

Over-the-Air Computation: Foundations, Technologies, and Applications

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Abstract—The rapid advancement of artificial intelligence technologies has given rise to diversified intelligent services, which place unprecedented demands on massive connectivity and gigantic data aggregation. However, the scarce radio resources and stringent latency requirement make it challenging to meet these demands. To tackle these challenges, over-the-air computation (AirComp) emerges as a potential technology. Specifically, AirComp seamlessly integrates the communication and computation procedures through the superposition property of multiple-access channels, which yields a revolutionary multiple-access paradigm shift from “compute-after-communicate” to “compute-when-communicate”. Meanwhile, low-latency and spectral-efficient wireless data aggregation can be achieved via AirComp by allowing multiple devices to access the wireless channels non-orthogonally. In this paper, we aim to present the recent advancement of AirComp in terms of foundations, technologies, and applications. The mathematical form and communication design are introduced as the foundations of AirComp, and the critical issues of AirComp over different network architectures are then discussed along with the review of existing literature. The technologies employed for the analysis and optimization on AirComp are reviewed from the information theory and signal processing perspectives. Moreover, we present the existing studies that tackle the practical implementation issues in AirComp systems, and elaborate the applications of AirComp in Internet of Things and edge intelligent networks. Finally, potential research directions are highlighted to motivate the future development of AirComp.

Index Terms—Over-the-air computation, massive connectivity, wireless data aggregation, integrated communication and computation.

I. INTRODUCTION

While the fifth-generation (5G) wireless networks are being deployed worldwide, the preliminary outlook for the next generation of networks, referred to as the sixth-generation (6G) wireless communication systems, has been initiated by both academia and industry. Since 6G is still in its infancy, a number of envisioned applications, system requirements,

and innovative techniques are spotlighted and discussed in a growing body of works [1]–[5]. A prevailing view among these works is that 6G will propel the transition of wireless networks from the Internet of Everything (IoE) to the Intelligent Internet of Everything (IIoE), which gives rise to numerous intelligent services with the advancement of emerging artificial intelligence (AI) techniques, especially deep learning and reinforcement learning. Meanwhile, massive connectivity and tremendous information exchange are imperative to fulfill the needs of data analysis and model training for intelligent services, which, however, poses significant challenges to current resource-constrained wireless communication systems. Accordingly, efficient and scalable multiple-access protocols are required to enable ubiquitous connections for rapid growing devices.

In retrospect, the increasing demand for wireless connections has stimulated the rapid development of multiple-access strategies over the previous decades. The existing multiple-access strategies can generally be divided into two categories, i.e., *orthogonal* and *non-orthogonal* schemes. Specifically, orthogonal multiple access (OMA) schemes entitle each of the devices to have an exclusive occupation of specifically allocated resources, which leads to interference-free transmission and results in a simple transceiver design. Depending on the allocated orthogonal domains, several OMA schemes have been extensively adopted in current wireless communication systems, including frequency-division multiple access (FDMA), time-division multiple access (TDMA), code-division multiple access (CDMA) [6], and orthogonal frequency-division multiple access (OFDMA) [7]. Since the available resources are divided into orthogonal blocks, OMA strictly limits the total number of simultaneously scheduled devices under limited radio resources. To circumvent such a restriction, non-orthogonal multiple access (NOMA) schemes become potential candidates for future massive access networks [8], such as power-domain NOMA [9]–[18], code-domain NOMA [19], space-division multiple access (SDMA) [20], and rate-splitting multiple access (RSMA) [21], [22], in which one resource block can be shared for simultaneous transmissions of multiple devices at the cost of introducing co-channel interference. Several techniques have been proposed for tackling the interference caused by overlapped resource allocation, such as successive interference cancellation (SIC), message passing, and multi-antenna beamforming. Assisted by these interference cancellation methods, NOMA is capable of yielding significant gains in massive connectivity, spectral

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efficiency, and communication latency at the cost of complexifying the transceiver design [8], [19].

The aforementioned multiple-access strategies are primarily designed according to the Shannon-Hartley theorem [23], with an objective to achieve reliable data transmission. As these protocols disregard the concrete tasks for which the data is used, they can be deemed as *task-agnostic* multiple access, where the transmission and use of data are treated as two separate and independent parts [24]. In practice, supporting the tasks of intelligent services usually demands the functional computation results of dispersed data for the final decision, such as climate early-warning by sensing environmental indicators, autonomous control by accessing the states of different agents, and model refreshment by aggregating distributed local training updates. Accordingly, the functional computation can be perceived as establishing a connection between the task and the dispersed data, which stimulates the exploration of *task-oriented* multiple access by integrating the communication and computation during wireless transmission. Besides, low communication latency is another essential requirement in intelligent services to guarantee the quality and timeliness of the corresponding applications, which motivates the low-latency design of future multiple-access strategies. *Over-the-air computation* (AirComp) becomes a promising candidate to realize such an integration and enable efficient wireless data aggregation (WDA) with massive connectivity [24].

The kernel of AirComp is to achieve the integration of communication and computation via the waveform superposition property of multiple-access channels (MACs) [25], where the concurrent signals over the same radio channel can be naturally aggregated over the air. Unlike conventional multiple-access schemes considering co-channel interference as a disruptor of wireless transmission, AirComp treats the interference as a contributor to the functional computation and reduces the complexity of interference cancellation for decoding each of the data, thereby achieving high spectral efficiency and low communication latency. The idea of data fusion with the waveform superposition property has been previously adopted to tackle the chief executive officer (CEO) problem [26], which aims at estimating a remote common source from multiple noisy observations in wireless sensor networks. Furthermore, the authors in [27] and [28] demonstrate that reconstructing the source information from superimposed uncoded signals achieves an optimal scaling-law for the CEO problem in a Gaussian sensor network. In contrast to the CEO problem that intends to realize a single-source estimation, AirComp desires to achieve a functional estimation/computation of the data from multiple sources, which is fundamentally different from the CEO problem in terms of the problem objective and the system architecture.

The study on AirComp is initiated by the seminal work [25], which pioneers the use of computation coding for reliable functional computation over MACs. Different from conventional information-theoretic coding strategies, computation coding does not force the transmit signals to be represented in digital bits [25], leading to the subsequent development of AirComp being divided into two branches, i.e., *coded (digital)*

and *uncoded (analog)* AirComp¹. Meanwhile, two novel performance metrics, namely *computation rate* and *computation accuracy*, are introduced to measure the reliability of the function reconstruction for AirComp systems, which respectively evaluate the maximum number of functions calculated per channel use and the distortion between the estimated and the desired function values. Under the guidance of these two metrics, several analyses and transceiver design for AirComp over multifarious network architectures have been developed recently. In the meantime, the unprecedented growth of Internet of Things (IoT) and edge intelligent networks prompts a growing enthusiasm for AirComp, motivating the researchers to further explore its implementation in emerging applications and potential in driving the evolution of wireless networks. Consequently, the research on AirComp has become active and numerous remarkable studies have recently emerged.

A. Contributions

In this paper, for the first time in literature, we provide a holistic and systematic survey of existing studies on AirComp, while discussing the existing challenges and envisioning future research directions. The main contributions of this paper are summarized as follows:

- Foundations of AirComp are elaborated in terms of the mathematical form and communication design, which establish the theoretical bases of AirComp and demonstrate the feasibility of AirComp in wireless communications.
- Existing works on AirComp over different network architectures are comprehensively reviewed and the major issues in each network architecture are discussed to highlight future research opportunities.
- The analysis and optimization for AirComp are outlined from the information theory and signal processing perspectives, where the system performance and transceiver design are presented in accordance with different network settings.
- Practical implementation issues of AirComp are discussed along with the solutions developed by existing studies, which illustrate the confronting challenges when moving from theoretical research to practical applications.
- Advantages of AirComp when applied to IoT and edge intelligent networks are introduced, and future research directions are presented to further promote the development of AirComp.

