

”Introducing PetriRL: An Innovative Framework for JSSP Resolution Integrating Petri nets and Event-based Reinforcement Learning”

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Abstract

Quality scheduling in industrial job shops is crucial. Although neural networks excel in solving these problems, their limited explainability hinders their widespread industrial adoption. In this research, we introduce an innovative framework for solving job shop scheduling problems (JSSP). Our methodology leverages Petri nets to model the job shop, not only improving explainability but also enabling direct incorporation of raw data without the need to preprocess JSSP instances into disjunctive graphs. The Petri net, with its controlling capacities, also governs the automated components of the process, allowing the agent to focus on critical decision-making, particularly resource allocation. The integration of event-based control and action masking in our approach yields competitive performance on public test benchmarks. Comparative analyses across a wide spectrum of optimization solutions, including heuristics, metaheuristics, and learning-based algorithms, highlight the competitiveness of our approach in large instances and its superiority over all competitors in small to medium-sized scenarios. Ultimately, our approach not only demonstrates a robust ability to generalize across various instance sizes but also leverages the Petri net’s graph nature to dynamically add job operations during the inference phase without the need for agent retraining, thereby enhancing flexibility.

Keywords: Job Shop Scheduling, Reinforcement learning, Petri nets, Maskable PPO, Event-based control, Flexible manufacturing.

1. Introduction

Remaining competitive and successfully navigating dynamic markets necessitate agility and adaptability. To reach this goal, the Flexible Manufacturing System (FMS) aims to serve as a bridge between conventional mass production and tailored manufacturing, enabling the mass production of products while accommodating customization. The key to achieving this flexibility lies in the implementation of high-quality scheduling processes[1]. Flexible manufacturing systems can be formulated using a job shop scheduling problem (JSSP). JSSP is a classic combinatorial optimization problem in the field of operations research and production planning. The JSSP involves scheduling a set of jobs through a set of machines, where each job consists of a sequence of operations and each operation must be processed on a specific machine [2].

Petri nets are mathematical modelling languages used for the description and analysis of systems that exhibit concurrent and distributed behaviour, which makes them widely used for modelling and analysing the behaviour of systems in various domains, including manufacturing, communication protocols, and software systems [3][4]. Petri nets stand out as an excellent choice for representing and analysing the dynamic behaviour of systems such as job shops, their dual functionality as both a control and modelling tool makes them exceptionally well-suited. On one side, the graphical representation capabilities of Petri nets contribute to a better understanding of the scheduling process, thereby improving overall explainability. On the other side, their control and simulation capabilities enable seam-

less collaboration with optimization algorithms to effectively address and solve the Job Shop Scheduling Problems[5]. The extensive body of literature available further solidifies Petri nets as a favourable option as a modelling tool. Various methods have been proposed to address JSSPs with both single and multiple goals. Exact methods, such as those outlined in [6][7], offered optimal solutions, but they are accompanied by drawbacks like high computational demands and limited action space. Approximate solutions, such as heuristics seen in [8] and [9], overcome computational challenges by employing strategies to simplify complex tasks and quickly converge to a solution. Neural networks also belong to approximate solutions using iterative approaches to solving JSSPs. Compared to heuristics the data-driven approach of neural networks minimizes the reliance on domain knowledge. Their ability to learn intricate patterns and process extensive datasets has led to successful applications in Scheduling Problems [10], as evidenced by studies such as in [11] [12] [13]. However, the ”black box” nature of neural networks poses a significant challenge, impeding their widespread adoption in industries. To overcome this challenge, unlocking the full potential of neural networks involves emphasizing explainability, facilitating knowledge transfer, and enhancing generalization. By addressing these aspects, the industrial adoption of neural networks can be promoted, making them more accessible and beneficial for real-world applications.

The contribution of this study can be summarized as follows:

1. We introduce PetriRL, an innovative framework that leverages Petri nets and actor-critic-based reinforcement

learning to address job-shop scheduling problems. This approach enhances explainability thanks to Petri nets effective graphical representation.

2. We propose a flexible solution, thanks to the graph nature of the Petri net. This enables the real-time addition of operations into existing jobs or the introduction of entirely new jobs during the inference process without the need to retrain the agent.
3. We enhance efficiency by integrating event-based control and action space masking, limiting the needed agent-environment interaction to solve the JSSP.
4. We eliminate the requirement for laborious preprocessing and casting of the JSSP instances into disjunctive reducing complexity and computation requirements, allowing efficient input of the raw instances in our system.
5. We conduct a comparative study on public benchmarks, comparing our results to heuristics, metaheuristics, and other learning-based approaches, proving our algorithm's competitive performances. Furthermore, we delve into ablation studies to analyze the contribution of individual elements of the framework.

The paper is structured as follows: Section 2 provides an overview of related works. In Section 3, we introduce the PetriRL framework, beginning with the mathematical formulation of Petri nets and the development of the environment. We then delve into key framework elements, including event-based control, action space masking, and reward shaping. Section 4 encompasses the testing of our approach on public benchmarks, results discussion, and an ablation study. Following this, we explore the generalization and flexibility capacity of our solution. Section 5 serves as the conclusion, summarizing the paper and presenting future perspectives.

2. Literature Review

One of the promising approaches to tackle scheduling challenges involves employing Petri nets alongside optimization methods like heuristics, meta-heuristics, and iterative learning approaches such as reinforcement learning. In this literature review, we commence with an exploration of the applications of Petri nets in production scheduling, followed by the meta-heuristics and iterative optimization tools such as reinforcement learning applications in solving JSSPs. Finally, we investigate the applications of graph neural networks before concluding with an identification of the research gap.

Petri nets serve as a mathematical modelling language crucial for describing and analyzing the dynamic behaviour of concurrent systems. Their widespread use in scheduling production systems is attributed to several advantages over alternative tools.

Notably, Petri nets excel in representing numerous states concisely, capturing precedence relations, and structural interactions, and effectively modelling critical aspects such as deadlocks, conflicts, and buffer sizes [3]. Various extensions of Petri nets have been introduced to enhance their modelling capabilities. Stochastic Petri nets handle uncertain events, offering a framework to represent and analyze probabilistic elements in JSSP applications [14]. Colored Petri nets introduce a multi-token approach, allowing diverse types of tokens within a single net allowing several jobs in a JSSP to share the same structure resulting in a concise network [15]. Timed Petri nets incorporate time-related aspects, facilitating the modelling of time delays and temporal relationships, a critical requirement for JSSP application [16]. The versatility of Petri nets is evident in their application across three distinct domains documented in the literature. Firstly, the associated reachability graphs of Petri nets play a pivotal role in predicting and controlling system states, notably in scenarios such as deadlock prevention [17]. Secondly, the underlying mathematical foundation of Petri nets enables qualitative and quantitative analyses of systems, providing valuable insights into their behaviour. Lastly, they serve as simulation tools, working collaboratively with optimization tools like heuristics, metaheuristics, and iterative approaches to enhance system performance.

Heuristics are popular optimization tools in JSSP applications thanks to their simplicity and reduced computational requirements compared to exact solutions. However, these simplifications come at the cost of suboptimal solutions and a heavy reliance on domain knowledge, hindering their generalization. To tackle this problem, Some meta-heuristics draw inspiration from nature, for example, Genetic Algorithms (GA), and Ant Colony Optimization (ACO), as discussed in [18] and [19], taking high-level strategies to solve JSSPs. While these approaches are not problem-specific and can be applied to a wide range of problems, they are still sensitive to initial conditions and require extensive tuning. Reinforcement learning (RL) belongs to the iterative optimization tools. RL has proven effective in dynamic environments, As shown in the findings in [20], where 85% of evaluated RL implementations significantly improved scheduling performance in JSSPs. In addressing the agility requirements for the dynamic and flexible job shop scheduling problem (DFJSP), [21] implemented a deep Q-network (DQN) agent to intelligently select the best dispatching rule at each rescheduling point. Another approach involved combining the duelling double Deep Q-network with prioritized replay (DDDQNPR) outperformed heuristic dispatching rules, particularly in dynamic environments with uncertain processing times [22]. Furthermore, [23] conducted experiments using multi-agent actor-critic techniques to solve JSSPs through the application of the Deep Deterministic Policy Gradient (DDPG) algorithm.

The graph-based approach gained prominence thanks to its ability to model complex relationships and adaptability to changing environments. [24] proposed a Graph Neural Network (GNN) to embed the JSSP states, this resulted in a size-agnostic policy network effectively enabling generalization on large-scale instances. [25] leveraged GNN capability to comprehend the

spatial structure of the problem, enabling adaptation to diverse JSSP instances and surpassing the performance of scenario-specific algorithms. Meanwhile, [26] introduced GraSP-RL to minimize makespan in a complex injection moulding production setting, achieving superior results compared to meta-heuristics like tabu search and genetic algorithms, all without the necessity for additional training. A further optimization came from focusing on the most pertinent features instead of indiscriminate aggregation of information, [27] implemented a Gated-Attention model in which a gating mechanism is implemented to modulate the flow of learned features outperforming existing heuristics and state-of-the-art DRL baselines. The success of the transformer-based models [28] in large language models LLMs application led to their widespread across many applications. In paper [29], the authors used node2vec algorithm to learn the JSSP instance's disjunctive graph characteristics in the form of a sequence, followed by a transformer architecture based on a multi-head attention mechanism to generate a solution. This approach not only offered long-range dependency thanks to the attention mechanism but also parallel computing capacities vital to solving large-scale problems.

