Differentially Private Fair Binary Classifications

Hrad Ghoukasian, Shahab Asoodeh
Department of Computing and Software, McMaster University
{ghoukash, asoodeh}@mcmaster.ca

Abstract

In this work, we investigate binary classification under the constraints of both differential privacy and fairness. We first propose an algorithm based on the decoupling technique for learning a classifier with only fairness guarantee. This algorithm takes in classifiers trained on different demographic groups and generates a single classifier satisfying statistical parity. We then refine this algorithm to incorporate differential privacy. The performance of the final algorithm is rigorously examined in terms of privacy, fairness, and utility guarantees. Empirical evaluations conducted on the Adult and Credit Card datasets illustrate that our algorithm outperforms the state-of-the-art in terms of fairness guarantees, while maintaining the same level of privacy and utility.

I. Introduction

Machine learning algorithms are increasingly utilized in high-stake decision-making processes, highlighting the need to thoroughly assess their trustworthiness. Two main topics in the context of trustworthy machine learning are privacy and fairness. Machine learning models are required to, on the one hand, protect the privacy of individuals in the training datasets, and on the other hand, avoid leading to discrimination against any demographic subgroups. Differential privacy (DP) [1], [2] is the de-facto standard for privacy-preserving machine learning algorithms deployed in practice (e.g., [3]–[6]). Informally speaking, a randomized algorithm is differentially private if its output distribution does not significantly change by changing an entry in the dataset (yielding a pair of neighboring datasets). However, unlike privacy, there is no universal definition of fairness. Therefore, how fairness is defined or algorithmically enforced depends on the particular context of the problem. A widely recognized concept of fairness is statistical parity [7], also known as demographic parity. This definition implies that predictions of a classification model should be independent of the *sensitive* attributes, e.g., gender or race. While privacy and fairness have been extensively studied separately in the literature, the intersection of them has recently gained attention.

In the literature on fair classification algorithms, incorporating sensitive attributes is known as "fairness through awareness" [8]. While the choice to use or not use sensitive attributes is specific to each problem, [9] demonstrated that in scenarios where it is legally and ethically tenable to use these attributes, training separate classifiers (decoupled classifiers) for each group can outperform the performance of a single optimal classifier in terms of both accuracy and fairness (without privacy included). [10] also showed that employing decoupled classifiers does not negatively impact any group in terms of average performance metrics in the information-theoretic regime where the underlying data distribution is known. However, a significant challenge emerges when considering privacy. It has been empirically demonstrated that differentially private mechanisms can exacerbate unfairness [11], [12]. This necessitates the implementation of specific strategies to mitigate the negative effects of DP on fairness guarantees. Combining the above idea of decoupled classification with differential privacy, [13] proposed a "stratification" technique to reduce the disparity in differentially private mechanisms. This technique involves applying a DP mechanism separately to different subgroups and then recombining the respective results to derive overall statistics for the entire dataset. It was demonstrated that a naive stratification approach can produce highly accurate estimates for populationlevel statistics without requiring an extra privacy budget. Building on this foundation, our pipeline is structured to first apply DP and subsequently address fairness using a post-processing step. This order of application is crucial because DP methods introduce random noise into the process, which can alter the predictions of our model. If the model met fairness standards before we applied DP, the introduction of this noise could lead to a situation where those fairness standards are no longer met.

One line of work explores the relationship between DP and different notions of fairness, examining their compatibility and determining how fairness deteriorates DP guarantees and vice versa [11], [12], [14]–[18]. Another research direction at the intersection of privacy and fairness centers around developing classification algorithms that simultaneously guarantee DP and a specific notion of fairness. Despite these remarkable recent advances, most such works suffer from inherent limitations. For instance, some of these methods are tailored only to certain types of classification models, such as logistic regression, and are not universally applicable to all desired models [19]–[21]. Other studies focus exclusively on providing privacy with respect to sensitive attributes of individuals in the dataset, neglecting the privacy of other features. More specifically, some methods ensure that an individual's sensitive attributes remain confidential, but they do not provide the same guarantees for non-sensitive attributes, leaving them potentially vulnerable to leakage [20], [22]–[24]. For instance, [22] introduced two effective differentially private fair classification algorithms which provide privacy guarantees only with respect to the sensitive attributes. This was achieved through the use of randomized response for sensitive attributes. They used randomized response mechanism (for achieving the *local* version of DP), and thus their method does not seem to naturally adapt to provide privacy guarantees for all features, especially when dealing with continuous and potentially high-dimensional non-sensitive attributes. Lastly, although most of these methods have shown practical effectiveness, they often lack theoretical guarantees regarding

TABLE I		
RENCHMARK METHODS IN DP-FAIR CLASSIFICATION		

Reference	Applicable to any model	Theoretical guarantee	Privacy w.r.t. all features
[19]	Х	×	✓
[20] (post-proc.)	✓	✓	Х
[20] (in-proc.)	Х	✓	✓
[21]	Х	Х	✓
[22]	✓	✓	Х
[25]	✓	Х	✓
[26]	✓	Х	✓
[23]	✓	X	Х
[24]	✓	Х	Х
[27]	✓	Х	✓
[28]	✓	Х	✓
[29] (FairDPSGD)	✓	Х	✓
[29] (FairPATE)	✓	Х	✓
This Work	✓	✓	✓

utility and the extent of fairness violation [19], [21], [23]–[29]. A summary of the characteristics of these methods can be found in Table I.

Our objective is to develop a binary classification algorithm with provable DP and fairness guarantees that addresses the limitations highlighted in Table I. Inspired by the success of decoupled classifiers and the effectiveness of stratification in reducing disparate impact in differentially private mechanisms (as noted in [9], [10], [13]), our approach involves training separate classifiers for each sub-population. These classifiers then go into a post-processing step to generate a single classifier. Following the method used in [30], our goal is to apply a post-processing technique that achieves statistical parity, ensuring that only minimal changes are made to the predictions of the original classifiers. We begin by slightly modifying the approach described in [31], initially developed for non-private settings (Algorithm 1). Then, we introduce Algorithm 2, a new method for binary classification that is both differentially private and fair. This new algorithm comes with theoretical guarantees for utility, fairness, and differential privacy. More precisely, our contributions are as follows:

- We establish a lower bound for the sum of prediction changes across a pair of subgroups under statistical parity (without privacy) and propose an algorithm (based on [31, Algorithm 1]) that attains it (Algorithm 1). Theoretical guarantee of this algorithm is given in Theorem 2.
- We then propose a differentially private version (Algorithm 2) of Algorithm 1 and derive its theoretical guarantees for fairness and utility —both with high probability and in expectation— in Theorem 3 and Proposition 4.
- Through several experiments on two well-known datasets (namely, Adult and Credit Card), we empirically demonstrate that Algorithm 2 achieves competitive accuracy when compared to the state-of-the-art DP-Fair classification method, DP-FRMI [28]. In particular, we show that for a given level of accuracy and privacy, our algorithm provides a significantly better fairness guarantee across both datasets.

II. NOTATION AND BASIC DEFINITIONS

We consider a binary classification setting where there is a joint distribution μ over the triplet T=(X,A,Y), where $X\in\mathcal{X}\subset\mathbb{R}^d$ is the feature vector of non-sensitive attributes, $A\in\{0,1\}$ is the sensitive attribute, and $Y\in\{0,1\}$ is the target output. We use $\mu(Y)$ to denote the marginal distribution of Y from a joint distribution μ over Y and some other random variables. Denote the marginal distribution of input X by μ^X . For $a\in\{0,1\}$, we use μ_a to mean the conditional distribution of (X,Y) conditioned on A=a, and μ_a^X to mean the marginal distribution of input X given A=a. For any group-aware classifier $h:\mathcal{X}\times\{0,1\}\to\{0,1\}$ and $a\in\{0,1\}$, we also use $h_a(\cdot)$ to denote the restriction of h on A=a, respectively, i.e., $h_a(\cdot):=h(\cdot,a)$. Finally, the probabilistic inequality $X\leq_\eta Y$ for a pair of random variables (X,Y) denotes the mathematical statement that $\mathbb{P}(X>Y)\leq\eta$.

We now formally define differential privacy and statistical parity.

Definition 1. (Differential privacy). A randomized mechanism $M: \mathcal{D} \to \mathcal{R}$ with domain \mathcal{D} and range \mathcal{R} is (ε, δ) -differentially private $((\varepsilon, \delta)\text{-DP})$ if for any pair of neighboring datasets D and D' that differ in exactly one record, and for any subsets of outputs $S \subseteq \mathcal{R}$, we have,

$$\mathbb{P}(M(D) \in S) \le e^{\varepsilon} \mathbb{P}(M(D') \in S) + \delta.$$

Definition 2. (Statistical parity). Given a joint distribution μ , the statistical parity gap of a binary classifier $\hat{Y} = \hat{h}(X, A)$ with $\hat{h}: \mathcal{X} \times \{0,1\} \to \{0,1\}$ is

$$\Delta_{SP}(\hat{h}) := |\mu_0(\hat{Y} = 1) - \mu_1(\hat{Y} = 1)|.$$

We say that $\hat{Y} = \hat{h}(X, A)$ satisfies γ -statistical parity if

$$\Delta_{SP}(\hat{h}) \leq \gamma.$$

III. MAIN RESULTS

In this section, we develop a framework for designing a binary classification algorithm with provable DP and fairness guarantees. We begin in Section III-A by detailing our approach to achieve fairness and the specified utility target in a non-private setting. Then in Section III-B, we expand this framework to include DP. Regardless of private or non-private settings, as outlined in Introduction, our framework involves partitioning the dataset according to sensitive attributes. We first develop a separate classifier for each subgroup. We then implement a post-processing technique that carefully combines those decoupled classifiers in a way to achieve statistical parity, with the objective of minimally perturbing the original classifiers' predictions.