B. Organization and Notations

The structure of this paper is outlined in Fig. 1. Section II introduces the basics of AirComp, including the mathematical form and communication design. Section III presents a literature review on AirComp over different network architectures. The analysis and design from information theory and signal processing perspectives for AirComp are highlighted in

¹In the rest of this paper, coded AirComp and digital AirComp are interchangeable, and uncoded AirComp and analog AirComp are interchangeable as well.

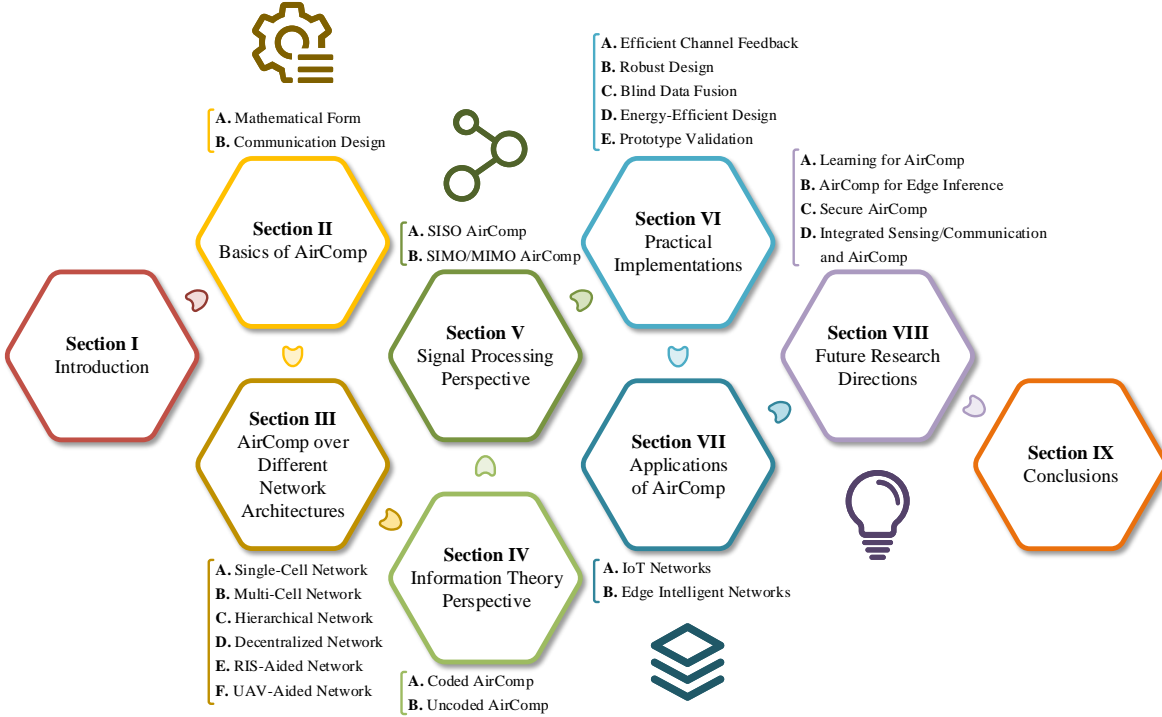


Fig. 1. Organization of the paper.

Section IV and Section V, respectively. Practical implementation issues are discussed in Section VI, and applications of AirComp in IoT and edge intelligent networks are presented in Section VII. Section VIII illustrates the potential research topics of AirComp. Finally, this paper is concluded in Section IX.

Notations: Italic and bold lower-case letters denote scalars and column vectors, respectively. \mathbb{R} , \mathbb{C} , and \mathbb{N}_+ denote real, complex, and positive integer domains, respectively. $\mathbb{P}[\cdot]$ is the statistical probability and $\mathbb{E}[\cdot]$ represents the statistical expectation. $(\cdot)^T$ denotes the transpose and $|\cdot|$ returns the absolute value of a scalar.

II. BASICS OF AIRCOMP

In this section, we present the basics of AirComp from the perspectives of *mathematical form* and *communication design*, which reveal the unique features and benefits of AirComp.

A. Mathematical Form

The main purpose of AirComp is to merge the concurrently transmitted data to compute a class of so-called nomographic functions, whose definition is given in the following.

Definition 1 (Nomographic Function [29], [30]). *A function $f(\cdot) : \mathbb{R}^K \rightarrow \mathbb{R}$ that can be represented in the form of*

$$f(x_1, x_2, \dots, x_K) = \psi \left(\sum_{k=1}^K \varphi_k(x_k) \right) \quad (1)$$

is called a nomographic function, where $\psi(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$ denotes the post-processing function at the receiver and $\varphi_k(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$

denotes the pre-processing function for data $x_k \in \mathbb{R}$ at device k , $\forall k \in \{1, 2, \dots, K\}$.

According to (1), the data generated at different devices can go through three procedures to enable the computation of a specific nomographic function at the receiver [31]: 1) pre-processing of data x_k at device k via function $\varphi_k(\cdot)$; 2) summation of pre-processed data $\varphi_k(x_k)$ via the waveform superposition property of MACs; and 3) post-processing of the aggregated data at the receiver via function $\psi(\cdot)$. Although the waveform superposition is a summation procedure, applying particularly designed data pre-processing functions at the transmitters and post-processing functions at the receiver enables AirComp to go beyond the computation of simple linear functions (e.g., averaging and summation) [29], [32]. Examples of computable nomographic functions via AirComp are presented in Table I.

Moreover, the authors in [33], [34] demonstrate that $2K + 1$ nomographic functions are sufficient to represent any continuous function of K variables, which can be summarized as follows.

Theorem 1 (Nomographic Representation [30], [33], [34]). *Every continuous function $\bar{f}(\cdot) : \mathbb{R}^K \rightarrow \mathbb{R}$ can be represented as the summation of no more than $2K + 1$ nomographic functions, i.e.,*

$$\begin{aligned} \bar{f}(x_1, x_2, \dots, x_K) &= \sum_{j=1}^{2K+1} f_j(x_1, x_2, \dots, x_K) \\ &= \sum_{j=1}^{2K+1} \psi_j \left(\sum_{k=1}^K \varphi_{j,k}(x_k) \right) \end{aligned} \quad (2)$$

TABLE I
EXAMPLES OF COMPUTABLE NOMOGRAPHIC FUNCTIONS VIA AIRCOMP

Name	Pre-processing: $\varphi_k(x_k)$	Post-processing: $\psi(y)$	Target function: $f(x_1, x_2, \dots, x_K)$
Arithmetic mean	x_k	y/K	$(1/K) \sum_{k=1}^K x_k$
Weighted sum	$\alpha_k x_k$	$y / (\sum_{k=1}^K \alpha_k)$	$\sum_{k=1}^K \alpha_k x_k / (\sum_{k=1}^K \alpha_k)$
Geometric mean	$\ln(x_k)$	$\exp(y/K)$	$(\prod_{k=1}^K x_k)^{1/K}$
Euclidean norm	x_k^2	$y^{1/2}$	$\sqrt{\sum_{k=1}^K x_k^2}$
Polynomial	$\alpha_k x_k^{\beta_k}$	y	$\sum_{k=1}^K \alpha_k x_k^{\beta_k}$