In exploring the existing body of literature, a noticeable gap emerges, notably the requirement for a more explainable modelling tool of the JSSP environment, a streamlined approach to input JSSP instances into the framework, enhancing generability, flexibility, and the efficient utilization of data samples. Integrating Petri nets with reinforcement learning offers a promising approach for tackling JSSPs. Avoiding the additional step of converting JSSP instances into disjunctive graphs thus reducing complexity and computation. Using an actor-critic agent alongside the Petri net brings benefits like improved sample efficiency and change adaptability. Therefore, in the next section, we introduce PetriRL, a framework for JSSP resolution, capitalizing on Petri nets and actor-critic reinforcement learning.

3. PetriRL: a JSSP solving framework

In this section, we start by defining the mathematical formulation of the JSSP's Petri net. Subsequently, we construct the environment, incorporating the Petri net formulation and seamlessly aligning it with other key components of the framework, including event-based control, advantage estimation, and action space masking.

3.0.1. The Petri net formulation

Coloured Petri nets are a powerful approach, to fold processes that share a similar structure but with different properties in a compact manner [30]. This proves especially advantageous in applications related to the JSSPs, where different jobs share the same shop floor. A marked-colored Petri net is defined by:

$$\text{CPN} = (P, T, A, \Sigma, C, N, E, G, I)$$

with:

- $P = \{p_1, p_2, \dots, p_n\}$: set of places,
- $T = \{t_1, t_2, \dots, t_n\}$: set of transitions,

- $A = \{a_1, a_2, \dots, a_n\}$: set of arcs,
- $\Sigma = \{c_1, c_2, \dots, c_m\}$: set of colors,
- $C : T \cup P \rightarrow \Phi(\Sigma)$, $\Phi(\Sigma) \subset \Sigma$ is the color function,
- $N : A \rightarrow (P \times T) \cup (T \times P)$ is the node function,
- $E : A \rightarrow e$ arc expression function,
- $G : T \rightarrow \{0, 1\}$ is the transitions guard function,
- $I : P \rightarrow$ initiation sequence is the initialization function.

Following the introduction of the generic coloured Petri net model, we proceed to tailor the formulation specifically for our JSSP application.

- $P = P_j \cup P_r \cup P_m$:job,ready and machines places,
- $T = T_s \cup T_a \cup T_d$: select,allocate and deliver transitions,
- $\Sigma = \{c_1, c_2, \dots, c_m\}$: m is the number of machines,
- $C : T_a \rightarrow \Phi(\Sigma)$: only allocation transitions are colored,

For any node in $P \cup T$, we define $\pi(n)$: the set of upstream nodes, $\sigma(n)$: the set of downstream nodes, and $M(p)$ is a function returning the number of tokens in a given place also called marking. When a transition $t \in T$ fires the new marking is given by :

$$\tilde{M}(p) = \begin{cases} M(p) - 1 & \forall p \in \pi(t) \\ M(p) + 1 & \forall p \in \sigma(t) \\ M(p) & \text{otherwise} \end{cases} \quad (1)$$

Depending on whether the transition is autonomous such as deliver transitions T_d , controllable such as selection transitions T_s , or a combination of controllable and coloured transitions such as the case of allocation transition T_a , the transition guard function \mathbf{G} is given by:

$$G(t \in T_d) = \begin{cases} 1 & \text{,if } \forall p \in \pi(t) | M(p) \geq 1 \\ 0 & \text{,otherwise} \end{cases} \quad (2)$$

$$G(t \in T_s) = \begin{cases} 1 & \text{,if control signal} = \text{True} \\ & \text{and } \forall p \in \pi(t) | M(p) \geq 1 \\ 0 & \text{, otherwise} \end{cases} \quad (3)$$

$$G(t \in T_a) = \begin{cases} 1 & \text{,if control signal} = \text{True} \\ & \text{and } \exists p_i, p_j \in \pi(t) | M(p_i) \text{ and } M(p_j) \geq 1 \\ & \text{and } \forall \tau \in \bigcup_{\forall p \in \pi(t)} \text{Tokens}(p), C(\tau) \equiv C(t) \\ 0 & \text{, otherwise} \end{cases} \quad (4)$$

Finally, the initiation function \mathbf{I} is extracted directly from the JSSP instance. Every job operations sequence in the instance is converted into an ordered list of tokens. The tokens bear the colour of the destination machine on top of additional features such as the job it belongs to and the processing time.

3.1. The Petri net environment

After introducing the mathematical representation of JSSP’s Petri net, this section delves into defining the environment with which the reinforcement learning agent can interact. The section begins with a description of the environment’s mechanisms, followed by key components such as observations, and action space, and concludes with a discussion of the advantages offered by the environment.

3.1.1. Environment description

In the proposed setting, as illustrated in Fig 1, various concepts from Petri net formalism are applied to model a job shop environment aimed at addressing Job Shop Scheduling Problems. This environment can be segmented into three main components: input, processing, and delivery.

Commencing with the input phase, each job is comprised of an ordered sequence of operations converted into tokens. The operation tokens bear the colour of the destination machine and feature such as processing time and the job they belong to. The job operation queue is connected to a controllable selection transition $t \in T_s$. The RL agent triggers a selection transition to select a given job queue. Subsequently, the first operation token in the queue is transferred to the ready operations’ place waiting to be allocated to a machine.

The processing section comprises the allocation transition, machine processing places, and corresponding idle check places for the machines. The allocation transition $t \in T_a$ is colour-specific, allowing it to only consume tokens with a compatible colour. This ensures that the job operation is processed on the correct machine. Machine idle check-places act as safeguards, ensuring that a machine can only process one operation at once. The allocation transition is enabled and can be triggered by the RL agent only if it satisfies the guard function $G(t \in T_a)$ aka a token with the correct colour is available, and the machine is in an idle state. Upon firing the allocation transition, the operation token is transferred to the machine processing location. Since the machine processing place is a timed one, the token must spend the processing time specified by its features before becoming available for the delivery transition.

Finally, the delivery process is automated, with autonomous delivery transitions $t \in T_d$ firing promptly upon the availability of tokens. Once fired the token is moved from the processing place to the delivery place. The environment has an internal clock that governs all time-related aspects such as processing durations, idle duration, job queue waiting duration, remaining times, machine entry date, token logging, etc. For every token transfer from one place to another logging is saved in the token history. Ultimately, by extracting the entry and departure times of the token in the processing places, the solution plan can be derived, as illustrated in Fig 2.

Following the concept explained above, we have developed a custom Gym environment [31]. In the upcoming sections, we elaborate on various aspects of this environment, including observation, termination condition, action space, and rewards followed by the advantages offered by the proposed environment. The environment is publicly available as a package on the official repository for Python packages under the name **jsspetri**.

3.1.2. Observation

The observation provides the agent with information about the current state of the environment. A state is a representation of the relevant aspects of the environment that the agent needs to consider to make decisions. The quality and completeness of observations directly impact the agent’s ability to understand and learn from the environment.

The observation is structured in our environment as a one-dimensional vector comprising three components. Firstly, it includes information about the state of the machines, where the remaining processing time for each machine is incorporated. If a machine is idle, the remaining processing time is equal to minus one. Secondly, the number of delivered pieces for each machine. Termination is determined by confirming that all job queues and machines are empty. Finally, the observation encompasses the job operation waiting queue, introducing ”observation depth” as a hyper-parameter. This parameter dictates the number of future operations to be included in the observation. A depth of one implies focusing solely on the job operations in the first row of each job queue. As the observation depth increases, the agent looks deeper in the future.

3.1.3. Action space

The action space is defined by the agent’s margin of manoeuvre. In the proposed framework, the agent role is limited to allocating a selected job operation to a specific processing machine. Once the job is allocated, the remaining is automated. The delivery transition operates autonomously, triggering promptly upon the token’s availability following the token finishing the required processing duration in the processing machine. This simultaneously delivers the piece and flags that the machine is now idle. Since the allocation transition is coloured, the allocation transition is enabled only if a token with a matching colour is waiting in the ready operations and the machine is idle as dictated by the transition guard function equation (4).

The choice of adding a standby action is a delicate one. Adding a standby option is necessary to allow the agent to opt for long-term strategies, by choosing to leave a machine idle for a given time in favour of allocating a better option in the future. This also comes with the risk of the agent always choosing the standby action as a default choice in a problem called reward hacking. This was solved in the section 3.3 where we discuss dynamic action space masking.

