SoK: Let The Privacy Games Begin! A Unified Treatment of Data Inference Privacy in Machine Learning

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Abstract-Deploying machine learning models in production may allow adversaries to infer sensitive information about training data. There is a vast literature analyzing different types of inference risks, ranging from membership inference to reconstruction attacks. Inspired by the success of games (i.e. probabilistic experiments) to study security properties in cryptography, some authors describe privacy inference risks in machine learning using a similar game-based style. However, adversary capabilities and goals are often stated in subtly different ways from one presentation to the other, which makes it hard to relate and compose results. In this paper, we present a game-based framework to systematize the body of knowledge on privacy inference risks in machine learning. We use this framework to (1) provide a unifying structure for definitions of inference risks, (2) formally establish known relations among definitions, and (3) to uncover hitherto unknown relations that would have been difficult to spot otherwise.

1. Introduction

Since the pioneering studies of attribute inference [21, 67] and membership inference [36, 54], research on the inference risks of deploying machine learning (ML) models has bloomed. There is a growing interest in understanding and mitigating the leakage of information about training data under various threat models that capture different adversarial capabilities (e.g., observing model outputs, model parameters, or transcripts of iterative optimization methods) and goals (e.g., membership inference [54], attribute inference [21, 67], property inference [22, 41, 57, 72], and data reconstruction [3, 10]).

An emerging trend in the literature is to capture threat models using *privacy games*. This originates from the seminal work of Wu et al. [67] on formalizing attribute inference. A privacy game is a probabilistic experiment where an *adversary* interacts with a *challenger*. The challenger drives the experiment, invoking the adversary to provide them with information and to allow them to make certain choices, possibly while interacting with oracles controlled by the challenger. The adversary eventually produces a guess for a confidential value. This experiment defines a probability space where the success of the adversary can be measured in terms of the probability of their guess being correct.

The use of games for privacy in ML is inspired by the well-established use of games to define and reason about security properties in cryptography. Cryptographic games are used to standardize and compare security definitions [24, 56], and to structure [6] and even mechanize proofs of security [4, 7]. In comparison, the use of privacy games in the ML literature is still in its infancy:

(1) there are no well-established standards for gamebased definitions,

(2) relationships between different privacy games have only been partially explored, and

(3) games are rarely used as an integral part of proofs, despite being especially convenient for this task.

This has resulted in many game variants in the literature that attempt to formalize the same adversary goal but have subtle yet important differences. This fragmentation leads to confusion and hinders progress—for membership inference alone, we found variants that differ in details that can change their meaning and substantially alter results.

To address this problem, we present the first systematization of knowledge about privacy inference risks in machine learning, going above and beyond the problem left open since 2016 by Wu et al. [67] of merely devising rigorous game-based definitions. Concretely,

• We break down the *anatomy* of game-based privacy definitions for ML systems into individual components: adversary's capabilities and goals, ways of choosing datasets and challenges, and measures of success (Section 2).

• Based on this anatomy, we propose a *unified representation* of five fundamental privacy risks as games: membership inference, attribute inference, property inference, differential privacy distinguishability, and data reconstruction (Section 3).

• Using the game-based framework, we *establish and rigorously prove relationships* between the above risks. Similarly to the study of *concrete security* in cryptography [5], we define a quantitative notion of *reduction* between privacy properties. Using this notion, we prove a minimal set of relations among the above five privacy risks. This allows us to establish, for every possible ordered pair of risks

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Figure 1. Relations among adversary goals. A solid arrow from node A to B means that security against A (i.e. a non-trivial advantage bound) implies security against B. A struck-through arrow from A to B means that security against A does not imply in general security against B; we show this separation with a construction that is secure against A but completely insecure against B. Dashed arrows are implied by solid arrows. Labels over solid arrows refer to the theorem showing the relationship.

A, B, either a reduction showing that security against A implies security against B, or a separation result showing the impossibility of a generic reduction from A to B. Figure 1 summarizes the conclusions of this systematization effort.

• We present a *case study* (Section 5), where we prove that a scenario described as a variant of membership inference in the literature can actually be decomposed into a combination of membership and property inference. Importantly, in this case we exploit *code-based* reductions, structured as a sequence of games; i.e., our arguments rely on transforming code with a formal semantics. This way of conducting proofs has seen great success in cryptography. However, before our work, it had not reached the same level of rigor when reasoning about privacy inference risks in ML.

Scope of this SoK. The focus of this SoK is to formalize and systematize game-based definitions that capture the risk of leaking information about the training data of ML models. We used the following methodology to identify existing game-based definitions from the literature: starting from the seminal works of Wu et al. [67] and Yeom et al. [68], we systematically surveyed *all peer-reviewed publications* as of August 2022 that cite either of these works. We examined all of these papers and collected all game-based definitions of attacks that aim to infer information about the training data of ML models. While our focus is on systematizing games that appear in the literature, we additionally showcase that our framework is expressive enough to capture attacks for which so far there has been no game-based formulation.

Summary of contributions. We propose a unifying gamebased framework for formalizing privacy inference risks of training data in ML, which we use to systematize definitions from the literature and to establish relations between them. Our work aims to reduce ambiguity and increase rigor when reasoning and communicating about ML privacy, and gives a solid foundation to future research and decision-making.

2. Anatomy of a Privacy Game

Privacy games are parametrized by an adversary (\mathcal{A}) and a training pipeline specifying the training algorithm (\mathcal{T}) , data distribution (\mathcal{D}) , and training dataset size (n). A challenger simulates the ML system; the adversary uses their capabilities—defined by a threat model—to interact with the system and infer information about the training dataset.

Game 1: Membership Inference						
Input: $\mathcal{T}, \mathcal{D}, n, \mathcal{A}$						
1 $S \sim \mathcal{D}^n$ // samp	ble n i.i.d. points from distribution \mathcal{D}					
2 $b \sim \{0, 1\}$	// flip a fair coin					
3 if $b = 0$ then						
4 $z \sim S$ // sample	e a challenge point uniformly from S					
5 else						
6 $z \sim D$	// sample a challenge point from ${\cal D}$					
7 end						
8 $\theta \leftarrow \mathcal{T}(S \cup \{z\})$	// train a model θ					
9 $\tilde{b} \leftarrow \mathcal{A}(\mathcal{T}, \mathcal{D}, n, \theta, z)$	// adversary guesses $b = \tilde{b}$					

Game 1 formalizes the now standard membership inference experiment of Yeom et al. [68], which we use as a running example. First, the challenger samples a training dataset S (line 1). Then, the challenger flips a fair coin b(line 2); depending on the outcome, they either sample a challenge point z from the training dataset S, or from the data distribution \mathcal{D} (lines 3–7). We discuss alternatives for choosing training datasets and challenges in Section 2.2.

The challenger then trains a target model θ (line 8), and asks the adversary to make a guess \tilde{b} for b (line 9). In this

game, the adversary is given the training algorithm (\mathcal{T}), data distribution (\mathcal{D}), dataset size (n), target model (θ), and the challenge point (z). We discuss different possibilities for the adversary's capabilities in Section 2.2 and Section 2.3.

The success of the adversary in making a correct guess $(\tilde{b} = b)$ is measured with respect to the baseline of a random guess. Any advantage over this baseline indicates leakage of membership information. We discuss other ways to quantify the adversary's success in Section 2.4.

We now discuss in more detail the building blocks of games described above and highlight common choices.

2.1. Adversary Goals

We identify five *adversary goals* from the literature that enable an adversary to directly infer information about the training dataset of an ML model. We describe these goals informally below, and formalize them as games in Section 3.

Membership Inference (MI). The adversary aims to determine whether a specific *record* [68] or *subject* [40, 58] (an entity who may contribute more than one record) was present in the training dataset of the target model. A successful MI attack could reveal a user's sensitive condition depending on their participation in a study/model, e.g., discovering correlations of a disease [54, 68].

Attribute Inference (AI). The adversary aims to use the model to infer unknown attributes of a record in the training dataset given partial information about the record [68]. A successful AI attack can result in the reconstruction of missing sensitive attributes from a target input.

Property Inference (PI). The adversary aims to learn sensitive *statistical* properties of the target model's training distribution. For example, in a malware classifier, the training dataset may have been generated using a particular testing environment, and it may benefit the adversary to learn certain properties of this environment [22]. From an auditing perspective, property inference could be used to assess the training dataset for harms (e.g., under-representation) [72].

Differential Privacy Distinguishability (DPD). The adversary aims to determine which of a pair of adjacent datasets (e.g. differing in the data of one record) of their choosing was used to train the target model. This definition recasts differential privacy in a game-based setting by making the adversary explicit. This definition is connected to differential privacy and can thus be used to estimate the privacy budget of training pipelines [42, 45, 70].

Dataset Reconstruction (RC). The adversary aims to reconstruct samples from the training dataset of a target model [3, 9, 10]. A successful RC attack can partially reconstruct the training dataset, potentially violating confidentiality requirements. **Beyond Training Data Inference.** Other adversary goals, such as model stealing [46, 60], hyperparameter stealing [64], and poisoning [41, 62], are beyond the scope of this SoK, because they do not enable the adversary to directly infer information about the training data. However, the *effects* of these other goals are readily captured by our game-based analysis. For example, a successful model stealing that is used as a precursor to membership inference can be represented by changing the adversary access from black-box to white-box (Section 2.3). Similarly, a poisoning attack can be modeled with an adversarially chosen dataset, as described in Section 2.2.

2.2. Selecting Challenges and Datasets

An important aspect of any privacy game is how the challenges and datasets are selected. In Game 1, the challenge point is a single record z; in other games, the challenge could comprise multiple points or even a data distribution. For the discussion below, we simplify the language by talking about a single challenge point. We now discuss the three the main approaches from the recent literature:

Randomly sampled. The challenge is sampled from a distribution by the challenger as part of the game [30, 65, 68]. A randomly sampled challenge provides a measure of *average case* privacy. While average case privacy measures the risk for average users, outlier users can be more vulnerable.

Externally provided. The challenge is provided as a parameter of the game [30, 40]. This approach may be used to quantify the privacy of specific points while leaving other individuals possibly more vulnerable, i.e., it provides *individual case* privacy.