We consider two classifiers, $h_0^*: \mathcal{X} \to \{0,1\}$ and $h_1^*: \mathcal{X} \to \{0,1\}$, each trained on subgroups specified by the sensitive attribute A with values 0 and 1, respectively. These classifiers are designed to maximize accuracy without initially considering fairness constraints. In a non-private setting, h_0^* and h_1^* are standard classifiers, whereas in private settings, they are considered classifiers learned by DP guarantees. Our post-processing method aims to derive a fair classifier $\hat{h}: \mathcal{X} \times \{0,1\} \to \{0,1\}$ from h_0^* and h_1^* . Following the methodology of [30], our objective is to achieve this goal by minimally perturbing the predictions of the original classifiers. We thus seek fair \hat{h} that optimizes the utility measured in terms of the sum $\mathbb{P}_{\mu_0^X}(\hat{h}_0(X) \neq h_0^*(X)) + \mathbb{P}_{\mu_1^X}(\hat{h}_1(X) \neq h_1^*(X))$. The reason for adopting the prediction changes over μ_0^X and μ_1^X (as opposed to the combined distribution μ^X) is as follows: when the demographic subgroups are imbalanced in the overall population, relying only on prediction changes across the combined distribution μ^X can be misleading. This approach may hide significant prediction shifts of the less-represented group, which would be more apparent if we examined the sum of the prediction changes within each subgroup's distribution (μ_0^X and μ_1^X). In other words, it might be possible to have small overall prediction changes across the combined distribution μ^X while minority groups are experiencing substantial changes in their predictions [31].

In Section III-A, we explore the non-private scenario, discussing Algorithm 1 that ensures statistical parity and optimal utility. In Section III-B, we extend the algorithm to take privacy into consideration. In particular, we propose Algorithm 2, which outputs a classifier that guarantees DP and achieves statistical parity while maintaining minimal changes in the predictions of the original classifiers, both in expectation and with high probability. To discuss our utility metric under the constraint of statistical parity, we rely on the following proposition.

Proposition 1. Let $h_0^*: \mathcal{X} \to \{0,1\}$ and $h_1^*: \mathcal{X} \to \{0,1\}$, be arbitrary classifiers trained on subgroups specified by the sensitive attribute A=0 and A=1, respectively. If a predictor $\hat{Y}=\hat{h}(X,A)$ satisfies γ -statistical parity, then

$$\mathbb{P}_{\mu_0^X}(\hat{h}_0(X) \neq h_0^*(X)) + \mathbb{P}_{\mu_1^X}(\hat{h}_1(X) \neq h_1^*(X)) \geq \left| \mathbb{P}_{\mu_0^X}(h_0^*(X) = 1) - \mathbb{P}_{\mu_1^X}(h_1^*(X) = 1) \right| - \gamma.$$

A. Fair Post-Processing Without Privacy

Let $h_0^*: \mathcal{X} \to \{0,1\}$ and $h_1^*: \mathcal{X} \to \{0,1\}$ be decoupled classifiers, that is they are trained on subgroups associated with the sensitive attribute A=0 and A=1, respectively. h_0^* and h_1^* can be any arbitrary classifiers. In the context of our analysis, they can be considered as the best available classifiers trained on μ_0 and μ_1 . Our goal is to develop an optimal fair classifier $\hat{h}: \mathcal{X} \times \{0,1\} \to \{0,1\}$ using h_0^* and h_1^* . More precisely, we seek \hat{h} that satisfies $\Delta_{SP}(\hat{h})=0$ while attaining the lower bound in Proposition 1 for $\gamma=0$:

$$\mathbb{P}_{\mu_0^X}(\hat{h}_0(X) \neq h_0^*(X)) + \mathbb{P}_{\mu_1^X}(\hat{h}_1(X) \neq h_1^*(X)) = \left| \mathbb{P}_{\mu_0^X}(h_0^*(X) = 1) - \mathbb{P}_{\mu_1^X}(h_1^*(X) = 1) \right|.$$

We present Algorithm 1 to address this objective. This algorithm, which is a slightly modified version of the algorithm in [31], constructs the fair classifier h_{Fair}^* . The original algorithm in [31] builds a fair optimal classifier assuming oracle access to the Bayes optimal classifiers h_0^* and h_1^* . Our ultimate goal, to be discussed in the next section with Algorithm 2, is to identify a classifier that is both private and fair. However, the assumption of having access to Bayes optimal classifiers is not practical in DP settings, primarily due to the necessity of introducing noise. Consequently, in Algorithm 1, h_0^* and h_1^* are considered to be any arbitrary classifiers trained on subgroups specified by the sensitive attribute A=0 and A=1 (not necessarily the Bayes optimal classifiers).

Theorem 2. The classifier h_{Fair}^* constructed by Algorithm 1 satisfies perfect statistical parity ($\Delta_{SP}(h_{Fair}^*) = 0$) and is optimal in terms of the sum of prediction changes compared to classifiers h_0^* and h_1^* , that is

$$\mathbb{P}_{\mu_0^X}(h_{\mathit{Fair}0}^*(X) \neq h_0^*(X)) + \mathbb{P}_{\mu_1^X}(h_{\mathit{Fair}1}^*(X) \neq h_1^*(X)) = \left| \mathbb{P}_{\mu_0^X}(h_0^*(X) = 1) - \mathbb{P}_{\mu_1^X}(h_1^*(X) = 1) \right|.$$

Algorithm 1 Optimal Fair Binary Classifier

Input: Classifiers h_0^* and h_1^*

- Output: A randomized classifier $h^*_{\mathrm{Fair}}: \mathcal{X} \times \{0,1\} \to \{0,1\}$ 1: Compute $\alpha = \mathbb{P}_{\mu_0^X}(h_0^*(X) = 1)$ and $\beta = \mathbb{P}_{\mu_1^X}(h_1^*(X) = 1)$. W.L.O.G. assume $\alpha \geq \beta$
- 2: For (x, a), randomly sample s from the uniform distribution U(0, 1)

$$h_{\text{Fair}}^*(x,a) := \begin{cases} a = 0: & \begin{cases} 0 & \text{if } h_0^*(x) = 0 \text{ or } h_0^*(x) = 1 \text{ and } s > \frac{\alpha + \beta}{2\alpha} \\ 1 & \text{if } h_0^*(x) = 1 \text{ and } s \leq \frac{\alpha + \beta}{2\alpha} \\ a = 1: & \begin{cases} 0 & \text{if } h_1^*(x) = 0 \text{ and } s > \frac{\alpha - \beta}{2(1 - \beta)} \\ 1 & \text{if } h_1^*(x) = 1 \text{ or } h_1^*(x) = 0 \text{ and } s \leq \frac{\alpha - \beta}{2(1 - \beta)} \end{cases}$$

B. Fair Post-Processing With Privacy

Next, we delve into a private version of Algorithm 1. It is worth noting that achieving $(\varepsilon, 0)$ -DP is incompatible with non-trivial guarantees of fairness and utility (see, e.g., [14], [18] for more details). As a result, our objective is to design an (ε, δ) -DP version of Algorithm 1 with comparable fairness and utility guarantees, provided that $\delta > 0$.

First, notice that this goal is not feasible just by replacing the input classifiers h_0^* and h_1^* with some differentially private decoupled classifiers $h_{\varepsilon_0,\delta_0}^*$ and $h_{\varepsilon_1,\delta_1}^*$. These classifiers can be learned by applying an existing differentially private learning method —most notably, differentially private stochastic gradient descent (DP-SGD) [32]— to each demographic subgroup. This is because Algorithm 1 involves some computations over the dataset (e.g., in computing α and β in line 1). This therefore necessitates some changes in Algorithm 1 for constructing its differentially private version. Thus, our goal can be formulated as follows: Given decoupled classifiers $h_{\varepsilon_0,\delta_0}^*$ and $h_{\varepsilon_1,\delta_1}^*$, that are (ε_0,δ_0) -DP and (ε_1,δ_1) -DP, respectively, we wish to generate a fair classifier $h^*_{\varepsilon,\delta,\mathrm{Fair}}$ with the following properties:

- 1) $h_{\varepsilon,\delta,\mathrm{Fair}}^*$ is (ε,δ) -DP with some ε and δ (depending on $\varepsilon_0,\varepsilon_1,\delta_0$, and δ_1),
 2) $h_{\varepsilon,\delta,\mathrm{Fair}}^*$ satisfies γ -statistical parity with γ being a positive value close to zero with high probability and in expectation,
- 3) Among all classifiers satisfying the same level of statistical parity gap as $h_{\varepsilon,\delta,\text{Fair}}^*$, the classifier $h_{\varepsilon,\delta,\text{Fair}}^*$ performs comparably with the optimal classifier in terms of utility, both with high probability and in expectation. Optimality here is defined based on minimizing the total number of prediction changes across the distributions μ_0^X and μ_1^X , relative to the predictions made by $h_{\varepsilon_0,\delta_0}^*$ and $h_{\varepsilon_1,\delta_1}^*$.

To this goal, we present Algorithm 2 for learning such $h_{\varepsilon,\delta,\mathrm{Fair}}^*$. Recall that Algorithm 1 requires estimates of the proportions of data points in each subgroup classified as label one (denoted by α and β). Algorithm 2 is designed to privately estimate these quantities using the Laplace mechanism, whose privacy guarantee is well-understood. The following theorem delineates the performance of Algorithm 2 in terms of its achievable privacy and fairness guarantees as well as bounds on its utility.

