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# Targeted collapse regularized autoencoder for anomaly detection: black hole at the center

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## Abstract

Autoencoders have been extensively used in the development of recent anomaly detection techniques. The premise of their application is based on the notion that after training the autoencoder on normal training data, anomalous inputs will exhibit a significant reconstruction error. Consequently, this enables a clear differentiation between normal and anomalous samples. In practice, however, it is observed that autoencoders can generalize beyond the normal class and achieve a small reconstruction error on some of the anomalous samples. To improve the performance, various techniques propose additional components and more sophisticated training procedures. In this work, we propose a remarkably straightforward alternative: instead of adding neural network components, involved computations, and cumbersome training, we complement the reconstruction loss with a computationally light term that regulates the norm of representations in the latent space. The simplicity of our approach minimizes the requirement for hyperparameter tuning and customization for new applications which, paired with its permissive data modality constraint, enhances the potential for successful adoption across a broad range of applications. We test the method on various visual and tabular benchmarks and demonstrate that the technique matches and frequently outperforms alternatives. We also provide a theoretical analysis and numerical simulations that help demonstrate the underlying process that unfolds during training and how it can help with anomaly detection. This mitigates the black-box nature of autoencoder-based anomaly detection algorithms and offers an avenue for further investigation of advantages, fail cases, and potential new directions.

## 1 Introduction

Anomaly detection is the task of identifying instances of data that significantly deviate from normal<sup>1</sup> observations. The task has many important applications including fault detection [45, 55, 31, 61, 16, 10], medical diagnosis [17, 23, 27, 51], fraud detection [44, 39, 40, 63, 24], performance and

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<sup>1</sup>The term “normal” in this text indicates “not anomalous.”

throughput anomaly detection and bottleneck identification [26, 11, 8], and network intrusion detection [1, 3, 15, 5, 53, 33, 37, 56], among others. Considering that anomalous samples are rare and can often highly vary, anomaly detection is typically done through learning regularities present in normal data. Subsequently, samples that reside far outside the normal class representation are regarded as anomalous.

Deep autoencoders [25, 6] are powerful tools to learn unsupervised representations of high dimensional data and have been central to many algorithms among various modern techniques for anomaly detection [48, 64, 20, 19]. The premise for using autoencoders in this context is that when trained on normal samples, they fail to effectively reconstruct anomalous samples as by definition they lie outside the training distribution. It therefore is possible to devise an anomaly scoring based on the reconstruction error. This premise, however, does not always hold and there indeed can be cases where an anomalous input can be well reconstructed [42, 65].

Various strategies have been proposed in the literature to improve the performance of autoencoders by including additional components and more sophisticated training procedures. Some examples of this include adding a memory module [20, 19, 58], visual and latent discriminators [58, 42], classifier [42], adversarial training [58, 42], and negative mining [42]. The added modules and involved training procedures can lead to increased costs associated with the utilization of such models, potentially making them less accessible to various applications and more challenging to troubleshoot.

In this work, we adopt two principles and aim to shed light on the mechanisms by which they enhance the utility of autoencoders in anomaly detection. First, we promote feature compactness in the latent space, ensuring that normal samples exhibit similar representations; secondly, we require feature descriptiveness so that the latent representation retains relevant information for effective separation between normal and anomalous samples. Based on these principles, we propose an autoencoder regularization that penalizes the norm of bottleneck representation. This is a simple and computationally efficient modification especially given the lower dimensionality of the latent space. We will show that this simple adjustment makes the anomaly detection performance of an encoder-decoder architecture match or surpass that of many deep techniques that incorporate the complex modifications mentioned earlier. We also theoretically analyze a simplified case and use the results to offer an explanation regarding the effects of norm minimization on learning dynamics and how it might help with anomaly detection.

The rest of the paper is organized as follows: in Section 2 we discuss related work on anomaly detection and autoencoder regularization. Section 3 discusses the proposed methodology and theoretical analysis, as well as empirical demonstrations for a simple case. In Section 4 we present anomaly detection results on benchmark datasets. Section 5 concludes the paper.

## 2 Related Work

Regularized autoencoders have shown improvements over their unregularized counterparts, resulting in better performance on various applications due to the advantages they introduce [4, 21, 34, 52, 46, 28]. Sparse autoencoders [34] regularize the network to encourage sparsity in hidden layers, which serves as an information bottleneck. Denoising autoencoders [52] promote robustness to corrupted data and better expressive features by training the model to reconstruct a denoised version of the noisy input. Expanding on a similar intuition, contractive autoencoders [46] promote feature robustness to variations in the neighborhood of training instances by penalizing the norm of the Jacobian matrix of the encoder’s activations with respect to input samples. Variational autoencoders [28] regularize the training by introducing the prior that additionally incentivizes samples to follow a standard Gaussian distribution in latent space and thus learns an approximation of data distribution parameters rather than arbitrary functions for encoding and reconstruction. These variants of autoencoders have been extensively used in applications of anomaly detection [48, 64] including fault detection and industrial health monitoring [55, 31, 61, 16], network intrusion, anomaly, and cyber-attack detection [3, 15, 5, 53, 33, 37, 56, 38], video anomaly detection [36], medical diagnosis [27], geochemical exploration [54], and high-energy physics searches [18].