Adversarially chosen. The challenge is selected by the adversary during the game [12, 42, 45]. Since the adversary can select the most advantageous challenge based on the information provided, this provides a measure of *worst case* privacy, i.e., measuring the risks for all users including outliers. For example, a membership inference adversary could choose a challenge that is an outlier w.r.t. the training data distribution, so that a target classification model is unlikely to classify it correctly unless it is included in the training dataset. This setting is usually considered when a system is audited to identify its possible risks.

Additional considerations. When the challenge is externally provided or adversarially chosen, the parameters of the game cannot completely determine a correct adversary guess. Otherwise, security statements that universally quantify over adversaries are void because the quantification includes adversaries with a hardcoded correct guess. This is similar to the difficulty of defining collision resistance of hash functions [49]. **Selecting datasets.** The training dataset can also be selected using any of the three options above: it can be randomly sampled by the challenger during the game, externally provided, or (partially) chosen by the adversary. The latter can be used to represent the case where the model has been trained on (poisoned) data contributed by potentially malicious users [41, 62].

2.3. Adversary Access

Depending on the scenario, the adversary may have different levels of access to the target model, training algorithm, training distribution, and training dataset. This allows the game to capture different *threat models*, which should ideally match the known or assumed capabilities of real-world adversaries. Most games assume one of two settings: *black-box* or *white-box* access.

Black-box. In this scenario, the adversary only has query access to the target model (e.g., a cloud-hosted model with an inference API) [11]. To formalize this setting, we give the adversary access to the model through the oracle **Oracle** $\mathcal{O}^{\theta}(\mathbf{x})$: return $\theta(x)$. This allows the adversary to query the model θ on inputs of their choosing and observe the model responses, but does not reveal internal workings of the model, such as its architecture or weights. Depending on the scenario/API, the oracle can return a confidence for each label, or only the highest-confidence label [16, 37]. The latter setting matches inference APIs that do not reveal confidence values, like some email spam classifiers or autocompletion systems. Additionally, the oracle can also be instrumented to only emit responses for queries satisfying a predicate p, which can be stateful to enforce a bound N on the number of allowed queries, e.g.,

Oracle
$$\mathcal{O}^{\theta}(x)$$

 $q_{\theta} \leftarrow q_{\theta} + 1$
if $q_{\theta} \le N$ then return $\arg \max \theta(x)$ else return \bot

Grey-box. In between the black-box and white-box settings, it is also possible to consider a range of *grey-box* threat models in which the adversary has more than black-box but less than full white-box access to the target model. For example, the adversary could have knowledge of the model architecture, or be given some of the model's hyperparameters, knowledge of the public base model used for fine-tuning the target model, but not the actual model weights [51, 54]. Such extra information can be the output of a hyperparameter stealing attack [64].

White-box. The white-box setting represents the strongest adversary, who has full direct access to the target model i.e., $\mathcal{A}(\theta, ...)$. This obviously provides the adversary with all the capabilities of the black-box setting, but also allows the adversary to inspect the internals of the model including its trained weights [35, 50]. For instance, a model deployed on client's devices gives white-box access to potential adversaries. Alternatively, perfect parameter stealing (both model

weights and hyperparameters) attacks on black-box models will enable an adversary to operate in a white-box setting.

Auxiliary information. In addition to having access to the target model, the adversary may have auxiliary information/data that could be useful for certain attacks. For example, some MI attacks assume the adversary has access to auxiliary data distributed similarly to the target model's training data, e.g., for building shadow models.

Time, memory, and resources budget. Most game-based formulations do not explicitly limit the resources available to an adversary. It could be important to consider resource-limited adversaries that can only issue a specific number of queries to an oracle, or can use a certain amount of memory, or are otherwise computationally bounded. Intuitively, limiting these resources can reduce the effectiveness of an attack. These limitations can either be specified outside the game as constraints on the adversary, or enforced by instrumenting the code of the game (as in Oracle O^{θ} above), or incorporated into the measure of success.

2.4. Measuring Adversary Success

There are various ways of quantifying the adversary's success in games. We discuss commonly used metrics next.

2.4.1. Attack Success Rate. The *attack success rate* (ASR) measures the expected number of times the adversary succeeds (i.e., wins the game) over multiple rounds. ASR is arguably the most intuitive and widespread metric for quantifying adversary success; for example, it matches the attacker's *accuracy* in membership inference.

However, the main drawback of ASR is that it does not take into account the baseline success probability for a given task. For example, if we evaluate an ML model's resilience to attribute inference, the prior distribution of that attribute will play a role in the adversary's success; trivially, if the attribute can only take one value, any adversary can achieve 100% ASR, but this will not be a meaningful measure. Similarly, the prior probability that an example belongs to the training set affects the membership inference accuracy.

Ideally, the metric should quantify the success of an adversary *relative to a suitable baseline*. The baseline should represent the *a priori* adversary success rate; that is, it should quantify the adversary's success rate if they used only their prior knowledge and had no access to the model.

2.4.2. Adversary Advantage. The notion of *advantage* is a commonly used metric in cryptography, which relates an adversary's success rate to a baseline. This gives a better intuition of how much an adversary gains by having access to the model (in any of the forms defined in Section 2.3). In general terms, suppose the adversary is trying to infer some variable p; this could be the membership of a data record or the value of a coin toss in a standard cryptographic game. If $\Pr[\mathcal{A} = p]$ is an adversary's success rate (probability to guess p correctly), and G is the baseline success rate, the

advantage can be expressed as [13]: $\operatorname{Adv}(\mathcal{A}) = \frac{\Pr[\mathcal{A}=p]-G}{1-G}$. This metric quantifies the adversary's advantage on a scale of [0, 1] relative to the baseline *G*; 0 is random guessing (i.e., no advantage over the baseline) and 1 is a perfect attack. When the secret information *p* is binary with a uniform prior, G = 1/2, and this leads to the familiar expression: $\operatorname{Adv}(\mathcal{A}) = 2 \Pr[\mathcal{A} = p] - 1$. Advantage is commonly used as a metric for ML privacy attacks. For example, Yeom et al. [68] define the MI advantage for an adversary \mathcal{A} as follows:

$$\mathsf{Adv}^{\mathsf{MI}}(\mathcal{A}, \mathcal{T}, n, \mathcal{D}) = 2\Pr[\mathsf{Exp}^{\mathsf{MI}}(\mathcal{A}, \mathcal{T}, n, \mathcal{D}) = 1] - 1,$$

where Exp^{MI} is a membership inference game (Game 2).

Providing an adequate baseline may be difficult because it may not be possible to accurately model the adversary's knowledge. This issue can sometime be bypassed by careful design of the privacy game. For example, instead of asking the adversary to reconstruct an arbitrary attribute's value, the game can be designed such that the adversary must distinguish between two equally-likely values of the attribute.

2.4.3. Beyond Advantage. Average-case metrics, such as ASR, fail to capture inference risks for individuals or subpopulations. For example, a MI attack against a model may achieve roughly 50% accuracy (with a 50% baseline) on average across the population, yet the same attack may perform better when targeting specific individuals or subpopulations [12, 34]. Having raised similar concerns, Carlini et al. [11] suggested that an adversary should be considered successful if it *reliably* succeeds even on small number of test cases, which can be expressed in terms of their true and false positive rates. This, they argue, gives a better indication of the risks posed by the various attacks in practice.

In this paper, we focus on advantage as a metric, since it has the following benefits: (1) it has an easy interpretation – it represents the gain of an adversary from having access to the system under scrutiny versus an adversary who only has prior knowledge; (2) it is directly related to other metrics, such as ASR (which can be derived directly from it), true and false positive rates (e.g., [68]), and Differential Privacy [13, 30]; (3) if the attacker's challenge is binary (e.g., distinguishing between members and non-members), the advantage computed when assuming the two choices have a uniform prior gives a bound for any other prior [13]. Nevertheless, given a game formulation, it is generally easy to derive many metrics of interest: e.g., ROC curves with their respective areas under the curve (AUC), F1-score, and TPR values for fixed FPR thresholds [11].

2.4.4. Consequences of Success. The anatomy presented can be used to specify threat models and quantify the chances that an adversary successfully achieves their goal. However, the *consequences* of a successful attack depend less on the threat model but rather on the adversary's goal (Section 2.1) and on the design of the ML system, e.g., the sensitivity of the training data. For example, the consequences of successful membership inference will be the same irrespective of whether it was performed in a blackbox or white-box setting.

3. Formalization and Games

In this section we present privacy games for each of the five adversary goals introduced in Section 2.1. Due to space limitations, some variants appear in the appendix. We summarize the notation in Table 1. We summarize the threat models considered in all games in Table 2.

TABLE 1. COMMON NOTATION USED ACROSS THE DIFFERENT GAMES.

Symbol	Description
\mathcal{A}'	The adversary's algorithm to choose a challenge point or dataset
\mathcal{A}	The adversary's algorithm to predict the challenge objective
z	A point consisting of data and label i.e., $z = (x, y)$, where x is the features/data and y is the label.
S	A dataset consisting of multiple points, i.e., $S = \{z_0, \cdots z_{ S -1}\}$
\mathcal{D}	A distribution over data points, usually used for sampling datasets i.e., $S \leftarrow \mathcal{D}$
${\mathcal T}$	The training algorithm to train an ML model
θ	A trained ML model, i.e., $\theta \leftarrow \mathcal{T}(S)$

3.1. Membership Inference

Membership inference (MI) aims to predict the participation of an entity in the training dataset of the model. The first (record-level) membership inference attack on supervised learning was proposed by Shokri et al. [54] against ML-based classifiers. Subsequent work has explored membership inference attacks with differing degrees of access to the model (e.g., white-box [35, 50] or label-only attacks [16, 37]), against different types of models (e.g., generative models [15, 26, 28], image segmentation [27], contrastive learning [39], recommender systems [71], and Graph Neural Networks (GNN) [66]), and under entirely different threat models ([29, 51, 53]).

In this section, we present MI variants that have been formalized as privacy games. We divide the MI games into two categories depending on whether they focus on a single input (*record-level*) or a *user* represented by a complete dataset (*user-level*).

3.1.1. Record-level Membership Inference. The most widely-used MI game was proposed by Yeom et al. [68], which we introduced as Game 1 in Section 2. This game simulates an adversary with full access to the model—they have the model on their device so can analyze its structure, parameters, and internal activations when executed. This game is used to measure average case privacy against MI. Several variants of the basic MI game have been considered in the literature; some are semantically equivalent (e.g., [30, 34]), whilst others alter the semantics of the game. We now systematize these variants based on their adversarial capabilities using the *anatomy* presented in Section 2.

