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Abstract

With the increasing awareness of privacy and the deployment of legislations in various multi-agent system application domains such as power systems and intelligent transportation, the privacy protection problem for multi-agent systems is gaining increased traction in recent years. This article discusses some of the representative advancements in the filed.

1 Introduction

All distributed algorithms for multi-agent systems require the sharing of information across the agents. The information sharing, although crucial to fulfill the coordination objective in multi-agent systems, also poses a threat for the privacy of participating agents in applications involving sensitive data. For example, in the rendezvous problem where a group of robots use distributed optimization to cooperatively find an optimal assembly point, participating robots may want to keep their initial positions private, which is particularly important in unfriendly environments (Zhang et al., 2019). In sensor network based localization, the positions of sensor agents should be kept private in sensitive (hostile) environments as well (Zhang and Wang, 2017; Zhang et al., 2019; Huang et al., 2015a). In fact, without an effective privacy mechanism in place, the results in Zhang et al. (2019); Huang et al. (2015a); Burbano-L et al. (2019) show that a participating agent's position can be easily inferred by an adversary or other participating agents in distributed-optimization based rendezvous and localization approaches. In multi-agent social networks, the opinions of individuals should also be kept private in many scenarios (Ye et al., 2019). Another example underscoring the importance of privacy protection in multi-agent systems is distributed machine learning where exchanged data may contain sensitive information such as medical records or salary information (Yan et al., 2012). In fact, recent results in Zhu et al. (2019) show that without a privacy mechanism in place, an adversary can use shared information to precisely recover the raw data used for training (pixel-wise accurate for images and token-wise matching for texts).

Although plenty of privacy mechanisms have been developed in the computer science domain, including differential privacy (Dwork et al., 2014), cryptography, secure-multiparty computation, etc, those mechanisms are developed for *static* data. Therefore, when directly applied to multi-agent systems involving *dynamics*, those privacy mechanisms usually fall short due to excessive computation/communication overhead or loss of algorithmic accuracy. In the past few years, plenty of efforts have been devoted to privacy protection in multi-agent systems. This article discusses some of the typical results in the control domain. It is worth noting that due to the vast amount of publications in this area in the past several years, our discussions do not pretend to be exhaustive and we apologize to anyone whose work is left out or not given the attention it deserves.

We consider two types of adversaries:

An honest-but-curious adversary is an agent who follows all protocol steps correctly but is curious and collects received data in an attempt to learn some information about other participating agents.

An eavesdropper is an external attacker who knows the network topology, and is able to wiretap communication links and access exchanged messages.

Generally speaking, an eavesdropper is more disruptive than an honest-but-curious agent in terms of information breaches because it can snoop messages exchanged on many channels whereas the latter can only access the messages destined to it. However, an honest-but-curious agent does have one piece of information that is unknown to an external eavesdropper, i.e., the internal state information of agent *i* is available to the adversary if agent *i* is an honest-but-curious agent.

We will consider three typical algorithms that underpin most multi-agent applications, i.e., the static average consensus, the dynamic average consensus, and distributed optimization. We will use agents and nodes interchangeably.

2 Privacy protection for static average consensus

2.1 Problem formulation

Static average consensus

Usually, the static average consensus is also called average consensus. Following the convention in Olfati-Saber et al. (2007a), we represent a network of *m* nodes as a graph G = (V, E, L) with node set $V = \{v_1, v_2, \dots, v_m\}$, edge set $E \subset V \times V$, and the adjacency matrix $L = [L_{ij}[k]]$ denoting coupling weights which satisfy $L_{ij}[k] > 0$ if $(v_i, v_j) \in E$ and 0 otherwise. Here *k* is time index, denoting that $L_{ij}[k]$ could be timevarying. The set of neighbors of a node v_i is denoted as $\mathbb{N}_i = \{v_i \in V | (v_i, v_j) \in E\}$ and its cardinality is denoted as $|\mathbb{N}_i|$.