In summary, the action space encompasses all combinations of jobs and machine transitions, with the addition of the standby action. The size of the action space is subject to dynamic adjustments based on the action mask which selectively enables specific actions based on the Petri net’s transition guard function.

3.1.4. Environment advantages

The suggested environment presents several advantages, particularly better explainability and, eliminating the need for extensive preprocessing.

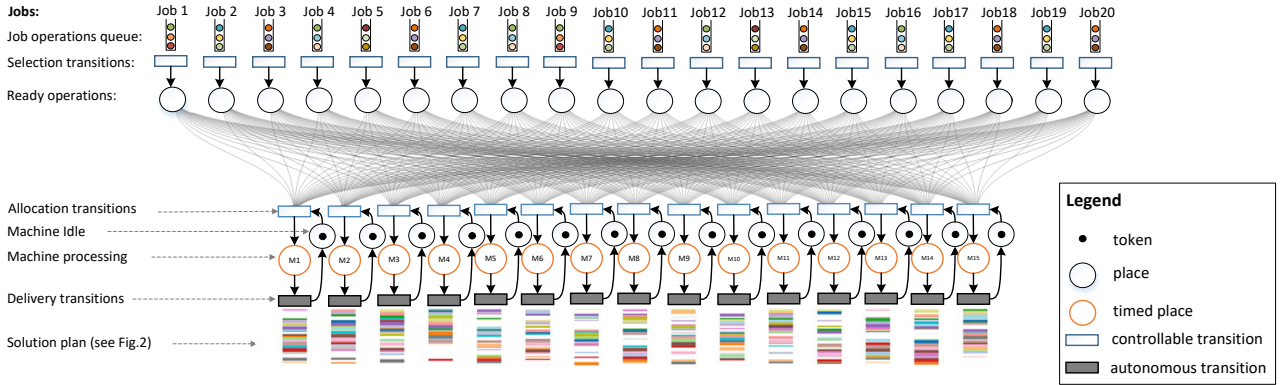


Figure 1: PetriRL framework graphical representation example for a 20 jobs 15 machines shop floor.

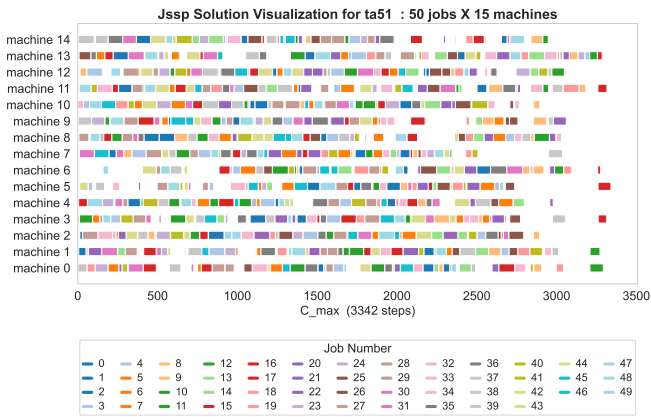


Figure 2: Planning solution for Taillard benchmark instance "ta51": 50 jobs x 15 machines.

In many of the competing learning-based reinforcement learning solutions to solve the JSSP, preprocessing data is required [29][27][24]. This preprocessing involves transforming each JSSP instance into a disjunctive graph representation. Once represented in the graph format, feature extraction techniques including methods like node2vec [32], and attention mechanisms are utilized. These techniques aim to extract features that are more tailored to the characteristics of the employed optimizer. Unless the used optimizer is a graph neural network, this additional step could be omitted. The disjunctive graph representing the JSSP contains two types of information. First, structural and routing information is encoded into the edges of the disjunctive graphs. Second, the nodes contain features such as processing times. Since the structure is shared between all the jobs, the Petri net's graph nature can be leveraged to encode the structural and routing part of the input. This allows us to remove the need for transforming JSSPs into disjunctive graphs and replace them with a list of coloured tokens. Compared to disjunctive graphs, representing JSSPs with a list of coloured tokens offers a more efficient, flexible, and streamlined approach.

Another advantage of Petri nets is their capability to serve as a comprehensive end-to-end control solution. Our approach consisted of delegating a fraction of the control to the Petri nets by utilizing tools such as timed places, autonomous transitions, and synchronization, to streamline the automated parts of the process. This allows the agent to concentrate solely on essential decision-making and allocation tasks through the controlled transitions. In essence, the agent delegates automated sections of the process to the Petri net, enabling more focus on decision-making tasks. This balanced share of control is a key element for efficiency and explainability

3.2. Event-based control

Event-based control refers to a paradigm where learning updates are triggered by specific events or occurrences rather than a fixed time interval[33][34]. In traditional RL, updates often occur at regular time intervals or after completing a specific number of steps. In contrast, event-based control adapts its learning schedule based on events in the environment. This approach can be advantageous in dynamic and unpredictable environments where the occurrence of important events is not uniform over time. In our case study, the environment and agent environment interaction is only triggered if one machine or more is available.

Our approach involves on-demand agent intervention. When all machines are in use or no operation can be assigned to an idle machine, the algorithm remains within the environment, incrementing only the internal clock. If at least one machine is available, it prompts the agent to either allocate a job to the idle machine or choose to stand by. The standby option is only possible if an idle machine is present, allowing the agent to strategically opt for long-term planning without making the standby action the default choice. Moreover, decoupling the agent's step count from the environment's internal step clock offers two primary advantages: improved reward assignment and more efficient utilization of environment-agent interactions.

First, reward is one of the critical parts of reinforcement learning. On top of the use of makespan projection for advantage estimation discussed in 3.4 which ensures consistent credit as-

signment to the agent, event-based control plays a pivotal role in achieving appropriate credit assignment. Without event-based control, the agent faces the challenge of consistently incurring negative rewards when all machines are occupied. This leads to agent confusion and dilutes the infrequent occurrence of positive rewards. The second advantage of employing event-based control lies in the optimal utilization of the available training environment-agent interactions. With event-based control implemented, the agent engages with the environment on demand, ensuring that the agent receives a relevant reward. This guarantees the efficient use of every training step, in contrast to the non-event-based approach where a significant portion of interactions may be wasted on selecting the stand-by action due to all machines being occupied. Empirical results validated this observation. Comparing the total time steps during training with the internal environment clock reveals that only 9% of the steps are utilized for allocation decision-making.

In summary, the adoption of event-based control significantly reduces the required agent-environment interactions to solve the JSSP. This is achieved by prompting the agent to make decisions on demand whenever a machine is idle. The benefits extend beyond enhanced learning due to improved credit assignment, it is also more efficient. When combined with the action masking strategy detailed in the subsequent paragraph, our approach ensures the effective utilization of available agent-environment interactions.

3.3. Maskable proximal policy optimization (PPO)

Reinforcement learning excels in training agents to make sequential decisions, but in complex environments, the sheer number of possible actions can hinder learning efficiency. Action masking emerges as a vital technique to address this challenge [35]. The authors of the paper [36] outlined four strategies for managing invalid actions: a control setup where no action are used, assigning an action penalty, applying naive action masking, or using action masking. The First strategy involves assigning a penalty upon choosing an invalid action such as implemented in [37]. In the second approach called naive action masking, the action is removed directly from the action space without directly modifying the policy. Since the action is not chosen after its exclusion, the gradient gradually diminishes the future probability of the invalid action. The last strategy is action masking, in this approach, actions are masked by directly manipulating the raw outputs of the policy also called logits before softmax normalization. By assigning a significantly negative value to the logits of the action to be masked, the probability of the invalid action becomes negligible.

The findings in [36] assert that masking removal is possible only in small action spaces, a finding confirmed also by our results in the ablation study. Naive action masking, however, exhibits drawbacks such as an elevated average Kullback–Leibler (KL) divergence between the target and current policy, leading to training instability and a volatile, sensitive behaviour. Lastly, the Invalid actions penalty proves impractical for scalability; the results indicate a performance collapse with an increase in the environment size compared to action masking. Furthermore, setting the penalty value for an invalid action is a challenging

hyper-parameter to tune and integrate with the overall environment reward. Due to these considerations, we choose to directly act on the policy as an invalid actions handling policy. The corresponding pseudo-code can be found in Algorithm 1.

Algorithm 1 Proximal Policy Optimization with Action Masking using Petri nets’s guard function.