MI^{skew}. Jayaraman et al. [33] propose a game that explicitly captures the adversary's background knowledge about the

Game 2: MI	MI ^{skew} MI ^{Adv} MI ^{BB}					
Input: $\mathcal{T}, \mathcal{D}, n, \mathcal{A}$	$,p$ $,\mathcal{A}'$					
1 $S \sim D^{-1}$ 2 $b \sim \{0, 1\}$	$b \sim 0 \oplus_p 1$					
3 $z\sim \mathcal{D}$	$z \leftarrow \mathcal{A}'(\mathcal{T}, \mathcal{D}, n)$					
4 if $b = 0$ then						
5 $z' = z$						
6 else						
7 $z' \sim D$	$z' \sim \mathcal{D} \setminus S$					
8 end						
9 $\theta \leftarrow \mathcal{T}(S \cup \{z'\})$	$\theta \leftarrow \mathcal{T}(S \cup \{z\})$					
10 $\tilde{b} \leftarrow \mathcal{A}(z, \mathcal{T}, \mathcal{D}, n,$	$\theta) \qquad \tilde{b} \leftarrow \mathcal{A}\big(z', \mathcal{O}^{\theta}(.), \mathcal{D}, n\big)$					
11 Oracle $\mathcal{O}^{\theta}(\mathbf{x})$:return $\theta(x)$						

target input through the value of p. The standard membership inference game is a special case of this game with p = 0.5, making members and non-members equally likely.

MI^{Adv}. Chang and Shokri [12] present a game that strengthens the adversary by allowing it to select the challenge point, instead of it being randomly sampled (Game 2, line 3). Intuitively, this game is used to measure the worst-case scenario's privacy, i.e., resilience against this variant would mean all users – even outliers – are protected against MI.

 MI^{BB} . This variant by Carlini et al. [11] introduces two changes. First, it assumes a black-box instead of white-box threat model (line 10 in Game 2), which captures scenarios in which the model is hosted in the cloud or run in a trusted execution environment. Second, it explicitly excludes the training dataset when sampling non-members (line 7). This is in contrast to the standard MI game, where nonmembers are sampled from the complete distribution and *may* be contained in the training dataset *S*. This modelling inaccuracy can simplify proofs and is negligible in practice when the size of the training dataset is small relative to the complete distribution (|S| << |D|).

 $\mathsf{MI}^{\mathsf{Pois}}$ and $\mathsf{MI}^{\mathsf{Diff}[z]}$. Tramèr et al. [62] introduce two more variants of Game 3. They follow the same definition of nonmembers and assume a black-box threat model, but with a different formalization. Abstractly, these variants consider a universe containing different elements/inputs the adversary needs to guess (\mathcal{U}). For membership inference, this universe is set to $\mathcal{U} = \{z, \bot\}$, which encodes membership inference of a target example z. Compared to the $\mathsf{MI}^{\mathsf{BB}}$ game, the basic game introduced in Tramèr et al. [62] ($\mathsf{MI}^{\mathsf{Diff}[z]}$) considers different training set sizes depending on z. In one case, the model is trained on $\{S \cup z\}$ and in the other, only on $\{S \cup \bot\}$. This does not make a significant difference in practice, as

the size of the training dataset is significantly larger than the difference in size between these two cases. The other variant lnf^{Pois} allows the adversary to statically poison the training dataset, capturing the ability to adversarially select or contribute to the base training dataset (Section 2.2).

Game 3: $MI^{Diff z }$ MI^{Pois}	
Input: $\mathcal{T}, \mathcal{D}, \mathcal{U}, n, \mathcal{A}$, \mathcal{A}', n' $S \sim \mathcal{D}^n$	
$\widetilde{z} \sim \widetilde{\mathcal{U}} \setminus S$	
$ \begin{array}{c} S' \leftarrow \mathcal{A}'(I, \mathcal{D}, \mathcal{U}, n') \\ \theta \leftarrow \mathcal{T}(S \cup \{z\} \mid \cup S') \end{array} $	// S' = n'
$\tilde{z} \leftarrow \mathcal{A}(\mathcal{T}, \mathcal{D}, \mathcal{U}, n, \mathcal{O}^{\theta}(.) \boxed{, n', S'})$	
Oracle $\mathcal{O}^{\theta}(\mathbf{x})$: return $\theta(x)$	

Other variants. There exist other MI games that explore different ML settings. Humphries et al. [30] change the sampling of the challenge point to be from different distributions (Game 11); we use this variant as the basis for our case study in Section 5. Tang et al. [59] strengthen the adversary by allowing them to control the complete scope of members and non-members (Game 17 in the appendix), simulating an MI game with an adversarially chosen distribution. Finally, a recent MI game captures machine unlearning [23], where the adversary tries to identify which target point was deleted from the model. All of these variants can be explained in terms of the categories of our anatomy, so we omit them for conciseness.

3.1.2. User-level Membership Inference. Privacy laws such as GDPR require generalizing the goal of MI. Instead of focusing on a single record, the interest is now the complete data of an individual. For instance, an auditor would be interested in learning if a user's data – usually modeled as a dataset of multiple instances – was used to train a target model. User-level membership inference was introduced to model such scenarios. Mahloujifar et al. [40] formalize user-level MI, as described in Game 4, where S^* denotes the target user's dataset, and aux the auxiliary/side information known by the adversary, e.g., the model architecture.

This game focuses on the complete dataset and not a specific record. It presents an easier task than basic membership inference, since the adversary has a complete dataset to perform MI. In other words, if the adversary finds only one member of the target dataset S^* to be a member, then they can win the game with high probability (depending on the overlap in users/distributions).

3.2. Attribute Inference

In attribute inference (AI) attacks, the adversary aims to infer a sensitive attribute of a target record, i.e., can the adversary complete a given incomplete record using the

Game 4: MI ^{User}
Input: $S^*, m, \mathcal{T}, \mathcal{D}, n, \mathcal{A},$ aux
$b \sim \{0, 1\}$
$\mathcal{D}_1,\ldots,\mathcal{D}_m\sim\mathcal{D}$
for $i=1,\ldots,m-1$ do
$S_i \leftarrow \mathcal{D}_i^n$
end
if $b = 0$ then
$S_m = S^*$
else
$S_m \leftarrow \mathcal{D}_m^n$
end
$\theta \leftarrow \mathcal{T}(\bigcup_{i=1}^m S_i)$
$ ilde{b} \leftarrow \mathcal{A}ig(S^*, \mathcal{O}^ heta(.), m, n, \mathcal{D}, auxig)$
Oracle $\mathcal{O}^{\theta}(\mathbf{x})$:return $\theta(x)$

model? Yeom et al. [68] were the first to formalize AI as a game. Recently the scope of AI expanded to other settings [32, 73]. We start with Yeom's formalization in Game 5, where $\varphi(z)$ denotes the adversary's knowledge about the point z, and π the sensitive attribute/feature extracting function i.e., if t represents the sensitive features targeted in the attack, then $\pi(z) = t$.

Game 5: [AI] MInv
Input: $\mathcal{T}, \mathcal{D}, n, \mathcal{A}$
$S \sim \mathcal{D}^n$
$\theta \leftarrow \mathcal{T}(S)$
$b \sim \{0, 1\}$
$z \sim D$
if $b = 0$ then
$\left[z \sim S \right]$
end
$ ilde{a} \leftarrow \mathcal{A}(arphi(z), \mathcal{T}, \mathcal{D}, n, heta)$

Intuitively, the AI experiment is similar to the basic membership inference experiment (Game 2) with the exception of the information the adversary is given and the adversary's target, i.e., the winning condition. Here the adversary is given the non-target features/attributes of the challenge $(\varphi(z))$ and aims to infer the target attributes $(\pi(z))$. The game is won if the adversary is able to correctly predict the attributes, i.e., $\tilde{a} = \pi(z)$. A poisoning variant of the attribute inference game can be easily realized by changing the AI game to include data provided by the adversary when training the target model, as shown in Game 3 (MI^{Pois}).

Model inversion. Another adversary goal with a similar aim to Al is model inversion [21, 65, 67]. While AI focuses only on the training dataset, model inversion targets the whole distribution. Hence, model inversion does not necessarily represent a privacy risk: a successful attack may lead to the adversary "learning" records that were not part of the

training dataset (or do not even exist). We illustrate the difference using our game-based framework in Game 5.

Model inversion attacks were first introduced by Fredrikson et al. [21] and subsequently formalized by Wang et al. [65] (Mlnv in Game 5). As shown in Game 5, the difference between attribute inference and model inversion is in how the challenge (z) is sampled: in Al it is sampled from the training dataset, whilst in Mlnv it is sampled from the distribution \mathcal{D} . This means that model owners who are concerned only with the privacy of the training dataset would use Al games, whilst those who are concerned about leakage of the training distribution would use Mlnv games.

3.3. Reconstruction

Reconstruction attacks aim to recover entire examples in the training data of a model. Reconstruction attacks have been studied in various settings, including Graph Neural Networks [73], image classification [52] and text generation [9, 10, 69]. A distilled scenario, where the adversary learns the training data of the target model except for a target example was first formalized by Balle et al. [3] using the following game where ℓ is a reconstruction loss function:

Input: $S, z \mid \mathcal{D}, n, \pi$; $\mathcal{T}, \mathcal{A}, aux, \ell, \eta$	
$S \sim \mathcal{D}^{n-1}$	
$\begin{array}{c} z \sim \pi \\ \theta \leftarrow \bar{\mathcal{T}}(S \cup \{z\}) \\ \tilde{z} \leftarrow A(S, \mathcal{T}, \theta; zw) \end{array}$	

Definition 1 (Balle et al. [3], Definition 2). A training pipeline is (η, γ) -reconstruction robust with respect to a prior π if for any dataset S and any reconstruction adversary A,

$$\Pr\left[\mathsf{RC}^{Ran}: \ell(z, \hat{z}) \le \eta\right] \le \gamma$$

As the game shows, the adversary is given everything, e.g., the model (θ) , training algorithm (\mathcal{T}) , and training dataset (S), except for one point which they need to reconstruct. A slight variation (RC^{Ran}) of this game for simplifying proofs is constructed by randomly sampling the target point z and training dataset S.

