# Parameter and State Estimation in Queues and Related Stochastic Models: A Bibliography

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#### Abstract

This is an annotated bibliography on estimation and inference results for queues and related stochastic models. The purpose of this document is to collect and categorise works in the field, allowing for researchers and practitioners to explore the various types of results that exist.

#### Our focus is on papers that deal with mathematical queueing models as well as related stochastic models motivated by queues.

We attempted to make this bibliography exhaustive, yet there are possibly some papers that we have missed. As it is updated continuously, additions and comments are welcomed.

Note that this bibliography is also a companion to our survey of parameter and state estimation in queues [20].

Additional works not mentioned in this bibliography include the following related categories: (i) Methods for parameter estimation of point processes (not involving queues). (ii) Methods for parameter estimation of stochastic matrix analytic models (not involving queues). (iii) Optimal control of queues including bandit problems where state or parameter estimation is not directly considered. (iv) Inference, estimation and tomography of communication networks not directly modelled as queueing networks. (v) Analysis of queues with parameter uncertainty, and robustness not directly involving inference. (vi) Road traffic modelling not involving an explicit congestion or queueing model.

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## 1 Chronological order with brief descriptions

## 1955

Cox [86]: An overview paper of queueing theory outlining the philosophy of estimating parameters of input processes vs. performance processes.

## 1957

**Benes** [40]: Transient  $M/M/\infty$  full observation over a fixed interval.

Clarke [77]: M/M/1 MLE with full observation. The first paper. Sampling until "busy time" reaches a pre-assigned value yields closed-form MLEs.

## 1961

Billingsley [48]: Book on inference of Markov chains.

## 1965

Cox [87]: An overview paper on parameter estimation, separate analysis of input and service mechanism and problems connected with the sampling of queueing process.

Kovalenko [204]: On recovering the characteristics of a system from observations of the outgoing flow (in Russian)

**Newell [252]:** A review of approximation methods for queues with application to the fixed-cycle traffic light.

Wolff [336]: Large sample theory for birth-death queues.

## 1966

Lilliefors [217]: Confidence intervals for standard performance measurements based on parameter error.

## 1967

**Greenberg** [139]: Different ways of determining for how long to observe a stationary M/M/1 queue (e.g. fixed number of arrivals, fixed total observation time, etc.

## 1968

**Daley [92]:** The serial correlation coefficients of queue sizes in a stationary GI/M/1 queue are studied.

**Daley [93]:** The serial correlation coefficients of a (stationary) sequence of waiting times in a stationary M/M/1, M/G/1 and GI/G/l queueing system are studied.

## 1970

**Brown [55]:** Estimating the G in  $M/G/\infty$  with arrival and departure times without knowing what customers they related to.

Ross [284]: Discusses identifying the distributions of GI/G/k uniquely based on observation of the queueing process.

## $\boldsymbol{1971}$

**Pakes [256]:** The serial correlation coefficients of waiting times in the stationary GI/M/1 queue are studied. (completing [92] work)

## 1972

Goyal and Harris [136]: MLE for queues with Poisson arrivals with state-dependent general service times when queue sizes are observed at departure points. Jenkins [179]: Compares the asymptotic variance of two estimators for M/M/1. Muddapur [245]: Adds a prior distribution to Clarke's [77] approach.

#### 1973

Harris [150]: An overview paper presented the statistical analysis of queueing systems with an emphasis on the estimation of input and service parameters and/or distributions. Neal and Kuczura [248]: Presents accurate approximation and asymptotic approximations (by using renewal theory) for the variance of any differentiable functions of different traffic measurements.

**Reynolds** [278]: Bayesian approach for estimation of birth death parameters.

#### $\mathbf{1974}$

Aigner [4]: Compares properties of various estimators for M/M/1 with cross-sectional data.

Brillinger [53]: Estimates parameter for a linear time invariant model that generalizes the  $G/G/\infty$  queue.

## 1975

Keiding [190]: Analyses asymptotic properties of the MLE for a birth-and-death process with linear rates (both birth and death).

#### 1979

**Thiagarajan and Harris [313]:** Exponential goodness of fit test for the service times of M/G/1 based on observations of the queue lengths and/or waiting times.

#### **1980**

Dave and Shah [98]: MLE of an M/M/2 queue with heterogenous servers. Gordon and Dowdy [134]: Investigates the effect of parameter estimation errors on the performance in closed product form queueing networks.

#### 1981

Basawa and Prabhu [37]: Estimates for non-parametric and parametric models of single server queues over a horizon up to the nth departure epoch. Also the m.l.e's of the mean inter-arrival time and mean service time in an M/M/1 observed over a fixed time-interval. Walrand [320]: Proposes an elementary justification of the filtering formulas for a Markov chain and an analysis of the arrival and departure processes at a ./M/1 queue in a quasi-reversible network.

**Grassmann [137]:** This paper shows that in the  $M/D/\infty$  queueing system with service time S, the optimal way to estimate the expected number in the system is by sampling the system at time  $0, S, 2S, \dots, kS$ .

#### **1982**

Halfin[144]: Finds the minimum-variance linear estimator for the expected value of a stationary stochastic process observed over a finite time interval, whose covariance function is a sum of decaying exponentials.

Schruben and Kulkarni [286]: Studies the interface between stochastic models and actual systems for the special case of M/M/1 queue.

#### **1983**

Hernàndez-Lerma and Marcus [157]: Adaptive control of an M/G/1 queueing system, where the control chooses the service rate to minimize costs.

#### $\mathbf{1984}$

Baras, Dorsey, and Makowski [31]: Stability, parameter estimation and adaptive control for discrete-time competing queues.

Eschenbach [117]: Briefly describes methods and results in the statistical analysis of queueing systems.

Edelman and McKellar [113]: Comments on [98].

Hernàndez-Lerma and Marcus [158]: Adaptive control of priority assignment in a multi-class queue.

Machihara [224]: The carried traffic estimate errors for delay system models are analyzed with emphasis on the analysis of the effect of the holding time distribution on the estimate errors.

Warfield and Foers [328]: A Bayesian method for analysing teletraffic measurement data is discussed.

Woodside, Stanford and Pagurek [337]: Presents optimal mean square predictors for queue lengths and delays in the stationary  $\rm GI/M/m$  queue, based on a queue length measurement.

#### $\boldsymbol{1985}$

Armero [14]: The posterior distribution of traffic intensity and the posterior predictive distribution of the waiting time and number of customers for a  $M/M/1/\infty$  FIFO queue are obtained given two independent samples of arrival and service times.

Warfield and Foers [329]: Bayesian analysis for traffic intensity in M/M/c/K type models and in retrial models.

#### 1986

Subba Rao and Harishchandra [304]: Large normal approximation test based for the traffic intensity parameter in GI/G/s queues.