- 1: Initialize policy parameters θ , value function parameters ϕ
- 2: Set hyperparameters, including the clipping parameter ϵ
- 3: **for** iteration = 1, 2, . . . **do**
- 4: Collect trajectories using the current policy π_θ
- 5: Compute advantages \hat{A}_t and returns R_t

Update the Value Function:

- 6: Compute the value function loss:

$$L^{VF}(\theta) = \mathbb{E} \left[\left(V_{\theta_{\text{old}}}(s_t) - V_\theta(s_t) \right)^2 \right]$$

- 7: Optimize the value function parameters θ
- 8: Compute the Mask using the Petri net’s guard function:

$$\text{Mask}[i] = G(a_i) \quad \forall a_i \in T_a$$

- 9: Compute the actor’s policy logits:

$$z_t(a_i) = \begin{cases} \text{policy_network}(s_t, a_i) & \text{if } G(a_i) = 1 \\ -\infty & \text{else} \end{cases}$$

- 10: Convert the logits into a probability distribution over actions and calculate the action probability ratio:

$$\pi_\theta(a_i|s_t) = \sum_{a'} \frac{e^{z_t(a')}}{e^{z_t(a')}} \quad , \rho_t = \frac{\pi_\theta(a_i|s_t)}{\pi_{\theta_{\text{old}}}(a_i|s_t)}$$

- 11: Compute Clipped surrogate objective:

$$L^{CLIP}(\theta) = \mathbb{E}_t \left[\min \left(\rho_t \hat{A}_t, \text{clip}(\rho_t, 1 - \epsilon, 1 + \epsilon) \hat{A}_t \right) \right]$$

- 12: The final PPO loss (C_1 is a balancing hyper-parameter) :

$$L(\theta) = L^{CLIP}(\theta) + C_1 L^{VF}(\theta)$$

- 13: Optimize the policy parameters θ (α is the learning rate):

$$\theta \leftarrow \theta - \alpha \nabla_\theta L(\theta)$$

- 14: **end for**
-

The event-based control and the action space masking share similarities. Both approaches aim to eliminate unnecessary steps and promote efficacy and performance. Like event-based control eliminating unnecessary agent-environment interaction, masking the action space prevents wasting training steps on non-valid actions. To estimate the benefits of implementing such techniques we logged the average of valid actions in our environment, it turns out that at any given time a fraction constituting only 10 % of the action is enabled. This can be explained by the synchronization property of Petri nets, synchronisation means that different conditions must be simultaneously satisfied for the transition to be enabled. for example, to enable the allocation transition, the machine needs to be idle, and a compatible operation is in the job operations waiting queue, which explains the low ratio of enabled action. If not implemented the agent will

waste nine out of ten interaction steps on non-valid action and receive a penalty. When paired with event-based control, the incorporation of action masking leads to a significant performance enhancement of 30 to 50 times. This notable improvement is demonstrated in our ablation study, where the exclusive application of action-making reduced the necessary steps to solve the 15x15 JSSp from 6200 to 240 steps.

3.4. Advantage estimation

The reward function is one of the cornerstones in the reinforcement learning paradigm. The reward plays a crucial role in shaping the RL agent policy by offering a guiding signal, which the agent iteratively uses to improve the policy. Sparse reward is a situation where the agent does not receive frequent feedback from the environment which poses a challenge for learning. In this case, the agent might struggle to map the actions with the outcome and hinder this capacity to form a good policy. To address the issue, researchers and practitioners often employ techniques such as reward shaping or curriculum learning, experience replay[38][39] and synthetically increase the feedback frequency.

In our JSSP case study, the selected performance metric and reward function are centred around minimizing the makespan, which is the time required to complete all predefined jobs. Since makespan inherently constitutes a sparse reward, as it only provides feedback at the end of task completion, challenges related to sparse rewards emerge. To address this issue, we opt for reward shaping. However, this solution brings its own set of drawbacks, including a loss of generalization, a reliance on domain expertise, and the potential for failure in dynamic environments. Moreover, if the shaped function becomes too specific, there is a risk of reward hacking, where the agent may discover shortcuts to maximize reward but may deviate from the true objectives in the process.

The solution involves maintaining a broad objective, such as minimizing makespan, while actively increasing the frequency of feedback. This approach prevents overfitting, preserves generalization properties, and ensures consistent feedback. In Algorithm 2 the projected makespan is calculated at each time step based on the chosen action. During training, the projected makespan is calculated both before and after taking action at each agent-environment interaction. The difference serves as an advantage estimation the agent uses to refine the policy.

The algorithm consistently refines its assessment of completion times for each job, and then the makespan is defined as the maximum time among all the job’s completion times. The process begins with estimating the completion time of operations currently in progress in machine places. This estimation is based on the operation process duration conveyed by the token feature and the environment’s internal clock. Subsequently, an optimistic strategy is employed, assuming that all remaining operations in the job queue will be optimally processed.

The algorithm starts with an optimistic estimation of the makespan since all the constraints such as machine sharing among jobs are initially ignored. Then with every action, the agent will iteratively refine its estimate until the job operation waiting for the queue is empty and the estimate converges to the

Algorithm 2 Makespan Estimation for Advantage Calculation

```

Initialize competition time for every job:
1: for  $j$  in all jobs do
2:   completion_time $_j$   $\leftarrow$  0
3: end for

Estimate the completion time for operations in the process:
4: for  $m$  in all machines do
5:   if machine is idle then
6:     completion_time $_j$   $\leftarrow$  internal-clock
7:   else
8:     completion_time $_j$   $\leftarrow$  internal-clock + remaining time
9:   end if
10: end for

Assume optimal processing of operation in the waiting queue:
11: for  $j$  in all jobs do
12:   for  $op$  in job operations queue do
13:     completion_time $_j$   $\leftarrow$  completion_time $_j$  +
14:       op_processing_time  $\times$  waiting_penalty
15:   end for
16: end for
17: Makespan  $\leftarrow$   $\max_{1 \leq j \leq n_{\text{jobs}}}(\text{completion\_time}_j)$ 
18: return estimated Makespan

```

actual makespan. A challenge emerges during the initialization phase, where the highest reward occurs when all operations are in the waiting queue. To address this issue and promote job allocation, a penalty is introduced on operations in the waiting queue. In our implementation, we set a waiting penalty of two, signifying that the processing time of a given operation is doubled if the operation happens to be in the waiting queue during the make-span estimation step. This approach not only incentivizes the agent to clear the waiting queue promptly but also resolves the initiation problem. The efficacy of this strategy and its contribution to overall results is demonstrated in the ablation study.

4. Results and discussion

4.1. Training

The algorithm was tested on public Taillard Benchmark instances [40] ranging in sizes from 15x15 to 100x20 (jobs x machines). The benchmark offers eight groups, every group is composed of ten instances of identical sizes but with different values giving a total of 80 instances. The agent was trained on the first instance in each group and tested on the remaining nine. We used the Maskable PPO (policy proximal optimization) from the sb3-contrib library which belongs to openAi stable baselines3 [41], a set of reliable implementations of reinforcement learning algorithms in PyTorch, as our agent.

The experiments were conducted on a machine equipped with an NVIDIA Quadro RTX 5000. The models were implemented using the PyTorch deep learning framework through the stable baselines libraries, and all experiments were performed on a

system running Windows 11. We trained the different instance-size group agents on a fixed number of 3×10^5 steps. In Fig 3, we report the training and deployment times of the agents for varying instance sizes.

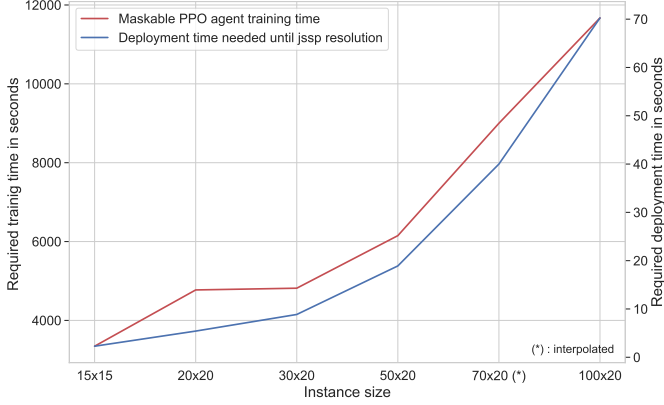


Figure 3: **Training and deployment time for different instances using PetriRL framework.** The red line represents the agent training time requirement for different instance sizes, the corresponding y-axis on the right, and the blue line represents the time requirement to solve a JSSP during the inference phase, corresponding y-axis to the left.

In Figure 4, we examine the training dynamics of the agent. To illustrate the agent’s learning behaviour across various sizes, we focus on a middle-sized instance ”ta51” (50x20) as a representative middle ground. Four metrics are selected for the analysis of the training process: episode mean length, mean reward, approximate KL divergence, and entropy loss. The episode mean length and reward serve as indicators for overall training performance, while the KL divergence is employed to evaluate training stability and the entropy to analyze the exploration-exploitation tendency.

The episode length is one of the most critical metrics in this research as it directly translates to the number of agent-environment interactions needed to solve the JJSP. In sub-figure (a), we observe a consistent, monotonic decrease in the episode length. The absence of plateaus is indicative of the agent not being stuck in suboptimal policies, affirming the effectiveness of the reward function in guiding the agent. During the later stages of training, the episode length stabilizes, suggesting the agent converges to a consistent policy. Notably, within the highlighted red rectangle in sub-fig (a), we identify an ”inflexion point” consistently present in the agent’s training for different size instances. In the context of JSSP planning, this inflexion point may denote the transition from a draft solution where the agent only aims to deliver all the pieces to a refined tuning phase. The episode reward also exhibits a monotonic rise after 50×10^3 steps confirming the tendency of the agent to learn a better policy resulting in a better reward collection.