The random baseline for this game would be an adversary that constructs \hat{z} by randomly sampling it from the distribution \mathcal{D} . This RC^{Ran} form circumvents having adversaries that hard-code the target input z. A similar situation was previously discussed in the formalization of collision resistance for hash functions [49]. This game can be adapted to simulate a poisoning-capable adversary by introducing an adversarially chosen dataset S' that is used in addition to S when training the model (as demonstrated in Game 3).

Data reconstruction against language models. Recently, multiple works have focused on large language models and evaluated reconstruction attacks against them. These works can be categorized as untargeted [10] or targeted [9] attacks. Untargeted attacks aim to reconstruct *any* training data from the generative model, whilst targeted attacks aim to reconstruct *specific* training data records, which may have been inserted as canaries during training. To demonstrate the flexibility of privacy games, we formalize an example from each category, as shown in Game 7.

For the untargeted category (RC^{UnTarg}), we formalize a black-box untargeted data reconstruction attack by Carlini et al. [10] tailored to large generative language models. A successful adversary in the untargeted game RC^{UnTarg} generates training inputs from the given model. The authors use the fraction of examples output by the attack that is in the training dataset as a measure of attack success.

For the targeted category ($\mathbb{RC}^{\mathsf{Targ}}$), we formalize the canary-based attack by Carlini et al. [9]. In this attack, canaries are inserted in the training data as a way of measuring unintended memorization; canaries are specified by a format sequence $s[\cdot]$ made of fixed tokens and *holes* to be filled with values sampled from a randomness space \mathcal{R} ; e.g., s = "the random number is $(\mathcal{RC})(\mathcal{RC})(\mathcal{RC})$ " with \mathcal{R} being the space of 9-digit decimal numbers. A successful adversary in the targeted game can reconstruct inputs with a specific pattern. Carlini et al. [9] define *exposure* of canaries in terms of the reduction in their guessing entropy given the model, and use exposure to measure the success of canary reconstruction attacks.

Game 7: RC ^{UnTarg} RC ^{Targ}	
Input: $\mathcal{T}, \mathcal{D}, n, \mathcal{A}$, \mathcal{R}, s, nr	
$S \sim \mathcal{D}^n$	
$r \sim \mathcal{R}$	
$\theta \leftarrow \mathcal{T}(S \mid \bigcup \{s[r]\}^{nr} \mid)$	
$\tilde{S} \leftarrow \mathcal{A}(\mathcal{T}, \mathcal{D}, \mathcal{O}^{\theta}(.), s)$	
Oracle $\mathcal{O}^{\theta}(\mathbf{x})$: return $\theta(x)$	

Intuitively, if a user wants to explore the strongest possible attack to reconstruct a single input, i.e., an adversary with the knowledge of everything except for an input, then the RC^{Ran} game would be the best choice. Otherwise, if they want to measure what is *currently* being leaked from their model, they should use the RC^{UnTarg}. Finally, the user should use the targeted version (RC^{Targ}) if their interest is measuring the *possible* leakage of a specific model.

Similar to MI, there exist other variants of reconstruction that target machine unlearning [23], i.e., the adversary aims to reconstruct a deleted challenge point from the model.

3.4. Distribution Inference Attack

Distribution inference attacks do not focus on a specific data record or user, but instead aim at inferring the participation of data points from a target distribution. A distribution inference attack can be used for two goals. The first is the property inference attack, where the adversary is interested in learning about the distribution of a specific sensitive property in the training dataset (e.g., gender or race). The second is subject-level distribution inference, where the distributions belong to different subjects participating in training. The adversary's goal is to identify a subject's participation in the training, with access to that subject's data distribution, not exact samples used in training.

3.4.1. Property Inference Attack. Property inference attack was first proposed by Ganju et al. [22] in the whitebox setting and by Zhang et al. [72] in the black-box setting. Zhou et al. showed them to be effective against generative models, namely, GANs [74]. Suri et al. [57] later formalize this attack (Game 8), defining \mathcal{G}_0 , \mathcal{G}_1 as two functions that transform the underlying distribution.



A generalization (PI^{Gen}) of this game replaces the transforming functions with two distributions (Game 8). This formalization of the game is only useful to simplify proofs; PI and PI^{Gen} are equivalent. PI^{Gen} was recently extended beyond the binary scenario of two distributions, by generalizing the adversary's task to identify which distribution out of *R* multiple distributions was used by the victim [25].

Another variant of this game (Pl^{Pois}) introduces active adversaries with the ability to poison the victim's training data by injecting adversarially crafted data. Poisoning attacks for property inference were first introduced by Mahloujifar et al. [41]; while they consider a blackbox threat model, we use a white-box one to simplify the visualization of Game 8. Chaudhari et al. [14] recently proposes stronger attack strategies for the poisoning scenario. These attacks increase the inference risk significantly, since the data injected by the adversary is crafted to maximize property leakage. A potential use-case for this threat model corresponds to multi-party learning, where a malicious party may introduce poisoned data via its data contributions to maximize property leakage for data from other participants.

3.4.2. Subject-Level Distribution Inference. Subject-level distribution inference aims at broadening the scope of user-level membership inference by not assuming access to the

user's *exact* data (potentially) used for training models. Instead, it only requires the adversary/auditor to have access to the target user's (or subject's) distribution. Marathe et. al. [43] propose the subject-level attack as a differential privacy setting and Suri et al. [58] present it as a subjectlevel membership inference attack. However, it is analogous to a subject-level distribution inference as it infers the participation of a particular distribution that belongs to subject. We formalize the subject-level inference using Game 9.

 $\begin{array}{c} \textbf{Game 9: } \mathsf{MI}^{\mathsf{Subj}} \\ \hline \mathbf{Input:} \ m, \mathcal{T}, \mathcal{D}^*, \mathcal{D}, n, \mathcal{A} \\ 1 \ b \sim \{0, 1\} \\ 2 \ \mathcal{D}_1, \dots, \mathcal{D}_m \sim \mathcal{D} \\ 3 \ \textbf{for} \ i = 1, \dots, m-1 \ \textbf{do} \\ 4 \ | \ S_i \sim \mathcal{D}_i^n \\ 5 \ \textbf{end} \\ 6 \ \textbf{if} \ b = 0 \ \textbf{then} \\ 7 \ | \ S_m \sim \mathcal{D}^{*n} \\ 8 \ \textbf{else} \\ 9 \ | \ S_m \sim \mathcal{D}_m^n \\ 10 \ \textbf{end} \\ 11 \ \theta \leftarrow \mathcal{T}(\bigcup_{i=1}^m S_i) \\ 12 \ \tilde{b} \leftarrow \mathcal{A}(\mathcal{T}, \mathcal{D}^*, \mathcal{D}, n, \theta) \end{array}$

As the game shows, this is a distribution inference attack, as the adversary seeks to infer which distribution the training data is sampled from. However, conceptually, the adversary performs membership inference, as its final aim is with respect to a certain user's membership. Hence, a successful adversary can identify if a user's data was used to train the target model without the knowing which exact inputs were used; i.e., with access to only the user's complete distribution and not the dataset as presented in Game 4.

3.5. Differential Privacy (DP) Distinguisher

Differential Privacy (DP) distinguisher games simulate the differential privacy settings, i.e., where the adversary is trying to distinguish between two neighboring datasets while having access to all training data except for one record. Prior work has introduced the DP distinguisher property (DPD) to audit the differential privacy of machine learning models in practice [31, 45, 61, 70], i.e., empirically measuring and quantifying the privacy of a given target model or pipeline. The difference between the DP distinguisher game DPD and the membership inference one MI is that, in the former, the adversary gets to choose the training dataset except for one point.

Similar to the basic DP distinguisher game, the adversary in this game gets to know the complete training dataset except for the challenge point. However, in this game, the adversary does not choose the training dataset or the challenge point as shown in Game 10. The difference between these two games is how the dataset and challenges are sampled, i.e., DPD is adversarially chosen while DPD^{SMI} is externally chosen. As mentioned in Section 2.2 this controls

Game 10: DPD DPD ^{SMI}
Input: $\mathcal{T}, \mathcal{D}, n, \mathcal{A}, \overline{S, z_0, z_1}$
$S, z_0, z_1 \leftarrow \mathcal{A}_{choose}(\mathcal{T}, \mathcal{D}, n)$
$b \sim \{0, 1\}$
$\theta \leftarrow \mathcal{T}(S \cup \{z_b\})$
$ ilde{b} \leftarrow \mathcal{A}(\mathcal{T}, \mathcal{D}, n, \theta, S, z_0, z_1)$

the scope of the measured privacy, e.g., worst or individual case privacy.

4. Relations and Proofs

In this section we establish relationships between privacy games. To this end, we define a new notion of *reduction* between games and use it to translate attacks (and guarantees) between the five fundamental games from the previous section, or show that no such connection exists. We conclude with a case study showing how these privacy games can be used to express new games in the literature.

4.1. Reductions for Privacy Games

Inspired by notions of reduction from complexity theory and cryptography [1], we introduce *reductions between privacy games* as a means of comparing risks for different kinds of inference. Whilst reductions in these fields are traditionally based on asymptotic behavior or security parameters, the reductions we define here are closer to those used in *concrete security proofs*, in that they rely on constants to quantify the loss incurred in the reduction.

Definition 2. We say that a game $Game_1$ is reducible to a game $Game_2$ if there is a constant c > 0 such that, for any adversary A against $Game_2$, there exists an adversary B against $Game_1$ for which

$$\mathsf{Adv}_{\mathsf{Game}_1}(\mathcal{B}) \ge c \cdot \mathsf{Adv}_{\mathsf{Game}_2}(\mathcal{A}),$$

We denote this using the shorthand $Game_1 \preceq_c Game_2$ and sometimes drop the constant c.

The intuition behind the shorthand is that $Game_1$ is *at* most as hard to break as $Game_2 - modulo$ the constant c. This intuition holds for c around or larger than 1. For $c \ll 1$, however, the lower bound on $Adv_{Game_1}(\mathcal{B})$ can get close to 0, in which case the intuition may be misleading.