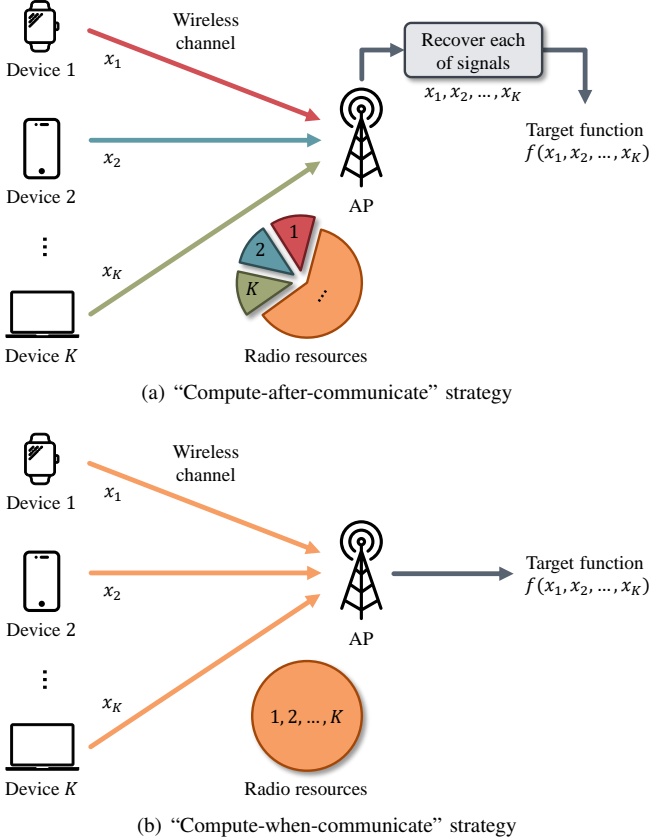


Fig. 2. Illustration of multiple-access strategies in wireless networks.

where both pre-processing functions $\varphi_{j,k}(\cdot)$ and post-processing functions $\psi_j(\cdot)$ are continuous, $\forall j \in \{1, 2, \dots, 2K + 1\}$, $\forall k \in \{1, 2, \dots, K\}$.

Theorem 1 reveals that any continuous function with K variables is computable via AirComp by utilizing at most $2K + 1$ channel uses with appropriately designed continuous pre- and post-processing functions. This further consolidates the mathematical foundation of AirComp and makes it applicable to compute various desired functions in different practical scenarios.

B. Communication Design

In 1940s, the Shannon's landmark work [23] presented a systematic analysis for point-to-point communications and proposed the Shannon-Hartley theorem for measuring the reliability of wireless data transmission. Under this guidance,

various communication schemes have been developed to guarantee the accurate reception of each message before further processing. In particular, each device that has data to transmit is assigned an orthogonal resource block. After successfully decoding the individual signal of each device, the access point (AP), serving as the fusion center (FC), can compute desired functions for further processing, which follows the principle of "compute-after-communicate", as shown in Fig. 2(a). Nevertheless, a vast number of emerging applications in wireless networks are interested in achieving efficient function computation of dispersed data generated by multiple devices, e.g., the arithmetic mean of sensing data in monitoring systems [35] and the weighted-sum of local updates in distributed learning systems [36]. In such scenarios, separated computation and communication may lead to low spectral efficiency and large communication delay, especially for large-scale wireless networks consisting of massive devices. Besides, recovering each data also causes high computational complexity at the receiver, which in turn increases the energy consumption and hardware implementation complexity.

AirComp addresses the aforementioned issues by adopting the "compute-when-communicate" strategy, as shown in Fig. 2(b). Instead of treating other concurrently transmitted signals as interference, AirComp aims to harness the co-channel interference to achieve the desired function computation without decoding each of the signals as in conventional multiple-access schemes. Specifically, as the concurrent signals occupy the same radio channel for data transmission, the number of required radio resources is independent of the number of devices, which achieves a much higher spectral efficiency than OMA schemes. Besides, since the signals concurrently transmitted by synchronized devices can be naturally superimposed over MACs, the AP is able to directly obtain the desired nomographic function from the received signal within one time slot in a symbol level, while saving the time for one-by-one decoding. Although NOMA schemes also allow multiple devices to access the same radio channel for simultaneous transmission as AirComp, the receiver needs to implement the interference cancellation techniques to reliably recover each of the received signals for maximizing the classic transmission rate, while AirComp directly processes the superposed signals for minimizing the distortion of function computation. In addition, by dividing the target computation task $f(\cdot)$ into $K + 1$ distributed tasks $\{\psi(\cdot), \varphi_1(\cdot), \dots, \varphi_K(\cdot)\}$ according to (1), the AP and each device merely need to perform lightweight signal processing, which effectively reduces the

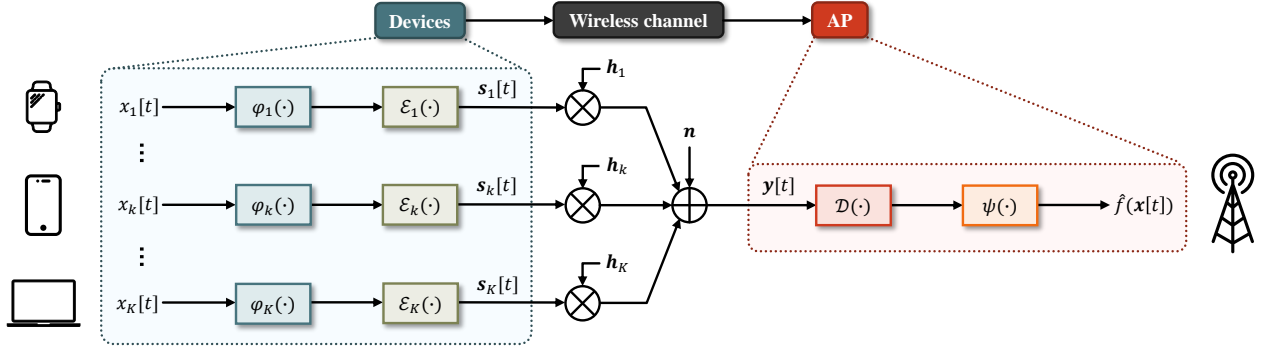


Fig. 3. Workflow of AirComp. Herein, $x_k[t] \in \mathbb{R}$ denotes the data generated at device $k \in \{1, 2, \dots, K\}$, $\mathcal{E}_k(\cdot) : \mathbb{R} \rightarrow \mathbb{C}^{\bar{L}}, \forall \bar{L} \in \mathbb{N}_+$, denotes the encoder of device k that converts the pre-processed data, $\varphi_k(x_k[t])$, into an \bar{L} -length channel input signal $\mathbf{s}_k[t] = [s_{k,(t-1)\bar{L}+1}, \dots, s_{k,t\bar{L}}]^T$, and $\mathcal{D}(\cdot) : \mathbb{C}^{\bar{L}} \rightarrow \mathbb{R}$ represents the corresponding decoder. Besides, \otimes realizes the element-wise multiplication, $\mathbf{h}_k \in \mathbb{C}^{\bar{L}}$ denotes the channel coefficient vector, $\mathbf{n} \in \mathbb{C}^{\bar{L}}$ is the additive noise, and $\mathbf{y}[t]$ represents the superimposed signal received at the AP.

computational complexity at the AP [37]. Note that, if a target function is decomposed into more than N different basic nomographic functions as in (2), AirComp becomes spectral-inefficient as compared with conventional multiple-access schemes due to more channel uses being required to accomplish the function computation [24]. The workflow of AirComp is illustrated in Fig. 3. With the aforementioned key differences from the conventional multiple-access schemes, we shall discuss three important aspects of AirComp that are critical for communication design.

1) *Taxonomy*: To achieve the desired function computation over MACs, AirComp can be implemented via both *coded* and *uncoded* communications depending on the adopted modulation strategy, which are elaborated as follows.