We represent the state variable of a node *i* as $x_i[k]$. For the sake of expositional simplicity, we assume scalar states. But the results are easily extendable to the case where the state is a vector. To achieve average consensus, namely convergence of all states $x_i[k]$ (*i* =

1, 2, \cdots , m) to the average of initial values, i.e., $\frac{\sum_{i=1}^{m} x_i[0]}{m}$, the update rule is formulated as (Olfati-Saber et al., 2007b)

$$x_i[k+1] = x_i[k] + \varepsilon \sum_{v_j \in \mathbb{N}_i} L_{ij}[k](x_j[k] - x_i[k])$$
(1)

where ε resides in the range $(0, \frac{1}{\Lambda}]$ with Δ defined as

$$\Delta \triangleq \max_{i=1,2,\cdots,m} |\mathbb{N}_i| \tag{2}$$

It has been well known that static average consensus can be achieved if the network is connected and there exists some $\eta > 0$ such that $\eta \le a_{ij}[k] < 1$ holds for all $k \ge 0$ (Nedić et al., 2010).

Privacy in static average consensus

In the static average consensus problem, the sensitive information are the initial values of individual agents. Namely, agent *i* should avoid its initial value $x_i[0]$ from being inferrable by honest-but-curious adversaries (i.e., other participating agents) and eavesdroppers (i.e., external observers).

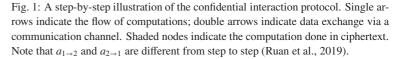
2.2 Literature review

In general, existing privacy solutions for the static average consensus problem are based on the following mechanisms:

Partially

homomorphic encryption based approaches Since commonly used encryption schemes rely on a trusted party to manage encryption and decryption keys, they are not appropriate for fully decentralized multi-agent systems. To the contrary, homomorphic encryption schemes allow computations to be performed on encrypted data without first having to decrypt it, and hence can be implemented in a fully decentralized setting without any trusted party to manage encryption and decryption keys. Homomorphic encryption schemes can be divided into two different categories, fully homomorphic encryption schemes and partially homomorphic encryption schemes. Although fully homomorphic encryption mechanisms allow any functions of unbounded depth to be evaluated in the encrypted domain, such approaches are extremely heavy in computation and communication and hence are rarely used in practice. Partially homomorphic encryption schemes can only allow functions of certain types, such as addition or multiplication, to be evaluated in the encrypted domain. However, their communication and communication overheads are manageable in many low-cost computing platforms, making them widely usable in practice. Some of the most popular partially homomorphic encryption schemes include RSA (Rivest et al., 1978), ElGamal (ElGamal, 1985), and Paillier (Paillier, 1999).

 v_1 v_2 Initial State $x_1, a_{1\to 2}, (k_{p1}, k_{s1}),$ $x_2, a_{2 \to 1}, (k_{p2}, k_{s2})$ $\mathcal{E}(-x_1, k_{p1})$ $\mathcal{E}(-x_2, k_{p2})$ Encrypt the Negative $\mathcal{E}_2(-x_2)$ $\mathcal{E}_1(-x_1)$ State (with its own key) Transmit the State $\mathcal{E}_2(-x_2), k_{p2}$ $\mathcal{E}_1(-x_1), k_{p1}$ and Public Key $\mathcal{E}(x_1, k_{p2})$ $\mathcal{E}(x_2, k_{p1})$ Encrypt the State $\mathcal{E}_2(x_1)$ $\mathcal{E}_1(x_2)$ (with received key) $\mathcal{E}_2(x_1)\mathcal{E}_2(-x_2)$ $\mathcal{E}_1(x_2)\mathcal{E}_1(-x_1)$ Compute the Difference $\mathcal{E}_2(x_1 - x_2)$ $\mathcal{E}_1(x_2 - x_1)$ (in ciphertext) $\mathcal{E}_2(x_1 - x_2)^{a_{1 \to 2}}$ $\mathcal{E}_1(x_2 - x_1)^{a_{2 \to 1}}$ Multiply the Weight $\mathcal{E}_2(a_{1\to 2}(x_1 - x_2))$ $\mathcal{E}_1(a_{2\to 1}(x_2 - x_1))$ (in ciphertext) Transmit the Result $\mathcal{E}_1(a_{2\to 1}(x_2 - x_1))$ $\mathcal{E}_2(a_{1\to 2}(x_1 - x_2))$ Back to Sender $\mathcal{D}(\cdot, k_{s1})$ $\mathcal{D}(\cdot, k_{s2})$ Decrypt the Result $a_{2\to 1}(x_2 - x_1)$ $a_{1\to 2}(x_1 - x_2)$ $a_{1\to 2}(\cdot)$ $a_{2 \to 1}(\cdot)$ Multiply the Weight Δx_{12} $\Delta x_{21} =$ (in plaintext) $a_{1\to 2}a_{2\to 1}(x_2-x_1)$ $a_{2\to 1}a_{1\to 2}(x_1-x_2)$