Resilience to attacks. Reductions between privacy games imply that attacks against one game translate into attacks against the other. An equivalent reading is that resilience of one game against attacks implies resilience of the other.

Definition 3. A game Game is *p*-resilient if for all adversaries A against Game,

$$\mathsf{Adv}_{\mathsf{Game}}(\mathcal{A}) < p$$

TABLE 2. AN OVERVIEW OF DIFFERENT GAMES AND THEIR CORRESPONDING THREAT MODELS. \checkmark means the game requires this assumption, - indicates the game does not require this assuming, and x denotes that it is not applicable.

Game	Adversary Access		Challenge			Training Dataset			Adversary Interest		
	Black-box	White-box	Rand	Adv	Param	Rand	Adv	Param	Record	Object	Distribution
			Membe	rship Iı	nference G	ames					
Game 2 (MI)[30, 34, 68]	-	\checkmark	\checkmark	-	-	\checkmark	-	-	\checkmark	-	-
Game 2 (MI ^{skew})[33]	-	\checkmark	\checkmark	-	-	\checkmark	-	-	\checkmark	-	-
Game 2 (MI ^{BB})[11]	\checkmark	-	\checkmark	-	-	\checkmark	-	-	\checkmark	-	-
Game 2 (MI ^{Adv})[12]	-	\checkmark	-	\checkmark	-	\checkmark	-	-	\checkmark	-	-
Game 11(MI ^{MM})[30]	-	\checkmark	\checkmark	-	-	\checkmark	-	-	\checkmark	-	-
Game 17 (MI ^{Dist})[59]	\checkmark	-	\checkmark	-	-	-	\checkmark	-	\checkmark	-	-
Game 4(MI ^{User})[40]	\checkmark	-	-	-	\checkmark	\checkmark	-	-	-	\checkmark	-
Game $3(MI^{Diff z })[62]$	\checkmark	-	\checkmark	-	-	\checkmark	-	-	\checkmark	-	-
Game 3(MI ^{Pois}) [62]	\checkmark	-	\checkmark	-	-	-	\checkmark	-	\checkmark	-	-
		Attribut	e Inferen	ce and	Model Inv	ersion G	ames				
Game 5 (AI)[68]	-	\checkmark	\checkmark	-	-	\checkmark	-	-	\checkmark	-	-
Game 5 (MInv)[65]	-	\checkmark	\checkmark	-	-	\checkmark	-	-	\checkmark	-	-
			Data R	Reconstr	uction Ga	mes					
Game 6 (RC)[3]	-	\checkmark	\checkmark	-	-	-	-	\checkmark	\checkmark	-	-
Game 7 (RC ^{UnTarg})[10]	\checkmark	-	\checkmark	-	-	\checkmark	-	-	\checkmark	-	-
Game 7 (RC ^{Targ})[9]	\checkmark	-	\checkmark	-	-	-	\checkmark	-	\checkmark	-	-
Distribution Inference Games											
Game 8(PI)[57]	-	\checkmark	х	х	х	\checkmark	-	-	-	-	\checkmark
Game 8(PI ^{Pois})[41]	-	\checkmark	х	х	х	-	\checkmark	-	-	-	\checkmark
Game 9(MI ^{Subj}) [58]	-	\checkmark	\checkmark	-	-	\checkmark	-	-	-	-	\checkmark
		Differe	ntial Priv	acy (DI	P) Disting	iisher Ga	ames				
Game 10 (DPD)[42, 45]	-	\checkmark	-	\checkmark	-	-	\checkmark	-	\checkmark	-	-
Game 10 (DPD ^{SMI})[3, 30]	-	\checkmark	-	-	\checkmark	-	-	\checkmark	\checkmark	-	-

Proposition 1. Let $Game_1 \leq_c Game_2$. If $Game_1$ is presilient then $Game_2$ is p/c-resilient.

Proof. By contradiction: If there is an attack on $Game_2$ with advantage more than p/c, then there is one on $Game_1$ with advantage more than p.

Proofs of resilience are rare in the ML privacy literature. The prime example are results that establish upper bounds on the advantage of a DP distinguisher when the model is trained with differential privacy [20, 30, 68]. The tightest such bound is the following result by [30]:

Proposition 2. Let \mathcal{T} be an (ϵ, δ) -differentially private training algorithm. Then

$$\mathsf{Adv}_{\mathsf{DPD}}(\mathcal{A}) \leq \frac{1 - e^{-\epsilon} + 2\delta}{e^{\epsilon} + 1}$$

Therefore, any game to which the DP distinguisher inference game can be reduced (see Figure 1 for an overview) inherits the security benefits of training with differential privacy via Propositions 1 and 2.

Separation Results. No reductions exist between several games. For them, we show separation results of the form $Game_1 \not\preceq Game_2$. We establish such results by showing that there is an instance of $Game_1$ that is resilient to attacks whereas its $Game_2$ counterpart is not, and use Proposition 1 to conclude that no reduction exists.

4.2. Overview of Relations between Games

Figure 1 shows all the relations between five fundamental privacy games. Each node in this figure (and in the following theorems) refers to the basic game-based definition of the corresponding inference risk, i.e. MI, AI, RC and DPD, with the exception of property inference, which refers to its generalized definition, i.e., PI^{Gen}.

As expected, PI is fully disconnected, i.e., there exists a separation result between it and every other game. This can be attributed to the PI adversary's goal of learning distributions, rather than exact datasets/inputs as in the other games. RC and DPD have the strongest threat models (i.e., require knowledge or control of the entire training dataset except for one point) and hence are unsurprisingly the hardest to reduce from other games. Finally, MI and AI are the most reducible due to their weak threat models. For this reason, we use the MI game as the anchor for our proofs. We now present results for the minimum necessary set of edges (solid lines) required to imply all other relations. We leave the proofs to the Appendix.

4.3. Reductions

There is a symmetry between the MI and AI properties, as they can be reduced in both directions. However, the two are clearly different due to the constants involved when using a membership inference oracle to perform an attribute inference attack. The following theorems, as proved by Yeom et al. [68], show the relation in both directions. **Theorem 4.1** (MI \leq_1 AI [68]). For any adversary \mathcal{A}_{AI} against attribute inference, there exists an adversary \mathcal{A}_{MI} against membership inference such that

$$\mathsf{Adv}_{\mathsf{MI}}(\mathcal{A}_{\mathsf{MI}}) = \mathsf{Adv}_{\mathsf{AI}}(\mathcal{A}_{\mathsf{AI}})$$

Theorem 4.2 (Al $\leq_{1/m}$ MI [68]). For any adversary \mathcal{A}_{MI} against membership inference, there exists an adversary \mathcal{A}_{AI} against attribute inference such that

$$\mathsf{Adv}_{\mathsf{AI}}(\mathcal{A}_{\mathsf{AI}}) = \frac{1}{m} \cdot \mathsf{Adv}_{\mathsf{MI}}(\mathcal{A}_{\mathsf{MI}})$$

where m is the number of possible values for the target attribute.

Unlike membership inference, the DP distinguisher has the strongest threat model: as Figure 1 indicates, DPD can be reduced against all other properties with the exception of PI. We present the necessary theorems below. We remark that the remaining one (RC \leq Al) is implied from the other results, as shown in Figure 1.

Balle et al. [3] showed that a reconstruction attack can be turned into a DP distinguisher. We formalize this via an explicit advantage notion.

Theorem 4.3 (DPD \leq RC). For any adversary A_{RC} against data reconstruction, there exists an adversary A_{DPD} against DP distinguisher such that

$$\operatorname{Adv}_{\operatorname{DPD}}(\mathcal{A}_{\operatorname{DPD}}) = 2p - 1$$

where z_0 and z_1 are the two different points in the S_0 and S_1 datasets of the DPD game, ℓ is a loss function that satisfies the triangle inequality, and A_{RC} achieves error $< \ell(z_0, z_1)/2$ with probability p. This theorem considers neighboring datasets with replacement for the DPD game.

Further, we note that that the DP distinguisher game can be reduced to membership inference.

Theorem 4.4 (DPD \leq MI). For any adversary A_{MI} against membership inference, there exists an adversary A_{DPD} against the DP distinguisher such that

$$\mathsf{Adv}_{\mathsf{DPD}}(\mathcal{A}_{\mathsf{DPD}}) = \mathsf{Adv}_{\mathsf{MI}}(\mathcal{A}_{\mathsf{MI}})$$

Finally, we show that a membership inference adversary can be used to mount a data reconstruction attack, with a constant depending on the size of the target distribution.

Theorem 4.5 (RC $\leq_{1/|\mathcal{D}\setminus S|}$ MI). For any adversary \mathcal{A}_{MI} against membership inference, there exists an adversary \mathcal{A}_{RC} against data reconstruction such that

$$\mathsf{Adv}_{\mathsf{RC}}(\mathcal{A}_{\mathsf{RC}}) \geq \frac{1}{|\mathcal{D} \setminus S|} \cdot \mathsf{Adv}_{\mathsf{MI}}(\mathcal{A}_{\mathsf{MI}})$$

4.4. Separation Results

Having the weakest threat model results in the impossibility of transferring the resilience against membership inference to other properties except for attribute inference. We present all separation results below: **Theorem 4.6** (MI \preceq PI). Resilience against membership inference does not imply resilience against property inference

Theorem 4.7 (MI $\not\leq$ DPD). Resilience against membership inference does not imply resilience against DP distinguisher.

Theorem 4.8 (MI $\not\preceq$ RC). Resilience against membership inference does not imply resilience against data reconstruction.

Finally, we show the remaining separation property to imply all missing separation results with respect to the property inference game.

Theorem 4.9 (PI \preceq MI). Resilience against property inference does not imply resilience against membership inference.

5. Reduction Case Study: Mixture Model Membership Inference

We present a case study where we showcase the expressiveness and rigour of privacy games. In particular we show that a novel variant of membership inference can actually be decomposed into a combination of standard membership and property inference. This complex relationship goes beyond the direct reductions presented in Section 4. In our proofs we exploit *code-based* reductions, structured as a sequence of games; i.e., our arguments rely on transforming the code with a formal semantics.

The game we target is due to Humphries et al. [30], who use it to model membership inference attacks in the presence of dependencies in the training data. In their game (MI^{MM} in Game 11), the training data distribution follows a twostage *mixture model*. Examples in the training dataset and the target example are chosen independently from two data distributions, \mathcal{D}_k and $\mathcal{D}_{k'}$, which are chosen uniformly at random without replacement from K possible distributions.