The approximate KL (Kullback-Leibler) divergence measures the difference between the new policy and the old policy. It helps assess one key property of the proximal policy optimization algorithm: ensure that the policy update is not too drastic, preventing instability. In the sub-fig (c) KL-loss curve, we notice that the value spikes in the beginning indicate an aggressive pol-

icy update which correlates with the exploration phase. Starting from 100×10^3 steps, the KL-loss value stabilizes around zero which confirms that the policy updates are relatively small and stable. The entropy loss represents the randomness in the policy, high entropy implies exploration, while low entropy suggests exploitation. The steady rise of the entropy loss in sub-fig (d) during the agent training suggests that the policy is becoming more and more deterministic as the agent gradually shifts to exploitation.

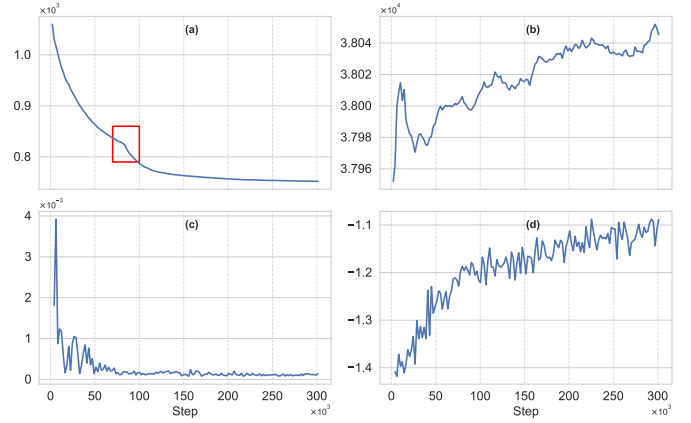


Figure 4: **Agent training performances, for instance ”ta51”(50x 20).** (a) the episode length, (b) the episode mean reward, (c) the Kullback-Leibler divergence evolution, (d) the entropy loss. The agent is trained for $3e5$ steps using the algorithm 1

In conclusion, key observations emerge from the training performance graphs. The episode’s mean length consistently decreases, signifying effective learning. The absence of plateaus indicates the agent avoids suboptimal policies, and an identified ”inflexion point” may signal a shift toward solution refinement. The episode reward shows a continuous monotonic rise, reflecting ongoing improvement in policy learning. Approximate KL divergence initially spikes during exploration but stabilizes later, suggesting consistent, small policy updates, and the entropy loss steadily rises, indicating a shift towards a deterministic policy.

4.2. Experimental results and analysis

Even though finite optimization offers exact solutions, their exorbitant computing cost renders them infeasible to solve bigger problems, paving the way for alternative less optimal solutions. Ranging from domain-specific heuristics to higher-level strategy metaheuristics and iterative approaches. In this section, we compare our results with a wide range of approaches, ranging from heuristics and metaheuristics to learning-based algorithms. The use of the Taillard instance across all the competing algorithms provided a solid benchmarking baseline. The list of competing algorithms can be found in Table 1.

In our study, we focus on six heuristics: SPT, LPT, SRM, SRPT, SSO, and LSO and two families of metaheuristics: tabu search and genetic algorithms. We assess our findings by comparing results with four metaheuristic benchmarks: TMIIG (Tabu-mechanism with the Improved Iterated Greedy algorithm), an enhanced variant within the tabu search family; CQGA (Coevolutionary Quantum Genetic Algorithm), a fusion of evolutionary

Heuristics [29]	
SPT	Job with the shortest processing time
LPT	Job with the longest processing time
SRM	Job with the shortest remaining machining time
SRPT	Job with the shortest remaining processing time
SSO	Job with the shortest processing time of subsequent operation
LSO	Job with the longest processing time of subsequent operation
Metaheuristics	
TMIG	Tabu with the improved iterated greedy algorithm [42]
CQGA	Coevolutionary quantum genetic algorithm [43]
HGSA	Genetic algorithm combined with simulated annealing [44]
GA-TS	Hybrid genetic algorithm and tabu search [9]
Learning-based	
GIN	Graph Isomorphism Network [24]
GAM	Gated-Attention Model [27]
DGERD	Disjunctive Graph Embedded Recurrent Decoding Transformer [29]

Table 1: Contending algorithms.

and genetic algorithms; HGSA, a blend of genetic algorithms with the simulated annealing heuristic; and GA-TS, a hybrid genetic algorithm and tabu search aiming to leverage the strengths of both approaches.

The comparison results between our approach and the heuristics and metaheuristics are summarized in Table 3. This comparison is conducted across 16 Taillard instances, with increasing complexity ranging from 15 machines x 15 jobs to 100 machines x 20 jobs. The makespan serves as the performance assessment metric for all algorithms. The algorithm with the best performance, resulting in a lower makespan for each instance, is denoted in bold in the table, while the runner-up is underlined.

The findings indicate that, although the runner-up algorithm varies depending on the instance, our approach consistently remains the top performer across all instance sizes, up to 50x20 instances. The optimality gap ranged from 2% up to 22% compared to the second-best approach. The Hybrid Genetic Algorithm and Tabu Search exhibited competitive results, overtaking PetriRL’s only in the 100x20 instances by 6%. The optimality gap is calculated based on the results of the runner-up as the baseline as follows:

$$\text{Optimality Gap} = -\frac{(C_{\max} - C_{\max(\text{baseline})})}{C_{\max(\text{baseline})}} \quad (5)$$

Following the assessment of heuristics and metaheuristics, our examination extends to more closely related iterative learning-based methodologies. We juxtapose our findings with three reinforcement learning frameworks: GIN (Graph Isomorphism Network), employing a graph neural network approach. GAM (Gated-Attention Model), employs an attention mechanism-based approach. Finally, DGERD (Disjunctive Graph Embedded Recurrent Decoding Transformer), adopts a transformer-based approach. Notably, all the competing learning-based algorithms employ the strategy of representing the Job Shop Scheduling Problem (JSSP) as a disjunctive graph—a limitation we will

delve into in a subsequent discussion.

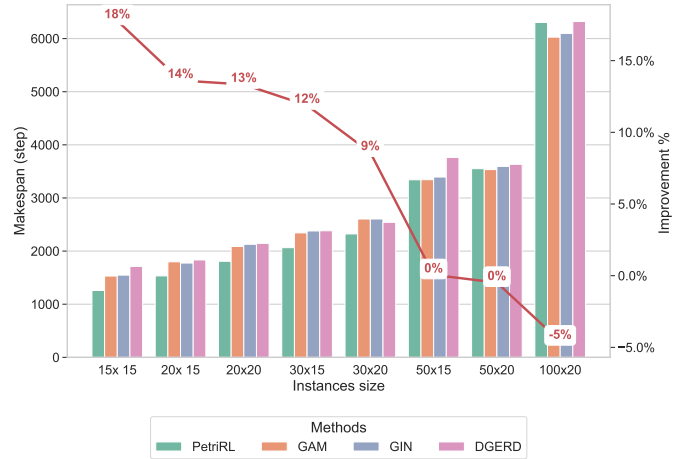


Figure 5: Comparison of the experimental results using PetriRL and learning-based approaches. In barplot is the makespan using different algorithms for different instance sizes, the associated y-axis is on the left. In the line plot is the improvement in percentage when using PetriRL compared to the second-best performer, the associated y-axis is on the right.

In line with the observations made during the heuristics and metaheuristics comparison, the results, as depicted in Fig. 5, reaffirm the superior performance of our approach. It demonstrates comparable efficacy in addressing mid-sized 50-job problems, exhibiting an optimality gap ranging from 9% to 18% for small to mid-sized instances to only be slightly surpassed on the larger 100 jobs instances.

Our approach provides a more effective solution to modelling the entire shop floor as a Petri net. This offers the advantage of passing the production line’s structure to the algorithm only once, in the form of a Petri net. This contrasts with the redundant implicit introduction of the floor structure when using disjunctive graphs. Notably, as long as a shop floor shares the same number of machines and job lines, it is represented by the same Petri net model. The sole input to our system consists of coloured tokens in the job operation queue, containing the processing time. This not only presents a more naturally efficient method for modelling the job shop but also provides dynamic planning capabilities, a topic we will further explore in this section 4.4.

In conclusion, our findings indicate that our approach surpassed various algorithms, spanning from heuristics to metaheuristics and learning-based methods. While it is noteworthy that our solution was outperformed in larger instances, it provides a more interpretable and intuitively modelled representation of a job shop. This eliminates the tedious task of depicting the job shop scheduling problem as a disjunctive graph for every instance input, as is required by competing learning-based approaches.

4.3. Ablation study

In this section, we conduct an ablation study to understand the contribution of individual components of the PetriRL framework to the results. We highlight three major components whose combination contributes to the positive performance of the framework: action masking, reward shaping, and event-based control.