To improve the performance of autoencoders in anomaly detection, some recent works propose using external memory to learn and record prototypical normal patterns [20, 19, 58]. The encoding is then used as a query to read the most relevant items from the memory based on an attention mechanism. The combined memory items are then used for reconstruction. [58] demonstrates that by incorporating

bidirectional generative adversarial networks (BiGANs) [12] with adversarial training and adding a cycle-consistency loss, memory units improve and tend to lie on the boundary of the convex hull of normal data encodings.

One-Class GAN (OCGAN) [42] focuses on ensuring that anomalous samples are poorly reconstructed by bounding the latent space and ensuring various regions of it will only represent examples of the normal class and only normal class samples can be generated from it through the decoder. To do this, the algorithm bounds the latent space and incorporates adversarial training with latent and visual discriminators, as well as a classifier that is used to ensure generated samples belong to the normal class. In addition, [42] leverages negative mining to actively seek and improve upon regions of latent space that produce samples of poor quality.

Besides these works that use adversarial training mainly to aid with the reconstruction-based anomaly score and autoencoder performance, there are many works based on GANs and adversarial training that explicitly use the discriminator output to complement or entirely substitute the reconstruction error [49, 62, 59, 2]. There are also techniques that attempt to directly learn the anomaly score. This can, for instance, be based on training an ordinal regression neural network that gives different scores to sample pairs given the pair composition [39]. Differently, likelihood-based techniques operate on the premise that normal and anomalous instances respectively correspond to high and low-probability events and therefore the model is optimized to, for instance, obtain high probability for training samples using a softmax or a noise contrastive estimate (NCE) objective [22, 9].

### 3 Methods

#### 3.1 Problem Statement

Semi-supervised anomaly detection (sometimes also referred to as unsupervised anomaly detection) can be formally defined as the following: given a dataset of  $m$ -dimensional normal samples  $\mathcal{X} = \{x^{(i)}\}_{i=1}^n$  (i.e. samples that are believed to belong to the distribution representing the normal class), obtain an anomaly scoring function  $a(\cdot) : \mathbb{R}^m \mapsto \mathbb{R}$  that assigns larger scores to anomalous samples and smaller scores to normal samples.

Deep methods incorporate neural networks to parameterize various functions within this process. Reconstruction-based methods mainly use autoencoders to parameterize and learn intermediate variable calculations (i.e. compression and reconstruction functions) toward evaluating the scoring function. A typical reconstruction-based anomaly score formulation for query sample  $x_q$  is

$$a(x_q) = \|\hat{x}_q - x_q\|_2; \quad \hat{x}_q = De(En(x_q; \theta_e); \theta_d), \quad (1)$$

where  $En(\cdot; \theta_e)$  and  $De(\cdot; \theta_d)$  are the encoder and decoder, respectively parameterized by  $\theta_e$  and  $\theta_d$ . It is then expected that normal samples (i.e. samples from the same distribution as training) incur smaller reconstruction errors compared to anomalous, out-of-distribution samples. Prior research [42, 65], however, shows that the assumptions on reconstruction quality might be dubious and the effectiveness of anomaly scores can vary depending on the specific type of anomaly. If the anomalous sample shares some critical features with normal samples, it may still be well reconstructed.

#### 3.2 Targeted Collapse Regularized Autoencoders

Considering the limitation of autoencoders, we propose an approach which we call Toll (Targeted collapse) that incorporates a simple adjustment applied to both the training and the anomaly score. Instead of solely penalizing the reconstruction error during training, we propose adding a regularization term that penalizes the norm of the latent representation. Figure 1 illustrates the approach.

For a training sample  $x$  the loss becomes

$$\mathcal{L} = \|\hat{x} - x\|_2 + \beta \|z\|_2, \quad (2)$$

where  $z = En(x; \theta_e)$  and  $\beta$  is a hyperparameter that determines the trade-off between the two terms. We modify the anomaly score accordingly to include the bottleneck representation norm:

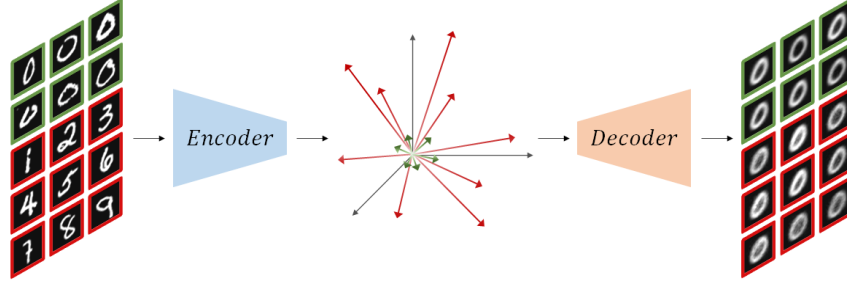


Figure 1: Overview of anomaly detection with Toll. Normal and anomalous samples and their associated latent representations and reconstructions are respectively represented by green and red outlines and vectors. During training, in addition to minimizing reconstruction error, latent representation norms are also minimized, with hyperparameter  $\beta$  specifying the trade-off between the two terms. To use the trained model for anomaly detection, the overall loss associated with a sample is used as the anomaly score. It is expected that anomalous samples incur larger values of combined reconstruction error and latent representation norm as specified by  $\beta$ . (Figure best viewed in color.)