Game 11: $[MI^{MM}]$ G_0	
Input: $\mathcal{T}, \mathcal{D}, n, \mathcal{A}$	
$k \sim \{1, \dots, K\}$	
$k' \sim \{1, \dots, K\} \setminus \{k\}$	
$S \sim \tilde{\mathcal{D}}_k^n$	
$ heta \leftarrow \mathcal{T}(S)$	
$b \sim \{0, 1\}$	
if $b = 0$ then	
$z \sim S$ $z \sim \mathcal{D}_k$	
else	
$z\sim \mathcal{D}_{k'}$	
end	
$ ilde{b} \leftarrow \mathcal{A}(\mathcal{T}, \mathcal{D}, n, \theta, z)$	

We show that MM can be decomposed into a property inference goal (inferring the training data distribution) and

Input: $\mathcal{T}, \mathcal{D}, n, \mathcal{A}$ $k \sim \{1, \dots, K\}$ $k' \sim \{1, \dots, K\} \setminus \{k\}$ $z \sim \mathcal{D}_k$ $b \sim \{0, 1\}$ if b = 0 then $\mid S \sim \mathcal{D}_k^n$ else $\mid S \sim \mathcal{D}_{k'}^n$ end $\theta \leftarrow \mathcal{T}(S)$ $\tilde{b} \leftarrow \mathcal{A}(\mathcal{T}, \mathcal{D}, n, \theta, z)$

a membership inference goal (inferring whether a target example has been sampled from the training data distribution \mathcal{D}_k or from the training dataset S).

Theorem 5.1. For any adversary \mathcal{A} against $\mathsf{MI}^{\mathsf{MM}}$, there exist adversaries $\mathcal{A}^{i}_{\mathsf{MI}}$ and $\mathcal{A}^{i}_{\mathsf{PI}}$ such that

$$\mathsf{Adv}_{\mathsf{MI}^{\mathsf{MM}}}(\mathcal{A}) \leq \max_{i \in \{1, \dots, K\}} \mathsf{Adv}_{\mathsf{MI}_i}(\mathcal{A}^i_{\mathsf{MI}}) + \mathsf{Adv}_{\mathsf{PI}_i}(\mathcal{A}^i_{\mathsf{PI}})$$

where MI_i is the standard membership inference experiment with training data distribution D_i , and PI_i the property to infer is whether the training data distribution is D_i or a distribution sampled uniformly at random from $\mathcal{D} \setminus D_i$.

Proof. Let \mathcal{A} be an adversary against $\mathsf{MI}^{\mathsf{MM}}$. Consider G_0 shown alongside $\mathsf{MI}^{\mathsf{MM}}$ in Game 11. Its only difference w.r.t. $\mathsf{MI}^{\mathsf{MM}}$ is that when b = 0, the example z is freshly sampled from the training data distribution \mathcal{D}_k rather than from the training dataset S. Conditioned on b = 0 and k = i, distinguishing between G_0 and $\mathsf{MI}^{\mathsf{MM}}$ is as difficult as winning a membership inference experiment. We show this using a black-box reduction: fixing k = i, we construct an adversary $\mathcal{A}^i_{\mathsf{MI}}$ that uses \mathcal{A} as an oracle to guess the challenge bit b in experiment MI_i (see Experiment 13). $\mathcal{A}^i_{\mathsf{MI}}$ simply forwards its inputs $\mathcal{T}, n, \theta, z$ to \mathcal{A} , passing it in addition the set of distributions \mathcal{D} .

Game 13: MI_i
Input: $\mathcal{T}, \mathcal{D}, n, \mathcal{A}$
$S\sim \mathcal{D}_i^n$
$ heta \leftarrow \mathcal{T}(S)$
$b \sim \{0, 1\}$
if $b = 0$ then
$z \sim S$
else
$z\sim {\cal D}_i$
end
$\tilde{b} \leftarrow \mathcal{A}^i_{MI}(\mathcal{T}, \mathcal{D}_i, n, \theta, z)$

Experiment MI^{MM} conditioned on b = 0 and k = i is equivalent to MI_i conditioned on b = 0. Likewise, experi-

Adversary 14: \mathcal{A}^i_{MI}	
Input: $\mathcal{T}, \mathcal{D}_i, n, \theta, z$	
return $\mathcal{A}(\mathcal{T}, \boldsymbol{\mathcal{D}}, n, \theta, z)$	

ment G_0 conditioned on b = 0 and k = i is equivalent to MI_i conditioned on b = 1. Hence,

$$\operatorname{Adv}_{\mathsf{MI}_{i}}(\mathcal{A}_{\mathsf{MI}}^{i}) = \Pr\left[\mathsf{MI}_{i}:\neg \tilde{b} \mid \neg b\right] - \Pr\left[\mathsf{MI}_{i}:\neg \tilde{b} \mid b\right]$$
$$= \Pr\left[\mathsf{MI}^{\mathsf{MM}}:\neg \tilde{b} \mid \neg b, k = i\right] - \Pr\left[G_{0}:\neg \tilde{b} \mid \neg b, k = i\right] (1)$$

Experiment MI^{MM} conditioned on b = 1 is equivalent to G_0 conditioned on b = 1, and so we have

$$\begin{aligned} \mathsf{Adv}_{\mathsf{MI}^{\mathsf{MM}}}(\mathcal{A}) &= \Pr\left[\mathsf{MI}^{\mathsf{MM}} : \neg \tilde{b} \mid \neg b\right] - \Pr\left[\mathsf{MI}^{\mathsf{MM}} : \neg \tilde{b} \mid b\right] \\ &= \frac{1}{K} \sum_{i=1}^{K} \Pr\left[\mathsf{MI}^{\mathsf{MM}} : \neg \tilde{b} \mid \neg b, k = i\right] - \Pr\left[\mathsf{MI}^{\mathsf{MM}} : \neg \tilde{b} \mid b, k = i\right] \\ &= \frac{1}{K} \sum_{i=1}^{K} \mathsf{Adv}_{\mathsf{MI}_{i}}(\mathcal{A}^{i}_{\mathsf{MI}}) + \Pr\left[G_{0} : \neg \tilde{b} \mid \neg b, k = i\right] - \end{aligned}$$
(2)
$$\Pr\left[G_{0} : \neg \tilde{b} \mid b, k = i\right] \end{aligned}$$

where the last equation follows from (1).

We reformulate G_0 in a semantics-preserving manner as G_1 (see Experiment 12). To see why both formulations are equivalent, note that conditioned on b = 0, in both experiments S and z are sampled from the same distribution chosen uniformly from \mathcal{D} , while conditioned on b = 1, S and z are sampled each from one of two distributions sampled without replacement from \mathcal{D} . Since b is identically sampled in both experiments, both experiments result in the same joint distribution of b, S, z, k, and therefore \tilde{b}, b, k :

$$\Pr\left[G_0:\neg \tilde{b} \mid \neg b, k=i\right] = \Pr\left[G_1:\neg \tilde{b} \mid \neg b, k=i\right]$$
(3)

$$\Pr\left[G_0:\neg \tilde{b} \mid b, k=i\right] = \Pr\left[G_1:\neg \tilde{b} \mid b, k=i\right]$$
(4)

Next, we show using a black-box reduction that distinguishing between the case when b = 0 and b = 1 in G_1 conditioned on k = i is as hard as guessing the challenge bit in the property inference experiment Pl^i shown in Experiment 15. To do this, we construct an adversary $\mathcal{A}_{\mathsf{Pl}}^i$ that uses \mathcal{A} as a black-box. $\mathcal{A}_{\mathsf{Pl}}^i$ perfectly simulates the inputs to \mathcal{A} in G_1 by forwarding its own inputs and freshly sampling z from \mathcal{D}_i .

$$\operatorname{Adv}_{\mathsf{PI}}(\mathcal{A}_{\mathsf{PI}}^{i}) = \Pr\left[\mathsf{PI}_{i}:\neg \tilde{b} \mid \neg b\right] - \Pr\left[\mathsf{PI}_{i}:\neg \tilde{b} \mid b\right]$$
$$= \Pr\left[G_{1}:\neg \tilde{b} \mid \neg b, k = i\right] - \Pr\left[G_{1}:\neg \tilde{b} \mid b, k = i\right] \quad (5)$$

Putting Equations (2)–(5) together we obtain

$$\mathsf{Adv}_{\mathsf{MI}^{\mathsf{MM}}}(\mathcal{A}) = \frac{1}{K} \sum_{i=1}^{K} \mathsf{Adv}_{\mathsf{MI}_{i}}(\mathcal{A}_{\mathsf{MI}}^{i}) + \mathsf{Adv}_{\mathsf{PI}_{i}}(\mathcal{A}_{\mathsf{PI}}^{i})$$

Game 15: PI_i
Input: $\mathcal{T}, \mathcal{D}, n, \mathcal{A}$
$b \sim \{0, 1\}$
if $b = 0$ then
$S \sim \mathcal{D}_i^n$
else
$k' \sim \{1, \dots, K\} \setminus \{i\}$
$S\sim \mathcal{D}^n_{k'}$
end
$\theta \leftarrow \mathcal{T}(S)$
$ ilde{b} \leftarrow \mathcal{A}_{PI}(\mathcal{T}, \boldsymbol{\mathcal{D}}, n, heta)$

Adversary 16: \mathcal{A}_{Pl}^i	
Input: $\mathcal{T}, \mathcal{D}, n, \theta$	
$z\sim {\cal D}_i$	
return $\mathcal{A}(\mathcal{T}, \mathcal{D}, n, \theta, z)$	

6. Discussion

We discuss strategies for choosing privacy games, their current and future uses, and their limitations.

6.1. Selecting games to use

With the variety of privacy games in the literature, it is natural to ask whether there is a *best* or *canonical* game that should be used instead of others. We believe this is not the case, i.e., no single game is the best choice in all circumstances because subtle differences in threat scenarios can lead to vastly different privacy evaluations (see, e.g., [45]). Instead, we recommend that users of games leverage the building blocks we provide in this paper to design games that accurately capture their application-specific threat models. For a given threat model, however, some differences between modelling choices are less important (e.g., in MIA, whether one samples non-members from the full distribution or explicitly excludes them from the training set), and we highlight this distinction throughout the paper.