Table 2: Comparison of Makespan using PetriRL and various Heuristic and Metaheuristic. Algorithms[29][9]

Inst	Size	Gap	PetriRL	Heuristics						Metaheuristics			
				SPT	LPT	SRM	SRPT	SSO	LSO	TMIIG	CQGA	HGSA	TS-GA
ta01	15x15	2%	1258	1872	1812	2163	2148	2148	1957	1486	1486	1324	1282
ta02	15x15	10%	1240	1913	1562	1814	2114	1905	1759	1528	1528	1442	1373
ta11	20x15	5%	1533	2273	2117	2353	2442	2343	2216	2011	2044	1713	1622
ta12	20x15	9%	1554	2527	2213	2459	2160	2253	2187	2166	2166	1718	1706
ta21	20x20	22%	1683	2488	2691	3071	2955	2610	2647	2973	2973	2331	2331
ta22	20x20	22%	1683	2510	2515	2796	2726	2636	2522	2582	2852	2280	2169
ta31	30x15	17%	2064	2993	2589	3101	3156	2916	2478	3161	3161	2731	2730
ta32	30x15	18%	2159	3050	2624	3166	3272	2890	2634	3432	3432	2934	2890
ta41	30x20	19%	2323	3105	3155	3482	3232	3058	2873	4274	4274	3198	3100
ta42	30x20	28%	2179	3772	3356	3641	3624	3528	3096	4177	4177	3020	3017
ta51	50x15	13%	3342	4456	3881	4174	4443	4418	3844	6129	6129	4105	4064
ta52	50x15	10%	3338	4179	3891	4588	4371	4059	3715	5725	5725	3992	3910
ta61	50x20	15%	3551	4500	4467	5024	5041	4520	4188	6397	6397	5536	5502
ta62	50x20	15%	3570	4933	4416	4764	4821	4757	4217	6234	6234	5302	5301
ta71	100x20	-6%	6303	7830	6949	7916	8118	7594	6754	8077	8077	5964	5962
ta72	100x20	-5%	5894	7611	6675	7607	7639	7077	6674	7880	7880	5596	5594

Instance	Size	Gap	PetriRL	DGERD	GIN	GAM
ta01	15x15	18%	1258	1711	1547	1530
ta11	20x15	14%	1533	1833	1774	1797
ta21	20x20	13%	1808	2145	2128	2086
ta31	30x15	12%	2064	2382	2378	2342
ta41	30x20	9%	2323	2541	2603	2603
ta51	50x15	0%	3342	3762	3393	3343
ta61	50x20	0%	3551	3633	3593	3534
ta71	100x20	-5%	6303	6321	6097	6027

Table 3: Comparison of PetriRL to reinforcement learning-based algorithms.

In Fig 6 two main environments were considered to conduct the ablation study. First a relatively small environment in the form of the Taillard Benchmark number "01" representing a JSSP problem of 15 jobs and 15 machines, the second is a larger environment in the form of the Taillard Benchmark number "61" representing a JSSP problem of 50 jobs and 20 machines.

The controlling parameter is the number of agent-environment interactions needed to reach a termination state which also coincides with the actual length of the episode. Since no maximum number of steps limit was introduced, the only possible termination scenario is the delivery of all pieces and resolving the JSSP. The decision to employ episode length as a control parameter in the ablation study is deliberate, as it stands as a neutral metric unaffected by the choice of the reward function. Given that the reward function is a component of the ablation study, episode length emerges as a prominent performance criterion for analysis. All the agents were trained for a fixed number of steps, namely 3×10^5 steps. The evolution of the episode length during the training using the reference setup in the small and large environment, respectively shown in the sub-graphs (a) and (b) can be found in Fig 6. The reference setup is defined by the use of a masked version of the proximal policy optimization

PPO, event-based control, and reward shaping combined.



Figure 6: Ablation study conducted using "ta01" (15x15) and "ta61" (50x20) instances. (a)-(b) is the reference performance using event-based control, action-masking, and reward shaping, (c)-(d) the reward shaping is removed and replaced by a fixed reward, (e)-(f) the event-based control is omitted, (g)-(h) no action making is employed.

To evaluate the impact of the shaped reward function, we replaced the dynamic advantage estimation with a fixed penalty of minus one for each step, encouraging the agent to minimize the number of steps taken to solve the JSSP. Examining the results in the small environment (c) compared to the reference case (a), it is evident that the agent can still learn despite the use of a fixed reward. However, this comes at the cost of efficiency and stability. A comparison of the two sub-graphs reveals that in the reference case, the episode length stabilizes at $50 \cdot 10^3$

steps, whereas using the fixed reward, it takes $100 \cdot 10^3$ steps. Additionally, the minimum episode length achieved with the fixed reward is 250 interactions, compared to 225 in the reference case. Furthermore, the overall smoother learning curve in the reference case highlights that, although the reward function may not be crucial in a small environment, its usage brings numerous benefits. The situation in the large environment is different, comparing the sub-graphs (b) and (d) we notice that the agent is no longer capable of learning using the fixed reward, in this case, the use of reward shaping is vital. This can be explained by the fact that in a small environment, the agent can reach the terminal state without guidance by randomly sampling actions, but in a larger environment, this becomes highly unlikely. Furthermore, the monotonic rise of the length of episodes in (b) despite the agent’s ability to reach termination states indicates that the fixed reward does not bring too much information about how good or bad the policy is each time step.

After analyzing the impact of reward shaping on performance, we explore the benefits of training the agent in an event-based environment. In sub-figures (e) and (f), we analyse the episode length during agent training for both small and large instances. In this scenario, the agent-environment interaction is not event-based, meaning the agent must take an action regardless of machines availability. In a small environment, this results in a significant increase in episode length which can be attributed to poor credit assignment. Without event-based control, the agent will constantly be forced to choose the standby action since no machine is available. At the same time the advantage function, based on an estimation of makespan, continues to evolve. Consequently, the same "state, action" pair receives different rewards, confusing the agent and impeding the learning process. This phenomenon is more pronounced in the large instance, as depicted in sub-figure (f) where the agent requires $23 \cdot 10^3$ steps to solve the JSSP, compared to $1 \cdot 10^3$ in the reference case.

After evaluating the impact of event-based control, our attention turns to the examination of action masking. Similar to the previous analysis, we investigate the influence of action masking in both small and large environments, depicted in sub-graphs (g) and (h) respectively. In the case of a small environment, we observe that the agent can achieve fragile stability. However, it necessitated a remarkably high 6200 interactions compared to the 225 steps in the reference scenario. This discrepancy can be attributed to the absence of a mask, which increases the likelihood of the agent selecting non-enabled actions, leading to wasted steps. Executing a non-enabled transition maintains the environment in the same state, resulting in a null advantage value. Consequently, the step provides no new information to the agent, hindering policy enhancement. In the larger environment, which results are shown in (h) the side effect of not using a mask is more accentuated as a consequence the agent is not able to learn. This is due to the much larger action space and the smaller probability of choosing an enabled action, especially in the early exploration phase.

In summary, the ablation study underscores the significance of action masking, event-based control, and reward shaping within the PetriRL framework. While reward shaping may be less

critical in smaller scenarios, it does positively impact overall performance. The same cannot be said for action masking and event-based control. Removing either of them in the smaller instances leads to a substantial increase in episode length by 10 to 20 times. In larger environments, all three elements are essential pillars for agent training, as the removal of any one element renders the agent unable to learn.

4.4. Generalisation and flexibility

In this section, we are studying the generalization and flexibility capability of the algorithm. Although the maximum makespan is the main performance metric in the previous paragraph, its dependence on the individual processing times in the different instances is a non-objective metric to compare performances. For example, an instance with long individual operation processing times will most probably end in a longer makespan despite the best agent effort to find the optimal planning. The episode length is chosen as a performance metric to mitigate this problem. Independently of the instance processing time, the episode length can be a good indication of whether the agent policy transfer to another instance is successful or not. The algorithm is put to the test using Taillard Benchmark instances. The instances can be grouped into ten instances, where the instances share the same characteristics such as the number of jobs and machines, but different processing time values. To assess the generalization capacity, local and global approaches are tested.

In the local approach, we test generalization capacity on instances of identical sizes, a dedicated agent is trained to solve instances in the same-size group. Shown in Fig 7 are the results of the generalization capacity for the local agents. First, the bar plots represent the makespan of the different JSSP instances, instances with identical sizes share the same colour in the graph, for example, all instances ranging from "ta01" to "ta10" are 15 jobs X 15 machines. In every size group, the agent was only trained on one of the instances denoted in Fig 7 by a star on the top. In the second y-axis, a line graph (in red) is used to plot the agent-environment interaction count needed to solve the JSSP. We observe a robust stable performance of the agent during deployment across all the instances in every group shown by the stability in the episode length. The max episode length variance in every group was **7 steps**.