#### 1987

Bhat and Rao [46]: A first major survey on statistical analysis of queueing systems. Mcgrath, Gross and Singpurwalla [236]: Attempts to illustrate Bayesian approach through M/M/1 and M/M/1/K examples.

McGrath and Singpurwalla [237]: This is part II to [236] (without Gross). Here the focus is on integrating the "Shannon measure of information" (cross-entropy) in the analysis. **Ramalhoto** [269]: Discusses estimation of generalizations of  $GI/G/\infty$ , i.e. random translations whose distribution is parameterized by a certain function,  $h(\cdot)$ .

#### 1988

**Basawa and Prabhu [38]:** Estimation of GI/G/1 with exponential family densities. Full observation over [0, T] where T is a stopping time. Several T's considered and asymptotic properties compared.

Chen, Harrison, Mandelbaum, Van Ackere, Wein [64]: Empirical evaluation of a queueing network model for semiconductor wafer fabrication.

Harishchandra and Rao [149]: Inference for the  $M/E_k/1$  queue.

Jain and Templeton [174]: Estimation of GI/M/1 (and GI/M/1/m with m known) parameters where the arrival rate is either  $\lambda$  or  $\lambda_1$  depending on the queue level.

Nozari and Whitt [254]: Propose an indirect approach to estimate average production intervals (the length of time between starting and finishing work on each product) using work-in-process inventory measurements.

#### **1989**

Fendick and Whitt [120]:Proposes measurements and approximations to describe the variability of offered traffic to a queue (the variability of the arrival process together with the service requirements) and predicts the average workload in the queue (which assumed to have a single server, unlimited waiting space and a work-conserving service discipline).

**Glynn and Whitt [131]:** Using the little's  $L = \lambda W$  and generalizations to infer L from W and the oppositive.

Hantler and Rosberg [148]: Parameter estimation of M/M/c queue with parameters in stochastic varying environment, first doing the constant invariant derivation and then using in conjunction with Kalman filter for the time-varying case.

Jain and Templeton [175]: Sequential analysis view for  $M/E_k/1$  queues.

Sengupta [291]: Present an algorithm for computing the steady-state distribution of the waiting time and queue length of the stable GI/K/l queue.

## 1990

Gaver and Jacobs [126]: Transient M/G/1 inference.

Gawlick [127]: Applies the (QIE) to ethernet data.

Larson [209]: This deals with "State Reconstruction" as opposed to "parameter inference" in what is called the "Queue Inference Engine" (QIE). This is the first of many papers on the idea of using transactional data to reconstruct an estimate of the queue length process. Rubin and Robson [285]: Inference and estimation of number of arrivals for a queueing system with losses due to bulking and a server that works a fixed shift and stays to work after the shift. Small sample analysis as opposed to asymptotic properties.

## 1991

Hall and Larson [146]: Modifies (extends) the QIE [209] to finite queues and to a case where there is data about exceeding a certain level.

Jain and Templeton [176]: Confidence intervals for estimation for M/M/2 with heterogenous servers.

Jain [169]: Compares confidence intervals for  $\rho$  using several methods and sampling regimens in  $M/E_k/1$  queues.

**Larson [210]:** An addendum to [209] reducing the computational complexity from  $O(N^5)$  to  $O(N^3)$ .

Thiruvaiyaru, Basawa and Bhat [315]: Large sample theory for MLEs of Jackson networks.

## **1992**

Asmussen [21]: proves that the stationary waiting time in a GI/PH/1 queue with phase-type service time is phase-type.

Asmussen and Bladt [22]: The Matrix-exponential distribution is introduced and some of its basic structural properties are given. Further, an algorithm for computing the waiting time distribution of a queue with matrix-exponential service times and general inter-arrival times is given. This algorithm is a slight generalization of the algorithm for computing the waiting time distribution of GI/PH/1 queues.

Basawa and Bhat [33]: Presents sequential analysis methods for the traffic intensity of single server queues.

Bertsimas and Servi [41]: Improves on the  $O(n^5)$  algorithm in [209] to  $O(n^3)$ . Also presents an on-line algorithm for estimating the queue length after each departure and includes time-varying Poisson generalizations.

Daley and Servi [94]: Continues the track of the QIE, using taboo probabilities.

Heyde [159]: Quasi-likelihood estimation methods for stationary processes and queueing examples.

Jain [170]: Derives the relative efficiency of a parameter for the M/G/1 queueing system based on reduced and full likelihood functions. In addition, Monte Carlo simulations were carried out to study the finite sample properties for estimating the parameters of an M/G/1 queueing system.

Kumar [208]: Studies the bias in the means of average idle time and average queue length estimates, over the interval [0, t], in a transient M/M/1 queue.

**Singpurwalla [296]:** A discussion paper about [314]. The same issue for QUESTA also has a rejoinder for the discussion.

Thiruvaiyaru and Basawa [314]: Discusses empirical Bayes estimation for variations of M/M/1 queues and Jackson networks.

#### 1993

**Daley and Servi [95]:** Discuss a fairly general Markov chain setting for describing a stochastic process at intermediate time points r in  $r \in (0, n)$  conditional on certain known behaviour of the process both on the interval and at the endpoints 0 and n.

Glynn, Melamed and Whitt [129]: Constructs confidence intervals for estimators and perform statistical tests by establishing a joint central limit theorem for customer and time averages by applying a martingale central limit theorem in a Markov framework.

#### **1994**

Armero and Bayarri [17]: Presents a Bayesian approach to predict several quantities in an M/M/1 queue in equilibrium.

Armero and Bayarri, M.J. [16]: Bayesian "prediction" in M/M/1 queues is considered. The meaning is Bayesian inference for steady state quantities such as the distribution of queue lengths.

Armero [15]: Another Bayesian inference paper.

Chandra and Lee [63]: Presents Bayesian methods for inferring customer behavior from transactional data in telecommunications systems.

Chen, Walrand and Messerschmitt [68]: Perhaps the first "probing" paper. Deals with arrivals in a deterministic service time queue and estimates the Poisson arrival rates based on probe delays.

Jang and Liu [177]: Presents a new queueing formula applicable in manufacturing which uses variables easier to estimate than the variance such as the number of machine idle periods.

Jones and Larson [182]: Develops an efficient algorithm for event probabilities of order statistics and uses it for the queue inference engine ([209]).

**Pitts [263]:** Analysis of non-parametric estimation of the GI/G/1 queue input distributions based on observation of the waiting time.

#### 1995

Duffield, Lewis, O'Connell, Russell, and Toomey [108]: Estimates directly the thermodynamic entropy of the data-stream at an input-port. From this, the quality-of-service parameters can be calculated rapidly.

Jain [171]: Change point detection in an M/M/1 queue.

Muthu and Sampathkumar [246]: The maximum likelihood estimates of the parameters involved in a finite capacity priority queueing model are obtained. The precision of the maximum likelihood estimates is studied using likelihood theory for Markov processes.