6.2. Current uses of privacy games

The use of privacy games has become prevalent in the literature on machine learning privacy. As of today, there have been two main applications: The first has been to support the *empirical evaluation* of machine learning systems against a variety of threats (e.g., [45, 55, 57, 67]). The second has been to *compare* the strength of privacy properties and attacks. Reductions enable us to translate *provable guarantees* against attacks from one property to another. Previously, only a few of the reduction and separation results in Figure 1 were known; in this paper we establish the remaining connections between representative games for each of the fundamental adversary goals.

6.3. Future uses of privacy games

We believe there are other promising uses for games in the future, of which we highlight two:

Communicating privacy properties. Reasoning about the privacy risks of ML models is not the exclusive purview of researchers. Other personas, e.g., privacy managers, need to make decisions about the compliance of training pipelines with regulatory or contractual constraints. Based on our experience, privacy managers currently base their reasoning on (1) empirical privacy evaluations, (2) formal guarantees given by mechanisms such as DP-SGD, and (3) informal texts such as the *Opinion 05/2014* [2] of the European Commission's Article 29 Working Party, and they are faced with the daunting task of combining these pieces into a coherent picture to assess the privacy risks of specific applications.

Privacy games can help with this task: by making the threat model and all assumptions about dataset creation and training explicit, they can disambiguate interpretations and can abstract a full application scenario with respect to its (provable *and* empirical) privacy properties. Indeed, based on our initial experience, privacy games facilitate discussing privacy goals and guarantees with stakeholders making guidelines and decisions around ML privacy.

Mechanization of proofs. An advantage of the game-based formalism is that games can be given an unambiguous semantics as probabilistic programs. This enables reasoning about games using program logics and manipulating them using program transformations. Reusable program transformations (e.g., procedure inlining) and proof techniques (e.g., conditioning on events) arise naturally and make proofs more amenable. As we show in Section 4 and Section 5, our proofs exhibit some of these common patterns.

We envisage techniques and frameworks to reason about game-based cryptographic proofs (e.g., EasyCrypt, FCF) being repurposed to reason about privacy games. The apparent complexity of privacy games compared to cryptographic games is not an obstacle since most proofs manipulate training algorithms, models, and data as abstract objects with minimal structure. The main challenge for mechanizing proofs about privacy games is that, unlike cryptographic games, privacy games sometimes require reasoning about continuous distributions (e.g., Gaussian noise in DP-SGD).

6.4. Limitations of privacy games

Privacy games encompass sequential probabilistic programs; however, they are not an immediate fit for expressing concurrent computations. This prevents the direct application of games to important scenarios such as federated learning (FL). Intuitively, this is due to the hardness of modeling the various possible parallel interactions between the different parties. The situation is similar for cryptographic games, where process calculi are used instead of games for modeling more complex multi-party interactions [8, 44]. It is an open question whether these calculi could also be used in the context of concurrent ML scenarios, such as FL.

7. Related Work

Alternative Formalisms. Games are not the only way to express privacy properties. We discuss two alternatives, informal and formal:

• A key example for a *formal* privacy property is Differential Privacy [19]. The definition of Differential privacy is *relational*, in that it compares the probability of events in two alternative worlds: one in which a participant contributes their data and one in which they don't. The DP definition abstracts from many details that are relevant for threat modelling, such as adversary capabilities, goals, and background knowledge, as well as the way datasets are created. This has led to disagreements in the literature about the consequences of differential privacy (see [63]).

• A key example for an *informal* description of privacy properties is the Opinion 05/2014 on Anonymization Tech*niques* [2] that complements the EU General Data Protection Regulation (GDPR) with practical recommendations for the use of anonymization techniques to meet the requirements set out by the regulator. In this influential document, the authors distill three classes of privacy risks (namely: singling out, linkability, and inference) to users' data. They discuss the suitability of different techniques-including kanonymity and Differential Privacy-for the mitigation of these risks, but the discussion (necessarily) remains inconclusive due to the lack of precise definitions.

Game-based definitions address shortcomings of both alternatives: They make the threat model and underlying assumptions explicit, which helps disambiguate interpretations.

Surveys and Taxonomies on Privacy. Several papers provide taxonomies of privacy attacks against different kinds of machine learning algorithms, and in different scenarios, such as as centralized and distributed [17, 38, 48]. Papernot et al. [47] focus on systematizing the possible attack surfaces of standard machine learning pipelines. Desfontaines and Pejo [18] systematize the different variants of differential privacy definitions. Before attacks against machine learning models were demonstrated, Li et al. [36] proposed a unifying framework for membership and differential privacy definitions mainly applicable to database systems.

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Appendix

1. Game Variants

We now present concrete variant of the MI in Game 17.

Game 17: MI ^{Dist}
Input: $\mathcal{T}, \mathcal{D}, n, \mathcal{A}, \mathcal{A}'$
$S^{2n} \leftarrow \mathcal{A}'(\mathcal{T}, n, \mathcal{D})$
$B \sim \{0, 1\}^{2n} : \sum B = n$
$S = \{ z_i; \forall i \in [2n], B[i] = 0 \}$
$\theta \leftarrow \mathcal{T}(S)$
$i \sim [2n]$
z = S[i]
$\tilde{b} \leftarrow \mathcal{A}(z, \theta(z), \mathcal{T}, S^{2n}, n)$

The adversary wins the MI^{Dist} game (Game 17) if they are able to predict the value of the bit vector B at the specific location i, i.e., $B[i] = \tilde{b}$.

2. Proofs of MI

2.1. MI ∠ RC.

Theorem 4.8 (MI \preceq RC). Resilience against membership inference does not imply resilience against data reconstruction.

Proof. The impossibility result in this setting stems from a system that satisfies the membership inference property while being vulnerable to an RC adversary. Let distribution $\mathcal{D} = \mathcal{N}(\mu, \sigma^2)$, and model $\theta : \theta(S) = \frac{1}{|n|} \sum_{x \in S}$. Since the mean of samples from the Gaussian distribution would also be distributed similarly, it is theoretically impossible to learn if a given number is a member or not. Moreover, ast n increases, the model's output will be close to μ . Thus, the adversary's membership inference advantage will be negligible. However, if an adversary has access to n - 1 of the training data samples, it can exactly reconstruct the missing member z as $n \cdot \theta(x_1, ..., x_n) - \sum S_{n-1}$, leading to perfect advantage.

2.2. MI ∠ DPD.

Theorem 4.7 (MI \preceq DPD). Resilience against membership inference does not imply resilience against DP distinguisher.

Proof. We show that there are training pipelines that are arbitrarily resilient against membership inference attacks but completely insecure against DP distinguishing attacks.

We construct a training pipeline $(\mathcal{T}, \mathcal{D}, n)$ such that the MI advantage of an adversary against it is at most $1/\sqrt{n}$, and so vanishes as *n* grows. Yet, we exhibit a DP distinguisher against the pipeline that achieves perfect advantage.

Let $\mathcal{D} = \text{Bernoulli}(p)$ and $\mathcal{T}(S) = \sum_{x \in S} x$. Consider Game 18. If the adversary were only given z_0 , this game would be equivalent to the standard MI game (Game 1). Since the adversary is given strictly more information, any

Game 18: Membership Inference

Input: $\mathcal{T}, \mathcal{D}, n, \mathcal{A}$	
$b \sim \{0, 1\}$	
$S \sim \mathcal{D}^{n-1}$	
$z_0, z_1 \sim \mathcal{D}$	
$\theta \leftarrow \mathcal{T}(S \cup \{z_b\})$	
$\tilde{b} \leftarrow \mathcal{A}(\mathcal{T}, \mathcal{D}, n, \theta, z_0, z_1)$	

bound on its advantage in this game would also bound the MI advantage of adversaries against the training pipeline. The adversary must distinguish between two simple hypotheses:

- $H_0: \theta \sim \text{Binomial}(n-1,p) + z_0$
- $H_1: \theta \sim \text{Binomial}(n-1,p) + z_1$

When $z_0 = z_1$, these coincide and the advantage of the adversary is 0. Otherwise, without loss of generality, assume $z_b = b$. By the Neyman-Pearson lemma, a likelihood ratio test yields the most powerful test for a significance α (i.e., Type-I error, false positive rate). Let f and F be the probability mass and cumulative distribution function of Binomial(n - 1, p), respectively. The likelihood ratio is

$$\Lambda(\theta = k) = \begin{cases} \infty & \text{if } k = 0\\ 0 & \text{if } k = n\\ \frac{f(k)}{f(k-1)} = \frac{(n-k)p}{k(1-p)} & \text{otherwise} \end{cases}$$

The test rejects H_0 when $\Lambda(\theta) < c$, for some c. The false positive rate α (the probability of rejecting H_0 when H_0 is true) is

$$\begin{aligned} \Pr_{H_0}(\Lambda(\theta) < c) &= \Pr_{H_0}\left(\frac{(n-k)p}{k(1-p)} < c\right) \\ &= \Pr_{H_0}\left(k > \frac{np}{p+c(1-p)}\right) \\ &= 1 - F\left(\frac{np}{p+c(1-p)}\right) \end{aligned}$$

The false negative rate β is

$$\Pr_{H_1}(\Lambda(\theta) \ge c) = \Pr_{H_1}\left(\frac{(n-k)p}{k(1-p)} \ge c\right)$$
$$= \Pr_{H_1}\left(k \le \frac{np}{p+c(1-p)} - 1\right)$$
$$= F\left(\frac{np}{p+c(1-p)} - 1\right)$$

Now, take p = 0.5 and assume that $n \ge 4$ and that n is even so that the mode of Binomial(n-1,p) is n/2. The

MI advantage of the adversary is

$$\begin{aligned} \mathsf{Adv}^{\mathsf{MI}}(\mathcal{A}) &= \frac{1}{2}(f(0) + f(n-1) + (1 - \alpha - \beta)) \\ &= f(0) + \frac{1}{2}f\left(\frac{np}{p + c(1-p)}\right) \\ &\leq \frac{1}{2^{n-1}} + \frac{f(n/2)}{2} \\ &\leq \frac{1}{2\sqrt{n}} + \frac{1}{2\sqrt{n}} = \frac{1}{\sqrt{n}} \end{aligned}$$

On the other hand, a DP distinguisher \mathcal{A} that chooses $z_0 = 0, z_1 = 1$, an arbitrary S, and that guesses $\tilde{b} = \theta - S$, has perfect advantage $Adv^{DPD}(\mathcal{A}) = 1$.