$$a(x_q) = \|\hat{x}_q - x_q\|_2 + \beta \|z_q\|_2. \quad (3)$$

The logic behind the adoption of this regularization is rather straightforward: due to the underlying similarity across normal samples, they should reside closely together in the latent space. A similar inductive bias is, for instance, introduced in the design of Deep Support Vector Data Description (DSVDD) [47], where the encoder is tasked with learning a mapping of normal data into a hypersphere with center  $c$  and radius  $R$  with minimal volume, outside of which anomalies are expected to reside. Unlike in that work, we do not impose a domain constraint for the resulting distribution, but rather only push the encodings towards the origin. This addition stipulates that not only is the autoencoder forced to efficiently reconstruct the normal data, but also to map them such that they form a compact latent representation, emphasizing shared characteristics among the normal samples, thereby capturing their prevalent patterns.

Just as constraining the dimensionality of the bottleneck representation results in retaining only the most relevant information, penalizing the vector norm can serve as a selective filter for information, encouraging the encoder to identify similar attributes within the normal class, thus highlighting characteristics that are likely to differ for anomalous samples.

Not all mappings leading to small representation norms are useful in this sense. An encoder’s latent space representations can be arbitrarily scaled down uniformly to reduce the norms. As another example, one can simply obtain a trivial function that maps any input to the origin. In fact, [47] explicitly discusses the phenomena (referred to in that work as “hypersphere collapse” given the constraint on latent representations) and demonstrates how it imposes restrictions on the choice of activation function and inhibits the ability to incorporate the hypersphere center and network biases as trainable parameters and instead offers an empirical strategy to pick a prespecified fixed center.

With gradient descent training, the latter scenario (collapsed representation) is avoided, noting that such a representation is not an optima: a collapsed latent space results in a strictly positive reconstruction error. As such, during training the encoder is taken away from trivial collapse in the parameter space. As for the former case, in the following we will provide a theoretical analysis of the learning dynamics for a simplified case that also shows we do not merely retrieve a scaled autoencoder bottleneck representation. In fact, the learning dynamics analysis and subsequent numerical simulations demonstrate that the regularization nicely complements the plain autoencoder and increases its utility for anomaly detection, which our results in Section 4 also corroborate.

### 3.3 Analysis of Learning Dynamics

Consider matrix  $X_{m \times n}$  containing the samples of a zero mean dataset, where  $n$  is the number of points and  $m$  is the sample dimensionality. The encoder is simplified as a single linear layer with no bias. The weights are denoted by  $W_{d \times m}$  that map data to a  $d$ -dimensional latent space. The dataset projection can be written as

$$Z = WX. \tag{4}$$

Consider the case where only the projection norm constitutes the loss. We have:

$$\mathcal{L}_{norm} = \frac{1}{2n} \sum_{i,j} Z_{ij}^2, \tag{5}$$

where  $Z_{ij}$  denotes the element at row  $i$  and column  $j$  of  $Z$ . We would like to see how  $Z$  evolves during training. We study the dynamics via gradient flow, the continuous analog of gradient descent with vanishing step sizes (corresponding to an infinitesimal learning rate).

**Lemma 1** *The weight matrix  $W$  under gradient flow evolves by*

$$\dot{W}(t) = -W(t)S, \tag{6}$$

where  $S = \frac{1}{n}XX^T$  is the empirical covariance matrix.

The proof of Lemma 1 is straightforward using the chain rule and is provided in Appendix A.1.

**Theorem 1** *The solution to the system in Eq. 6 is*

$$W(t) = W(0)U \exp(-\Lambda t)U^T, \tag{7}$$

where  $W(0)$  are the initial weights and  $\Lambda$  and  $U$  are the eigenvalues and eigenvectors of  $S$ , respectively. As a result, the latent space evolution is given by

$$Z(t) = W(0)U \exp(-\Lambda t)U^T X. \tag{8}$$

The proof for the theorem is in Appendix A.2. To interpret the evolution of latent representations, consider tentatively that  $W(0) = I_m$ , the  $m$ -dimensional identity matrix. Then we have  $Z = U \exp(-\Lambda t)U^T X$ . In the beginning,  $Z(0) = X$ . As time progresses, all dimensions collapse, each with an exponential shrink factor equal to the associated eigenvalue. In other words, the dimensions featuring higher variability collapse faster. Now considering a general weight matrix, we obtain a fixed projection of the shrinking dynamics, as if the input data is shrinking and being projected to a latent space through a fixed mapping.

Let us now consider the scenario in which only the reconstruction error constitutes the loss of a linear autoencoder. It is known (as shown e.g. in [43]) that a linear autoencoder with a  $d$ -dimensional bottleneck and trained using mean squared error loss tends to encode data in the space spanned by the  $d$  principal components (i.e. eigenvectors associated with the  $d$  largest eigenvalues of covariance), fully discarding low eigenvalue directions. Consequently, it produces a low-rank approximation of the data at reconstruction.