Theorem 3. The classifier $h_{\varepsilon,\delta,Fair}^*: \mathcal{X} \times \{0,1\} \to \{0,1\}$ constructed by Algorithm 2 satisfies the following three properties:

- (Privacy guarantee) $h_{\varepsilon,\delta,Fair}^*$ satisfies (ε,δ) -DP with $\varepsilon = \sum_{i=0}^3 \varepsilon_i$ and $\delta = \delta_0 + \delta_1$,
- (Fairness guarantee) We have:

$$\Delta_{SP}(h_{\varepsilon,\delta,\mathit{Fair}}^*) \leq_{\eta_0 + \eta_1} \frac{\log(1/\eta_0)}{n_0 \varepsilon_2} + \frac{\log(1/\eta_1)}{n_1 \varepsilon_3},$$

• (Utility guarantee) Let $err^*(h_{\varepsilon_0,\delta_0}^*,h_{\varepsilon_1,\delta_1}^*)$ be defined as:

$$\min_{\substack{\hat{h}:\mathcal{X}\times\{0,1\}\to\{0,1\}\\\Delta_{SP}(\hat{h})\leq\Delta_{SP}(h_{\varepsilon,\delta,Fair}^*)}} \left[\mathbb{P}_{\mu_0^X}(\hat{h}_0(X)\neq h_{\varepsilon_0,\delta_0}^*(X)) + \mathbb{P}_{\mu_1^X}(\hat{h}_1(X)\neq h_{\varepsilon_1,\delta_1}^*(X)) \right].$$
(1)

Then, we have:

$$\begin{split} \mathbb{P}_{\mu_0^X}(h_{\varepsilon,\delta,\mathit{Fair}_0}^*(X) \neq h_{\varepsilon_0,\delta_0}^*(X)) + \mathbb{P}_{\mu_1^X}(h_{\varepsilon,\delta,\mathit{Fair}_1}^*(X) \neq h_{\varepsilon_1,\delta_1}^*(X)) \\ \leq & \eta_0 + \eta_1 \ \mathit{err}^*(h_{\varepsilon_0,\delta_0}^*,h_{\varepsilon_1,\delta_1}^*) + \frac{5}{2} \Big(\frac{\log(1/\eta_0)}{n_0 \varepsilon_2} + \frac{\log(1/\eta_1)}{n_1 \varepsilon_3} \Big). \end{split}$$

The privacy guarantee of $h_{\varepsilon,\delta,\text{Fair}}^*$ has two components: the privacy guarantees of the input classifiers and the Laplace mechanism employed to privately estimate $\bar{\alpha}$ and $\bar{\beta}$ (line 2 in Algorithm 2). Thus, the privacy guarantee in the above theorem follows directly from a composition result (e.g., basic composition [33]). However, the analysis pertaining to fairness and utility guarantees is rather long and thus deferred to the appendix. The fairness guarantee demonstrates, as expected, that perfect statistical parity can no longer be achievable when requiring privacy as well. Nevertheless, the theorem shows that $h^*_{\varepsilon,\delta,\mathrm{Fair}}$ satisfies γ -statistical parity where $\gamma > 0$ is a small constant provided that n_0 and n_1 are sufficiently large (compared to $1/\varepsilon_2$

Algorithm 2 Private and Fair Binary Classifier with Utility Gap Guarantee

Input: Classifiers $h_{\varepsilon_0,\delta_0}^*$ and $h_{\varepsilon_1,\delta_1}^*$, datasets $D_0 = (X_i,A_i=0,Y_i)_{i=1}^{n_0}$ and $D_1 = (X_j,A_j=1,Y_j)_{j=1}^{n_1}$, privacy parameters ε_2 , and ε_3

Output: Classifier $h^*_{\varepsilon,\delta,\mathrm{Fair}}: \mathcal{X} \times \{0,1\} \to \{0,1\}$ with $\varepsilon = \sum_{i=0}^3 \varepsilon_i$ and $\delta = \delta_0 + \delta_1$ 1: Compute $\bar{\alpha} = \frac{1}{n_0} \sum_{i=1}^{n_0} h^*_{\varepsilon_0,\delta_0}(X_i)$ and $\bar{\beta} = \frac{1}{n_1} \sum_{j=1}^{n_1} h^*_{\varepsilon_1,\delta_1}(X_j)$

- Sample l_0 from $\text{Lap}(\frac{1}{n_0 \varepsilon_2})$ and l_1 from $\text{Lap}(\frac{1}{n_1 \varepsilon_3})$. Define $\tilde{\alpha} = [\bar{\alpha} + l_0]_0^1$ and $\tilde{\beta} = [\bar{\beta} + l_1]_0^1$, where $[\cdot]_0^1$ denotes the
- 3: For (x,a), randomly sample s from the uniform distribution U(0,1)
- 4: Construct $h_{\varepsilon,\delta,\mathrm{Fair}}^*$ as follows: if $\tilde{\alpha} \geq \beta$, then

projection onto [0, 1].

$$h^*_{\varepsilon,\delta,\mathrm{Fair}}(x,a) := \begin{cases} \mathbbm{1}[h^*_{\varepsilon_0,\delta_0}(x)=1]\mathbbm{1}[s \leq \frac{\tilde{\alpha}+\tilde{\beta}}{2\tilde{\alpha}}] & \text{if } a=0\\ \mathbbm{1}[h^*_{\varepsilon_1,\delta_1}(x)=0]\mathbbm{1}[s \leq \frac{\tilde{\alpha}-\tilde{\beta}}{2(1-\tilde{\beta})}] + \mathbbm{1}[h^*_{\varepsilon_1,\delta_1}(x)=1] & \text{if } a=1 \end{cases}$$

if $\tilde{\alpha} < \tilde{\beta}$, then

$$h^*_{\varepsilon,\delta,\mathrm{Fair}}(x,a) := \begin{cases} \mathbbm{1}[h^*_{\varepsilon_1,\delta_1}(x) = 1]\mathbbm{1}[s \leq \frac{\tilde{\alpha} + \tilde{\beta}}{2\tilde{\beta}}] & \text{if } a = 1\\ \mathbbm{1}[h^*_{\varepsilon_0,\delta_0}(x) = 0]\mathbbm{1}[s \leq \frac{\tilde{\beta} - \tilde{\alpha}}{2(1 - \tilde{\alpha})}] + \mathbbm{1}[h^*_{\varepsilon_0,\delta_0}(x) = 1] & \text{if } a = 0 \end{cases}$$

return $h_{\varepsilon,\delta,\mathrm{Fair}}^*$

and $1/\varepsilon_3$). We remark that the assumption of large n_0 and n_1 was implicitly made in [31], in which they ignored the error in estimating the proportion of label one in each subgroup. Finally, the utility guarantee presented in the theorem indicates that the necessary perturbations in the final prediction by $h_{\varepsilon,\delta,\mathrm{Fair}}^*$ is almost identical to what is expected by the optimal classifier having the same level of statistical parity.

Rather than aiming to achieve a small statistical parity gap and a small utility gap with high probability, we could alternatively focus on the average guarantees for fairness and utility, as expounded by the next result.

Proposition 4. The classifier $h_{\varepsilon,\delta,Fair}^*: \mathcal{X} \times \{0,1\} \to \{0,1\}$ constructed by Algorithm 2 satisfies the following two properties:

- (Fairness guarantee) $\mathbb{E}\left[\Delta_{SP}(h_{\varepsilon,\delta,Fair}^*)\right] \leq \frac{1}{n_0\varepsilon_2} + \frac{1}{n_1\varepsilon_3}$,
- (Utility guarantee) We have

$$\mathbb{E}\Big[\mathbb{P}_{\mu_0^X}(h_{\varepsilon,\delta,\mathit{Fair}_0}^*(X) \neq h_{\varepsilon_0,\delta_0}^*(X)) + \mathbb{P}_{\mu_1^X}(h_{\varepsilon,\delta,\mathit{Fair}_1}^*(X) \neq h_{\varepsilon_1,\delta_1}^*(X))\Big] \leq \mathbb{E}\big[\mathit{err}^*(h_{\varepsilon_0,\delta_0}^*,h_{\varepsilon_1,\delta_1}^*)\big] + \frac{5}{2}\Big(\frac{1}{n_0\varepsilon_2} + \frac{1}{n_1\varepsilon_3}\Big),$$

where $err^*(\cdot,\cdot)$ was defined in (1).

IV. EXPERIMENTS

In this section, we seek to empirically compare Algorithm 2 with the current state-of-the-art differentially private fair classifier, namely, DP-FERMI [28]. To this goal, we focus on two benchmark datasets from the UCI machine learning repository [34]: the Adult and Credit Card datasets, both with binary sensitive attributes and target labels.

In our experiments, we assessed two privacy configurations: $(\varepsilon = 3, \delta = 10^{-5})$ and $(\varepsilon = 9, \delta = 10^{-5})$. When applying Algorithm 2 to the Adult and Credit Card datasets, we set $\varepsilon_2 = \varepsilon_3 = 0.05$ and $\varepsilon_2 = \varepsilon_3 = 0.1$, respectively. We then feed Algorithm 2 with the decoupled classifiers, $h_{\varepsilon_0,\delta_0}^*$ and $h_{\varepsilon_1,\delta_1}^*$ with parameters (ε_0,δ_0) and (ε_1,δ_1) chosen in a way as to satisfy $\varepsilon = \sum_{i=0}^{3} \varepsilon_i$ and $\delta = \delta_0 + \delta_1$ for each specified (ε, δ) pair. We trained these classifiers via DP-SGD. The training was conducted using Opacus [35], an open-source PyTorch library designed for training deep learning models with differential privacy. Accordingly, we chose the standard deviations of Gaussian noise in DP-SGD to be 4.17 and 2 for the Adult dataset, and to 5.92 and 3.31 for the Credit Card dataset, in order to achieve their respective privacy parameters ($\varepsilon = 3, \delta = 10^{-5}$) and $(\varepsilon = 9, \delta = 10^{-5})$. Across all experiments of DP-SGD, we consistently applied a clipping constant of 2 and a learning rate of 0.01. To achieve the most precise computation of privacy parameters, we utilized the PRV accountant [36], the state-ofthe-art accounting method, to determine the values for $(\varepsilon_0, \delta_0)$ and $(\varepsilon_1, \delta_1)$ for the classifiers $h_{\varepsilon_0, \delta_0}^*$ and $h_{\varepsilon_1, \delta_1}^*$. Additionally, to evaluate the variance in accounting methodologies, we conducted privacy parameter computations using two alternative methods: moments accountant method [32] and GDP accountant [37], [38], as detailed in the appendix.

