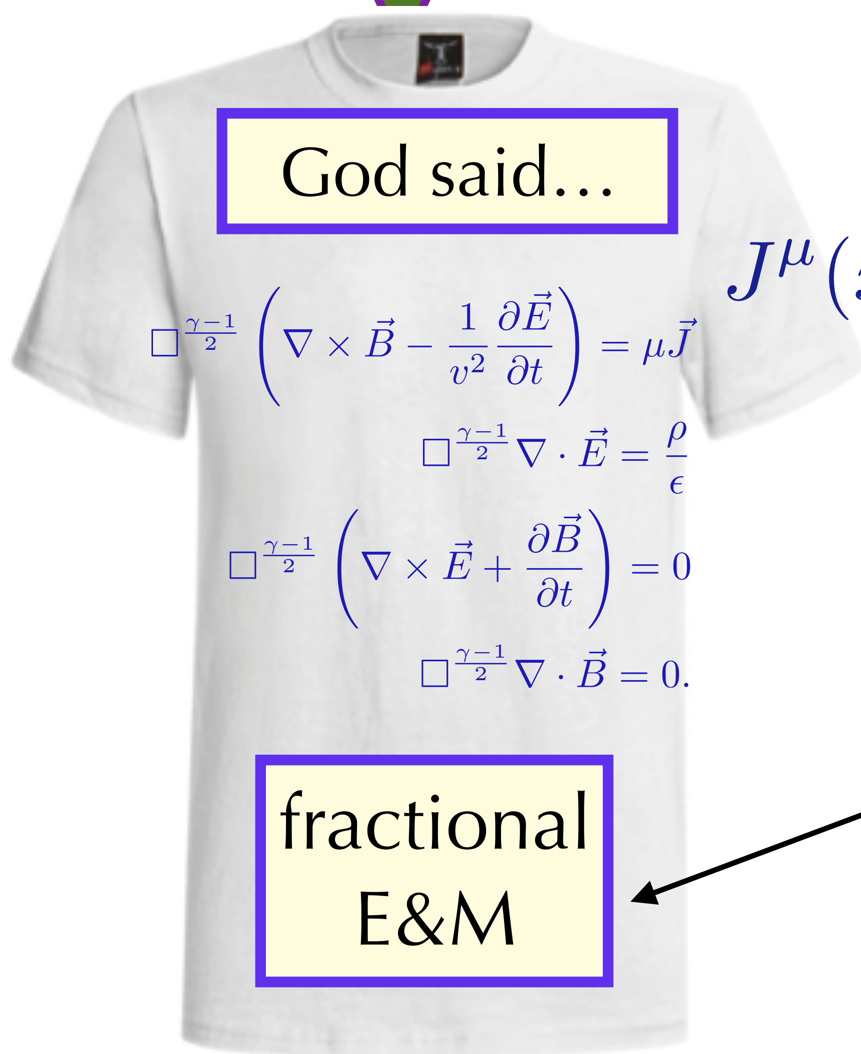
$$[A_\mu] = d_A \neq 1$$

Pippard Kernel

$$J^\mu(x) = -\int d^d x' C_{\mu\nu}(|x - x'|) A^\nu$$

$$[J] \neq d - 1$$

$$[A] \neq 1$$



God said...

$$\Box^{\frac{\gamma-1}{2}} \left(\nabla \times \vec{B} - \frac{1}{v^2} \frac{\partial \vec{E}}{\partial t} \right) = \mu \vec{J}$$

$$\Box^{\frac{\gamma-1}{2}} \nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$

$$\Box^{\frac{\gamma-1}{2}} \left(\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} \right) = 0$$

$$\Box^{\frac{\gamma-1}{2}} \nabla \cdot \vec{B} = 0.$$

fractional E&M

$$\omega = ck$$

in SC!