2.3. MI ∠ PI.

Theorem 4.6 (MI \preceq PI). *Resilience against membership inference does not imply resilience against property inference*

Proof. We prove this separation result by presenting a property inference attack against a system that satisfies the membership inference MI property. We set the property of interest to be determining if the dataset consists of all 0 or 1 and the target system to be a counting mechanism.

We first create two distributions $(\mathcal{D}_0, \mathcal{D}_1)$ consisting of 0s and 1s respectively. The challenger flips a coin *b* and samples a dataset with size *M* from the selected distribution \mathcal{D}_b . Next, they release the sum of these numbers and releases it to the adversary. This system satisfies membership inference as all inputs for each distribution are the same, hence it is impossible to identify the membership status of a specific input from \mathcal{D}_b . However, a property inference adversary can trivially win by checking if the result is 0 or *M*, hence a system can be resilient against the membership inference property but vulnerable to the property inference one.

3. Proofs of DPD

3.1. DPD \leq RC.

Theorem 4.3 (DPD \leq RC). For any adversary A_{RC} against data reconstruction, there exists an adversary A_{DPD} against DP distinguisher such that

$$\operatorname{Adv}_{\operatorname{DPD}}(\mathcal{A}_{\operatorname{DPD}}) = 2p - 1$$

where z_0 and z_1 are the two different points in the S_0 and S_1 datasets of the DPD game, ℓ is a loss function that satisfies the triangle inequality, and A_{RC} achieves error $< \ell(z_0, z_1)/2$ with probability p. This theorem considers neighboring datasets with replacement for the DPD game.

Proof. Let A_{RC} be an oracle that can win against the data reconstruction game. We construct an adversary $A_{DPD\rightarrow RC}$ (Game 19) that uses A_{RC} to guess the challenge bit *b* in the DP distinguisher game.

Adversary 19: ADPD BC

Input: $\mathcal{T}, n, \mathcal{D}$	
$S \sim \mathcal{D}^{n-1}$	
$z_0, z_1 \sim \mathcal{D}: z_0 eq z_1$	
$S_0 = \{S \cup z_0\}$	
$S_1 = \{S \cup z_1\}$	
$\theta \leftarrow DPD(S_0, S_1)$	
$\tilde{z} \leftarrow \mathcal{A}_{RC}(\theta, S, \mathcal{T})$	
if $\ell(z_0, \tilde{z}) < \ell(z_1, \tilde{z})$ then	
return 0	
end	
return 1	

Analysis.

$$\ell(z_0, \tilde{z}_0) < \ell(z_0, z_1)/2$$
 (6)

$$\ell(z_0, z_1) \le \ell(z_0, \tilde{z}_0) + \ell(z_1, \tilde{z}_0) (\triangle \text{ inequality})$$
(7)

$$\ell(z_0, z_1) < \ell(z_0, z_1)/2 + \ell(z_1, \tilde{z}_0) (\leftarrow 6)$$
(8)

$$\ell(z_0, z_0) < \ell(z_1, z_0) (\leftarrow 6)$$
(9)

where $\tilde{z}_0 \leftarrow \mathcal{A}_{\mathsf{RC}}(\theta, S, \mathcal{T}) : \theta \leftarrow \mathcal{T}(S_0)$

 $\mathsf{Adv}^{\mathsf{DPD}}$

$$= \Pr \left[\mathcal{A}_{\mathsf{DPD}\to\mathsf{RC}} = 0 | b = 0 \right]$$
(10)
$$- \Pr \left[\mathcal{A}_{\mathsf{DPD}\to\mathsf{RC}} = 0 | b = 1 \right]$$
$$= \Pr \left[\ell(z_0, \tilde{z}) < \ell(z_1, \tilde{z}) | \mathcal{A}_{\mathsf{RC}}(\theta \leftarrow \mathcal{T}(S_0), S, \mathcal{T}) \right]$$
(11)
$$- \Pr \left[\ell(z_0, \tilde{z}) < \ell(z_1, \tilde{z}) | \mathcal{A}_{\mathsf{RC}}(\theta \leftarrow \mathcal{T}(S_1), S, \mathcal{T}) \right]$$
$$= 2p - 1 (\leftarrow 9)$$
(12)

Cost: Single access to RC oracle (A_{RC}). Note: We considering the notion of neighbouring datasets with replacement for this proof.

3.2. DPD \leq MI.

Theorem 4.4 (DPD \leq MI). For any adversary A_{MI} against membership inference, there exists an adversary A_{DPD} against the DP distinguisher such that

$$\mathsf{Adv}_{\mathsf{DPD}}(\mathcal{A}_{\mathsf{DPD}}) = \mathsf{Adv}_{\mathsf{MI}}(\mathcal{A}_{\mathsf{MI}})$$

Proof. Let A_{MI} be an oracle that can win against the membership inference game. We construct an adversary $A_{DPD \rightarrow MI}$ (Game 20) that uses A_{MI} to guess the challenge bit *b* in the DP distinguisher game.

Adversary 20: $\mathcal{A}_{\text{DPD}\rightarrow\text{MI}}$	
Input: $\mathcal{T}, n, \mathcal{D}$	
$S \sim \mathcal{D}^{n-1}$	
$z_0, z_1 \sim \mathcal{D}: z_0 eq z_1$	
$S_0 = \{S \cup z_0\}$	
$S_1 = \{S \cup z_1\}$	
$\theta \leftarrow DPD(S_0, S_1)$	
$ ilde{b} \leftarrow \mathcal{A}_{MI}(z_0, \mathcal{T}, \mathcal{D}, n, heta)$	
return b	

Analysis.

$$\mathsf{Adv}^{\mathsf{DPD}} = \Pr\left[\mathcal{A}_{\mathsf{DPD}\to\mathsf{MI}} = 0|b=0\right]$$
(13)
$$\Pr\left[\mathcal{A}_{\mathsf{DPD}\to\mathsf{MI}} = 0|b=1\right]$$

$$-\Pr\left[\mathcal{A}_{\mathsf{DPD}\to\mathsf{MI}}=0|b=1\right]$$
$$=\Pr\left[\tilde{b}=0|b=0\right]-\Pr\left[\tilde{b}=0|b=1\right] \quad (14)$$

$$= \operatorname{Adv}^{\operatorname{MI}}$$
 (15)

Cost: Single access to MI oracle (A_{MI}).

4. Proofs of PI

4.1. PI ∠ MI.

Theorem 4.9 (PI \preceq MI). Resilience against property inference does not imply resilience against membership inference.

Proof. The separation result in this setting stems from a system that satisfies the property inference property while being vulnerable to a membership inference adversary.

Let \mathcal{D} be some distribution, and two distributions $\mathcal{D}_0, \mathcal{D}_1$ such that every $(z, 0)|z \sim \mathcal{D}$ belongs to \mathcal{D}_0 and every $(z, 1)|z \sim \mathcal{D}$ belongs to \mathcal{D}_1 . Now, consider a simple system that uniformly samples (based on a coin b) a distribution, and outputs the first dimension of a randomly sampled input from it $(z \sim \mathcal{D}_b)$. A membership inference adversary can thus win against this system as it outputs the point, i.e., $Adv_{MI} = 1$. For a property inference adversary, the advantage is clearly zero, since the part of the input output by the system is common to both distributions, and thus indistinguishable. Hence, we have a system that has perfect security against property inference, while being fully vulnerable to the membership inference one.

5. Proofs of RC

Note: All the proofs here assume/use the capability of the adversary to query all points in the target distribution.

5.1. RC $\leq_{1/|\mathcal{D}\setminus S|}$ MI.

Theorem 4.5 (RC $\leq_{1/|\mathcal{D}\setminus S|}$ MI). For any adversary \mathcal{A}_{MI} against membership inference, there exists an adversary \mathcal{A}_{RC} against data reconstruction such that

$$\mathsf{Adv}_{\mathsf{RC}}(\mathcal{A}_{\mathsf{RC}}) \geq \frac{1}{|\mathcal{D} \setminus S|} \cdot \mathsf{Adv}_{\mathsf{MI}}(\mathcal{A}_{\mathsf{MI}})$$

Proof. Let A_{MI} be an oracle that can win against the membership inference attack. We construct the adversary $A_{RC \to MI}$ (Game 21) which uses A_{MI} to reconstruct \tilde{z} and win against the data reconstruction game.

Adversary 21: $A_{RC\toMI}$
Input: θ , S , T ; aux, aux contains the distribution D
$\mathcal{D}' = \mathcal{D} \setminus S$
$ ilde{z}\sim\mathcal{D}'$
n = S
$\tilde{b} \leftarrow \mathcal{A}_{MI}(z, \mathcal{T}, S, n, \theta)$
if $\tilde{b} = 0$ then
return \tilde{z}
else
return ⊥
end

Analysis.

$$\mathsf{Adv}^{\mathsf{RC}} \ge \Pr\left[\tilde{z} = z\right] \cdot \Pr\left[\mathcal{A}_{\mathsf{RC} \to \mathsf{MI}} = \tilde{z}\right]$$
(16)

$$= \frac{1}{|\mathcal{D} \setminus S|} \cdot \Pr\left[\mathcal{A}_{\mathsf{RC} \to \mathsf{MI}} = \tilde{z}\right]$$
(17)

$$= \frac{1}{|\mathcal{D} \setminus S|} \cdot \left(\Pr\left[\mathcal{A}_{\mathsf{MI}} = 0|b=0\right] \right)$$
(18)
$$-\Pr\left[\mathcal{A}_{\mathsf{MI}} = 1|b=0\right] \right)$$

$$=\frac{1}{|\mathcal{D}\setminus S|}\cdot\mathsf{Adv}^{\mathsf{MI}}$$
(19)

Cost: Single access to MI oracle (A_{MI}) .

There exists another – better – adversary that instead of sampling a single point, can iterate over the distribution \mathcal{D}' . However, this adversary comes at a higher cost, i.e., the adversary would need to query the MI oracle for as many inputs till they find a member.