For comparability with DP-FERMI, we employed logistic regression models. DP-FERMI utilizes a loss function with a regularization constant λ to limit statistical parity violations. A higher λ imposes stricter penalties for fairness violations, often at the cost of reduced accuracy. DP-FERMI sets the desired ε and δ , and determines the necessary noise levels to achieve these

TABLE II $\mbox{Adult Dataset} \ (\varepsilon = 3, \delta = 10^{-5}) \mbox{ - PRV Accountant}$

Method	Accuracy	Statistical Parity Gap
Algorithm 2	0.7766	0.0089
DP-FERMI ($\lambda = 0.5$)	0.7998	0.1020
DP-FERMI ($\lambda = 1$)	0.7859	0.0462
DP-FERMI ($\lambda = 1.5$)	0.7822	0.0267
DP-FERMI ($\lambda = 1.8$)	0.7770	0.0182
DP-FERMI ($\lambda = 2.5$)	0.7673	0.0099

TABLE III $\text{Adult Dataset} \ (\varepsilon = 9, \delta = 10^{-5}) \text{ - PRV Accountant}$

Method	Accuracy	Statistical Parity Gap
Algorithm 2	0.7782	0.0095
DP-FERMI ($\lambda = 0.5$)	0.8091	0.0944
DP-FERMI ($\lambda = 1$)	0.7923	0.0413
DP-FERMI ($\lambda = 1.5$)	0.7810	0.0152
DP-FERMI ($\lambda = 1.7$)	0.7785	0.0130
DP-FERMI ($\lambda = 2.5$)	0.7693	0.0030

TABLE IV ${\it Credit Card Dataset} \ (\varepsilon = 3, \delta = 10^{-5}) \ \hbox{- PRV Accountant}$

Method	Accuracy	Statistical Parity Gap
Algorithm 2	0.7893	0.0093
DP-FERMI ($\lambda = 0.1$)	0.7899	0.0212
DP-FERMI ($\lambda = 0.25$)	0.7826	0.0195
DP-FERMI ($\lambda = 0.5$)	0.7777	0.0185
DP-FERMI ($\lambda = 1$)	0.7759	0.0105
DP-FERMI ($\lambda = 2.5$)	0.7669	0.0110

TABLE V ${\rm Credit\ Card\ Dataset\ } (\varepsilon = 9, \delta = 10^{-5}) \mbox{ - PRV\ Accountant}$

Method	Accuracy	Statistical Parity Gap
Algorithm 2	0.7969	0.0046
DP-FERMI ($\lambda = 0.25$)	0.7996	0.0188
DP-FERMI ($\lambda = 0.3$)	0.7971	0.0182
DP-FERMI ($\lambda = 0.5$)	0.7912	0.0172
DP-FERMI ($\lambda = 1$)	0.7895	0.0105
DP-FERMI ($\lambda = 2.5$)	0.7884	0.0066

privacy guarantees. All models, including Algorithm 2 and DP-FERMI, were trained for 50 epochs, except the Credit Card models using ($\varepsilon = 9, \delta = 10^{-5}$) parameters, which were trained for 100 epochs. Within DP-FERMI, learning rates were set at $\eta_{\theta} = 0.005$ and $\eta_{W} = 0.01$. We chose these parameters mainly because they were empirically shown in [28] to be optimal for the datasets under consideration. A uniform batch size of 1024 was maintained for all experiments.

While our theoretical framework focuses on the sum of prediction changes across subgroup distributions, for comparison purposes with DP-FERMI, we used overall accuracy as a utility metric. The final results are presented in Tables II, III, IV, and V. All of the results are averages over 10 trials. We remark that DP-FERMI offers two privacy options: one for sensitive attributes and another for all features. We used noise parameters for all-feature privacy in DP-FERMI to compare fairly with Algorithm 2.

In the Adult dataset, as shown in Table II, Algorithm 2 achieves accuracy 0.7766. Setting λ at 1.8, DP-FERMI achieves a comparable accuracy (0.7770) but exhibits a statistical parity gap twice as large as Algorithm 2's guarantee. Table III illustrates that Algorithm 2 and DP-FERMI with $\lambda=1.7$ achieve a similar accuracy, yet Algorithm 2 exhibits a smaller statistical parity gap (0.0095) compared to DP-FERMI (0.0130). Altogether, these tables empirically underscore that applying Algorithm 2 to the Adult dataset results in a better fairness guarantee while maintaining similar privacy and accuracy guarantees.

Similar trends are observed in the Credit Card dataset. Table IV displays Algorithm 2 achieving an accuracy of 0.7893, closely matched by DP-FERMI with an accuracy of 0.7899 at $\lambda=0.1$. However, the statistical parity gap of Algorithm 2 is less than half that of DP-FERMI (0.0093 compared to 0.0212). As shown in Table V, Algorithm 2 and DP-FERMI with $\lambda=0.3$ attain similar accuracy, yet Algorithm 2 exhibits a statistical parity gap of 0.0046, nearly a fourth of DP-FERMI's 0.0182. We provide comprehensive descriptions of the datasets, training processes, and additional results in Appendix B.

REFERENCES

- [1] Cynthia Dwork, Frank McSherry, Kobbi Nissim, and Adam Smith. Calibrating noise to sensitivity in private data analysis. In *Theory of Cryptography: Third Theory of Cryptography Conference, TCC 2006, New York, NY, USA, March 4-7, 2006. Proceedings 3*, pages 265–284. Springer, 2006.
- [2] Cynthia Dwork. Differential privacy. In International colloquium on automata, languages, and programming, pages 1-12. Springer, 2006.
- [3] Úlfar Erlingsson, Vasyl Pihur, and Aleksandra Korolova. Rappor: Randomized aggregatable privacy-preserving ordinal response. In *Proceedings of the 2014 ACM SIGSAC conference on computer and communications security*, pages 1054–1067. ACM, 2014.
- [4] Differential privacy team Apple. Learning with privacy at scale, 2017.
- [5] Daniel Kifer, Solomon Messing, Aaron Roth, Abhradeep Thakurta, and Danfeng Zhang. Guidelines for implementing and auditing differentially private systems. *ArXiv*, abs/2002.04049, 2020.
- [6] Ryan Rogers, Subbu Subramaniam, Sean Peng, David Durfee, Seunghyun Lee, Santosh Kumar Kancha, Shraddha Sahay, and Parvez Ahammad. Linkedin's audience engagements api: A privacy preserving data analytics system at scale. arXiv preprint arXiv:2002.05839, 2020.
- [7] Michael Feldman, Sorelle A Friedler, John Moeller, Carlos Scheidegger, and Suresh Venkatasubramanian. Certifying and removing disparate impact. In proceedings of the 21th ACM SIGKDD international conference on knowledge discovery and data mining, pages 259–268, 2015.
- [8] Cynthia Dwork, Moritz Hardt, Toniann Pitassi, Omer Reingold, and Richard Zemel. Fairness through awareness. In *Proceedings of the 3rd innovations in theoretical computer science conference*, pages 214–226, 2012.
- [9] Cynthia Dwork, Nicole Immorlica, Adam Tauman Kalai, and Max Leiserson. Decoupled classifiers for group-fair and efficient machine learning. In *Conference on fairness, accountability and transparency*, pages 119–133. PMLR, 2018.
- [10] Hao Wang, Hsiang Hsu, Mario Diaz, and Flavio P Calmon. To split or not to split: The impact of disparate treatment in classification. IEEE Transactions on Information Theory, 67(10):6733–6757, 2021.

- [11] Eugene Bagdasaryan, Omid Poursaeed, and Vitaly Shmatikov. Differential privacy has disparate impact on model accuracy. Advances in neural information processing systems, 32, 2019.
- [12] David Pujol, Ryan McKenna, Satya Kuppam, Michael Hay, Ashwin Machanavajjhala, and Gerome Miklau. Fair decision making using privacy-protected data. In Proceedings of the 2020 Conference on Fairness, Accountability, and Transparency, pages 189–199, 2020.
- [13] Lucas Rosenblatt, Julia Stoyanovich, and Christopher Musco. A simple and practical method for reducing the disparate impact of differential privacy. arXiv preprint arXiv:2312.11712, 2023.
- [14] Rachel Cummings, Varun Gupta, Dhamma Kimpara, and Jamie Morgenstern. On the compatibility of privacy and fairness. In Adjunct publication of the 27th conference on user modeling, adaptation and personalization, pages 309–315, 2019.
- [15] Paul Mangold, Michaël Perrot, Aurélien Bellet, and Marc Tommasi. Differential privacy has bounded impact on fairness in classification. In *International Conference on Machine Learning*, pages 23681–23705. PMLR, 2023.
- [16] Hongyan Chang and Reza Shokri. On the privacy risks of algorithmic fairness. In 2021 IEEE European Symposium on Security and Privacy (EuroS&P), pages 292–303. IEEE, 2021.
- [17] Tom Farrand, Fatemehsadat Mireshghallah, Sahib Singh, and Andrew Trask. Neither private nor fair: Impact of data imbalance on utility and fairness in differential privacy. In *Proceedings of the 2020 workshop on privacy-preserving machine learning in practice*, pages 15–19, 2020.
- [18] Sushant Agarwal. Trade-offs between fairness, interpretability, and privacy in machine learning. UWSpace, 2020. http://hdl.handle.net/10012/15861.
- [19] Depeng Xu, Shuhan Yuan, and Xintao Wu. Achieving differential privacy and fairness in logistic regression. In Companion proceedings of The 2019 world wide web conference, pages 594–599, 2019.
- [20] Matthew Jagielski, Michael Kearns, Jieming Mao, Alina Oprea, Aaron Roth, Saeed Sharifi-Malvajerdi, and Jonathan Ullman. Differentially private fair learning. In *International Conference on Machine Learning*, pages 3000–3008. PMLR, 2019.
- [21] Jiahao Ding, Xinyue Zhang, Xiaohuan Li, Junyi Wang, Rong Yu, and Miao Pan. Differentially private and fair classification via calibrated functional mechanism. In Proceedings of the AAAI Conference on Artificial Intelligence, volume 34, pages 622–629, 2020.
- [22] Hussein Mozannar, Mesrob Ohannessian, and Nathan Srebro. Fair learning with private demographic data. In *International Conference on Machine Learning*, pages 7066–7075. PMLR, 2020.
- [23] Cuong Tran, Ferdinando Fioretto, and Pascal Van Hentenryck. Differentially private and fair deep learning: A lagrangian dual approach. In Proceedings of the AAAI Conference on Artificial Intelligence, volume 35, pages 9932–9939, 2021.
- [24] Cuong Tran, Keyu Zhu, Ferdinando Fioretto, and Pascal Van Hentenryck. Sf-pate: scalable, fair, and private aggregation of teacher ensembles. arXiv preprint arXiv:2204.05157, 2022.
- [25] Depeng Xu, Wei Du, and Xintao Wu. Removing disparate impact on model accuracy in differentially private stochastic gradient descent. In Proceedings of the 27th ACM SIGKDD Conference on Knowledge Discovery & Data Mining, pages 1924–1932, 2021.
- [26] Cuong Tran, My Dinh, and Ferdinando Fioretto. Differentially private empirical risk minimization under the fairness lens. Advances in Neural Information Processing Systems, 34:27555–27565, 2021.
- [27] M. S. Esipova, A. A. Ghomi, Y. Luo, and J. C. Cresswell. Disparate impact in differential privacy from gradient misalignment. In *The Eleventh International Conference on Learning Representations*, 2023.
- [28] Andrew Lowy, Devansh Gupta, and Meisam Razaviyayn. Stochastic differentially private and fair learning. In *International Conference on Learning Representations*, 2023.
- [29] Mohammad Yaghini, Patty Liu, Franziska Boenisch, and Nicolas Papernot. Learning with impartiality to walk on the pareto frontier of fairness, privacy, and utility. arXiv preprint arXiv:2302.09183, 2023.
- [30] Ray Jiang, Aldo Pacchiano, Tom Stepleton, Heinrich Jiang, and Silvia Chiappa. Wasserstein fair classification. In Uncertainty in artificial intelligence, pages 862–872. PMLR, 2020.
- [31] Han Zhao and Geoffrey J Gordon. Inherent tradeoffs in learning fair representations. *The Journal of Machine Learning Research*, 23(1):2527–2552, 2022
- [32] Martin Abadi, Andy Chu, Ian Goodfellow, H Brendan McMahan, Ilya Mironov, Kunal Talwar, and Li Zhang. Deep learning with differential privacy. In Proceedings of the 2016 ACM SIGSAC conference on computer and communications security, pages 308–318, 2016.
- [33] Cynthia Dwork, Aaron Roth, et al. The algorithmic foundations of differential privacy. Foundations and Trends® in Theoretical Computer Science, 9(3-4):211-407, 2014.
- [34] M. Lichman. Uci machine learning repository, 2013.
- [35] Ashkan Yousefpour, Igor Shilov, Alexandre Sablayrolles, Davide Testuggine, Karthik Prasad, Mani Malek, John Nguyen, Sayan Ghosh, Akash Bharadwaj, Jessica Zhao, Graham Cormode, and Ilya Mironov. Opacus: User-friendly differential privacy library in pytorch. 2021.
- [36] Sivakanth Gopi, Yin Tat Lee, and Lukas Wutschitz. Numerical composition of differential privacy. Advances in Neural Information Processing Systems, 34:11631–11642, 2021.
- [37] Jinshuo Dong, Aaron Roth, and Weijie J Su. Gaussian differential privacy. arXiv preprint arXiv:1905.02383, 2019.
- [38] Zhiqi Bu, Jinshuo Dong, Qi Long, and Weijie J Su. Deep learning with gaussian differential privacy. *Harvard data science review*, 2020(23):10–1162, 2020.