Combining the effect of both loss terms, one can observe that regularizing a linear autoencoder using encoding norms counteracts the greedy elimination of information associated with dimensions featuring less pronounced variations. This can be advantageous since the dimensions featuring lower variability might in fact be important for the subsequent task (anomaly detection in this case). Figure 2 demonstrates this point. It shows contours of anomaly score when a linear autoencoder is trained on two-dimensional samples from a zero-mean bivariate Gaussian featuring a diagonal covariance with variances of 4 and 1 respectively for the first and second dimensions. Encoder and decoder are both single-layer architectures that map to and from a one-dimensional latent space. We can see that with regularization, anomaly score contours can reflect the normal class distribution much more closely. When only reconstruction is considered here, however, anomaly behavior needs to align with a strong implicit assumption for successful performance; the assumption being that the anomalous samples deviate from the distribution along the second principal component and their first principal attribute does not have a bearing on their anomalousness.

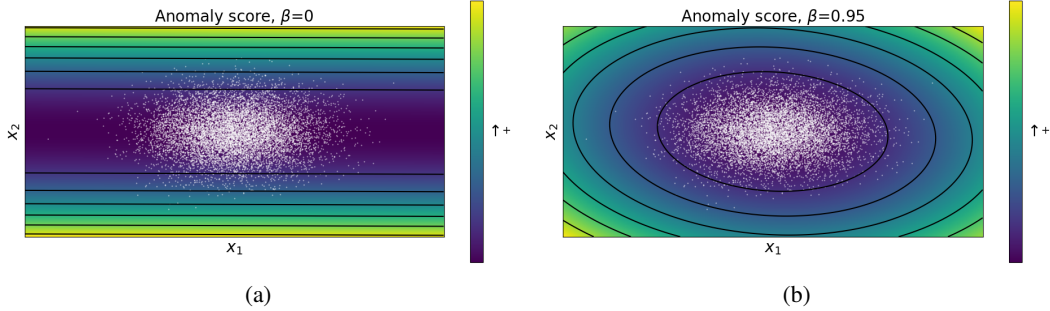


Figure 2: Contours of anomaly score output using an unregularized (a) and a norm-regularized (b) linear autoencoder in the 2D input data space. The autoencoder is trained to encode samples from a two-dimensional Gaussian distribution (bright points) to a one-dimensional latent space and decode back to the original dimensionality. To apply regularization, a  $\beta$  coefficient is multiplied by the regularization term, and  $1 - \beta$  is multiplied by the reconstruction error term. The unregularized autoencoder encodes the first principal component of the dataset ( $x_1$ ) and therefore reconstruction errors scale with the distance of points from the origin along  $x_2$ . Regularization prevents the complete vanishing of variation along  $x_2$  in the bottleneck and, for a good choice of  $\beta$ , the norm-regularized autoencoder yields a much more accurate anomaly score profile.

In the following (Theorem 2), we present the equations governing the learning dynamics for a linear autoencoder whose training is regularized via encoding norm. Encoder and decoder are linear layers with no bias whose weights are respectively  $W_1$  and  $W_2$ .  $\hat{X} = W_2W_1X$  is the reconstructed dataset. The loss function is a weighted sum of MSE reconstruction loss  $\mathcal{L}_{rec}$  and  $\mathcal{L}_{norm}$ :

$$\mathcal{L} = \left[ \frac{1}{2n} \sum_{i,j} (\hat{X}_{ij} - X_{ij})^2 \right] + \beta \left[ \frac{1}{2n} \sum_{i,j} Z_{ij}^2 \right]. \quad (9)$$

**Theorem 2**  $W_1$  and  $W_2$  under gradient flow evolve by

$$\dot{W}_1(t) = [W_2^T - W_2^T W_2 W_1 - \beta W_1] S, \quad (10a)$$

$$\dot{W}_2(t) = [I - W_2 W_1] S W_1^T. \quad (10b)$$

Proof of Theorem 2 can be found in Appendix A.3. Even in such a simplified case, the equations are nonlinear and coupled, making further analysis challenging. Hence, these equations remain unsolved in this work.

While the above analysis does not consider nonlinearities, it provides insight into the behavior of an anomaly detection scheme built based on autoencoding performance. Based on our observation, by properly selecting  $\beta$ , suitable expressions of data manifold can be retrieved for anomaly detection, with virtually no computational or memory overhead during training and inference. Also given the technique and the principle behind it, the algorithm can readily accommodate various data modalities.