Partially homomorphic encryption was first introduced to the control domain by Kogiso and Fujita (2015a) who first applied partially homomorphic encryption in a networked control system. Although plenty of results were reported following Kogiso and Fujita (2015a),

Privacy mechanisms		Typical relevant results	Comments
Partially homomorphic encryption	fully decentralized	Ruan et al. (2017), Ruan et al. (2019), Hadjicostis and Domínguez-García (2020), Yin et al. (2020), Yin et al. (2020), Yu et al. (2021),	overhead
	with a server	Kogiso and Fujita (2015b), Gao et al. (2021)	Heavy in computation/communication overhead
Decomposition	state decomposition	Wang (2019), Wang et al. (2021b), Zhang et al. (2022e), Zhang et al. (2022a), Chen et al. (2023b), Duan et al. (2023)	
	edge decomposition	Zhang et al. (2022e), Xiong and Li (2022)	
Dynamics based	directed graph	Gao et al. (2018a), Gao and Wang (2022), Gao et al. (2022)	Information theoretic privacy
	undirected graph	Gupta et al. (2019)	Information theoretic privacy
Differential privacy	decentralized	Nozari et al. (2017), He et al. (2018), Gao et al. (2018b), Wang et al. (2021a), Fiore and Russo (2019), He et al. (2020), Liu et al. (2020), He et al. (2019), Zhang et al. (2022c), Katewa et al. (2018), Zhang et al. (2022d), Chen et al. (2023a), Wang et al. (2023)	Lose accurate convergence
	with a server	Huang et al. (2012)	Lose accurate convergence
Observability based	undirected graph	Manitara and Hadjicostis (2013), Mo and Murray (2016), Kia et al. (2015), Alaeddini et al. (2017)	Restricted in interaction topology

Table 1: Privacy solutions for static average consensus

there is a major hurdle for applying such approaches in the static average consensus problem, where the interaction weights have to be symmetric in undirected interaction graphs. In fact, in static average consensus, whenever agent *i* has access to the value of the interaction term $a_{ij}[k](x_j[k] - x_i[k])$ and the interaction weights $a_{ij}[k]$, it can always infer the state value of its neighbor *j*. Ruan et al. (2017) and Ruan et al. (2019) first solved the problem by proposing a mechanism to make the interaction weight $a_{ij}[k]$ unknown to both agent *i* and agent *j*. The idea is to decompose the interaction weight for any pair of interacting agents into the product of two positive values which are private to the two agents, respectively. The idea is illustrated in Fig. 1, where we represent a pair of interacting agents as agent v_1 and agent v_2 for the sake of notational simplicity.

Decomposition based approaches

The decomposition based privacy mechanism was first proposed in our work (Wang, 2019). Its basic idea is to decompose each agent's state x_i into two sub-states x_i^{α} and x_i^{β} , with the initial values $x_i^{\alpha}[0]$ and $x_i^{\beta}[0]$ randomly chosen from the set of all real numbers under the constraint $x_i^{\alpha}[0] + x_i^{\beta}[0] = 2x_i[0]$ (see Fig. 2). The sub-state x_i^{α} succeeds the role of the original state x_i in inter-node interactions and it is in fact the only state value from node *i* that can be seen by its neighbors. The other sub-state x_i^{β} also involves in the distributed interaction by (and only by) interacting with x_i^{α} . So the existence of x_i^{β} is invisible to neighboring nodes of node *i*, although it directly affects the evolution of x_i^{α} . Taking node 1 in Fig. 2(b) for example, x_1^{α} acts as if it were x_1 in the inter-node interactions while x_1^{β} is invisible to nodes other than node 1, although it affects the evolution of x_1^{α} .