In the global approach, the agent is only trained on one of the largest available instances, in our case $100 \text{ jobs} \times 20 \text{ machines}$. Once trained, it can handle instances of any size up to $100 \text{ jobs} \times 20 \text{ machines}$. This capability is possible due to the inherent masking feature of the Petri Net, enabling the use of only a portion of the shop floor. Depending on the instance size, job queues are filled, and the unused job places remain vacant. Consequently, the corresponding selection and allocation transitions are automatically excluded from the action space, thus non affecting the decision-making. shown in Fig 4.4 are the results of the generalization capacity for the global agent where we contrast the performance of training a dedicated agent for each size group with that of using a single agent across all instances. The comparison, reveals a negligible average performance drop of 0.7% compared to using local agents proving a robust generalization capacity across all the instances.

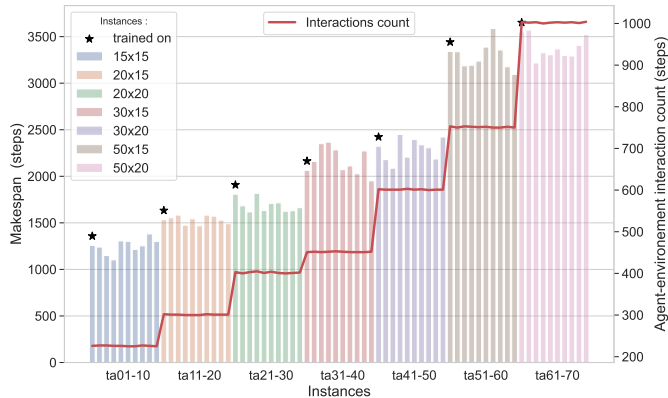


Figure 7: **Generability performance on different instances using local agents.** In barplot is the makespan obtained using different agents on different instance sizes and the line plot is the episode length evolution. Every agent is trained on one instance denoted by a (*) and tested on the rest of the instances in the same size group.

While the global approach offers undeniable advantages, such as enhanced flexibility, there exist situations where employing local agents proves more advantageous. Training an agent on a larger network comes with a computation price since updating the weights of the larger policy requires more computation. Moreover, to a lesser extent, the performance is also affected during inference since the agent is required to propagate the input through all the policy network’s layers during the forward pass despite the majority of the action space being masked in the output. In contrast, training a local agent for every size group will result in less computation and deployment time for the smaller instances as the agent will use a more adapted neural network size. In summary, prioritizing flexibility makes opting for a global agent the better-suited choice. On the other hand, if performance, particularly during inference, is more critical, using a dedicated agent is more efficient.

The proposed framed work is an object-oriented modelling approach, meaning that every machine, job, and token are object. This opens the possibility for the dynamic addition of job operations to in-process jobs, or dynamically changing the the number of production lines by adding complete jobs requiring new machines offering great flexibility. During the decision-making, depending on the observation depth, the agent only considers a set number of tokens in the job operation queue to choose an action. This means appending new operations to a job queue is also possible during inference. In the case, that the observation depth is one, an operation can be added at every time step without affecting the decision-making. For a single-agent approach, a given number of machines and job places are initially allocated. The system has the flexibility to adjust the number of utilized resources dynamically based on input parameters.

5. Conclusion

In this work, we proposed a novel job shop scheduling-solving framework. We employed Petri net not only as a graphical tool to represent the job shop promoting explainability but also as a con-

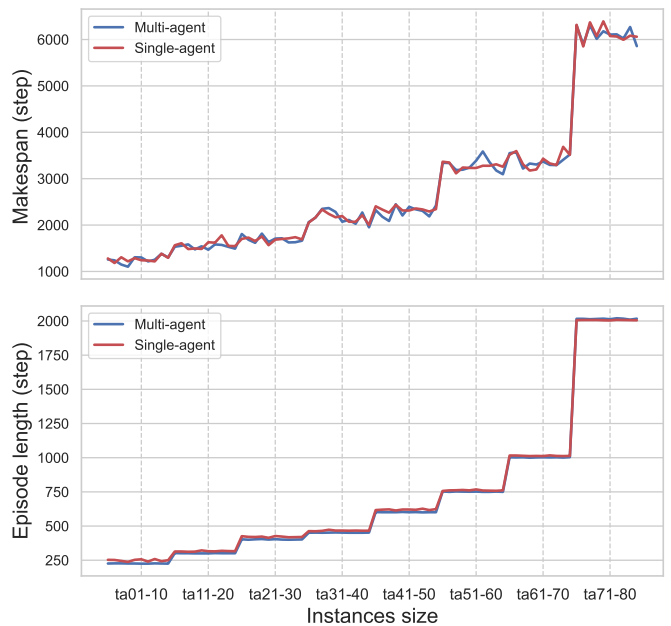


Figure 8: **Generability performance on different instances using a global agent.** The agent is only trained on the "ta80" (100×20) instance and tested on the rest of the Taillard benchmark instances. The global agent performance depicted in the red line is compared to the local agent performance depicted in the blue line.

trol tool to govern the autonomous components of the processes. Delegating the non-critical control parts to the Petri net allowed the RL agent to efficiently focus on allocation decision-making. The efficiency is further improved with the introduction of event-based control and action space masking. Thanks to the job shop’s main structure being encompassed in the Petri net model, we were able to eliminate the need for the laborious preprocessing step of casting the JSSP instances into a disjunctive graph. We compared our performance to a large spectrum of contenders ranging from heuristics, and metaheuristics, to learning-based algorithms. The results show that our approach offers competitive performance in large instances, and outperformed all the competition in small to medium-sized instances. We carried out an ablation study to determine the contribution of individual components of the PetriRL framework to the results, and we found that with varying degrees of importance, all of the action masking, reward shaping, and event-based control are vital for the agent’s successful training, especially in larger instances. Finally, we tested the generability and flexibility characteristics of our framework. In the initial phase, individual agents were trained for each size group, resulting in robust generalization within instances of the same size. In the subsequent phase, we extended the framework’s capabilities by training a single agent to handle all instances, regardless of size. This leveraged the intrinsic masking capability of Petri Nets through the transition guard function. The across-size generalization demonstrated robust performance with only a minimal drop compared to local generalization, yet offered greater flexibility. A key advantage of our proposed framework lies in its object-oriented modelling

approach, where each job operation is represented by a token. This allows to dynamically add operations in the middle of the process, and dynamically opening production lines without compromising performances. In essence, our framework provides a flexible, dynamic, efficient, and explainable solution for solving JSSPs.

In upcoming research, our focus will be on enhancing interpretability, a critical limitation of machine learning in industrial applications. While strides were made using Petri nets as an interpretable graphical representation, the inherent opacity of the decision-making process within the policy neural network leaves room for improvement.