APPENDIX

This appendix is organized into two sections: Appendix A, containing proofs of our theoretical results; and Appendix B, providing in-depth information about the experimental setup, datasets used, and supplementary experiments.

APPENDIX A

Proof of Proposition 1. We have classifiers $h_0^*: \mathcal{X} \to \{0,1\}$ and $h_1^*: \mathcal{X} \to \{0,1\}$ trained on subgroups specified by the sensitive attribute A with values 0 and 1, respectively. We can combine classifiers h_0^* and h_1^* to have a group aware classifier $h^*: \mathcal{X} \times \{0,1\} \to \{0,1\}$. Basically, $h^*(x,0) = h_0^*(x) \ \forall x \in \mathcal{X}$ and $h^*(x,1) = h_1^*(x) \ \forall x \in \mathcal{X}$. Let $Y^* = h^*(X,A)$ and $\hat{Y} = \hat{h}(X,A)$. Then for $a \in \{0,1\}$, we have:

$$d_{TV}(\mu_{a}(Y^{*}), \mu_{a}(\hat{Y})) = \left| \mu_{a}(Y^{*} = 1) - \mu_{a}(\hat{Y} = 1) \right|$$

$$= \left| \mathbb{E}_{\mu_{a}^{X}}[h_{a}^{*}(X)] - \mathbb{E}_{\mu_{a}^{X}}[\hat{h}_{a}(X)] \right|$$

$$\leq \mathbb{E}_{\mu_{a}^{X}} \left[\left| h_{a}^{*}(X) - \hat{h}_{a}(X) \right| \right]$$

$$= \mathbb{P}_{\mu_{a}^{X}} \left(Y^{*} \neq \hat{Y} \right). \tag{2}$$

Therefore, from (2) it follows:

$$d_{TV}(\mu_a(Y^*), \mu_a(\hat{Y})) \le \mathbb{P}_{\mu_a^X} \left(Y^* \ne \hat{Y}\right).$$

We assumed $\hat{Y} = \hat{h}(X, A)$ satisfies γ statistical parity $(\Delta_{SP}(\hat{h}) \leq \gamma)$. Thus, we have:

$$d_{TV}(\mu_0(\hat{Y}), \mu_1(\hat{Y})) = \left| \mu_0(\hat{Y} = 1) - \mu_1(\hat{Y} = 1) \right| = \Delta_{SP}(\hat{h}) \le \gamma.$$

Since $d_{TV}(\cdot,\cdot)$ is symmetric and satisfies the triangle inequality, we have:

$$d_{TV}(\mu_0(Y^*), \mu_1(Y^*)) \le d_{TV}(\mu_0(Y^*), \mu_0(\hat{Y})) + d_{TV}(\mu_0(\hat{Y}), \mu_1(\hat{Y})) + d_{TV}(\mu_1(\hat{Y}), \mu_1(Y^*))$$

$$\le d_{TV}(\mu_0(Y^*), \mu_0(\hat{Y})) + \gamma + d_{TV}(\mu_1(Y^*), \mu_1(\hat{Y})). \tag{3}$$

Combining (3) with (2), we have:

$$d_{TV}(\mu_0(Y^*), \mu_1(Y^*)) \leq \mathbb{P}_{\mu_0^X} \left(Y^* \neq \hat{Y} \right) + \mathbb{P}_{\mu_1^X} \left(Y^* \neq \hat{Y} \right) + \gamma$$

= $\mathbb{P}_{\mu_0^X} (\hat{h}_0(X) \neq h_0^*(X)) + \mathbb{P}_{\mu_1^X} (\hat{h}_1(X) \neq h_1^*(X)) + \gamma.$

Therefore, we can obtain:

$$\left| \mathbb{P}_{\mu_0^X}(h_0^*(X) = 1) - \mathbb{P}_{\mu_1^X}(h_1^*(X) = 1) \right| \leq \mathbb{P}_{\mu_0^X}(\hat{h}_0(X) \neq h_0^*(X)) + \mathbb{P}_{\mu_1^X}(\hat{h}_1(X) \neq h_1^*(X)) + \gamma.$$

Thus, we have:

$$\mathbb{P}_{\mu_0^X}(\hat{h}_0(X) \neq h_0^*(X)) + \mathbb{P}_{\mu_1^X}(\hat{h}_1(X) \neq h_1^*(X)) \geq \left| \mathbb{P}_{\mu_0^X}(h_0^*(X) = 1) - \mathbb{P}_{\mu_1^X}(h_1^*(X) = 1) \right| - \gamma.$$

which concludes the proof of Proposition 1.

Proof of Theorem 2. Let s in Algorithm 1 be the realization of the random variable S. First, we compute the statistical parity gap of the classifier h_{Fair}^* .

$$\mathbb{P}_{\mu} \left(h_{\text{Fair}}^{*}(X, A) = 1 | A = 0 \right) = \mathbb{P}_{\mu_{0}^{X}} \left(h_{0}^{*}(X) = 1 \right) \mathbb{P}(S \leq \frac{\alpha + \beta}{2\alpha}) = \alpha \left(\frac{\alpha + \beta}{2\alpha} \right) = \frac{\alpha + \beta}{2}.$$

$$\mathbb{P}_{\mu} \left(h_{\text{Fair}}^{*}(X, A) = 1 | A = 1 \right) = \mathbb{P}_{\mu_{1}^{X}} \left(h_{1}^{*}(X) = 1 \right) + \mathbb{P}_{\mu_{1}^{X}} \left(h_{1}^{*}(X) = 0 \right) \mathbb{P} \left(S \leq \frac{\alpha - \beta}{2(1 - \beta)} \right)$$

$$= \beta + (1 - \beta) \left(\frac{\alpha - \beta}{2(1 - \beta)} \right)$$

$$= \frac{\alpha + \beta}{2}.$$

Hence, we have:

$$\Delta_{SP}(h_{\mathrm{Fair}}^*) = \left| \frac{\alpha + \beta}{2} - \frac{\alpha + \beta}{2} \right| = 0.$$

Therefore, perfect statistical parity is satisfied. Now we show that h_{Fair}^* is optimal.

$$\mathbb{P}_{\mu_0^X}(h_{\mathrm{Fair}0}^*(X) \neq h_0^*(X)) = \mathbb{P}_{\mu_0^X}(h_0^*(X) = 1)\mathbb{P}\left(S > \frac{\alpha + \beta}{2\alpha}\right) = \alpha\left(\frac{\alpha - \beta}{2\alpha}\right) = \frac{\alpha - \beta}{2}.$$

$$\mathbb{P}_{\mu_1^X}(h_{\mathrm{Fair}1}^*(X) \neq h_1^*(X)) = \mathbb{P}_{\mu_1^X}(h_1^*(X) = 0)\mathbb{P}\left(S \leq \frac{\alpha - \beta}{2(1 - \beta)}\right)$$

$$= (1 - \beta)\left(\frac{\alpha - \beta}{2(1 - \beta)}\right) = \frac{\alpha - \beta}{2}.$$

Thus, we have:

$$\begin{split} \mathbb{P}_{\mu_0^X}(h_{\mathrm{Fair}_0}^*(X) \neq h_0^*(X)) + \mathbb{P}_{\mu_1^X}(h_{\mathrm{Fair}_1}^*(X) \neq h_1^*(X)) &= \frac{\alpha - \beta}{2} + \frac{\alpha - \beta}{2} = \alpha - \beta = |\alpha - \beta| \\ &= \left| \mathbb{P}_{\mu_0^X}(h_0^*(X) = 1) - \mathbb{P}_{\mu_1^X}(h_1^*(X) = 1) \right|. \end{split}$$

Therefore, h_{Fair}^* satisfies perfect statistical parity and attains the utility lower bound of Proposition 1.

Proof of Theorem 3.