Dynamics based approaches

There are two types of dynamics based privacy approaches for static average consensus. The first approach employs the robustness of dynamical systems stability to embed uncertainty based privacy without compromising convergence accuracy. For example, we know that for a scalar dynamical system $\dot{x} = ax$ where x is the state and a is a constant, it is always stable when a is negative, no matter what value a is. Employing this idea, we can introduce uncertainties in the coupling weights judiciously to enable privacy protection without compromising

the accuracy of convergence. This idea is first employed in Ruan et al. (2017, 2019) with the assistance of encryption and then generalized in Gao et al. (2018a); Gao and Wang (2022) without the assistance of encryption. The second dynamics based privacy approach for static average consensus is to add temporally or spatially corrected noises, which dates back at least to Abbe et al. (2012). This approach has been employed for privacy protection in static average consensus in Mo and Murray (2016), Manitara and Hadjicostis (2013), and Gupta et al. (2019), among others.

Differential privacy based approaches Differential privacy is a privacy framework initially proposed for protecting static datasets. Intutively speaking, differential privacy requires that for a mechanism performed on a dataset, when the dataset is changed in at most one entry, the output distribution of the mechanism is not changed significantly. The most commonly used definition of differential privacy is called ϵ -differential privacy, which is defined as follows Dwork et al. (2014):

Definition. (ϵ -differential privacy Huang et al. (2012)). For a given $\epsilon > 0$, a static average consensus algorithm is ϵ -differentially private if for any two sets of initial states \mathcal{P} and \mathcal{P}' that differ in at most one agent's initial value (usually called adjacent initial states), any set of observation sequences $O_s \subseteq \mathbb{O}$ (with \mathbb{O} denoting the set of all possible observation sequences), we always have

$$\mathbb{P}[\mathcal{R}_{\mathcal{P}}(O_s)] \le e^{\epsilon} \mathbb{P}[\mathcal{R}_{\mathcal{P}'}(O_s)] \tag{3}$$

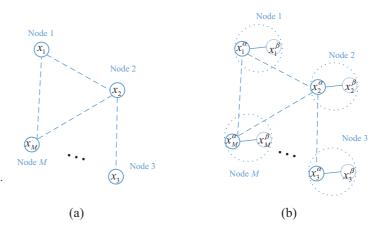


Fig. 2: State-decomposition based privacy-preserving average consensus (Wang, 2019). (a) Before state decomposition (b) After state decomposition

where \mathcal{P} denotes the mapping from initial states to observations under a given consensus algorithm and the probability \mathbb{P} is taken over the randomness over iteration processes.

Since differential privacy is defined under the probabilistic framework, it is usually achieved by injecting additive noises to shared messages. The first differentially private static average consensus approach was proposed in Huang et al. (2012) under the assistance of a central server. Fully decentralized solutions for differentially private static average consensus have been proposed in Nozari et al. (2017), He et al. (2018), and Katewa et al. (2018), among others.

Observation based privacy

This approach achieves privacy by making a certain state unobservable to some adversarial agents. However, given that the interaction graph has to be connected in static average consensus to ensure that all agents can converge to the same desired value, this approach can only achieve a very limited level of privacy protection.

3 Privacy protection for dynamic average consensus

3.1 Problem formulation

Dynamic average consensus

We consider a dynamic average consensus problem among a set of *m* agents $[m] = \{1, \dots, m\}$. We index the agents by 1, 2, \dots, m . Agent *i* can access fixed-frequency samples of its own reference signal $r_i \in \mathbb{R}^d$, which could be varying with time. Every agent *i* also maintains a state $x_i \in \mathbb{R}^d$. The aim of dynamic average consensus is for all agents to collaboratively track the average reference signal $\bar{r} \triangleq \frac{\sum_{i=1}^m r_i}{m}$ while every agent can only access discrete-time measurements of its own reference signal and share its state with its immediate neighboring agents.