## 4 Experiments

### 4.1 Datasets and Implementation Details

We present the results of Toll on three datasets: MNIST [30], CIFAR-10 [29], and Arrhythmia [13]. For MNIST and CIFAR-10 we use the same protocol as that used in e.g. [47, 42, 58]. MNIST and CIFAR-10 are visual datasets and both have 10 classes and have respectively 60000 and 50000 images in their training set and 10000 images in test sets. Figure 3 shows sample images from all classes

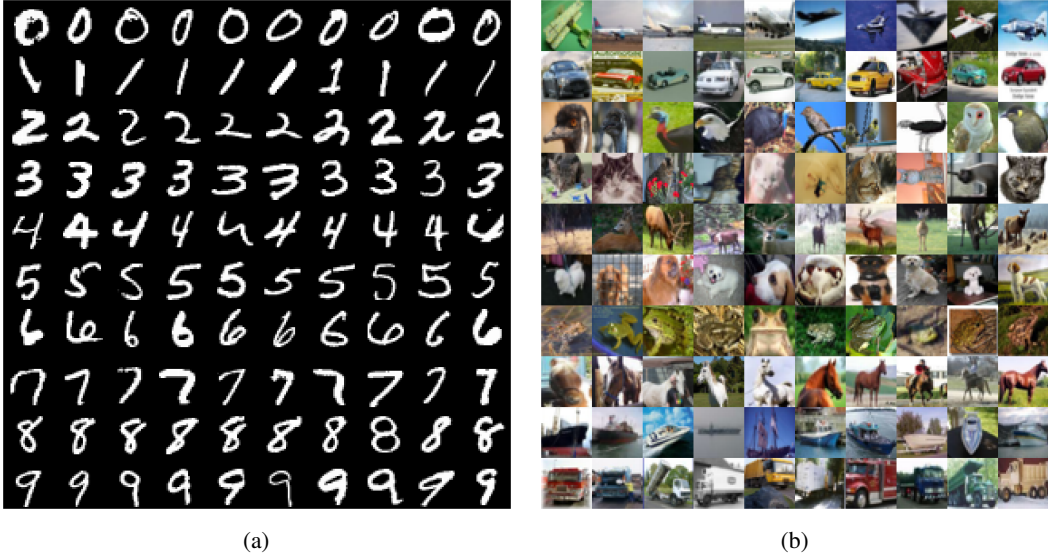


Figure 3: Samples from MNIST (a) and CIFAR-10 (b) datasets. Rows show different classes.

in MNIST and CIFAR-10. We also set aside 10000 samples from the training set for validation. In both datasets, we perform 10 experiments, each time considering one of the classes as the normal class and all the other classes as anomalous. Training is done on samples from the normal class of the associated experiment. (i.e. approximately 5000 samples for MNIST and 4000 for CIFAR-10).

The Arrhythmia dataset is a tabular dataset adapted from a dataset for the classification of cardiac arrhythmia. The dataset consists of a total of 452 samples and 274 attributes across 16 classes. In line with previous work, we designate classes 3, 4, 5, 7, 8, 9, 14, and 15 as anomalous and the remaining classes as normal. The anomalous samples make up approximately 15 percent of the dataset. We set aside 20 percent of the data for testing and consider another 30 percent for validation and use the remaining normal samples to train the model.

MNIST dataset was normalized to have a zero mean and a standard deviation of 1 for pixel values. CIFAR-10 images were preprocessed with global contrast normalization using the  $L^1$ -norm and rescaled to  $[0, 1]$  with min-max scaling. The encoder and decoder architectures used for CIFAR-10 experiments are similar to encoder and generator architectures in [14, 58]. The architectures for MNIST and Arrhythmia experiments are also based on [58]. See Appendix B for detailed architectures.

In all experiments, training is done through iterating over 2000 random batches with checkpoints considered at 20 batch intervals. The batch sizes for MNIST, CIFAR-10, and Arrhythmia are respectively 1000, 50, and 100. Adam optimizer with a fixed learning rate is used for training. Learning rates,  $\beta$  values, and the bottleneck dimensions are picked based on the validation set performance. We keep these hyperparameters unchanged across the 10 classes in MNIST and CIFAR-10, but we note that test performances further improve, in many instances considerably, if hyperparameters are picked specific to the normal class. The best-performing model on the validation set is selected for subsequent evaluation on the test set. Each experiment is repeated for 10 different random seeds and averaged results are reported.

We comply with the existing literature for evaluation. The metric used to evaluate results on MNIST and CIFAR-10 is area under ROC curve (AUC). For Arrhythmia, we set the score threshold such that we label the expected 15 percent of samples with the highest anomaly scores as anomalous. We then calculate the  $F_1$  score.

## 4.2 Results

Tables 1, 2, and 3 show anomaly detection results on MNIST, CIFAR-10, and Arrhythmia datasets. On all datasets, our approach achieves the highest performance with a considerable margin. On

Table 1: Mean and standard deviation of AUC in % on MNIST classes over 10 seeds. The highest performance in each row is in bold.