- Computing $\tilde{\alpha}$ and $\tilde{\beta}$ is done using the Laplace mechanism followed by a post-processing (projection). Therefore, these two mechanisms will satisfy ε_2 -DP and ε_3 -DP [33]. Also, learning $h_{\varepsilon_0,\delta_0}^*$ and $h_{\varepsilon_1,\delta_1}^*$ was done with privacy parameters (ε_0,δ_0) and (ε_1,δ_1) . By basic composition, we have $h_{\varepsilon,\delta,\mathrm{Fair}}^*$ satisfies (ε,δ) -DP with $\varepsilon=\sum_{i=0}^3 \varepsilon_i$ and $\delta=\delta_0+\delta_1$.
- Let $\alpha = \mathbb{P}_{\mu_0^X}(h_{\varepsilon_0,\delta_0}^*(X) = 1)$ and $\beta = \mathbb{P}_{\mu_1^X}(h_{\varepsilon_1,\delta_1}^*(X) = 1)$. Let $\bar{\alpha} = \alpha + e_0$ and $\bar{\beta} = \beta + e_1$. Following the methodology in [31], we assume e_0 and e_1 are sufficiently small, i.e., we have sufficiently large samples to empirically approximate the true value of α and β precisely. To prove the claims in the theorem, we assume $\tilde{\alpha} \geq \tilde{\beta}$. For the case that $\tilde{\alpha} < \tilde{\beta}$, we will have the same results by symmetry. Also, let $L_0 \sim \text{Lap}(\frac{1}{n_0\varepsilon_2})$ and $L_1 \sim \text{Lap}(\frac{1}{n_1\varepsilon_3})$. In Algorithm 2, we sample l_0 from L_0 and l_1 from L_1 . Similar to the proof of Theorem 2, let s in Algorithm 2 be the realization of the random variable s. By definition, we have:

$$\Delta_{SP}(h^*_{\varepsilon,\delta,\mathrm{Fair}}) = \big| \mathbb{P}_{\mu^X_0} \left(h^*_{\varepsilon,\delta,\mathrm{Fair}}(X,0) = 1 \right) - \mathbb{P}_{\mu^X_1} \left(h^*_{\varepsilon,\delta,\mathrm{Fair}}(X,1) = 1 \right) \big|.$$

We first compute $\mathbb{P}_{\mu_0^X}\left(h_{\varepsilon,\delta,\mathrm{Fair}}^*(X,0)=1\right)$:

$$\begin{split} \mathbb{P}_{\mu_0^X} \left(h_{\varepsilon,\delta,\mathrm{Fair}}^*(X,0) = 1 \right) &= \mathbb{P}_{\mu_0^X} (h_{\varepsilon_0,\delta_0}^*(X) = 1) \mathbb{P} \left(S \leq \frac{\tilde{\alpha} + \tilde{\beta}}{2\tilde{\alpha}} \right) \\ &= \alpha \frac{\tilde{\alpha} + \tilde{\beta}}{2\tilde{\alpha}} \\ &= \frac{\alpha}{\tilde{\alpha}} \left(\frac{\tilde{\alpha} + \tilde{\beta}}{2} \right). \end{split}$$

We then compute $\mathbb{P}_{\mu_1^X}\left(h_{\varepsilon,\delta,\mathrm{Fair}}^*(X,1)=1\right)$:

$$\mathbb{P}_{\mu_1^X}\left(h_{\varepsilon,\delta,\mathrm{Fair}}^*(X,1)=1\right) = \mathbb{P}_{\mu_1^X}(h_{\varepsilon_1,\delta_1}^*(X)=1) + \mathbb{P}_{\mu_1^X}(h_{\varepsilon_1,\delta_1}^*(X)=0)\mathbb{P}\left(S \leq \frac{\tilde{\alpha}-\tilde{\beta}}{2(1-\tilde{\beta})}\right)$$
$$= \beta + (1-\beta)\left(\frac{\tilde{\alpha}-\tilde{\beta}}{2(1-\tilde{\beta})}\right) = \beta + \frac{(1-\beta)}{(1-\tilde{\beta})}\left(\frac{\tilde{\alpha}-\tilde{\beta}}{2}\right).$$

For each realization of $\tilde{\alpha}$ and $\tilde{\beta}$, let $\bar{\alpha}=\alpha+e_0$, $\tilde{\alpha}=\bar{\alpha}+d_0$, $\bar{\beta}=\beta+e_1$, $\tilde{\beta}=\bar{\beta}+d_1$. Thus, we have:

$$\Delta_{SP}(h_{\varepsilon,\delta,\text{Fair}}^*) = \left| \frac{\alpha}{\tilde{\alpha}} \left(\frac{\tilde{\alpha} + \tilde{\beta}}{2} \right) - \left[\beta + \frac{(1-\beta)}{(1-\tilde{\beta})} \left(\frac{\tilde{\alpha} - \tilde{\beta}}{2} \right) \right] \right| \\
= \left| \frac{\tilde{\alpha} - e_0 - d_0}{\tilde{\alpha}} \left(\frac{\tilde{\alpha} + \tilde{\beta}}{2} \right) - \left[\tilde{\beta} - e_1 - d_1 + \frac{(1-\tilde{\beta} + e_1 + d_1)}{(1-\tilde{\beta})} \left(\frac{\tilde{\alpha} - \tilde{\beta}}{2} \right) \right] \right| \\
= \left| \left(\frac{\tilde{\alpha} + \tilde{\beta}}{2} \right) - (e_0 + d_0) \left(\frac{\tilde{\alpha} + \tilde{\beta}}{2\tilde{\alpha}} \right) - \tilde{\beta} + e_1 + d_1 - \left(\frac{\tilde{\alpha} - \tilde{\beta}}{2} \right) - (e_1 + d_1) \left(\frac{\tilde{\alpha} - \tilde{\beta}}{2(1-\tilde{\beta})} \right) \right| \\
= \left| (e_1 + d_1) \left(1 - \frac{\tilde{\alpha} - \tilde{\beta}}{2(1-\tilde{\beta})} \right) - (e_0 + d_0) \left(\frac{\tilde{\alpha} + \tilde{\beta}}{2\tilde{\alpha}} \right) \right| \\
\leq \left| (e_1 + d_1) \left(1 - \frac{\tilde{\alpha} - \tilde{\beta}}{2(1-\tilde{\beta})} \right) \right| + \left| (e_0 + d_0) \left(\frac{\tilde{\alpha} + \tilde{\beta}}{2\tilde{\alpha}} \right) \right| \\
\leq \left| (e_1 + d_1) \right| + \left| (e_0 + d_0) \right| \\
\leq \left| (e_1 + d_1) \right| + \left| (e_0 + d_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
 \leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
\leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right| \\
 \leq \left| (e_0 + e_0) \right| + \left| (e_0 + e_0) \right|$$

The last line follows from the fact that $0 \le \tilde{\alpha} \le 1$, $0 \le \tilde{\beta} \le 1$, and $\tilde{\alpha} \ge \tilde{\beta}$.

Assuming sufficiently large number of samples for each subgroup, which implies that e_0 and e_1 are sufficiently small, the values of $|e_0|$ and $|e_1|$ become negligible. Let l_0 and l_1 be the realization of the Laplace noises in Algorithm 2. We know that $|d_0| \leq |l_0|$ and $|d_1| \leq |l_1|$ since d_0 and d_1 are computed after projection. On the other hand, from [33], we know that if $L \sim \text{Lap}(\frac{\Delta_1^q}{\varepsilon})$, then:

$$\mathbb{P}\left[|L| \ge \left(\log \frac{1}{\eta}\right) \left(\frac{\Delta_1^q}{\varepsilon}\right)\right] \le \eta.$$

where Δ_1^q is the ℓ_1 -sensitivity of the query to which we add noise.

Given $L_0 \sim \operatorname{Lap}\left(\frac{1}{n_0\varepsilon_2}\right)$ and $L_1 \sim \operatorname{Lap}\left(\frac{1}{n_1\varepsilon_3}\right)$, we have: $|L_0| \leq_{\eta_0} \left(\log\frac{1}{\eta_0}\right) \left(\frac{1}{n_0\varepsilon_2}\right)$ and $|L_1| \leq_{\eta_1} \left(\log\frac{1}{\eta_1}\right) \left(\frac{1}{n_1\varepsilon_3}\right)$. Thus, with probability at least $(1-\eta_0)(1-\eta_1)$, we have:

$$|L_0| + |L_1| \le \left[\left(\log \frac{1}{\eta_0} \right) \left(\frac{1}{n_0 \varepsilon_2} \right) + \left(\log \frac{1}{\eta_1} \right) \left(\frac{1}{n_1 \varepsilon_3} \right) \right].$$

Since $(1 - \eta_0)(1 - \eta_1) \ge 1 - (\eta_0 + \eta_1)$ for $0 \le \eta_0, \eta_1 \le 1$, we have:

$$|L_0| + |L_1| \le_{\eta_0 + \eta_1} \left[\left(\log \frac{1}{\eta_0} \right) \left(\frac{1}{n_0 \varepsilon_2} \right) + \left(\log \frac{1}{\eta_1} \right) \left(\frac{1}{n_1 \varepsilon_3} \right) \right]. \tag{5}$$

Combining (4) and (5), it can be shown:

$$\Delta_{SP}(h_{\varepsilon,\delta,\text{Fair}}^*) \leq_{\eta_0 + \eta_1} \frac{\log\left(\frac{1}{\eta_0}\right)}{n_0 \varepsilon_2} + \frac{\log\left(\frac{1}{\eta_1}\right)}{n_1 \varepsilon_3}.$$

• We have:

$$\begin{split} \mathbb{P}_{\mu_0^X}(h_{\varepsilon,\delta,\mathrm{Fair}_0}^*(X) &\neq h_{\varepsilon_0,\delta_0}^*(X)) + \mathbb{P}_{\mu_1^X}(h_{\varepsilon,\delta,\mathrm{Fair}_1}^*(X) \neq h_{\varepsilon_1,\delta_1}^*(X)) \\ &= \mathbb{P}_{\mu_0^X}\left((h_{\varepsilon_0,\delta_0}^*(X) = 1) \text{ and } (h_{\varepsilon,\delta,\mathrm{Fair}_0}^*(X) = 0)\right) + \mathbb{P}_{\mu_1^X}\left((h_{\varepsilon_1,\delta_1}^*(X) = 0) \text{ and } (h_{\varepsilon,\delta,\mathrm{Fair}_1}^*(X) = 1)\right) \\ &= \alpha\left(\frac{\tilde{\alpha} - \tilde{\beta}}{2\tilde{\alpha}}\right) + (1-\beta)\left(\frac{\tilde{\alpha} - \tilde{\beta}}{2(1-\tilde{\beta})}\right) = \frac{\alpha}{\tilde{\alpha}}\left(\frac{\tilde{\alpha} - \tilde{\beta}}{2}\right) + \frac{(1-\beta)}{1-\tilde{\beta}}\left(\frac{\tilde{\alpha} - \tilde{\beta}}{2}\right) \\ &= \frac{\tilde{\alpha} - e_0 - d_0}{\tilde{\alpha}}\left(\frac{\tilde{\alpha} - \tilde{\beta}}{2}\right) + \frac{(1-\tilde{\beta} + e_1 + d_1)}{1-\tilde{\beta}}\left(\frac{\tilde{\alpha} - \tilde{\beta}}{2}\right) \\ &= (\tilde{\alpha} - \tilde{\beta}) + (e_1 + d_1)\left(\frac{\tilde{\alpha} - \tilde{\beta}}{2(1-\tilde{\beta})}\right) - (e_0 + d_0)\left(\frac{\tilde{\alpha} - \tilde{\beta}}{2\tilde{\alpha}}\right). \end{split}$$