We describe the local communication among agents using a weight matrix $L = \{L_{ij}\}$, where $L_{ij} > 0$ if agent *j* and agent *i* can directly communicate with each other, and $L_{ij} = 0$ otherwise. For an agent $i \in [m]$, its neighbor set \mathbb{N}_i is defined as the collection of agents *j* such that $L_{ij} > 0$. We define $L_{ii} \triangleq -\sum_{j \in \mathbb{N}_i} L_{ij}$ for all $i \in [m]$, where \mathbb{N}_i is the neighbor set of agent *i*.

Privacy in dynamic average consensus

In the dynamic average consensus problem, the sensitive information are the reference signals of individual agents. Namely, we have to make sure that the reference signal r_i of agent i is not inferable by honest-but-curious adversaries (i.e., other agents participating in the

dynamic average consensus problem) and eavesdroppers (i.e., external observers).

3.2 Literature review

Compared with the static average consensus problem, existing results on privacy protection for dynamic average consensus are relatively sparse (Zhang et al., 2022b). In fact, given that in many dynamic average consensus problems, the initial state $x_i[0]$ of agent *i* is usually set as the initial value of the reference signal $r_i[0]$, protecting the reference signal r_i includes protecting initial value as a special case. In fact, protecting the entire signal r_i is equivalent to protecting the values of r_i at infinitely many time instants, which makes privacy protection for dynamic average consensus much more challenging than privacy protection for static average consensus.

It is worth noting that in many applications of dynamic average consensus, such as distributed optimization where r_i is the gradient of agent *i*, many privacy solutions have been proposed. However, since we will specifically discuss privacy protection in distributed optimization in the next section, we do not consider those results in this section. We want to emphasize the results in Wang (2023) which proposed a robust dynamic average consensus algorithm that can ensure both differential privacy and accurate convergence:

Algorithm 1: Robust dynamic average consensus (Wang, 2023)

Parameters: Weakening factor $\chi^k > 0$ and stepsize $\alpha^k > 0$.

Every agent *i*'s reference signal is r_i^k . Every agent *i* maintains one state variable x_i^k , which is initialized as $x_i^0 = r_i^0$.

for
$$k = 1, 2, ...$$
 do

х

- a. Every agent *j* adds persistent DP-noise ζ_j^k to its state x_j^k , and then sends the obscured state $x_j^k + \zeta_j^k$ to agent $i \in \mathbb{N}_j$.
- b. After receiving $x_i^k + \zeta_j^k$ from all $j \in \mathbb{N}_i$, agent *i* updates its state as follows:

$$\sum_{i}^{k+1} = (1 - \alpha^{k})x_{i}^{k} + \chi^{k} \sum_{j \in \mathbb{N}_{i}} L_{ij}(x_{j}^{k} + \zeta_{j}^{k} - x_{i}^{k}) + r_{i}^{k+1} - (1 - \alpha^{k})r_{i}^{k}.$$
(4)

It is worth noting that recently Wang (2024) extended the result to the constrained consensus case where the state of every agent is constrained in a nonempty, closed, and convex set $X \subset \mathbb{R}^d$ (see details in Algorithm 2). However, it is worth noting that the problem in Wang (2024) is not a standard dynamic average consensus problem, since the final convergence point does not necessarily equal to the average reference signal therein.

Algorithm 2: Differentially-private constrained dynamic consensus (Wang, 2024)

Parameters: Weakening factor $\chi^k > 0$ and stepsize $\gamma^k > 0$.

Every agent *i*'s input is r_i^k . Every agent *i* maintains one state variable x_i^k , which is initialized randomly in *X*.

- a. Every agent *j* adds persistent DP-noise ζ_j^k to its state x_j^k , and then sends the obscured state $x_j^k + \zeta_j^k$ to agent $i \in \mathbb{N}_j$.
- b. After receiving $x_i^k + \zeta_i^k$ from all $j \in \mathbb{N}_i$, agent *i* updates its state as follows:

$$x_{i}^{k+1} = \Pi_{X} \left| x_{i}^{k} + \chi^{k} \sum_{j \in \mathbb{N}_{i}} w_{ij}(x_{j}^{k} + \zeta_{j}^{k} - x_{i}^{k}) + \gamma^{k} r_{i}^{k} \right|.$$
(5)

where Π_X denotes the Euclidean projection to the set *X*.