**Masuda [234]:** Provides sufficient conditions under which the intuition (based on partial observations) can be justified, and investigates related properties of queueing systems.

#### 1996

Basawa, Bhat, and Lund [34]: MLE for GI/G/1 based on waiting time data.

**Dimitrijevic** [102]: Considers the problem of inferring the queue length of an M/G/1 queue using transactional data of a busy period.

Manjunath and Molle [231]: Introduces a new off-line estimation algorithm for the waiting times of departing customers in an M/G/1 queue with FCFS service by decoupling the arrival time constraints from the customer departure times.

Massey, Parker, and Whitt [233]: Estimate the parameters of a nonhomogeneous Poisson process with linear rate over a finite interval, based on the number of counts in measurement subintervals.

Sohn [298]: Simple M/M/1 Bayesian parameter estimation.

Sohn [299]: Bayesian estimation of M/M/1 using several competing methods.

#### 1997

Armero and Bayarri [18]: Bayesian inference of  $M/M/\infty$ .

Basawa, Lund, and Bhat [36]: Extends [34] using estimating functions.

Bhat, Miller and Rao [45]: A survey paper, a decade after the previous Survey by Bhat and Rao, [46].

**Daley and Servi [96]:** Computes the distributions and moments of waiting times of customers within a busy period in an FCFS queuing system with a Poisson arrival process by exploiting an embedded Markov chain.

Glynn and Torres [130]: Deals with estimation of the tail properties of the workload process in both the M/M/1 queue and queues with more complex arrivals such as MMPP. Ho and Cassandras [160]: A survey on perturbation theory.

Pickands and Stine [261]: Discrete time  $M/G/\infty$  queue.

**Toyoizumi [316]:** Waiting time inference in G/G/1 queues in a non-parametric manner using "Sengupta's invariant relationship".

#### 1998

**Daley and Servi [97]:** Computes the distributions and moments of waiting times of customers within a busy period in an FCFS queuing system with a Poisson arrival process by exploiting an embedded Markov chain.

Ganesh, Green, O'Connell and Pitts [124]: Appears like a "visionary" paper on the use of non-parametric Bayesian methods in network management.

Insua, Wiper and Ruggeri [168]: Bayesian inference for M/G/1 queues with either Erlang or hyper-exponential service distributions.

Mandelbaum and Zeltyn [225]: Queuing inference estimation in networks.

Rodrigues and Leite [281]: A Bayesian inference about the traffic intensity in an M/M/1 queue, without worrying about nuisance parameters.

Sharma and Mazumdar [293]: Proposes several schemes that the call acceptance controller, at the entering node of an ATM network, can use to estimate the traffic of the users on the various routes in the network by sending a probing stream.

**Wiper [335]:** Perhaps complements [168] with analysis of G/M/c queues with the G being Erlang or hyper-exponential renewal processes.

## 1999

Acharya [3]: Analyses the rate of convergence of the distribution of MLEs in GI/G/1 queues with assumptions on the distributions as being from exponential families.

Conti [82]: Baysian inference for a Geo/G/1 Discrete time queue.

Bingham and Pitts [49]: Non-parameteric estimation in  $M/G/\infty$  queues.

**Bingham and Pitts [50]:** Estimates the arrival rate of an M/G/1 queue given observations of the busy and idle periods of this queue.

Jones [180]: Analyses queues in the presence of balking, using only the service start and stop data utilized in Larson's Queue Inference Engine.

Rodrigo and Vazquez [280]: Analyses a general G/G/1 retrial queueing systems from a statistical viewpoint.

**Sharma [292]:** Using the measurement tools available on the Internet, suggests and compares different estimators to estimate aggregate traffic intensities at various nodes in the network.

## 2000

Armero and Conesa [19]: Statistical analysis of bulk arrival queues from a Bayesian point of view.

**Duffield [107]:** Analyses the impact of measurement error within the framework of Large Deviation theory.

Glynn and Zeevi [132]: Estimates tail probabilities in queues.

Jain [172]: Sequential analysis.

Jain and Rao [173]: Investigates the problems of statistical inference for the GI/G/1 queueing system.

**Zheng and Seila [352]:** Construct estimators for the limiting expected number of customers in the queue (and several other performance measures) with better sampling properties in comparison to the existing estimators.

## 2001

Alouf, Nain and Towsley [6]: Probing estimation for M/M/1/K queues using moment-based estimators based on a variety of computable performance measures.

Huang and Brill [163]: Deriving the minimum variance unbiased estimator (MVUE) and the maximum likelihood estimator (MLE) of the stationary probability function of the number of customers in a collection of independent M/G/c/c subsystems.

Jang, Suh and Liu [178]: Presents a new GI/G/2 queueing formula which uses a slightly different set of data easier to obtain than the variance of inter-arrival time.

**Paschalidis and Vassilaras [259]:** Buffer overflow probabilities in queues with correlated arrival and service processes using large deviations.

## 2002

**Conti** [83]: Non-parametric statistical analysis of a discrete-time queueing system is considered and estimation of performance measures of the system is studied.

Conti and De Giovanni [85]: Considers performance evaluation of a discrete-time GI/G/1 queueing model with a focus on the equilibrium distribution of the waiting time.

Sohn [300]: Even though the title has "Robust", this paper appears to be a standard M/M/1 Baysian inference paper using the input data.

Zhang, Xia, Squillante and Mills [345]: A general approach to infer the per-class service times at different servers in multi-class queueing models.

#### 2003

Cao, Andersson, Nyberg, and Kihl [58]: Web server performance is modeled via an  $M/G/1/K^*PS$  queue for which the authors also carry out maximum likelihood estimation of the parameters.

Pichitlamken, Deslauriers, L'Ecuyer, and Avramidis [260]: This is a simulation modelling paper where the authors also carry out parameter estimation for the simulation model data via a real data set.

#### 2004

Ausín, Wiper and Lillo [24]: Bayesian inference of M/G/1 using phase type representations of the G.

**Conti** [84]: A Bayesian non-parametric approach to the analysis of discrete-time queueing models.

**Fearnhead [119]:** Using forward-backward algorithm to do inference for M/G/1 and Er/G/1 queues.

Hall and Park [145]: An  $M/G/\infty$  non-parametric paper.

Wang, Chen, and Ke [192]: Maximum likelihood estimates and confidence intervals of an M/M/R/N queue with balking and heterogeneous servers.

#### 2005

Bladt and Sørensen [51]: Likelihood inference for discretely observed Markov jump processes with finite state space.

Brown, Gans, Mandelbaum, Sakov, Shen, Zeltyn and Zhao [54]: Major paper dealing with telephone call centre data analysis.

Hei, Bensaou and Tsang [155]: Probing focusing on the inter-departure SCV of the probing stream in tandem finite buffer queues.

Mandjes and van de Meent [227]: Propose an approach to accurately determine the burstiness of a network link on small time-scales (for instance 10 ms), by sampling the buffer occupancy (for instance) every second.