- **Coded AirComp**: Coding is an essential method for mitigating the signal distortion for conventional point-to-point communications, and is able to provide reliable transmissions by following the Shannon-Hartley theorem. Different from conventional coding strategies being used to combat interference, coded AirComp allows multiple devices to simultaneously transmit signals relying on the code with a linear structure, e.g., nested lattice code [38], which enables the receiver to reliably recover linear functions from the integer combinations of transmitted codewords [39]. Besides, an one-bit over-the-air aggregation method based on the signSGD algorithm [40] has recently been proposed in [41], which applies one-bit quantization at transmitters to simply represent the data via sign-taking operation and adopts a majority-voting based decoder at the receiver for estimating the sign of superimposed signals. By employing the above schemes, coded AirComp is able to combat the channel noise but at the expense of reducing the computation accuracy due to the quantization error. In addition, due to the intrinsic digital modulation, coded AirComp can be directly embedded into modern digital communication systems and integrated into the existing standards, e.g., 4G Long-Term Evolution (LTE) and 5G New Radio (NR).
- **Uncoded AirComp**: In most of the recent studies, uncoded AirComp is a more popular paradigm than coded

AirComp. It has been demonstrated that the uncoded transmission is optimal for computing the sum of Gaussian sources over a Gaussian MAC when the channel bandwidth matches the need for the number of sources [25], but may perform worse than the coded AirComp with highly correlated Gaussian sources [42]. Since the signals are simply scaled at the transmitters and the receiver, uncoded AirComp can significantly reduce the complexity of the encoding and decoding operations compared with coded AirComp. By synchronizing different devices, the concurrently transmitted signals over the same radio channel can be superimposed over the air with the weights being proportional to their corresponding channel coefficients. However, as the signals are directly exposed to wireless channels, uncoded AirComp is more fragile than coded AirComp, leading to unavoidable signal distortion due to the channel fading and additive noise. Therefore, it is critical for uncoded AirComp to implement an appropriate transceiver design to reduce the signal distortion and achieve the magnitude alignment at the receiver, thereby realizing the data aggregation with desired coefficients.

The examples of coded and uncoded AirComp with two devices are shown in Figs. 4(a) and 4(b), respectively, which illustrate the superimposed signals respectively obtained by digital and analog modulation strategies in AirComp systems.

2) *Performance Metrics*: Since AirComp aims to achieve efficient functional computation rather than the reliable transmission of each data, the computation rate is regarded as an important metric to evaluate the performance of AirComp systems [25]. The definition of computation rate is given in the following.

Definition 2 (Computation Rate [30], [43]). *Let an (f, T, L) computation code for an MAC consist of*

- A target function $f(\cdot) : \mathbb{R}^K \rightarrow \mathbb{R}$.
- An encoder at each of the K transmitters to map T generated data $\{x_k[t]\}$ to L channel input symbols $\{s_{k,\ell}\}$ such that

$$(x_k[1], x_k[2], \dots, x_k[T]) \mapsto (s_{k,1}, s_{k,2}, \dots, s_{k,L}),$$

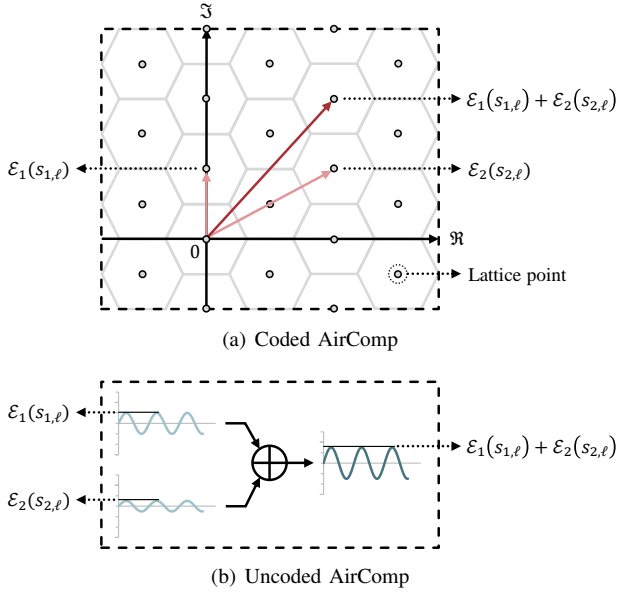


Fig. 4. Examples of coded and uncoded AirComp with two devices.

$$\forall k \in \{1, 2, \dots, K\}. \quad (3)$$

- A decoder at the receiver to obtain T estimates of the target function based on L channel output symbols $\{y_\ell\}$ such that

$$(y_1, y_2, \dots, y_L) \mapsto (\hat{f}(\mathbf{x}[1]), \hat{f}(\mathbf{x}[2]), \dots, \hat{f}(\mathbf{x}[T])) \quad (4)$$

$$\text{with } \mathbf{x}[t] = [x_1[t], x_2[t], \dots, x_K[t]]^\top.$$

Then, given an arbitrary but a fixed computation error requirement, $\epsilon > 0$, computation rate R_C , specifying the number of computed function values per channel use, is achievable if, for every rate

$$R = \frac{T}{L} = \frac{1}{L} \leq R_C \quad (5)$$

and $\delta \in (0, 1)$, there exists a sequence of (f, T, L) computation codes such that, for sufficiently large L ,

$$\mathbb{P} \left[\bigcup_{t=1}^T \left\{ \left| \hat{f}(\mathbf{x}[t]) - f(\mathbf{x}[t]) \right| \geq \epsilon \right\} \right] \leq \delta \quad (\text{Coded AirComp}) \quad (6)$$

or

$$\mathbb{E} \left[\left| \hat{f}(\mathbf{x}[t]) - f(\mathbf{x}[t]) \right|^2 \right] \leq \epsilon, \forall t \quad (\text{Uncoded AirComp}). \quad (7)$$

Compared to the maximum reliable transmission rate that quantifies the bits successfully transmitted per second, the computation rate specifies the maximum number of functions that can be computed with a single channel usage under a desired computation accuracy, regardless of the reliability of individual signal transmissions. Note that the computation code given in Definition 2 represents a signal mapping relationship, which can be either a digital coding that encodes the signal into multiple binary bits or an analog processing

that transmits a power-scaling signal over multiple channel blocks. In addition, the computation accuracy is another critical performance metric and is generally measured by the mean-squared error (MSE) given in the left hand side of (7), which quantifies the distortion between the estimated and the ground-truth values of the target function.

With the above performance metrics, various analyses and designs have been proposed to evaluate and optimize the performance of AirComp under different system setups, which will be elaborated in following sections.

3) *Synchronization*: Precise time and frequency synchronization among multiple devices are critical for ensuring that the signals transmitted from multiple devices can simultaneously arrive at the receiver, thereby realizing the accurate functional computation over the air. Fortunately, various well-studied synchronization techniques can be adopted to achieve synchronization for AirComp. For instance, the timing advance technique is commonly used in 4G LTE and 5G NR to achieve time synchronization by controlling the uplink transmission timing of each device [44]. Specifically, each device first evaluates its propagation delay through the timestamps of the reference time broadcast by the AP, and then compensates for the corresponding delay by advancing or retarding its uplink transmission. This ensures that the signals transmitted from all devices can simultaneously arrive at the AP. On the other hand, frequency synchronization is also required in AirComp to eliminate carrier frequency offset (CFO) in concurrently transmitted signals, which is critical to combine the signals in a desired manner. To this end, a primitive method, named AirShare, is proposed in [45] to realize distributed coherent transmissions by sharing a reference-clock with multiple devices to eliminate the CFO among them. Consequently, the above synchronization techniques enable AirComp to be implemented in practical wireless networks.

III. AIRCOMP OVER DIFFERENT NETWORK ARCHITECTURES

Nowadays, diverse applications impose different requirements on the functional computation, such as low-latency, multi-target, and multi-tiered computation, leading to the exploration of different network architectures to tackle these issues. In this section, we review the existing studies on AirComp over various wireless network architectures, including *single-cell*, *multi-cell*, *hierarchical*, *decentralized*, *reconfigurable intelligent surface (RIS)-aided*, and *unmanned aerial vehicle (UAV)-aided* networks, and discuss the major issues therein.