4 Privacy protection for distributed optimization

4.1 Problem formulation

Distributed optimization

We consider a network of *m* agents, interacting on a general directed graph. We describe a directed graph using an ordered pair $\mathcal{G} = ([m], \mathcal{E})$, where $[m] = \{1, 2, ..., m\}$ is the set of nodes (agents) and $\mathcal{E} \subseteq [m] \times [m]$ is the edge set of ordered node pairs describing the interaction among agents. For a nonnegative weighting matrix $L = \{L_{ij}\} \in \mathbb{R}^{m \times m}$, we define the induced directed graph as $\mathcal{G}_L = ([m], \mathcal{E}_L)$, where the directed edge (i, j) from agent *j* to agent *i* exists, i.e., $(i, j) \in \mathcal{E}_L$ if and only if $L_{ij} > 0$. For an agent $i \in [m]$, its in-neighbor set \mathbb{N}_i^{in} is defined as the collection of agents *j* such that $L_{ij} > 0$; similarly, the out-neighbor set $\mathbb{N}_i^{\text{out}}$ of agent *i* is the collection of agents *j* such that $L_{ji} > 0$.

The distributed optimization problem can be reformulated as follows:

$$\min_{\theta \in \mathbb{R}^d} F(\theta) \triangleq \frac{1}{m} \sum_{i=1}^m f_i(\theta)$$
(6)

Privacy mechanisms		Typical relevant results	Comments
Partially homomorphic encryption	fully decentralized	Zhang et al. (2018a), Zhang and Wang (2018)	Heavy in computation/communication overhead
	with a server	Lu and Zhu (2018), Alexandru et al. (2020)	Heavy in computation/communication overhead
Decomposition	state decomposition	Zhang et al. (2018b), Chen et al. (2023a), Sun et al. (2023)	
Dynamics based	coupling weight based	Zhang et al. (2018c), Gao et al. (2023a),	
	stepsize based	Wang and Poor(2022),Wang and Nedić (2023a)	Wang and Poor (2022) achieved informa- tion theoretic privacy
	quantization based	Wang and Başar (2022b)	Achieved differential privacy
Differential privacy	decentralized	Huang et al. (2015b), Zhang and Zhu (2016), Ding et al. (2021),Wang and Nedić (2023b), Xuan and Wang (2023), Wang and Başar (2023), (2023), Nozari et al. (2016), Mao et al. (2023), Wu et al. (2022), Zhao et al. (2022)	Wang and Nedić (2023b) maintains accu- rate convergence while ensuring differen- tial privacy
	with a server	Han et al.(2016),Hale and Egerstedt (2017)	Lose accurate convergence

Table 2: Privacy solutions for distributed optimization

where *m* is the number of agents, $\theta \in \mathbb{R}^d$ is a decision variable common to all agents, while $f_i : \mathbb{R}^d \to \mathbb{R}$ is a local objective function private to agent *i*.

It is worth noting that when the local objective function f_i is set as $f_i = ||\theta - \theta_i[0]||^2$, then the above distributed optimization problem reduces to the static average consensus problem (Zhang and Wang, 2018).

Privacy in distributed optimization

In most applications of distributed optimization, the sensitive information are contained in the objective function or gradient of participating agents. For example, in sensor network based target localization, the positions of sensors should be kept private in sensitive (hostile) environments (Zhang et al., 2019; Huang et al., 2015a). In existing distributed optimization based localization algorithms, the position of a sensor is a parameter of its objective function, and as shown in Zhang et al. (2019); Huang et al. (2015a); Burbano-L et al. (2019), it is easily inferable by an adversary using information shared in these distributed algorithms. The privacy problem is more acute in distributed machine learning where involved training data may contain sensitive information such as medical or salary information (note that in machine learning, together with the model, training data determines the objective function). In fact, as shown in our recent results (Wang and Başar, 2022b; Wang and Nedić, 2023b; Wang and Poor, 2022), in the absence of a privacy mechanism, an adversary can use information shared in distributed optimization to precisely recover the raw data used for training.