**Prieger [266]:** Shows that the MLE based on the complete inter-arrival and service times (IST) dominates the MLE based on the number of units in service (NIS), in terms of ease of implementation, bias, and variance.

Ross and Shanthikumar [282]: Estimating effective capacity in Erlang loss systems under competition.

Neuts [251]: Reflections on statistical methods for complex stochastic systems.

#### 2006

**Castellanos, Morales, Mayoral, Fried and Armero [62]:** Develops a Bayesian analysis of queueing systems in applications of the machine interference problem, like job-shop type systems, telecommunication traffic, semiconductor manufacturing or transport.

**Chick** [70]: A survey chapter on subjective probability and the Bayesian approach, specifically in Monte-Carlo simulation, yet gives some insight into queueing inference.

Chu and Ke [73]: Construction of confidence intervals of mean response time for an M/G/1 FCFS queueing system.

**Doucet, Montesano Jasra [106]:** Presents a trans-dimensional Sequential Monte Carlo method for online Bayesian inference in partially observed point processes.

Hansen and Pitts [147]: Non-parametric estimation of the service time distribution and the traffic intensity in M/G/1 queues based on observations of the workload.

Hei, Bensaou and Tsang [156]: Similar to [155] but here the focus is on inter-departure SCV of the two consecutive probing packets.

Ke and Chu [186]: Proposes a consistent and asymptotically normal estimator of intensity for a queueing system with distribution-free inter-arrival and service times.

Kim, Shiravi, and Min [203]: This paper deals with network congestion analysis via approximating processes and uses real network trace data for parameter fitting of self-similar network traffic using the index of dispersion for counts and coefficient of determination.

Liu, Heo, Sha and Zhu [221]: Proposes a queueing-model-based adaptive control approach for controlling the performance of computing systems.

Liu, Wynter, Xia and Zhang [222]: presents an approach for solving the problem of calibration of model parameters in the queueing network framework using inference techniques.

Rodrigo [279]: Analyse the M/G/1 retrial queue from a statistical viewpoint.

Wang, Ke, Wang and Ho [325]: Studies MLE and confidence intervals of an M/M/R queue with heterogeneous servers under steady-state conditions.

## 2007

Ausín, Lillo and Wiper [23]: Considers the problem of designing a GI/M/c queueing system.

Chu and Ke [74]: Proposes a consistent and asymptotically normal estimator of the mean response time for a G/M/1 queueing system, which is based on the empirical Laplace function.

Chu and Ke [75]: Estimation and confidence interval of mean response time for a G/G/1 queueing system using data-based recursion relation and bootstrap methods.

Morales, Castellanos, Mayoral, Fried and Armero [243]: Exploits Bayesian criteria for designing an M/M/c//r queueing system with spares.

**Park [257]:** The use of auxiliary functions in non-parametric inference for the  $M/G/\infty$  queueing model is considered.

**Ross, Taimre and Pollett [283]:** Estimation of rates in M/M/c queues using observations at discrete queues and MLE estimates of an approximate Orenstein Ullenbeck (OU) process.

## 2008

Ausín, Wiper and Lillo [25]: Bayesian inference for the transient behaviour and duration of a busy period in a single server queueing system with general, unknown distributions for the inter-arrival and service times is investigated.

Basawa, Bhat and Zhou [35]: Parameter estimation based on the differences of two positive exponential family random variables is studied.

Casale, Cremonesi and Turrin [60]: Proposed service time estimation techniques based on robust and constrained optimization.

Casale, Zhang and Smirni [61]: Several contributions to the Markovian traffic analysis. Choudhury and Borthakur [71]: Bayesian-based techniques for analysis of the M/M/1 queueing model.

**Dey [100]:** Bayes' estimators of the traffic intensity and various queue characteristics in an M/M/1 queue under quadratic error loss function have been derived.

Ke, Ko and Sheu [189]: Proposes an estimator for the expected busy period of a controllable M/G/1 queueing system in which the server applies a bicriterion policy during his idle period.

Ke, Ko and Chiou [188]: Presents a sensitivity investigation of the expected busy period for a controllable M/G/1 queueing system by means of a factorial design statistical analysis. Kim and Park [202]: Introduces methods of queue inference which can find the internal behaviours of queueing systems with only external observations, arrival and departure time. Sutton and Jordan [307]: Analysing queueing networks from the probabilistic modelling perspective, applying inference methods from graphical models that afford significantly more modelling flexibility.

Ramirez, Lillo and Wiper [270]: Considers a mixture of two-parameter Pareto distributions as a model for heavy-tailed data and use a Bayesian approach based on the birth-death Markov chain Monte Carlo algorithm to fit this model.

## 2009

**Baccelli, Kauffmann and Veitch [27]:** Points out the importance of inverse problems in queueing theory, which aim to deduce unknown parameters of the system based on partially observed trajectories.

Baccelli, Kauffmann and Veitch [28]: Evaluates the algorithm proposed in [27].

**Comert and Cetin** [81] Presents a real-time estimation of queue lengths from the location information of probe vehicles in a queue at an isolated and under-saturated intersection.

Chu and Ke [76]: Constructs confidence intervals of intensity for a queueing system, which are based on four different bootstrap methods.

**Duffy and Meyn [110]:** Conjectures and presents support for this: a consistent sequence of non-parametric estimators can be constructed that satisfies a large deviation principle.

Gorst-Rasmussen Hansen [135]: Proposes a framework based on empirical process techniques for inference about waiting time and patience distributions in multi-server queues with abandonment.

Heckmuller and Wolfinger [153]: Proposes methods to estimate the parameters of arrival processes to G/D/1 queueing systems only based on observed departures from the system.

**Ibrahim and Whitt [165]:** Studies the performance of alternative real-time delay estimators based on recent customer delay experiences.

Kiessler and Lund [193]: A note that considers traffic intensity estimation in the classical M/G/1 queue.

Ke and Chu [187]: Proposes a consistent and asymptotically normal estimator of intensity for a queueing system with distribution-free inter-arrival and service times.

Kraft, Pacheco-Sanchez, Casale and Dawson [205]: Proposes a linear regression method and a maximum likelihood technique for estimating the service demands of requests

based on the measurement of their response times instead of their CPU utilization.

Liu, Wu, Ma, and Hu [220]: Presents an approach to estimate time-dependent queue length even when the signal links are congested.

Mandjes and Żuraniewski [229]: Develops queueing-based procedures to (statistically) detect overload in communication networks, in a setting in which each connection consumes roughly the same amount of bandwidth.

Mandjes and van De Meent [228]: The focus is on dimensioning as the approach for delivering performance requirements of the network.

Nam, Kim and Sung [247]: Estimates the available bandwidth for an M/G/1 queueing system.

Novak and Watson [253]: Presents a technique to estimate the arrival rate from delay measurements, acquired using single-packet probing.