Normal	OC-SVM [50]	OCGAN [42]	VAE [28]	DCAE [35]	IF [32]	AnoGAN [49]	KDE [41]	DSVDD [47]	MEMAE [20]	MEMGAN [58]	Toll
0	98.6±0.0	<b>99.8</b>	99.7	97.6±0.7	98.0±0.3	96.6±1.3	97.1±0.0	98.0±0.7	99.3±0.1	99.3±0.1	<b>99.8±0.1</b>
1	99.5±0.0	<b>99.9</b>	<b>99.9</b>	98.3±0.6	97.3±0.4	99.2±0.6	98.9±0.0	99.7±0.1	99.8±0.0	<b>99.9±0.0</b>	<b>99.9±0.0</b>
2	82.5±0.1	94.2	93.6	85.4±2.4	88.6±0.5	85.0±2.9	79.0±0.0	91.7±0.8	90.6±0.8	94.5±0.1	<b>96.2±1.6</b>
3	88.1±0.0	96.3	95.9	86.7±0.9	89.9±0.4	88.7±2.1	86.2±0.0	91.9±1.5	94.7±0.6	95.7±0.4	<b>97.9±0.6</b>
4	94.9±0.0	97.5	97.3	86.5±2.0	92.7±0.6	89.4±1.3	87.9±0.0	94.9±0.8	94.5±0.4	96.1±0.4	<b>97.8±0.4</b>
5	77.1±0.0	98.0	96.4	78.2±2.7	85.5±0.8	88.3±2.9	73.8±0.0	88.5±0.9	95.1±0.1	93.6±0.3	<b>98.1±0.6</b>
6	96.5±0.0	99.1	99.3	94.6±0.5	95.6±0.3	94.7±2.7	87.6±0.0	98.3±0.5	98.4±0.5	98.6±0.1	<b>99.5±0.1</b>
7	93.7±0.0	98.1	97.6	92.3±1.0	92.0±0.4	93.5±1.8	91.4±0.0	94.6±0.9	95.4±0.2	96.2±0.2	<b>98.7±0.2</b>
8	88.9±0.0	93.9	92.3	86.5±1.6	89.9±0.4	84.9±2.1	79.2±0.0	93.9±1.6	86.9±0.5	93.5±0.1	<b>97.3±0.5</b>
9	93.1±0.0	98.1	97.6	90.4±1.8	93.5±0.3	92.4±1.1	88.2±0.0	96.5±0.3	97.3±0.2	95.9±0.1	<b>98.5±0.3</b>
Average	91.3	97.5	97.0	89.7	92.3	91.4	87.0	94.8	95.2	96.5	<b>98.4</b>

Table 2: Mean and standard deviation of AUC in % on CIFAR-10 classes over 10 seeds. The highest performance in each row is in bold.

Normal	OCGAN	VAE	DSVDD	DSEBM [60]	DAGMM [65]	IF	AnoGAN	ALAD [59]	MEMAE	MEMGAN	Toll
airplane	<b>75.7</b>	70.0	61.7±4.1	41.4±2.3	56.0±6.9	60.1±0.7	67.1±2.5	64.7±2.6	66.5±0.9	73.0±0.8	75.0±4.3
auto	53.1	38.6	65.9±2.1	57.1±2.0	48.3±1.8	50.8±0.6	54.7±3.4	38.7±0.8	46.4±0.1	52.5±0.7	<b>67.6±3.2</b>
bird	64.0	<b>67.9</b>	50.8±0.8	61.9±0.1	53.8±4.0	49.2±0.4	52.9±3.0	67.0±0.7	66.0±0.1	67.2±0.1	56.2±2.8
cat	62.0	53.5	59.1±1.4	50.1±0.4	51.2±0.8	55.1±0.4	54.5±1.9	59.2±0.3	52.9±0.1	57.3±0.2	<b>64.6±1.6</b>
deer	72.3	<b>74.8</b>	60.9±1.1	73.3±0.2	52.2±7.3	49.8±0.4	65.1±3.2	72.7±0.6	72.8±0.1	73.9±0.9	67.7±1.4
dog	62.0	52.3	65.7±0.8	60.5±0.3	49.3±3.6	58.5±0.4	60.3±2.6	52.8±1.2	52.9±0.2	65.0±0.2	<b>68.1±1.6</b>
frog	72.3	68.7	67.7±2.6	68.4±0.3	64.9±1.7	42.9±0.6	58.5±1.4	69.5±1.1	63.7±0.4	72.8±0.7	<b>74.7±2.5</b>
horse	57.5	49.3	<b>67.3±0.9</b>	53.3±0.7	55.3±0.8	55.1±0.7	62.5±0.8	44.8±0.4	45.9±0.1	52.5±0.5	65.8±1.3
ship	<b>82.0</b>	69.6	75.9±1.2	73.9±0.3	51.9±2.4	74.2±0.6	75.8±4.1	73.4±0.4	70.1±0.1	74.4±0.3	80.0±1.1
truck	55.4	38.6	73.1±1.2	63.6±3.1	54.2±5.8	58.9±0.7	66.5±2.8	39.2±1.3	48.2±0.2	65.6±1.6	<b>76.2±0.5</b>
Average	65.6	58.3	64.8	60.4	54.4	55.5	61.8	59.3	58.5	65.3	<b>69.6</b>

MNIST and CIFAR-10, it also achieves the highest performance for respectively ten and five out of ten total classes. As the tables show, norm regularization is a simple yet powerful way to improve anomaly detection performance.

### 4.3 Ablation Study

Table 4 compares the anomaly detection performance with and without norm minimization. The effect of regularization varies across different normal classes. This is due to the fact that different classes benefit to different extents from the regularization and that the single  $\beta$  value used across various classes is not necessarily a good fit for all. Even with this caveat, the contribution of norm minimization is significant at about 4 percent on average in this high-accuracy regime. For the given  $\beta$ , this effect is the most pronounced when considering digit “8” as the normal class. Figure 4 shows the effect of introducing the regularization at various intensities on performance.