Therefore, we have:

$$\begin{split} \mathbb{P}_{\mu_0^X}(h_{\varepsilon,\delta,\mathrm{Fair}_0}^*(X) \neq h_{\varepsilon_0,\delta_0}^*(X)) + \mathbb{P}_{\mu_1^X}(h_{\varepsilon,\delta,\mathrm{Fair}_1}^*(X) \neq h_{\varepsilon_1,\delta_1}^*(X)) - \mathrm{err}^*(h_{\varepsilon_0,\delta_0}^*,h_{\varepsilon_1,\delta_1}^*) \\ &= (\tilde{\alpha} - \tilde{\beta}) + (e_1 + d_1) \left(\frac{\tilde{\alpha} - \tilde{\beta}}{2(1 - \tilde{\beta})}\right) - (e_0 + d_0) \left(\frac{\tilde{\alpha} - \tilde{\beta}}{2\tilde{\alpha}}\right) - \mathrm{err}^*(h_{\varepsilon_0,\delta_0}^*,h_{\varepsilon_1,\delta_1}^*). \end{split}$$

From previous part, we know that:

$$\Delta_{SP}(h_{\varepsilon,\delta,\text{Fair}}^*) \le |e_0| + |e_1| + |d_0| + |d_1|.$$

From Proposition 1, we know that if $\Delta_{SP}(\hat{h}) \leq \gamma$, then:

$$\mathbb{P}_{\mu_0^X}(\hat{h}_0(X) \neq h_{\varepsilon_0,\delta_0}^*(X)) + \mathbb{P}_{\mu_1^X}(\hat{h}_1(X) \neq h_{\varepsilon_1,\delta_1}^*(X)) \geq \left| \mathbb{P}_{\mu_0^X}(h_{\varepsilon_0,\delta_0}^*(X) = 1) - \mathbb{P}_{\mu_1^X}(h_{\varepsilon_1,\delta_1}^*(X) = 1) \right| - \gamma.$$

For all classifiers $\hat{h}: \mathcal{X} \times \{0,1\} \to \{0,1\}$ that satisfy $\Delta_{SP}(\hat{h}) \leq \Delta_{SP}(h^*_{\varepsilon,\delta,\mathrm{Fair}})$, we have $\Delta_{SP}(\hat{h}) \leq |e_0| + |e_1| + |d_0| + |d_1|$. Thus, for all those classifiers we have:

$$\begin{split} \mathbb{P}_{\mu_0^X}(\hat{h}_0(X) \neq h_{\varepsilon_0,\delta_0}^*(X)) + \mathbb{P}_{\mu_1^X}(\hat{h}_1(X) \neq h_{\varepsilon_1,\delta_1}^*) \\ & \geq \left| \mathbb{P}_{\mu_0^X}(h_{\varepsilon_0,\delta_0}^*(X) = 1) - \mathbb{P}_{\mu_1^X}(h_{\varepsilon_1,\delta_1}^*(X) = 1) \right| - \left(|e_0| + |e_1| + |d_0| + |d_1| \right) \\ & = |\alpha - \beta| - \left(|e_0| + |e_1| + |d_0| + |d_1| \right). \end{split}$$

Therefore, by definition of $\operatorname{err}^*(h_{\varepsilon_0,\delta_0}^*,h_{\varepsilon_1,\delta_1}^*)$, we have:

$$\operatorname{err}^*(h_{\varepsilon_0,\delta_0}^*, h_{\varepsilon_1,\delta_1}^*) \ge |\alpha - \beta| - (|e_0| + |e_1| + |d_0| + |d_1|).$$

Therefore, it follows:

$$\begin{split} \mathbb{P}_{\mu_0^X}(h_{\varepsilon,\delta,\mathrm{Fair}_0}^*(X) \neq h_{\varepsilon_0,\delta_0}^*(X)) + \mathbb{P}_{\mu_1^X}(h_{\varepsilon,\delta,\mathrm{Fair}_1}^*(X) \neq h_{\varepsilon_1,\delta_1}^*(X)) - \mathrm{err}^*(h_{\varepsilon_0,\delta_0}^*,h_{\varepsilon_1,\delta_1}^*) \\ & \leq (\tilde{\alpha} - \tilde{\beta}) + (e_1 + d_1) \left(\frac{\tilde{\alpha} - \tilde{\beta}}{2(1 - \tilde{\beta})}\right) - (e_0 + d_0) \left(\frac{\tilde{\alpha} - \tilde{\beta}}{2\tilde{\alpha}}\right) - |\alpha - \beta| + (|e_0| + |e_1| + |d_0| + |d_1|) \\ & \leq \frac{1}{2}|e_1 + d_1| + \frac{1}{2}|e_0 + d_0| + (\tilde{\alpha} - \tilde{\beta}) - |\alpha - \beta| + (|e_0| + |e_1| + |d_0| + |d_1|) \\ & \leq (\tilde{\alpha} - \tilde{\beta}) - |\alpha - \beta| + \frac{3}{2} \left(|e_0| + |e_1| + |d_0| + |d_1|\right) \end{split}$$

$$= (\tilde{\alpha} - \tilde{\beta}) - |\tilde{\alpha} - e_0 - d_0 - \tilde{\beta} + e_1 + d_1| + \frac{3}{2} (|e_0| + |e_1| + |d_0| + |d_1|)$$

$$\leq \frac{5}{2} (|e_0| + |e_1| + |d_0| + |d_1|).$$

Assuming e_0 and e_1 are sufficiently small, and by the same argument of the second part, we can conclude that:

$$\begin{split} \mathbb{P}_{\mu_0^X}(h_{\varepsilon,\delta,\mathrm{Fair}_0}^*(X) \neq h_{\varepsilon_0,\delta_0}^*(X)) + \mathbb{P}_{\mu_1^X}(h_{\varepsilon,\delta,\mathrm{Fair}_1}^*(X) \neq h_{\varepsilon_1,\delta_1}^*(X)) \\ \leq_{\eta_0 + \eta_1} \mathrm{err}^*(h_{\varepsilon_0,\delta_0}^*,h_{\varepsilon_1,\delta_1}^*) + \frac{5}{2} \left(\frac{\log\left(\frac{1}{\eta_0}\right)}{n_0 \varepsilon_2} + \frac{\log\left(\frac{1}{\eta_1}\right)}{n_1 \varepsilon_3} \right). \end{split}$$

Proof of Proposition 4. Let $\alpha = \mathbb{P}_{\mu_0^X}(h_{\varepsilon_0,\delta_0}^*(X) = 1)$ and $\beta = \mathbb{P}_{\mu_1^X}(h_{\varepsilon_1,\delta_1}^*(X) = 1)$. Also, let $\bar{\alpha} = \alpha + e_0$ and $\bar{\beta} = \beta + e_1$. Similar to the proof of Theorem 3, following the strategy in [31], we assume e_0 and e_1 are sufficiently small, i.e., we have sufficiently large samples to empirically approximate the true value of α and β precisely. Let $L_0 \sim \text{Lap}(\frac{1}{n_0\varepsilon_2})$ and $L_1 \sim \text{Lap}(\frac{1}{n_1\varepsilon_3})$. We sample l_0 from L_0 and l_1 from L_1 . In fact, l_0 and l_1 are realizations of the Laplace noise. From proof of Theorem3, we know that for each realization of the noise we have:

$$\Delta_{SP}(h_{\varepsilon,\delta,\text{Fair}}^*) \le [|e_0| + |e_1| + |l_0| + |l_1|].$$

And

$$\left[\mathbb{P}_{\mu_0^X}(h_{\varepsilon,\delta,\mathrm{Fair}_0}^*(X) \neq h_{\varepsilon_0,\delta_0}^*(X)) + \mathbb{P}_{\mu_1^X}(h_{\varepsilon,\delta,\mathrm{Fair}_1}^*(X) \neq h_{\varepsilon_1,\delta_1}^*(X)) - \mathrm{err}^*(h_{\varepsilon_0,\delta_0}^*,h_{\varepsilon_1,\delta_1}^*)\right] \leq \frac{5}{2} \left[\left(|e_0| + |e_1| + |l_0| + |l_1| \right) \right].$$

We have:

$$\mathbb{E}\left[\Delta_{SP}(h_{\varepsilon,\delta,\mathrm{Fair}}^*)\right] \leq \mathbb{E}\left[|e_0| + |e_1| + |L_0| + |L_1|\right].$$

And

$$\mathbb{E}\Bigg[\mathbb{P}_{\mu_0^X}(h_{\varepsilon,\delta,\mathrm{Fair}_0}^*(X) \neq h_{\varepsilon_0,\delta_0}^*(X)) + \mathbb{P}_{\mu_1^X}(h_{\varepsilon,\delta,\mathrm{Fair}_1}^*(X) \neq h_{\varepsilon_1,\delta_1}^*(X)) - \mathrm{err}^*(h_{\varepsilon_0,\delta_0}^*,h_{\varepsilon_1,\delta_1}^*)\Bigg] \leq \frac{5}{2}\mathbb{E}\left[\left(|e_0| + |e_1| + |L_0| + |L_1|\right)\right].$$

Where both expectations are over the randomness of the Laplace noise. For $L \sim \text{Lap}(\frac{\Delta_1^q}{\varepsilon})$, we have $\mathbb{E}[|L|] = \frac{\Delta_1^q}{\varepsilon}$. Since $L_0 \sim \text{Lap}\left(\frac{1}{n_0\varepsilon_2}\right)$ and $L_1 \sim \text{Lap}\left(\frac{1}{n_1\varepsilon_3}\right)$, assuming e_0 and e_1 are sufficiently small, we have:

$$\mathbb{E}\left[\Delta_{SP}(h_{\varepsilon,\delta,\text{Fair}}^*)\right] \leq \left(\frac{1}{n_0\varepsilon_2}\right) + \left(\frac{1}{n_1\varepsilon_3}\right).$$

And

$$\mathbb{E}\left[\mathbb{P}_{\mu_0^X}(h_{\varepsilon,\delta,\mathrm{Fair}_0}^*(X)\neq h_{\varepsilon_0,\delta_0}^*(X)) + \mathbb{P}_{\mu_1^X}(h_{\varepsilon,\delta,\mathrm{Fair}_1}^*(X)\neq h_{\varepsilon_1,\delta_1}^*(X))\right] \leq \mathbb{E}\Big[\mathrm{err}^*(h_{\varepsilon_0,\delta_0}^*,h_{\varepsilon_1,\delta_1}^*)\Big] + \frac{5}{2}\left(\frac{1}{n_0\varepsilon_2} + \frac{1}{n_1\varepsilon_3}\right).$$

APPENDIX B

In this section, we provide additional details on our experiments, including datasets, pre-processing approach, and additional experimental results.