## 2010

**Chen and Zhou [66]:** Proposes a non-linear quantile regression model for the relationship between stationary cycle time quantiles and corresponding throughput rates of a manufacturing system.

**Duffy and Meyn [109]:** Deals with large deviations showing that in broad generality, that estimates of the steady-state mean position of a reflected random walk have a high likelihood of over-estimation.

Frey and Kaplan [122]: Introduces an algorithm for queue inference problems involving periodic reporting data.

**Pin, Veitch, and Kauffmann [262]:** This paper incorporates queueing theory results in the application of network tomography where statistical estimation of delays in a multicast tree using an EM algorithm is developed.

Gans, Liu, Mandelbaum, Shen, and Ye [125]: Studies operational heterogeneity of call center agents where the proxy for heterogeneity is agents' service times (call durations). Heckmueller and Wolfinger [152]: Proposes methods to estimate the parameters of arrival processes to G/D/1 queueing systems only based on observed departures from the system.

Pin, Veitch and Kauffmann [262]: Focuses on a specific delay tomographic problem on a multicast diffusion tree, where end-to-end delays are observed at every leaf of the tree, and mean sojourn times are estimated for every node in the tree.

Ramirez-Cobo, Lillo, Wilson and Wiper [272]: Presents a method for carrying out Bayesian estimation for the double Pareto lognormal distribution.

Sutton and Jordan [308]: Presents a viewpoint that combines queueing networks and graphical models, allowing Markov chain Monte Carlo to be applied.

Xu, Zhang and Ding [338]: Discusses testing hypotheses and confidence regions with correct levels for the mean sojourn time of an M/M/1 queueing system.

Zhang and Xu [346]: Discuss constructing confidence intervals of performance measures for an M/G/1 queueing system.

Zuraniewski, Mandjes and Mellia [354]: Explores techniques for detecting unanticipated load changes with a focus on large-deviations based techniques developed earlier in [229].

## 2011

Abramov [1]: Statistical bounds for certain output characteristics of the M/GI/1/n and GI/M/1/n loss queueing systems are derived on the basis of large samples of an input characteristic of these systems.

Amani, Kihl and Robertsson [7]: An applications paper to computer systems.

Ban, Hao, and Sun [30]: Studies how to estimate real time queue lengths at signalized intersections using the intersection travel times collected from mobile traffic sensors.

Chen, Nan, Zhou [65]: Investigates the statistical process control application for monitoring queue length data in M/G/1 systems.

Feng, Dube, and Zhang [121]: Considers estimation problems in  $G/G/\infty$  queue under incomplete information. Specifically, where it is infeasible to track each individual job in the system and only aggregate statistics are known or observable.

Veeger, Etman, Lefeber, Adan, Van Herk, and Rooda [318]: Predicting cycle time distributions for integrated processing workstations: an aggregate modelling approach.

Grübel and Wegener [141]: In  $M/G/\infty$  systems, considers the matching and exponentiality problems where the only observations are the order statistics associated with the departure times and the order in which the customers arrive and depart, respectively.

**Ibrahim and Whitt [166]:** Develops real-time delay predictors for many-server service systems with a time-varying arrival rate, a time-varying number of servers, and customer abandonment.

Mandjes and Zuraniewski [230]:  $M/G/\infty$  change point detection using large deviations. Manoharan and Jose [232]: Considers an M/M/1 queueing system with customer impatience in the form of random balking.

McCabe, Martin and Harris [235]: Presents an efficient probabilistic forecast of integervalued random variables that can be interpreted as a queue, stock, birth and death process or branching process.

Park, Kim and Willemain [258]: Proposes new approaches that can analyse the unobservable queues using external observations.

Sousa-Vieira [301]: Considers the suitability of the  $M/G/\infty$  process for modelling the spatial and quality scalability extensions of the H.264 standard in video traffic modelling.

Srinivas, Rao and Kale [303]: Maximum likelihood and uniform minimum variance unbiased estimators of measures in the M/M/1 queue are obtained and compared.

Sutton and Jordan [309]: A Bayesian inference paper by computer systems researchers.

#### 2012

Duffy and Meyn [111]: Large deviation asymptotics for busy periods for a queue.

Fabris-Rotelli, Kraamwinkel, and et al [118]: A historical and theoretical overview of G/M and M/G queueing processes.

Hu and Lee [162]: Consider a parameter estimation problem when the state process is a reflected fractional Brownian motion (RFBM) with a non-zero drift parameter and the observation is the associated local time process.

Jones [181]: Remarks on queue inference from departure data alone and the importance of the queue inference engine.

Kauffmann [185]: Proposes a new approach, on the basis of existing TCP connections and reaching therefore a zero probing overhead based on the theory of inverse problems in bandwidth sharing networks.

Kim and Whitt [195]: Statistical analysis with Little's law.

Kim and Whitt [194]: Contains supplementary martial to [195].

Kim and Whitt [197]: Estimating waiting times with the time-varying Little's law.

Kim and Whitt [196]: Appendix to [197].

McVinish and Pollett [238]: Uses estimating equations to get estimators for M/M/c queues and related models. Performance is compared to [283].

Mohammadi and Salehi-Rad [242]: Exploits the Bayesian inference and prediction for an M/G/1 queuing model with optional second re-service.

Nelgabats, Nov and Weiss [249]:  $M/G/\infty$  estimation.

**Ren and Li [277]:** Bayesian estimator of the traffic intensity in an M/M/1 queue is derived under a new weighted square error loss function.

**Ren and Wang [276]:** Similer to [277]. Bayesian estimators of the traffic intensity in an M/M/1 queue are derived under a precautionary loss function.

Whitt [331]: Fitting birth-and-death queueing model to data.

## 2013

Larson [211]: A brief review on QIE.

Acharya, Rodríguez-Sánchez and Villarreal-Rodríguez [2]: Presents the derivation of maximum likelihood estimates for the arrival rate and service rates in a stationary M/M/c queue with heterogeneous servers.

Chow [72]: Analysis of queueing model based on chaotic mapping.

Li, Chen, Li and Zhang [216]: Proposes a new algorithm based on the temporal–spatial queueing model to describe the fast travel-time variations using only the speed and headway time series that is measured at upstream and downstream detectors.

Wiler, Bolandifar, Griffey, Poirier, and Olsen [334]: This paper applies queueing theory and inference of model parameters to derive and validate a novel queuing theory-based model that predicts the effect of various patient crowding scenarios on patients.

Weerasinghe and Mandelbaum [330]: studies the tradeoff between blocking and abandonment, with cost accumulated over a random, finite time horizon of a controlled queueing system of the G/M/n/B+M type with many servers and impatient customers.