References

- [1] A. Kuhnle, L. Schäfer, N. Stricker, G. Lanza, Design, implementation and evaluation of reinforcement learning for an adaptive order dispatching in job shop manufacturing systems, 2212-8271 81 (2019) 234–239. doi: 10.1016/j.procir.2019.03.041.
- [2] Y. Fang, C. Peng, P. Lou, Z. Zhou, J. Hu, J. Yan, Digital-twin-based job shop scheduling toward smart manufacturing, IEEE Transactions on Industrial Informatics 15 (12) (2019) 6425–6435. doi:10.1109/tii.2019.2938572.
- [3] G. Tuncel, G. M. Bayhan, Applications of petri nets in production scheduling: a review, The International Journal of Advanced Manufacturing Technology 34 (7-8) (2007) 762–773. doi:10.1007/s00170-006-0640-1.
- [4] R. Zurawski, M. Zhou, Petri nets and industrial applications: A tutorial, IEEE Transactions on Industrial Electronics 41 (6) (1994) 567–583. doi: 10.1109/41.334574.
- [5] L. Hu, Z. Liu, W. Hu, Y. Wang, J. Tan, F. Wu, Petri-net-based dynamic scheduling of flexible manufacturing system via deep reinforcement learning with graph convolutional network, Journal of Manufacturing Systems 55 (2020) 1–14. doi:10.1016/j.jmsy.2020.02.004.
- [6] H. M. Wagner, An integer linear-programming model for machine scheduling, Naval Research Logistics Quarterly 6 (2) (1959) 131–140. doi:10.1002/nav.3800060205.
- [7] P. Brucker, B. Jurisch, B. Sievers, A branch and bound algorithm for the job-shop scheduling problem, 0166-218X 49 (1-3) (1994) 107–127. doi:10.1016/0166-218X(94)90204-6.
- [8] X. Li, M. Li, Multiobjective local search algorithm-based decomposition for multiobjective permutation flow shop scheduling problem, IEEE Transactions on Engineering Management 62 (4) (2015) 544–557. doi:10.1109/TEM.2015.2453264.
- [9] M. S. Umam, M. Mustafid, S. Suryono, A hybrid genetic algorithm and tabu search for minimizing makespan in flow shop scheduling problem, Journal of King Saud University - Computer and Information Sciences 34 (9) (2022) 7459–7467. doi:10.1016/j.jksuci.2021.08.025.
- [10] J. P. Usuga Cadavid, S. Lamouri, B. Grabot, R. Pellerin, A. Fortin, Machine learning applied in production planning and control: a state-of-the-art in the era of industry 4.0, Journal of Intelligent Manufacturing 31 (6) (2020) 1531–1558. doi:10.1007/s10845-019-01531-7.
- [11] M. Lubosch, M. Kunath, H. Winkler, Industrial scheduling with monte carlo tree search and machine learning, 2212-8271 72 (2018) 1283–1287. doi:10.1016/j.procir.2018.03.171.
- [12] B. Waschneck, A. Reichstaller, L. Belzner, T. Altenmüller, T. Bauernhansl, A. Knapp, A. Kyek, Optimization of global production scheduling with deep reinforcement learning, 2212-8271 72 (2018) 1264–1269. doi: 10.1016/j.procir.2018.03.212.
- [13] P. Priore, B. Ponte, J. Puente, A. Gómez, Learning-based scheduling of flexible manufacturing systems using ensemble methods, Computers & Industrial Engineering 126 (2018) 282–291. doi:10.1016/j.cie.2018.09.034.
- [14] I. Hatono, K. Yamagata, H. Tamura, Modeling and online scheduling of flexible manufacturing systems using stochastic petri nets, IEEE Transactions on Software Engineering 17 (2) (1991) 126–132. doi: 10.1109/32.67588.
- [15] K. Jensen, Coloured petri nets: A high level language for system design and analysis, International Conference on Application and Theory of Petri Nets (1989) 342–416.
- [16] H. M. Shih, T. Sekiguchi, A timed petri net and beam search based online fms scheduling system with routing flexibility, in: Proceedings. 1991 IEEE International Conference on Robotics and Automation, IEEE Comput. Soc. Press, 1991, pp. 2548–2553. doi:10.1109/ROBOT.1991.132010.
- [17] G. Mejia, J. P. Caballero-Villalobos, C. Montoya, Petri nets and deadlock-free scheduling of open shop manufacturing systems, IEEE Transactions on Systems, Man, and Cybernetics: Systems 48 (6) (2018) 1017–1028. doi:10.1109/TSMC.2017.2707494.
- [18] C. Yu, Q. Semeraro, A. Matta, A genetic algorithm for the hybrid flow shop scheduling with unrelated machines and machine eligibility, Computers & Operations Research 100 (2018) 211–229. doi:10.1016/j.cor.2018.07.025.
- [19] O. Engin, A. Güçlü, A new hybrid ant colony optimization algorithm for solving the no-wait flow shop scheduling problems, Applied Soft Computing 72 (2018) 166–176. doi:10.1016/j.asoc.2018.08.002.
- [20] M. Panzer, B. Bender, Deep reinforcement learning in production systems: a systematic literature review, International Journal of Production Research 60 (13) (2022) 4316–4341. doi:10.1080/00207543.2021.1973138.
- [21] S. Luo, Dynamic scheduling for flexible job shop with new job insertions by deep reinforcement learning, Applied Soft Computing 91 (2020) 106208. doi:10.1016/j.asoc.2020.106208.
- [22] B.-A. Han, J.-J. Yang, Research on adaptive job shop scheduling problems based on dueling double dqn, IEEE Access 8 (2020) 186474–186495. doi:10.1109/access.2020.3029868.
- [23] C.-L. Liu, C.-C. Chang, C.-J. Tseng, Actor-critic deep reinforcement learning for solving job shop scheduling problems, IEEE Access 8 (2020) 71752–71762. doi:10.1109/ACCESS.2020.2987820.
- [24] Zhang, Cong and Song, Wen and Cao, Zhiguang and Zhang, Jie and Tan, Puay Siew and Chi, Xu, Learning to dispatch for job shop scheduling via deep reinforcement learning, Advances in Neural Information Processing Systems 33 (2020) 1621–1632.
- [25] J. Park, J. Chun, S. H. Kim, Y. Kim, J. Park, Learning to schedule job-shop problems: representation and policy learning using graph neural network and reinforcement learning, International Journal of Production Research 59 (11) (2021) 3360–3377. doi:10.1080/00207543.2020.1870013.
- [26] M. S. A. Hameed, A. Schwung, Graph neural networks-based scheduler for production planning problems using reinforcement learning, Journal of Manufacturing Systems 69 (2023) 91–102. doi:10.1016/j.jmsy.2023.06.005.
- [27] Gebreyesus, Goytom and Fellek, Getu and Farid, Ahmed and Fujimura, Shigeru and Yoshie, Osamu, Gated-attention model with reinforcement learning for solving dynamic job shop scheduling problem, IEEE Transactions on Electrical and Electronic Engineering 18 (23) (2023) 932–944.
- [28] A. Vaswani, N. Shazeer, N. Parmar, J. Uszkoreit, L. Jones, A. N. Gomez, Ł. Kaiser, I. Polosukhin, Attention is all you need, Advances in Neural Information Processing Systems 30 (2017).
- [29] R. Chen, W. Li, H. Yang, A deep reinforcement learning framework based on an attention mechanism and disjunctive graph embedding for the job-shop scheduling problem, IEEE Transactions on Industrial Informatics 19 (2) (2023) 1322–1331. doi:10.1109/TII.2022.3167380.
- [30] K. Jensen, Coloured petri nets Basic concepts, analysis methods, and practical use, 2nd Edition, Springer, 1997.
- [31] G. Brockman, V. Cheung, L. Pettersson, J. Schneider, J. Schulman, J. Tang, W. Zaremba, Openai gym (2016).
- [32] Grover, Aditya and Leskovec, Jure, node2vec: Scalable feature learning for networks, Proceedings of the 22nd ACM SIGKDD international conference on Knowledge discovery and data mining (2016) 855–864.
- [33] D. Baumann, J.-J. Zhu, G. Martius, S. Trimpe, Deep reinforcement learning for event-triggered control, in: 2018 IEEE Conference on Decision and Control (CDC), IEEE, 2018, pp. 943–950. doi:10.1109/CDC.2018.8619335.
- [34] V. Narayanan, H. Modares, S. Jagannathan, F. L. Lewis, Event-driven off-policy reinforcement learning for control of interconnected systems, IEEE transactions on cybernetics 52 (3) (2022) 1936–1946. doi:10.1109/tcyb.2020.2991166.
- [35] D. Ye, Z. Liu, M. Sun, B. Shi, P. Zhao, H. Wu, H. Yu, S. Yang, X. Wu, Q. Guo, Q. Chen, Y. Yin, H. Zhang, T. Shi, L. Wang, Q. Fu, W. Yang, L. Huang, Mastering complex control in moba games with deep reinforce-

- ment learning, *Proceedings of the AAAI Conference on Artificial Intelligence* 34 (04) (2020) 6672–6679. doi:10.1609/aaai.v34i04.6144.
- [36] S. Huang, S. Ontañón, A closer look at invalid action masking in policy gradient algorithms, *The International FLAIRS Conference Proceedings* 35 (2022). doi:10.32473/flairs.v35i.130584.
- [37] T. G. Dietterich, Hierarchical reinforcement learning with the maxq value function decomposition, *Journal of Artificial Intelligence Research* 13 (2000) 227–303. doi:10.1613/jair.639.
- [38] B. J. de Moor, J. Gijsbrechts, R. N. Boute, Reward shaping to improve the performance of deep reinforcement learning in perishable inventory management, *European Journal of Operational Research* 301 (2) (2022) 535–545. doi:10.1016/j.ejor.2021.10.045.
- [39] A. Nair, B. McGrew, M. Andrychowicz, W. Zaremba, P. Abbeel, Overcoming exploration in reinforcement learning with demonstrations, in: *2018 IEEE international conference on robotics and automation (ICRA)*, IEEE, 2018, pp. 6292–6299.
- [40] E. Taillard, Benchmarks for basic scheduling problems, *European Journal of Operational Research* 64 (2) (1993) 278–285. doi:10.1016/0377-2217(93)90182-M.
- [41] Raffin, Antonin and Hill, Ashley and Gleave, Adam and Kanervisto, Anssi and Ernestus, Maximilian and Dormann, Noah, Stable baselines3, *The Journal of Machine Learning Research* 22 (1) (2021) 12348–12355.
- [42] J.-Y. Ding, S. Song, J. N. Gupta, R. Zhang, R. Chiong, C. Wu, An improved iterated greedy algorithm with a tabu-based reconstruction strategy for the no-wait flowshop scheduling problem, *Applied Soft Computing* 30 (2015) 604–613. doi:10.1016/j.asoc.2015.02.006.
- [43] G. Deng, M. Wei, Q. Su, M. Zhao, An effective co-evolutionary quantum genetic algorithm for the no-wait flow shop scheduling problem, *Advances in Mechanical Engineering* 7 (12) (2015) 168781401562290. doi:10.1177/1687814015622900.
- [44] H. Wei, S. Li, H. Jiang, J. Hu, J. Hu, Hybrid genetic simulated annealing algorithm for improved flow shop scheduling with makespan criterion, *Applied Sciences* 8 (12) (2018) 2621. doi:10.3390/app8122621.