## 5 Conclusion

In this work, we proposed a simple method for regularizing autoencoder training and modifying the anomaly score function based on minimizing the norm of bottleneck representations. The method comes from simple first principles and despite its simplicity, appropriately fits the purpose of anomaly detection, leading to a significant performance boost. The implementation is straightforward and the methodology does not introduce any constraints on the application or the associated data modality. We showcased the technique’s advantages by conducting comparisons of our method against well-established and intricate approaches to anomaly detection, as well as through ablation studies that evaluate the impact of the regularization effect. We also carried out an analytical investigation into

Table 3: Average  $F_1$  score in % on Arrhythmia dataset over 10 seeds. The highest performance is in bold.

IF	OC-SVM	DSEBM-e	AnoGAN	DAGMM	ALAD	RCGAN [57]	DSVDD	GOAD [7]	MEMAE	MEMGAN	Toll
53.03	45.18	46.01	42.42	49.83	51.52	54.14	34.79	52.00	51.13	55.72	<b>60.00</b>



Table 4: AUC comparison for MNIST classes over 10 seeds for an unregularized versus a regularized autoencoder. The highest performance in each row is in bold.

Normal	$\beta=0$	$\beta=1000$
0	99.1±0.1	<b>99.8±0.1</b>
1	<b>99.9±0.0</b>	<b>99.9±0.0</b>
2	91.4±0.3	<b>96.2±1.6</b>
3	93.4±0.2	<b>97.9±0.6</b>
4	94.3±0.3	<b>97.8±0.4</b>
5	95.1±0.1	<b>98.1±0.6</b>
6	98.5±0.1	<b>99.5±0.1</b>
7	96.4±0.1	<b>98.7±0.2</b>
8	82.9±0.2	<b>97.3±0.5</b>
9	96.2±0.1	<b>98.5±0.3</b>
Average	94.7	<b>98.4</b>

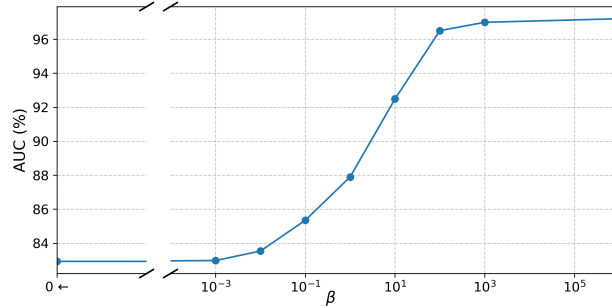


Figure 4: Effect of regularization intensity for digit “8” as normal class in MNIST. The AUC values shown are averaged over 3 random seeds.

the learning dynamics, exploring the phenomenological consequences of the proposed modification and how it complements the capabilities of autoencoders.

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## Appendix

### A Delayed Proofs

#### A.1 Proof of Lemma 1

The governing equation for weight dynamics in gradient flow is

$$\dot{W} = -\frac{\partial \mathcal{L}_{norm}}{\partial W}. \quad (11)$$

Using chain rule we can write

$$\begin{aligned} \frac{\partial \mathcal{L}_{norm}}{\partial W} &= \frac{\partial \mathcal{L}_{norm}}{\partial Z} \frac{\partial Z}{\partial W} \\ &= \frac{\partial \mathcal{L}_{norm}}{\partial Z} X^T \\ &= \frac{1}{n} Z X^T \\ &= \frac{1}{n} W X X^T \\ &= WS; \end{aligned} \quad (12)$$

therefore,

$$\dot{W}(t) = -W(t)S.$$

#### A.2 Proof of Theorem 1

The solution to the system in Eq. 6 can be expressed as

$$W(t) = W(0) \exp(-St). \quad (13)$$

If we write the eigendecomposition of the empirical covariance as  $S = U\Lambda U^T$  (considering the orthonormality of  $U$ ) and substitute above we get

$$W(t) = W(0) \exp(-U\Lambda U^T t), \quad (14)$$

which we can further evaluate as

$$W(t) = W(0)U \exp(-\Lambda t)U^T.$$

Using Eq. 4 we can subsequently evaluate representation dynamics:

$$Z(t) = W(0)U \exp(-\Lambda t)U^T X.$$

### A.3 Proof of Theorem 2

The governing equations for dynamics of  $W_1$  and  $W_2$  in gradient flow are

$$\dot{W}_1 = -\frac{\partial \mathcal{L}}{\partial W_1}, \quad (15a)$$

$$\dot{W}_2 = -\frac{\partial \mathcal{L}}{\partial W_2}. \quad (15b)$$

Using chain rule we have

$$\frac{\partial \mathcal{L}}{\partial W_1} = \frac{\partial \mathcal{L}}{\partial Z} \frac{\partial Z}{\partial W_1}, \quad (16a)$$

$$\frac{\partial \mathcal{L}}{\partial W_2} = \frac{\partial \mathcal{L}}{\partial \hat{X}} \frac{\partial \hat{X}}{\partial W_2}. \quad (16b)$$