#### $\mathbf{2014}$

Antunes, Jacinto and Pacheco [10]: estimation of the arrival rate and the service time moments of an M/G/1 queue with probing, i.e., with special customers (probes) entering the system. The probe inter-arrival times are i.i.d. and probe service times follow a general positive distribution. The only observations used are the arrival times, service times and departure times of probes. We derive the main equations from which the quantities of interest can be estimated. Two particular probe arrivals, deterministic and Poisson, are investigated.

Azriel, Feigin and Mandelbaum [26]: Proposes a new model called Erlang-S, where "S" stands for Servers where there is a pool of present servers, some of whom are available to serve customers from the queue while others are not, and the process of becoming available or unavailable is modelled explicitly.

Dinha, Andrewa and Nazarathy [103]: A conceptual and numerical contribution to design and control of speed-scaled systems in view of parameter uncertainty.

Edelmann and Wichelhaus [114]: Two different nonparametric estimation approach for discrete-time stochastic networks of  $\text{Geom}^X/\text{G}/\infty$  queues where the observation consists of the external arrival and external departure processes at the nodes over some time.

He, Li, Huang and Lei [151]: Considers the queuing system as a black box and derive a performance index for the queuing system by the principle of maximum entropy only on the assumption that the queue is stable.

Kannan and Jabarali [183]: This paper deals with maximum likelihood estimation parameters for a variant of an M/M/1 queue with vacations.

Kim and Whitt [199]: Considers different issues in testing the suitability of the nonhomogeneous Poisson process as an arrival process with service system data.

Kim and Whitt [198]: (in the following of [199]) shows that call center and hospital arrivals are well modelled by nonhomogeneous Poisson processes.

Senderovich, Weidlich, Gal, and Mandelbaum [288]: Establish a queueing perspective in operational process mining and demonstrate the value of queue mining using the specific operational problem of online delay prediction.

Yom-Tov and Mandelbaum [339]: Analyses a queueing model, where customers can return to service several times during their sojourn within the system.

## 2015

Bakholdina1 and Gortsev [29]: Focused on the problem of optimal estimation of the states of the modulated semi-synchronous integrated flow of events.

Burkatovskaya, Kabanova, and Vorobeychikov [319]: CUSUM algorithms for parameter estimation in queueing systems where the arrival process is a Markov-modulated Poisson process.

Cahoy, Polito, and Phoha [57]: Statistical analysis of fractional M/M/1 queue and fractional birth-death processes; the point processes governed by difference differential equations containing fractional derivative operators.

Chen and Zhou [67]: Propose the cumulative sum (CUSUM) schemes to efficiently monitor the performance of typical queueing systems based on different sampling schemes.

**Dong and Whitt [104]:** Explores a stochastic grey-box modelling of queueing systems by fitting birth-and-death processes to data.

**Dong and Whitt [105]:** Using a birth-and-death process to estimate the steady-State distribution of a periodic queue.

Efrosinin, Winkler, and Martin [115]: Considers the problem of estimation and confidence interval construction of a Markovian controllable queueing system with unreliable server and constant retrial policy.

Goldenshluger [133]: Non-parametric estimation of service time distribution of the M/G/1 queue from incomplete data on the queue.

Gurvich, Huang and Mandelbaum [143]: Proposes a diffusion approximation for a many-server Erlang-A queue.

Kim and Whitt [200]: Similar to [199]

Liu, Wu, and Michalopoulos [219]: Improves queue size estimation by proposing different ramp queue estimation algorithms.

Mohajerzadeh, Yaghmaee, and Zahmatkesh [241]: Proposed a method to prolong the network lifetime and to estimate the target parameter efficiently in wireless sensor networks.

Schweer and Wichelhaus [287] Estimation of the service time distribution in the discretetime  $GI/G/\infty$ -queue based solely on information on the arrival and departure processes is considered. The focus is put on the estimation approach via the so called "sequence of difference" and proving a functional central limit theorem for the resultant estimator.

Senderovich, Leemans, Harel, Gal, Mandelbaum, and van der Aalst [289]: Explores the influence of available information in the log on the accuracy of the queue mining techniques.

Senderovich, Weidlich, Gal, Mandelbaum [290]: Queue mining for delay prediction in multi-class service processes.

Srinivas and Kale [302]: Compares the Maximum Likelihood (ML) and Uniformly Minimum Variance Unbiased (UMVU) estimation for the M/D/1 queueing systems.

Sutartoa and Joelianto [305]: Presents an overview of urban traffic flow from the perspective of system theory and stochastic control.

Wang and Casale [326]: Proposes maximum likelihood (ML) estimators for service demands in closed queueing networks with load-independent and load-dependent stations.

Wang, Pérez, and Casale [327]: A software for parameter estimation.

Whitt [332]: Sequel to [104] and [106]. Establishes many-server heavy-traffic fluid limits for the steady-state distribution and the fitted birth and death rates in periodic Mt/GI/infty models.

Zhan, Li, and Ukkusuri [343]: In the context of transportation engineering, with the complete arrival and departure information, a car-following based simulation scheme is applied to estimate the real-time queue length for each lane.

## 2016

Amini, Pedarsani, Skabardonis, and Varaiya [8]: Queue-length estimation using realtime traffic data.

Antunes, Jacinto, Pacheco, and Wichelhaus [12]: Uses a probing strategy to estimate the time dependent traffic intensity in an  $M_t/G_t/1$  queue, where the arrival rate and the general service-time distribution change from one time interval to another, and derive statistical properties of the proposed estimator.

Anusha, Sharma, Vanajakshi, Subramanian, and Rilett [13]: Develops a modelbased scheme to estimate the number of vehicles in queue and the total delay.

**Comert [78]:** This paper, motivated by the field of traffic engineering, develops estimators for market penetration level and arrival rates based on queue lengths from probe vehicles at isolated traffic intersections.

Cruz, Quinino and Ho [89]: Uses a Bayesian technique, the sampling/importance resampling method to estimate the parameters of multi-server queueing systems in which inter-arrival and service times are exponentially distributed.

Ghorbani-Mandolakani and Salehi Rad [128]: Derives the ML and Bayes estimators of traffic intensity and asymptotic confidence intervals for mean system size of a two-phase tandem queueing model with a second optional service and random feedback and two heterogeneous servers.

Krishnasamy, Sen, Johari, and Shakkottai [206]: Considers regret analysis of a server allocation problem where service rates of servers are unknown. This analysis is in the context of multi-armed bandits.

Morozov, Nekrasova, Peshkova, and Rumyantsev [244]: Develops a novel approach to confidence estimation of the stationary measures in high performance multi-server queueing systems.

**Quinino and Cruz [267]:** Describes a Bayesian method for sample size determination for traffic intensity estimation.

Zammit, Fabri and Scerri [341]: A self-estimation algorithm is presented to jointly estimate the states and model parameters.

Zhang, Xu, and Mi [347]: Considers the hypothesis tests of performance measures for an  $M/E_k/1$  queueing system.