Adult Dataset: This dataset contains census information about individuals. The prediction task is to determine whether a person earns over \$50K a year. In this dataset, gender is considered as a sensitive attribute. For all experiments, we followed a pre-processing approach similar to [28]. After pre-processing, the dataset contains a total of 102 input features. The size of the dataset is around 48,000 entries.

Credit Card Dataset: This dataset includes financial data from bank users in Taiwan. The prediction task is to assess whether a person defaults on their credit card bills, essentially evaluating client credibility. Gender is taken as a sensitive attribute. We applied the same pre-processing method as in [28] for all experiments. After pre-processing, the dataset contained 85 input features, with a total of 30,000 entries.

 $\label{eq:table_vi} \text{Table VI}$ Adult Dataset ($\varepsilon=3, \delta=10^{-5})$ - Moments Accountant

Method	Accuracy	Statistical Parity Gap
Algorithm 2	0.7754	0.0054
DP-FERMI ($\lambda = 0.5$)	0.7998	0.1020
DP-FERMI ($\lambda = 1$)	0.7859	0.0462
DP-FERMI ($\lambda = 1.5$)	0.7822	0.0267
DP-FERMI ($\lambda = 1.9$)	0.7749	0.0126
DP-FERMI ($\lambda = 2.5$)	0.7673	0.0099

TABLE VII ${\rm Adult\ Dataset}\, (\varepsilon=9, \delta=10^{-5}) \ {\rm - \ Moments\ Accountant}$

Method	Accuracy	Statistical Parity Gap
Algorithm 2	0.7769	0.0078
DP-FERMI ($\lambda = 0.5$)	0.8091	0.0944
DP-FERMI ($\lambda = 1$)	0.7923	0.0413
DP-FERMI ($\lambda = 1.5$)	0.7810	0.0152
DP-FERMI ($\lambda = 1.8$)	0.7769	0.0113
DP-FERMI ($\lambda = 2.5$)	0.7693	0.0030

TABLE VIII $\text{Credit Card Dataset } (\varepsilon = 3, \delta = 10^{-5}) \text{ - Moments Accountant }$

Method	Accuracy	Statistical Parity Gap
Algorithm 2	0.7891	0.0108
DP-FERMI ($\lambda = 0.1$)	0.7899	0.0212
DP-FERMI ($\lambda = 0.25$)	0.7826	0.0195
DP-FERMI ($\lambda = 0.5$)	0.7777	0.0185
DP-FERMI ($\lambda = 1$)	0.7759	0.0105
DP-FERMI ($\lambda = 2.5$)	0.7669	0.0110

TABLE IX CREDIT CARD DATASET ($arepsilon=9, \delta=10^{-5}$) - Moments Accountant

Method	Accuracy	Statistical Parity Gap
Algorithm 2	0.7951	0.0060
DP-FERMI ($\lambda = 0.25$)	0.7996	0.0188
DP-FERMI ($\lambda = 0.35$)	0.7950	0.0174
DP-FERMI ($\lambda = 0.5$)	0.7912	0.0172
DP-FERMI ($\lambda = 1$)	0.7895	0.0105
DP-FERMI ($\lambda = 2.5$)	0.7884	0.0066

For DP-FERMI experiments, datasets were split into 75% training and 25% testing. However, for Algorithm 2, we needed to train two initial classifiers $h_{\varepsilon_0,\delta_0}^*$ and $h_{\varepsilon_1,\delta_1}^*$, followed by a post-processing step. For this, we divided the dataset into three parts: 50% for training the initial classifiers, and 25% each for post-processing and testing.

To calculate the statistical parity gap of output classifiers in all experiments, we refer to Definition 2. We need to compute:

$$\Delta_{SP}(\hat{h}) := |\mu_0(\hat{Y} = 1) - \mu_1(\hat{Y} = 1)|.$$

In practice, where we do not have access to true distributions μ_a , we use their empirical counterparts. Specifically, we calculate the empirical version of $\mu_a(\hat{Y}=1)$, i.e., we compute $\hat{\mathbb{P}}[\hat{Y}=1|A=a]$. Thus, the empirical version of the statistical parity gap is defined as:

$$\widehat{\Delta_{SP}}(\hat{h}) := \left| \hat{\mathbb{P}} \Big[\hat{Y} = 1 | A = 0 \Big] - \hat{\mathbb{P}} \Big[\hat{Y} = 1 | A = 1 \Big] \right|.$$

In this expression, $\hat{\mathbb{P}}[\hat{Y}=1|A=0]$ and $\hat{\mathbb{P}}[\hat{Y}=1|A=1]$ are computed on the test split of the dataset. The statistical parity gap values listed in the column of all tables are derived using the empirical statistical parity gap calculation.

We present the results of our experiments with privacy parameters computed using both the moments accountant and GDP accountant methods. For moments accountant method, we selected the standard deviations of Gaussian noise in DP-SGD to be 4.4 and 2.08 for the Adult dataset, and 6.2 and 3.46 for the Credit Card dataset. These values were chosen to achieve their respective privacy parameters of $(\varepsilon = 3, \delta = 10^{-5})$ and $(\varepsilon = 9, \delta = 10^{-5})$. To achieve the target privacy parameters using GDP accountant in Algorithm 2, we set specific noise variances for the Adult and Credit Card datasets. For the Adult dataset, with privacy parameters $(\varepsilon = 3, \delta = 10^{-5})$ and $(\varepsilon = 9, \delta = 10^{-5})$, the standard deviation of the Gaussian noise were set to 4 and 1.94, respectively. In the case of the Credit Card dataset, under identical privacy parameters, we used a Gaussian noise with a standard deviation of 5.7 and 3.24 in DP-SGD. All other parameters, including ε_2 , ε_3 , learning rate, number of epochs, batch size, and clipping constant, were kept consistent with those used in the experiments where the privacy guarantee was calculated using PRV accountant method (the experiments detailed in Tables II, III, IV, and V).

The results corresponding to moments accountant method are presented in Tables VI, VII, VIII, and IX. Additionally, the results corresponding to GDP accountant method are available in Tables X, XI, XII, and XIII. Note that the DP-FERMI experiment results remain the same; the only difference lies in the line corresponding to Algorithm 2.

It can be concluded that across all these experiments, similar to what was noted in Section IV, Algorithm 2 consistently offers a substantially better (smaller) statistical parity gap compared to DP-FERMI for fixed privacy and accuracy, regardless of varying privacy parameters, datasets (Adult and Credit Card), and privacy accounting methods employed.

TABLE X $\mbox{Adult Dataset} \ (\varepsilon = 3, \delta = 10^{-5}) \mbox{ - GDP Accountant}$

Method	Accuracy	Statistical Parity Gap
Algorithm 2	0.7801	0.0088
DP-FERMI ($\lambda = 0.5$)	0.7998	0.1020
DP-FERMI ($\lambda = 1$)	0.7859	0.0462
DP-FERMI ($\lambda = 1.5$)	0.7822	0.0267
DP-FERMI ($\lambda = 1.7$)	0.7795	0.0215
DP-FERMI ($\lambda = 2.5$)	0.7673	0.0099

TABLE XI $\mbox{Adult Dataset} \ (\varepsilon = 9, \delta = 10^{-5}) \mbox{ - GDP Accountant}$

Method	Accuracy	Statistical Parity Gap
Algorithm 2	0.7813	0.0048
DP-FERMI ($\lambda = 0.5$)	0.8091	0.0944
DP-FERMI ($\lambda = 1$)	0.7923	0.0413
DP-FERMI ($\lambda = 1.5$)	0.7810	0.0152
DP-FERMI ($\lambda = 1.8$)	0.7769	0.0113
DP-FERMI ($\lambda = 2.5$)	0.7693	0.0030

TABLE XII TABLE XIII CREDIT CARD DATASET ($\varepsilon=3,\delta=10^{-5}$) - GDP ACCOUNTANT CREDIT CARD DATASET ($\varepsilon=9,\delta=10^{-5}$) - GDP ACCOUNTANT

Method	Accuracy	Statistical Parity Gap
Algorithm 2	0.7884	0.0070
DP-FERMI ($\lambda = 0.1$)	0.7899	0.0212
DP-FERMI ($\lambda = 0.25$)	0.7826	0.0195
DP-FERMI ($\lambda = 0.5$)	0.7777	0.0185
DP-FERMI ($\lambda = 1$)	0.7759	0.0105
DP-FERMI ($\lambda = 2.5$)	0.7669	0.0110

Method	Accuracy	Statistical Parity Gap
Algorithm 2	0.7961	0.0068
DP-FERMI ($\lambda = 0.25$)	0.7996	0.0188
DP-FERMI ($\lambda = 0.35$)	0.7950	0.0174
DP-FERMI ($\lambda = 0.5$)	0.7912	0.0172
DP-FERMI ($\lambda = 1$)	0.7895	0.0105
DP-FERMI ($\lambda = 2.5$)	0.7884	0.0066