A. Single-Cell Network

For AirComp over canonical single-cell networks, the AP aims to aggregate the signals concurrently transmitted from multiple devices, where the intra-cell interference is harnessed to compute the desired function through the waveform superposition over MACs, as shown in Fig. 5. Accordingly, how to effectively exploit such intra-cell interference to achieve accurate and efficient functional computation becomes the main focus of the research on single-cell AirComp. To this

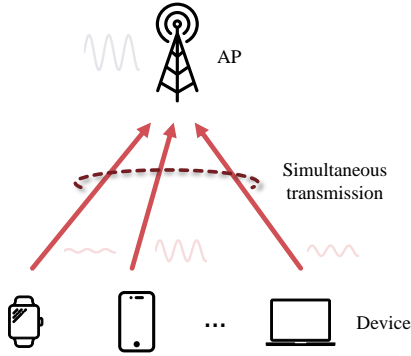


Fig. 5. AirComp over a single-cell network.

end, digital linear-structured codes and analog transceiver design are developed to achieve desired functional computation with coded and uncoded AirComp, respectively. For instance, linear-structured coding strategy are considered in [25], [30], [39], [46]–[48], where the achievable computation rates of coded AirComp are analyzed under different channel models, such as discrete linear [25], Gaussian [25], [39], constant [30], and fading [46], [47] MACs. To superimpose signals over the air with desired coefficients in uncoded AirComp, transceiver design for various scenarios is studied to align the magnitudes and phase shifts of different signals, such as optimal power control for single-input single-output (SISO) systems [37], [49], [50] and uniform-forcing design for single-input/multiple-input multiple-output (SIMO/MIMO) systems [31], [51]–[53]. Instead of considering narrow-band transmissions, wide-band AirComp is investigated in [54]–[57], where the target function consists of multiple sub-functions computed in each of subchannels. The authors in [58] and [59] also study the design for AirComp with correlated channels and signals, respectively.

On the other hand, instead of merely focusing on stand-alone AirComp, the co-existence of AirComp and traditional communication services needs to be considered in practical scenarios, where multiple heterogeneous services are supported in a single network over the same radio channel. In such systems, only partial intra-cell interference can be harnessed to compute the desired function via AirComp and other interference should be reduced while satisfying the transmission requirements of other devices, which underscores the importance of interference management for such situation. To tackle this issue, the authors in [60] propose a joint transceiver design for the AirComp and cellular co-existence communication system, which minimizes the distortion of AirComp while satisfying the signal-to-interference-plus-noise-ratio (SINR) requirements of traditional cellular wireless communications. Besides, the authors in [61] develop a power control strategy to balance the performance of AirComp and NOMA that are co-existed in a single-cell network. Nevertheless, the system design for co-existed AirComp and traditional wireless communications is still in an early phase of research, which needs to be further explored for future wireless networks with diverse service requirements.

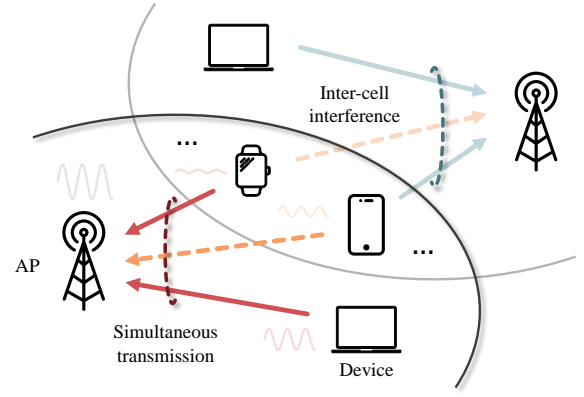


Fig. 6. AirComp over a multi-cell network.

B. Multi-Cell Network

Benefiting from high spectral efficiency and low communication latency, AirComp is promising to enable a wide range of data aggregation in large-scale wireless networks, which necessitates the multi-cell network to capacitate extensive connections while supporting the computation of different target functions characterized by different services, as shown in Fig. 6. Similar to the co-existence of multiple services in single-cell networks, the multi-cell setting requires interference management for dealing with multiple co-existed AirComp tasks in different cells, such that utilizing intra-cell interference within each cell to support function computation and alleviating inter-cell interference among different cells to reduce the computation distortion.

Interference management for conventional multi-cell wireless networks has been studied for many years [62]–[65]. An intuitive strategy is to avoid interference through orthogonal resource allocation, which has been exploited in [30] for the multi-cluster functional computation by activating each cluster in a time-division manner. Besides, among the emerging interference management techniques, interference alignment (IA) is a promising one to realize high transmission rates in interference channels by dividing the channel space into two subspaces for distinguishing the interference and desired signals [62]. Motivated by this, the authors in [66] present a simultaneous signal-and-interference alignment (SIA) scheme for a two-cell AirComp system. The SIA scheme requires two types of precoding, i.e., the IA precoding to restrict the inter-cell interference to the interference subspace and the signal alignment precoding to inverse the effective channel within the signal space. Unlike the conventional IA scheme where the space partitioning is related to the number of devices [62], the SIA scheme performs a symmetric subspace division regardless of the number of devices in multi-cell AirComp systems [66]. Another effective method for interference management is to optimize the weighted-sum multi-cell performance, such as maximizing the weighted-sum rate [64] and minimizing the weighted-sum transmit power [65] of multi-cell networks. Likewise, the authors in [67] present a cooperative power control scheme to minimize the weighted-sum MSE in a multi-cell AirComp system. In particular, the

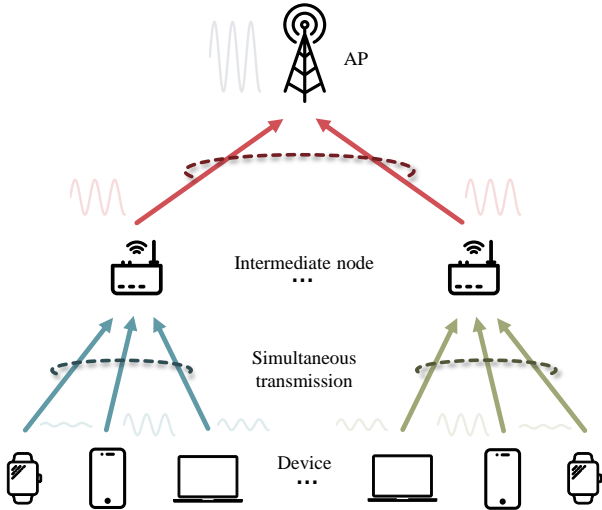


Fig. 7. AirComp over a hierarchical network.

Pareto boundary of the multi-cell MSE region is developed to quantify the performance trade-off among different cells, based on which a weighted-sum MSE minimization problem is formulated under a centralized scenario and can be solved via the profiling technique [68]. Subsequently, a distributed optimization method is presented in [67] to further reduce the communication overhead, in which the multi-cell power control problem reduces to separated single-cell optimizations by introducing interference temperature constraints that limits the inter-cell interference from one cell to another one. Moreover, the authors in [69] study the interference management for multi-cell AirComp with multi-antenna FCs.

The studies in [66], [67], [69] all assume a precise synchronization for the devices in all cells, which, however, is quite challenging to be achieved in a large-scale wireless network. To overcome this drawback, the authors in [70] consider the majority-voting based method in a multi-cell AirComp system to relieve the requirement on precise synchronization, where the aggregated data is determined by the sign of energy difference detected on two subchannels, rather than the specific amplitude and the phase of received signals. Although the majority-vote based scheme works well in federated learning (FL) scenario [41], this approach is difficult to extend to general scenarios that demand for accurate data aggregation. So far, it is still an open issue to study how to achieve accurate function computation in multi-cell AirComp systems with imperfect synchronization.