4.2 Literature review

In Table 2, we summarize typical existing results on privacy protection for distributed optimization. It is worth noting that since we focus on decentralized optimization, many other results based on cloud/server (see, e.g., Xiong et al. (2020)) are not included.

4.3 Typical algorithms

Algorithm 3: Differential-privacy-oriented distributed optimization	
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Parameters: Stepsize λ^k and weakening factor γ^k .

Every agent *i* maintains one state x_i^k , which is initialized with a random vector in \mathbb{R}^d .

for
$$k = 1, 2, ...$$
 do

a. Every agent *j* adds persistent DP-noise ζ_j^k to its state x_j^k , and then sends the obscured state $x_j^k + \zeta_j^k$ to agent $i \in \mathbb{N}_i^{\text{out}}$.

b. After receiving $x_i^k + \zeta_i^k$ from all $j \in \mathbb{N}_i^{\text{in}}$, agent *i* updates its state as follows:

$$x_i^{k+1} = x_i^k + \sum_{j \in \mathbb{N}_i^m} \gamma^k L_{ij} (x_j^k + \zeta_j^k - x_i^k) - \lambda^k \nabla f_i(x_i^k)$$

$$\tag{7}$$

c. end

The sequence $\{\gamma^k\}$ diminishes with time and is used to suppress the influence of persistent differential-privacy noise ζ_j^k on the convergence point of the iterates. The stepsize sequence $\{\lambda^k\}$ and attenuation sequence $\{\gamma^k\}$ have to be designed appropriately to guarantee the almost sure convergence of all $\{x_i^k\}$ to a common optimal solution θ^* . The persistent differential-privacy noise processes $\{\zeta_j^k\}$, $i \in [m]$ have zero-mean its variance is allowed to increase with time. In fact, allowing the variance to increase with time is key for our approach to enabling rigorous differential privacy while maintaining accurate convergence, even in the infinite time horizon. It is worth noting that an increasing noise variance will make the relative level between noise ζ_i^k and signal x_i^k increase with time. However, since the increase in noise variance can be outweighed by the decrease of γ^k , the actual noise fed into the algorithm, i.e., $\gamma^k \zeta_j^k$, still decays with time, which makes it possible for Algorithm 3 to ensure almost sure convergence to an optimal solution.

5 Privacy protection for other algorithms in multi-agent systems

We considered privacy protection in static average consensus, dynamic average consensus, and distributed optimization, which are the three most important primitives for coordination in multi-agent systems. In fact, the problem of privacy protection has also been addressed in many other algorithms for multi-agent systems. For example, distributed Nash equilibrium seeking is receiving increased traction in recent years due to its ability to capture the noncooperative relationship among agents in many multi-agent systems. To enable privacy protection in distributed Nash equilibrium seeking, plenty of efforts have been reported (see, e.g., Ye et al. (2021); Wang et al. (2022)). Two specific results worth mentioning are our recent results in Wang and Başar (2022a) and Wang and Nedić (2024) which enable differential privacy and accurate convergence simultaneously in aggregative games and general games, respectively. In addition, bipartite consensus is an algorithm for multi-agent systems which can model the dynamics in social networks. Recently, Zuo et al. (2022) and Wang et al. (2024) studied differential privacy for bipartite consensus. Furthermore, broadly speaking, networked control systems (Wang et al., 2008) and oscillator networks (Wang and Doyle III, 2011) can also be viewed as multi-agent systems (with heterogeneous agents and continuous-time interactions, respectively). Their privacy protection problem is also gaining increased attention recently (Cortés et al., 2016; Gupta and Chopra, 2018; Sultangazin et al., 2018; Darup et al., 2021; Rezazadeh and Kia, 2018).