#### 2017

Cruz, Quinino, and Ho [90]: Bayesian estimation of traffic intensity based on queue length in a multi-server M/M/s queue.

den Boer and Mandjes [99]: Considers convergence rates of Laplace-transform based estimators.

Gu, Qian, and Zhang [142]: A Bayesian probabilistic model along with an expectation-maximization extended Kalman filter (EM-EKF) algorithm is proposed for traffic state estimation of urban road networks using a link queue model.

Kim, Whitt, and Cha [201]: A data-driven model of an appointment-generated arrival process at an outpatient clinic.

Li, Tang, Yao, and Li [215]: Proposes a cycle-by-cycle queue length estimation method using only probe data without the foregoing assumption for signalized intersections.

Quinino and Cruz [268]: A Bayesian method is described for sample size determination for traffic intensity estimation of an M/M/1 queue.

Sutarto, Joelianto, and Nugroho [306]: Developing a stochastic model of queue length at a signalized intersection.

Whitt and Zhang [333]: Develops an aggregate stochastic model of an emergency department based on a careful study of data on individual patient arrival times and length of stay.

Zammit, Fabri, and Scerri [342]: Online state and multidimensional parameter estimation for a macroscopic model of a traffic junction based on the Expectation-Maximization algorithm and multidimensional Robbins-Monro stochastic approximation.

#### 2018

An, Wub, Xiaa, and Huanga [9]: This paper from the field of traffic engineering focuses on real-time queue length estimation in the context of signalized intersections.

Almeida and Cruz [5]: The Jeffreys prior is proposed to obtain the posterior and predictive distributions of some parameters of interest. Samples are obtained through simulation and some performance characteristics are analyzed.

Cruz, Almeida, D'Angelo, and van Woensel [88]: Investigating the finite-sample behaviour of some well-known methods for the estimation of single-server finite Markovian queues or, in Kendall notation, queues, namely, the maximum likelihood estimator, Bayesian methods, and bootstrap corrections.

**Ozawa [255]:** Analysis of the stability condition of a two-dimensional QBD process and its application to evaluate the efficiency of two-queue models.

**Polson and Sokolov [264]:** Develops an efficient particle learning algorithm for real time online inference of states and parameters. This requires a two-step approach, first resampling the current particles with a mixture predictive distribution and second propagation of states using the conditional posterior distribution.

Suyama, Quinino, and Cruz [310]: Estimators for the parameters of the Markovian multiserver queues are presented, from samples that are the number of clients in the system at arbitrary points and their sojourn times.

## 2019

Bhat and Basawa[44]: An overview of the literature on the use of the maximum likelihood method for estimating parameters in queueing models.

Emami, Sarvi, and Asadi Bagloee [116]: A neural network algorithm for queue length estimation based on the concept of k-leader connected vehicles.

Li, Okamura, and Dohi [212]: suppose an Mt/M/1/K queueing system whose job arrival follows a Non-homogeneous Poisson Process (NHPP) and propose a parameter estimation method for the NHPP approximately from the utilization data based on the maximum likelihood estimation (MLE) via the expectation maximization (EM) algorithm.

Mei, Gu, Chung, Li, and Tang [239]: A Bayesian approach for estimating vehicle queue lengths at signalized intersections using probe vehicle data.

Ravner, Boxma, and Mandjes [274]: Develops estimation schemes for a Lévy-driven queue by sampling the workload process at Poisson times.

Tan, Yao, Tang, and Sun [312]: Cycle-based queue length estimation by fusing realtime and historical probe vehicle trajectory data, through a statistical parameter estimation method, i.e., maximum likelihood estimation (MLE)

Zhao, Zheng, Wong, Wang, Meng, and Liu [351]: Various methods for queue length and traffic volume estimation using probe vehicle trajectories.

## 2020

Cruz, Almeida, D'Angelo, and van Woensel [88]: A bias-corrected version of MLE estimator of traffic intensity by the nonparametric bootstrap method for small and moderate samples.

Tan, Liu, Wu, Cao, and Tang [311]: Applying probe vehicle trajectory, Bayesian theory, and Fuzing license plate recognition data and vehicle trajectory data for lane-based queue length estimation at signalized intersections.

Van Phu and Farhi [317]: Estimation of urban traffic state with probe vehicles.

Zhang, Liu, Chen, Yu, and Wang [344]: Proposes a cycle-based end-of-queue estimation method for queue length using sampled vehicle trajectory data under relatively low probe vehicle penetration rates.

Wang, Huang, and Lo [324]: This paper from the field of traffic engineering Combines shockwave analysis and Bayesian networks for traffic parameter estimation at signalized intersections by considering queue spillback.

## 2021

Asanjarani, Nazarathy, and Taylor [20]: A broad literature survey of parameter and state estimation for queueing systems.

**Bassamboo and Ibrahim [39]:** Using a combination of queueing-theoretic analysis, reallife data analysis, and simulation, the performance of static and dynamic announcements are analysed, and an appropriate weighted average of them is derived.

**Basak and Choudhury [32]:** Finding a Bayes estimator of traffic intensity for an M/M/1 queueing model using data on queue size (number of customers present in the queue) observed at any random point in time.

Comert, Amdeberhan, Begashaw, and Chowdhury [79]: A Combinatorial Approach for Nonparametric Short-Term Estimation of Queue Lengths using Probe Vehicles.

Cruz, Santos, Oliveira, and Quinino [91]: Estimation of performance measures in a general bulk-arrival Markovian multi-server finite queue.

**Dieleman [101]:** The method of MLE is used in combination with Stochastic Approximation (SA) to calibrate the arrival parameter of a G/G/1 queue via waiting time data.

Ebert, Dutta, Mengersen, Mira, Ruggeri, and Wu [112]: Presents a likelihood-free parameter estimation for dynamic queueing networks with a case study of passenger flow in an international airport terminal.

Krishnasamy, Sen, Johari, and Shakkottai [207]: Considers regret analysis of a server allocation problem where service rates of servers are unknown. This analysis is in the context of multi-armed bandits. The work extends previous work: [206].

Lin, He, and Pang [218]: Queuing network topology inference using passive end-to-end measurements originated by a single source.

Mandjes and Ravner [226]: In this paper the authors devise hypothesis testing procedures for Lévy-driven storage systems by sampling of the storage level.

Singh, Acharya, Cruz, and Quinino [294]: Proposed a methodology to determine the sample size for an queueing system under the Bayesian setup by observing the number of customer arrivals during the service time of a customer.

Walton and Xu [322]: Provides an extensive review of reinforcement learning ideas with a view of queueing network control. The paper also connects ideas of adversarial learning to queuing network concepts framed in the context of information uncertainty.

Wang and Honnappa [323]: Studies inference for a Cox / G /  $\infty$  queue, sampled at discrete time points. Uses approximate inference for maximizing a lower bound for the associated finite dimensional distribution.

Zhao, Shen, and Liu [349]: A hidden Markov model for the estimation of correlated queues in probe vehicle environments.