C. Hierarchical Network

As stated in the previous subsection, multi-cell network is a remedy to tackle multiple co-existed services in large-scale networks. If a common task needs to be accomplished in a large-scale network, the hierarchical network with a multi-hop topology, as shown in Fig. 7, can be leveraged to address the challenges posed by single-hop networks with direct communications, such as the requirement for precise synchronization of all devices and the compensation for severe propagation loss caused by long distance. In such a hierarchical network,

intermediate nodes are required to be deployed between the devices and the AP to ensure the quality of the received signal via amplify/decode-and-forward (AF/DF) operations [71].

Recently, several works have exploited the hierarchical architecture to promote the performance of AirComp in large-scale networks [72]–[81]. Specifically, the authors in [72] develop a two-phase AF relaying protocol for hierarchical AirComp systems, where the first-phase data aggregation from the devices to the relays and second-phase data aggregation from the relays to the AP are both realized via AirComp. Instead of exploiting all relays deployed in the network, the authors in [73] propose a relay selection scheme based on the source-relay and relay-destination channel conditions, and the corresponding MSE outage probabilities are further derived under SISO and SIMO scenarios. To realize energy-efficient hierarchical AirComp, the authors in [74] and [75] propose to only select the devices that are located far away from the AP to leverage the relay to forward their data, while the others conduct direct communications with the AP. This scheduling method achieves an accurate function computation with less energy consumption as compared with the scenario that all devices exploit the relay to forward data. Besides, the authors in [76] and [77] resort to the cloud radio access network (C-RAN) for hierarchical AirComp, where the devices first transmit signals to distributed remote radio heads (RRHs) via AirComp and the RRHs then forward the aggregated data to the baseband unit (BBU) through capacity-limited fronthaul links. In addition, the authors in [78] and [79] study the relay-assisted AirComp by considering both direct and relay links, which demonstrates that additional performance gains can be obtained from direct links. On the other hand, the authors in [80] propose a multi-layer AirComp system for composite function computation by dividing nodes into groups and further into sub-groups in each layer. Herein, the source nodes in each sub-group upload their data to the destination node in the next layer to compute a sub-group function through AirComp, while, for different sub-groups in one group, they forward sub-function values to the same destination node in orthogonal time slots. Moreover, the authors in [81] consider a device-to-device (D2D) based multi-hop AirComp to support a MapReduce framework [82]. Since the D2D network is taken into account, the devices can be designated as the intermediate nodes of surrounding areas during multi-hop transmissions.

Meanwhile, one of the notable issues in relay-assisted hierarchical networks for uncoded AirComp is that the communication error induced at each relay in the first phase can be amplified together with the signals in the second phase, which makes it possible to obtain worse computation results as compared to the single-hop AirComp. With this in mind, the authors in [83] make a preliminary analysis on a hierarchical AirComp system with a single AF relay. The analysis suggests that a more accurate function computation can be obtained than conventional single-hop network only when the channel condition of the worst device-relay link is no worse than that of the worst device-AP link, as well as there are sufficient relay transmit power and/or good relay-AP channel. Such performance analyses for more complicated hierarchical AirComp systems need to be further investigated,

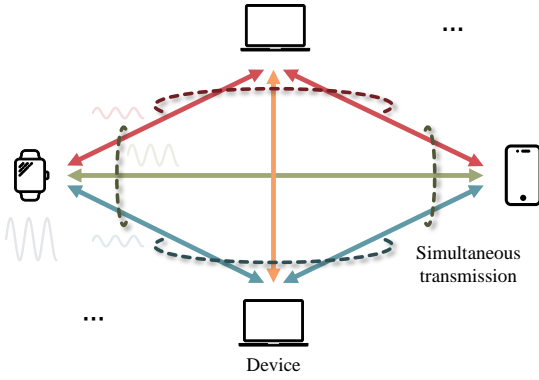


Fig. 8. AirComp over a decentralized network.

so as to provide the deployment guidance for relays or other types of intermediate nodes in practical systems.

D. Decentralized Network

Heretofore, the aforementioned literature on AirComp is studied in AP-centric networks, which relies on one specific receiver to aggregate the data from all devices. However, in some application scenarios, e.g., cooperative autonomous vehicles and swarm robotics, the behaviors of devices are characterized by time-sensitive functions, which require the corresponding functional computation to be finished with ultra-low latency [84]. This makes it much intractable to rely on a centralized server to coordinate massive dispersed devices in the above scenarios. Besides, the performance of an AP-centric system is generally hampered by the straggler devices with poor communication resources, which prolongs the communication latency and reduces the transmission reliability. Therefore, a decentralized system with D2D communications is preferred to tackle these issues, which enables devices to communicate with their neighbors at high rates with low energy consumption [85]–[87].

By deploying AirComp in decentralized networks, as shown in Fig. 8, we can realize efficient local observation gathering among adjacent devices for several applications, such as data consensus [88]–[95] and decentralized FL [96]–[98]. Specifically, the authors in [88] leverage AirComp to accelerate the gossip algorithm for average-based consensus, where each device is allowed to compute an average of multiple neighboring observations at once. A multi-cluster scenario for average consensus is further considered in [89], which is then extended to the consensus problem for arbitrary functions in [90]. Instead of assuming ideal/known channel coefficients, the authors in [91] develop an AirComp-assisted consensus protocol in a blind manner without accessing the channel conditions, which is then applied for the consensus-based formation control [93]. The authors in [94] develop a consensus-based privacy preserving strategy to distributively solve linear equations. Besides, the max-consensus problem with AirComp is studied in [92] and [95] under ideal and fading channels, respectively. Since FL can be regarded as a model consensus problem, decentralized AirComp is leveraged in [96]–[98] to facilitate the consensus procedures under lim-

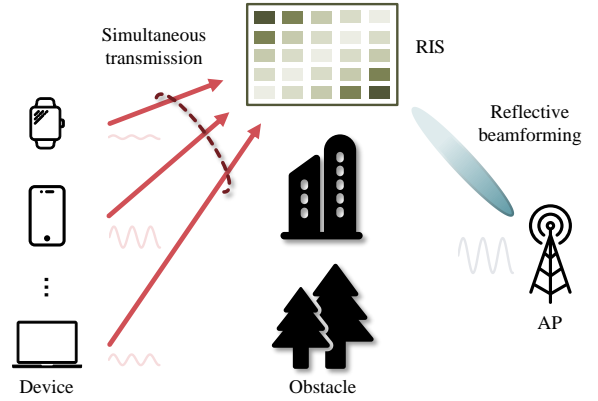


Fig. 9. AirComp over an RIS-aided network.

ited radio resources. Moreover, the authors in [99] propose a distributed AirComp framework to achieve efficient distributed optimization, which significantly reduces the communication latency with massive dispersed devices.

Although AirComp enables fast data aggregation by exploiting the intra-cell interference, the computation result may be deteriorated by the signals from non-neighboring devices that are severely distorted by the long-distance transmission, which should be avoided in decentralized AirComp. Except for considering the case where a device can receive signals from all other devices, the issue of non-neighboring interference in the existing studies is generally addressed by allocating different local computation task to orthogonal radio resources, leading to the computation efficiency being determined by the neighborhood size under limited radio resources. Hence, there is a trade-off between the computation efficiency and accuracy in decentralized AirComp, which is an open issue to be studied in the future. In addition, due to the device mobility, the network topology and neighboring relationships may change during the process of data consensus, whose impact on system performance also needs further investigation.