We now evaluate each derivative:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \hat{X}} &= \frac{1}{n} (\hat{X} - X) \\ &= \frac{1}{n} (W_2 W_1 X - X) \end{aligned} \quad (17a)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial Z} &= \frac{\partial \mathcal{L}_{rec}}{\partial Z} + \beta \frac{\partial \mathcal{L}_{norm}}{\partial Z} \\ &= W_2^T \frac{\partial \mathcal{L}}{\partial \hat{X}} + \beta \left( \frac{1}{n} W_1 X \right) \\ &= \frac{1}{n} W_2^T (W_2 W_1 X - X) + \beta \left( \frac{1}{n} W_1 X \right) \end{aligned} \quad (17b)$$

$$\frac{\partial Z}{\partial W_1} = X^T \quad (17c)$$

$$\frac{\partial \hat{X}}{\partial W_2} = Z^T = X^T W_1^T. \quad (17d)$$

Substituting in Eqs. 16a and 16b and considering Eqs. 15a and 15b we get

$$\dot{W}_1 = -\frac{1}{n} W_2^T (W_2 W_1 X - X) X^T - \beta \left( \frac{1}{n} W_1 X X^T \right), \quad (18a)$$

$$\dot{W}_2 = -\frac{1}{n} (W_2 W_1 X - X) X^T W_1^T. \quad (18b)$$

Substituting  $\frac{1}{n}XX^T$  as  $S$  and simplifying, we get

$$\dot{W}_1(t) = [W_2^T - W_2^T W_2 W_1 - \beta W_1]S,$$

$$\dot{W}_2(t) = [I - W_2 W_1]S W_1^T.$$

## B Architectures and Hyperparameters

Below are the detailed architectures and dataset-specific hyperparameter values for training. 2D convolution and transposed convolution layers with channel size  $c$ , kernel size  $k \times k$ , and stride  $s$  are denoted respectively as  $\text{Conv}(c, k, s)$  and  $\text{ConvT}(c, k, s)$ . Leaky ReLU activation with negative slope  $a$  is denoted as  $\text{LReLU}(a)$ . Linear layer with output size  $l$  is denoted as  $\text{Linear}(l)$ .

### B.1 MNIST

**Encoder:**  $\text{Conv}(64, 4, 1)$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.2)$  -  $\text{Conv}(128, 4, 1)$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.2)$  -  $\text{Conv}(256, 4, 2)$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.2)$  -  $\text{Conv}(512, 4, 1)$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.2)$  -  $\text{Conv}(64, 4, 1)$  -  $\text{Flatten}$  -  $\text{Linear}(128)$ .

**Decoder:**  $\text{Linear}(1024)$  -  $\text{Unflatten}(64, 4, 4)$  -  $\text{ConvT}(512, 4, 1)$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.2)$  -  $\text{ConvT}(256, 4, 1)$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.2)$  -  $\text{ConvT}(128, 4, 2)$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.2)$  -  $\text{ConvT}(64, 4, 1)$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.2)$  -  $\text{ConvT}(1, 4, 1)$ .

$$\beta = 1000 \quad , \quad lr = 10^{-4}.$$

### B.2 CIFAR-10

**Encoder:**  $\text{Conv}(32, 5, 1, \text{bias}=\text{False})$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.1)$  -  $\text{Conv}(64, 4, 2, \text{bias}=\text{False})$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.1)$  -  $\text{Conv}(128, 4, 1, \text{bias}=\text{False})$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.1)$  -  $\text{Conv}(256, 4, 2, \text{bias}=\text{False})$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.1)$  -  $\text{Conv}(512, 4, 1, \text{bias}=\text{False})$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.1)$  -  $\text{Conv}(512, 1, 1, \text{bias}=\text{False})$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.1)$  -  $\text{Conv}(64, 1, 1)$  -  $\text{Flatten}$ .

**Decoder:**  $\text{Unflatten}(64, 1, 1)$  -  $\text{ConvT}(256, 4, 1, \text{bias}=\text{False})$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.1)$  -  $\text{ConvT}(128, 4, 2, \text{bias}=\text{False})$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.1)$  -  $\text{ConvT}(64, 4, 1, \text{bias}=\text{False})$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.1)$  -  $\text{ConvT}(32, 4, 2, \text{bias}=\text{False})$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.1)$  -  $\text{ConvT}(32, 5, 1, \text{bias}=\text{False})$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.1)$  -  $\text{Conv}(32, 1, 1, \text{bias}=\text{False})$  -  $\text{BatchNorm}$  -  $\text{LReLU}(0.1)$  -  $\text{Conv}(3, 1, 1)$ .

$$\beta = 10 \quad , \quad lr = 10^{-2}.$$

### B.3 Arrhythmia

**Encoder:**  $\text{Linear}(256)$  -  $\text{BatchNorm}$  -  $\text{ReLU}$  -  $\text{Linear}(128)$  -  $\text{BatchNorm}$  -  $\text{ReLU}$  -  $\text{Linear}(16)$ .

**Decoder:**  $\text{Linear}(128)$  -  $\text{BatchNorm}$  -  $\text{ReLU}$  -  $\text{Linear}(256)$  -  $\text{BatchNorm}$  -  $\text{ReLU}$  -  $\text{Linear}(274)$ .

$$\beta = 10 \quad , \quad lr = 10^{-3}.$$