Zhao, Wong, Zheng, and Liu [350]: Maximum likelihood estimation of probe vehicle penetration rates and queue length distributions from probe vehicle data.

#### $\boldsymbol{2022}$

Antunes, Jacinto, and Pacheco [11]: A discussion on statistical inference in queueing networks with probing information.

**Comert and Bagashaw [80]:** Cycle-to-cycle queue length estimation from connected vehicles with filtering on primary parameters. This study improve accuracy of by enhancing the low level parameter estimators using filtering algorithms.

Li, Zheng, Okamura, and Dohi [213]: Hierarchical Bayesian Parameter Estimation of Queueing Systems using Utilization Data.

Li, Zheng, Okamura, and Dohi [214]: Parameter estimation of a MAP/M/1/K queueing system using utilization data. In particular, the parameters are estimated by using the

maximum likelihood estimation (MLE) method.

**Ravner [273]:** Considers an M/G/1 queue for which the workload process is observed periodically. The goal is to estimate the arrival rate  $\lambda$  and the parameters of the job-size distribution G.

Singh, Acharya, Cruz, and Quinino [295]: Bayesian inference and prediction in a queueing system.

Zhong, Birge, and Ward [353]: Considers a scheduling policy in time-varying multiclass many server queues with abandonment and proposes a Learn-Then-Schedule algorithm composed of a learning phase and an exploitation phase.

#### 2023

Chen, Liu, and Hong [69]: Proposes and studies an on-line learning algorithm for optimal control and pricing of a GI/GI/1 queue.

Bura and Sharma [56]: Maximum likelihood and Bayesian estimation for an M/M/1 queueing model with balking.

Carmeli, Yom-Tov and Boxma [59]: Analyzes fork-join networks and incorporates data driven estimation for such models.

Inoue, Ravner, and Mandjes [167]: Deals with parameter estimation of a queueing system with impatient customers and balking. A novel algorithm for the estimation of customer impatience is proposed and analyzed.

Luo, Deng, Chen, et. al. [223]: This paper from the field of traffic engineering focuses on queue length estimation based on probe vehicle data at signalized intersections using a shockwave approach in the model.

**Ravner and Wang [275]:** Parameter estimation of a queueing system where customers behave strategically, are able to choose their arrival times, and are at a Nash equilibrium.

# 2 Literature Analysis (up to 2017)

This section was last updated in 2017 and captures a classification of the papers up to 2017 via several categories. It also serves as a background companion to our survey paper, [20]. Up to 2017, the majority of the references were in the format of journal and/or conference articles. A few are surveys, textbooks, book chapters, Ph.D. theses, and significant related materials which are listed in the table below.

Surveys	[46] $[45]$ and the more recent $[20]$
Textbooks	[52] [240] [250] [265] [43] [340]
Book chapters	[44] –Chapter 2, [42] –Chapter 10, [140] –Section 6.7, [123] –Chapter 6, [321] –Chapter 10
Ph.D. theses	[138] [271] [164] [184] [161] [348]
Significant related materials	[47] (A book)

## 2.1 Classification by Model

Model	References
${ m M}/{ m M}/{ m 1}$	[77] [139] [93] [179] [4] [286] [236] [237] [208] [17][16] [234] [281] [300] [71] [307] [100] [66] [338] [232] [303] [276] [277] [67] [267] [352]
${ m M/M/2}$ (Heterogenous Servers)	[98] [113]
M/M/1/K	[236] [237] [6]

Model	References
${ m M/M/1/\infty}$ (FIFO)	[14]
M/M/c	[224] [148] [238] [325] [283] [238] [2] [339] [67]
M/M/c/N	[192] [243]
${ m M/M/\infty}$	[18] [229]
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	[270]
M/D/1	[253] [302]
$\mathbf{M}/\mathbf{E}_k/1$	[176] [347]
${ m M/G/\infty}$ (Random translation models in general)	[40] [55] [53] [269] [261] [49] [257] [229] [141] [230] [249] [330] [191]
M/G/1	[93] [313] [157] [158] [170] [149] [126] [231] [99] [102] [168] [50] [172] [24] [119] [62] [279] [74] [188] [189] [193] [247] [346] [65] [242] [10] [67] [133]
M/GI/1	[1]
${f E}_k/{f G}/1$	[119]
M/G/c	[95]

Model	References
${ m M/G/c/C}$	[163]
G/G/1	[316] [280] [75] [67]
${{{\rm GI}}/{{\rm M}}/{ m 1}}$ (state-dependent arrival rate)	[92] [256] [174]
${f GI/M/1/n}$	[1]
GI/M/c	[337] [23] [165]
${ m GI/G/1}$	[93] [38] [263] [34] [3] [173] [85] [35]
GI/G/2	[178]
GI/G/c	[202]
${f G}/{f G}/\infty$	[269]
${ m GI/K/1}$ (service time has Matrix-Exponential distribution)	[21] [22] [291] [297]
GI/GI/s	[104]
${f GI}/{f GI}/\infty$	[104]
${ m M/D/\infty}$ (Deterministic service)	[137]
M/D/1	[224]
G/D/1	[153] [152]

Model	References
MAP/D/1	[61]
Erlang Loss System	[282]
Jackson Networks or More General Networks	[315] [27]
Systems Where Little's Law Holds	[131]

## 2.2 Classification by Sampling Regime

Sampling Regime	References
Full Observation	[266]
Observation at Discrete Points	[99] [238] [283]
Probing	[293] [6] [12] [27] [68] [81] [153] [156] [155] [185] [201] [247] [253] [10]
Queue Inference Engine	[209] [127] [63] [102] [225] [122] [258] [96] [97] [154]

# 2.3 Classification by Statistical Paradigm

Statistical Paradigm	References
Bayesian	[245] [278] [328] [236] [237] [63] [314] [296] [16] [17] [15] [18] [168] [281] [82] [180] [19] [300] [24] [84] [62] [23] [243] [71] [25] [187] [272] [309] [242] [276] [277] [267]
Maximum Entropy	[151]
Emphasis on the way of selecting sampling time	[38]
Non-parametric	[82] [49] [180] [83] [84] [186] [257] [235] [133]
Change point detection	[171]
Adaptive Control	[157] [330]
Sequential Inference	[175] [33] [319]
Perturbation analysis	[160]
Large Deviations	[130]

# 2.4 Classification by Application

Statistical Paradigm	References
Telephone Call Centres	[54] [135] [165] [125] [198] [330] [26]
Manufacturing	[177] [62] [165] [66] [330]
Health Care	[198] [201] [242] [333] [339]
Transportation	[62] [216] [219] [305] [341] [13]
Economics	[266]
ATM	[108] [102] [83] [85] [62]
Communication/Telecommunication Networks	[127] [27] [29] [233] [241]
Network Traffic Modelling	[292] [227] [61] [220] [30] [301] [309]

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