E. RIS-Aided Network

The uncontrollable propagation environment is deemed as an unintentional adversary to wireless communications, and only the transceiver design at end nodes can be optimized to mitigate the adverse impact caused by the environment, e.g., channel deep fading [100]. Besides, the radio propagation is usually blocked by the obstacles like buildings and hills, which leads to significant energy decaying on signals, especially in high frequencies, e.g., millimeter and terahertz waves [101], [102]. To overcome these challenges, RIS emerges as a cost-effective technique for enhancing the reliability and efficiency of wireless communications via reconfiguring the wave propagation environment [103]–[112]. An RIS typically consists of several passive reflecting elements, which are capable of adjusting the phases of incident signals through an embedded controller, thereby generating a reflective beamforming for boosting the received signal power.

Inspired by this new communication paradigm, RIS-aided AirComp, as shown in Fig. 9, has been studied in several

works to further ameliorate the performance of AirComp. Specifically, as the simulation results shown in [113]–[115], the computation accuracy of AirComp can be significantly increased under the assistance of an RIS. Then, the authors in [116] and [117] present a double-RIS design to further extend the coverage area and reduce the signal distortion in AirComp systems. Due to the concern of excessive overhead for controlling the phase shifts at the RIS, the authors in [118] propose a two-stage beamforming design to minimize the time-average MSE of AirComp. In particular, the transmit power and receive beamforming are optimized at each time slot based on real-time low-dimensional cascaded channel state information (CSI), while the phase shifts at the RIS is updated once every few slots based on the channel statistics, thereby reducing the signaling overhead. Besides, the authors in [119] and [120] develop the robust design for inaccurate channel estimation in RIS-assisted AirComp systems. Moreover, RIS is also deployed to promote the performance of hierarchical AirComp [76], and co-existed AirComp and NOMA system [61].

The above studies highlight the potentials of RIS for enhancing the performance of AirComp, which makes it still interesting to investigate the design for AirComp assisted by single/multiple RIS(s) in more system settings, such as RIS-aided multi-hop AirComp by designating multiple RISs as intermediate nodes [121] and RIS-aided multi-cell AirComp by exploiting the RIS to improve the desired signal power from cell-edge devices while suppressing the inter-cell interference [122], [123]. In addition, since RIS can reflect signals with only phase and amplitude adjustments, the signals directly from the devices and reflected by the RIS may have a time lag when they arrive at the receiver, especially when multiple RISs are deployed to achieve multi-hop reflections. This makes it challenging to simultaneously synchronize direct and reflective signals for AirComp, which needs to be further studied in the future.

F. UAV-Aided Network

In most of existing studies on AirComp, the devices are assumed to be static or can only move within the area covered by static ground APs. However, large amounts of data are generally generated on mobile devices in practice, e.g., smart phones and vehicles, which can move over a wide area and even move out of the service area. Meanwhile, the infrastructure for wired/wireless communication networks can be unavailable due to unpredictable natural disasters and other catastrophic situations. Such scenarios prevent us from achieving efficient AirComp, and thus a flexible communication system is required to tackle these issues.

UAV is emerging as a promising cost-effective and flexible equipment to assist the terrestrial communication networks, which can establish line-of-sight (LoS) air-ground channels at a high altitude and promote the communication performance through the mobility control [124]–[126]. Therefore, the UAV-mounted AP can provide flexible network services for devices distributed in complicated environments, which has been exploited in [127]–[130] to facilitate AirComp, as

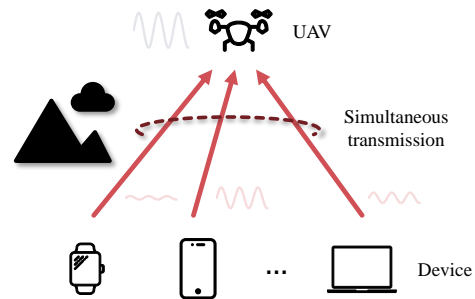


Fig. 10. AirComp over a UAV-aided network.

shown in Fig. 10. Specifically, the authors in [127] exploit the mobility of UAV to reduce the computation error of AirComp in backscatter sensor networks, where the UAV serves as both a power emitter and reader for efficient data aggregation from backscatter nodes. The authors in [128] develop an AirComp strategy with space-time line code, which enables a data-collection UAV to efficiently aggregate data from multiple sensing UAVs by fully exploiting the spatial diversity gain. Besides, the authors in [129] propose a joint trajectory and transceiver design for UAV-aided AirComp to minimize the time-average MSE, where the devices continuously move within the mission duration. Furthermore, the performance of UAV-enabled AirComp is analyzed in [130] to evaluate the impact of channel estimation and synchronization errors on the MSE. The above investigations reveal the advantages of deploying a UAV to assist AirComp. It is still interesting to explore more applications of UAVs in AirComp systems, e.g., hierarchical AirComp with multiple cooperative UAVs [131] and sustainable AirComp with UAV-enabled wireless power transfer (WPT) [132], as well as taking into account some critical issues, e.g., the estimation error on the movement of devices and UAV movement energy consumption, for practical implementations.

IV. INFORMATION THEORY PERSPECTIVE

Instead of quantifying the system performance in terms of the transmission rate to measure the maximum number of bits that can be successfully delivered per second, AirComp utilizes the computation rate to specify the maximum number of functions that can be reliably computed, which can be regarded as a reinterpretation of information theory from a functional computation perspective. In this section, we will review the existing works that analyze and maximize the computation rate for *coded* and *uncoded* AirComp from the information theory perspective. A summary of representative works can be previewed in Table II.

A. Coded AirComp

For coded AirComp, each device encodes its data into linear-structured codes, e.g., nested lattice code, which leads to the number of channel uses, L , being larger than the volume of generated data, T , such that achievable computation rate R_C is smaller than one according to Definition 2 in this scenario. Since the integer combination of linear-structured codes can

TABLE II
REPRESENTATIVE WORKS ON AIRCOMP FROM THE INFORMATION THEORY PERSPECTIVE

Category	Reference	Channel model	Feature
Coded AirComp	[25]	Discrete linear and Gaussian MACs	Pioneer introduction and definition of computation rate.
	[30]	Constant MACs with gains equal to one	Computation over a multi-cluster network.
	[39]	Gaussian MACs	Proposal of “compute-and-forward” relaying scheme.
	[46]	Fading MACs	Opportunistic transmission at devices.
	[47]	Uniform and non-uniform MACs	Comparison between coded AirComp and conventional OMA scheme.
	[48]	Fading MACs	CSI acquisition with automatic repeat request (ARQ) strategy.
	[54]	Fading MACs	Wide-band transmission with orthogonal frequency division multiplexing (OFDM).
	[55]	Fading MACs	Computation assisted by power-domain NOMA.
Uncoded AirComp	[133]	Fading MACs	Computation over a multi-layer network.
	[25]	Arbitrary MACs	/
	[43], [134]	Fading MACs	Coarse frame synchronization requirement on transmission.

still fall into the field of itself, the same decoder used for decoding individual data can also be leveraged to recover superimposed signals, which makes coded AirComp easily to be implemented in existing digital wireless communication systems.

Recently, several works have analyzed the computation rate of coded AirComp under different system settings. Specifically, the authors in [25] present the computation rates of coded AirComp over discrete linear and Gaussian MACs with joint source-channel computation coding, which outperform the scheme with separated source-channel coding in terms of the computation rate. The authors in [39] apply coded AirComp to the “compute-and-forward” relaying strategy, where the relays first obtain the linear functions of transmitted signals via coded AirComp and then send these function values to the receiver. Herein, the computation rate is analyzed based on the average probability of achieving a target computation accuracy at all relays, and then the coefficients for combining transmitted messages are designed to maximize the computation rate over additive white Gaussian noise (AWGN) channels [39]. In addition, the authors in [30] investigate the computation rate of coded AirComp over a multi-cluster network with constant channel gains, which demonstrates that the computation rate scales down by the total number of clusters if each cluster is activated in a time-division manner.