6 Typical Applications

6.1 Application in robot networks

We consider the distributed rendezvous problem where a group of robots want to agree on the nearest meeting point without revealing each other's trajectories (Huang et al., 2015a) (note that the position information of a robot is embedded in its local gradient function). Mathematically, this can be modeled as the problem $\min_{x \in \mathbb{R}^d} \sum_{i=1}^m f_i(x) = \sum_{i=1}^m \frac{1}{2} ||x - p_i||^2$, where p_i represents the initial position of node *i*. For the simplicity of exposition, we consider the d = 1 case but similar results can be obtained when $d \neq 1$. We consider a circle graph where an agent can only communicate with its two immediate neighbors. We use the privacy approach in Gao et al. (2023b) which employs uncertainties in inter-agent coupling to make one agent's gradient indistinguishable by adversaries from observations (shared information). Fig. 3 shows the two different gradients of agent 1 that can lead to the same observations, which clearly makes agent 1's gradients indistinguishable by adversaries.

6.2 Application in machine learning

We consider the decentralized training of a convolutional neural network (CNN). More specially, we consider five agents which collaboratively train a CNN using the MNIST dataset (LeCun et al., 1994) under the topology in Fig. 4. The MNIST data set is a large benchmark database of handwritten digits widely used for training and testing in the field of machine learning (Deng, 2012). Each agent has a local copy of the CNN. The CNN has 2 convolutional layers with 32 filters with each followed by a max pooling layer, and then two more convolutional layers with 64 filters each followed by another max pooling layer and a dense layer with 512 units. Each agent has access to a portion of the MNIST dataset, which was further divided into two subsets for training and validation,

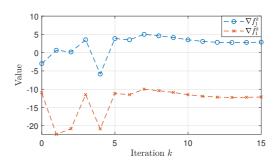


Fig. 3: The two different gradient functions of node 1 that lead to identical observations (Gao et al., 2023b).

respectively. We use the differentially private Algorithm 3 (Wang and Nedić, 2023b) to enable privacy, where the stepsize was set as $\lambda^k = \frac{1}{1+0.01k}$ and the weakening factor was set as γ^k as $\frac{1}{1+0.01k^{0.9}}$. The Laplace noise parameter

was set to $v^k = 1 + 0.01k^{0.3}$ to enable ϵ -differential privacy. The evolution of the training and testing accuracies averaged over 50 runs are illustrated by the solid and dashed blue curves in Fig. 5. To compare the convergence performance of this algorithm with the conventional distributed gradient descent algorithm under differential privacy noise, we also show the results of using the distributed gradient descent (DGD) algorithm in Nedić and Ozdaglar (2009) to train the same CNN using stepsize $\frac{1}{1+0.01k}$ under the same Laplace noise. The results are illustrated by the solid and dotted red curves in Fig. 5. It can be seen that Algorithm 3 has much better robustness to differential privacy noise. Moreover, to compare with the differential privacy approach for distributed optimization (PDOP) in Huang et al. (2015b), we also plot the results under PDOP in Huang et al. (2015b) under the same privacy budget ϵ . PDOP uses geometrically decaying stepsizes and noises to ensure a finite privacy budget. However, such fast-decaying stepizes turned out to be unable to train the complex CNN model (see training and testing accuracies in solid and dashed black curves in Fig. 5, respectively under $\lambda^k = 0.95^k$ and $v^k = 0.98^k$).

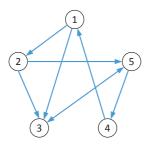


Fig. 4: The interaction graph.

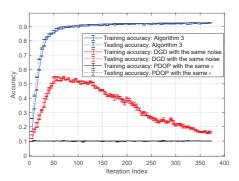


Fig. 5: Comparison of Algorithm 1 in Wang and Nedić (2023b) with the distributed gradient descent algorithm (DGD) in Nedić and Ozdaglar (2009) (under the same noise) and the differential-privacy approach for decentralized optimization PDOP in Huang et al. (2015a) (under the same privacy budget) using the MNIST image classification problem

7 Conclusions

We have discussed several typical approaches for privacy protection in multi-agent systems. In fact, all of the discussed results with superior performances are based on some kind of co-design of the privacy mechanism and coordination algorithms. Although different approaches have their respect advantages and disadvantages, and new privacy results have been continuously emerging from the control domain, we believe that only by cross fertilizing privacy results in computer science and control can we ensure effective privacy protection in multi-agent systems while retaining real-time and accuracy guarantees of coordination algorithms, which are essential for promoting multi-agent system applications in practical domains such as power systems and intelligent transportation.

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