Instead of assuming ideal MACs as in the above works, the authors in [46] study the coded AirComp over fading MACs, where an opportunistic in-network computation framework is proposed to ensure that the computation rate is not limited by deep-fading devices. Then, a long-term ergodic computation rate is analyzed in [46], which shows that a multi-user diversity gain can be achieved by scheduling the devices with large channel gains to participate in the aggregation. Besides, the authors in [47] analyze and compare the achievable computation rates of coded AirComp and conventional OMA scheme, which indicates that coded AirComp is not always superior to the OMA scheme under different settings in terms of the functional computation. Furthermore, the authors in [54] extend AirComp from narrow-band to wide-band fading

channels, where each sub-carrier is exploited to compute a sub-function via coded AirComp, and then the desired function is reconstructed from the received sub-functions. The computation rate of such a wide-band scenario is then derived in [54], followed by an optimal power control for improving the computation rate while achieving a non-vanishing property. Moreover, the authors in [55] leverage the power-domain NOMA to enable multiple sub-functions to be computed over non-orthogonal resource blocks in a wide-band scenario, which increases the computation rate and spectral efficiency. The outage probability and diversity order are developed in [55], which reveal that the device with the worst channel gain imposes a performance limitation as the power goes to infinity. Such a function division method is also utilized in a multi-hop network with a computation rate analysis [80]. In addition, the transceiver design is taken into account to maximize the achievable computation rate of hierarchical AirComp [133] and to balance the trade-off between the computation rate and the transmission delay for CSI acquisition [48].

B. Uncoded AirComp

Uncoded AirComp is able to simply map each pre-processed data to one channel input symbol with linear power scaling, which leads to the number of channel uses, L , being equal to the volume of generated data, T . Hence, the computation rate of uncoded AirComp can reach one, i.e., $R_C = 1$, according to Definition 2 over the ideal MACs that are free of fading and noise [43]. Beyond that, the authors in [25] analyze the computation rate of uncoded transmission for sending arbitrary function over arbitrary MACs, which shows that the corresponding lower bound for the minimum mean-squared error (MMSE) in function estimation exponentially increases with an attenuation factor of $1/R_C$. In addition, the authors in [134] propose a robust scheme for function computation with uncoded transmission, where each device encodes its data into a series of power-scaled random signal sequences and the receiver is equipped with an energy detector to obtain the sum of transmit energy, such that only a coarse block synchronization is required. The computation rate of

such a robust analog transmission design is given in [43] to capture the trade-off between the efficiency and reliability of the functional computation, which is determined by the function to be computed and the desired level of accuracy.

V. SIGNAL PROCESSING PERSPECTIVE

To achieve a desired magnitude alignment of superimposed signals, it is crucial to jointly design the transmitters and the receiver to compensate for the non-uniform channel fading between the AP and each of the devices, which stimulates the development of transceiver design with various advanced signal processing techniques. In this section, we shall introduce the transceiver design for AirComp systems under different wireless communication scenarios from a signal processing perspective. A summary of representative works reviewed in this section is provided in Table III.

A. SISO AirComp

The transceiver design in SISO AirComp is to find appropriate transmit and receive scaling factors in one-dimensional complex scalar field, which is more tractable than the multi-dimensional optimization in multi-antenna scenarios. To further simplify the transceiver design, the authors in [37] demonstrate that the phase of transmit scaling factor at each device can be designed to compensate for the phase induced by the complex receive scaling factor and the known channel coefficient, while its magnitude does not need to be changed and is merely constrained by the transmit power budget. Hence, the transceiver design in SISO AirComp can be reduced to the joint design of transmit power at the devices and the receive scaling factor at the AP, where the optimization is performed in the real number field.

By assuming that the perfect CSI is available and all devices are well synchronized, the optimal transceiver design under different system constraints has been studied on SISO AirComp systems [37], [49], [50]. Considering the peak-power constraint at each device, the optimal transmit power of each device follows a threshold structure, where the threshold is determined by the receive scaling factor (also known as the denoising factor) and the quality indicator that accounts for both the channel conditions and power constraints at different devices [37], [49]. It suggests that the device can utilize the channel-inversion policy to perfectly compensate for the channel fading if its quality indicator exceeds the threshold, otherwise the full-power transmission should be applied. Besides, a larger noise variance yields a higher threshold and asks for more devices to adopt full-power transmission, which requires the receiver to implement a smaller receive scaling factor to suppress the error caused by channel noise [49]. Instead of considering static channel conditions, the time-varying fading channel, a more general case, is also studied in [37], [49]. In particular, an optimal transceiver design of the time-varying case is proposed in [49], where the power control is shown to have a regularized channel-inversion structure for balancing the trade-off between the signal-magnitude alignment and the noise suppression. Besides, the authors in [37] investigate the ergodic performance of SISO AirComp in terms of the

time-averaging MSE and power consumption, which reveals the trade-off between the computation effectiveness and the energy efficiency. In addition to the peak-power constraints, the authors in [50] propose an optimal design for minimizing the MSE under the sum-power constraint, where the solution can be obtained in a closed form from the equivalent convex problem. Besides, the optimal design for sum-power minimization under the MSE requirement is also studied in [50].

Unlike the above works that consider a single radio channel, a wide-band AirComp system is studied in [56] to increase the computation accuracy. This is done by exploiting the multi-channel diversity, where each device broadcasts the signal over multiple selected subchannels under a transmit power constraint. The results in [56] indicate that, in order to enhance the aggregation accuracy, the devices prefer to choose a subset of channels with good conditions for their data transmission, instead of simply broadcasting signals over all subchannels. Besides, the analyses on MSE minimization within real and complex domains in [56] reveal that the optimization in real domain can separately process the real and imaginary parts of the signal and align them onto a one-dimensional signal space, which is able to yield a smaller MSE than the optimization in complex domain that uses a complex number to directly scale the complex distorted signal. In addition, the authors in [135] propose a multi-slot AirComp to reduce the signal distortion by distributing the transmissions over multiple slots. Similar to the broadband AirComp considered in [56], multi-slot AirComp enables devices to select a slot with good channel gain for their data transmissions, thereby escaping the deep fading that may be confronted with only single-slot transmission.

Different from the aforementioned works that assume independent channel fading and independent transmitted signals among different devices, the AirComp systems with correlated channels and correlated signals are studied in [58] and [59], respectively. In particular, the authors in [58] propose a scheme for AirComp in a general setting of fast fading channels that can be non-Gaussian and correlated. For correlated signals, the authors in [49] demonstrate that the corresponding MSE minimization problem can be transformed into a tractable upper bound minimization problem, which reduces to the optimization with independent signals as proposed in [37], [49]. To take a step further, the authors in [59] implement an optimal policy for AirComp with spatial-and-temporal correlated signals. Herein, the transceiver design is achieved through the Kalman filter in an iterative prediction-correction way, which is also valid for spatial-and-temporal independent signals [59]. Also, a low-complexity design is further proposed in [59] by applying a linear filter to the stored historical signals as the current computation output, which achieves the same performance as the optimal design with a sufficient filter length.

B. SIMO/MIMO AirComp

To further enhance the performance of AirComp, multi-antenna technology is a powerful tool to increase the diversity order and degrees of freedom in the transceiver design [31],

TABLE III
REPRESENTATIVE WORKS ON AIRCOMP FROM THE SIGNAL PROCESSING PERSPECTIVE

Antenna	Reference	Objective	Transceiver design	Feature
SISO	[37], [49]	Instantaneous MSE and time-average MSE	<ul style="list-style-type: none"> • Threshold-based structure for static MACs • Channel-inversion structure for time-varying MACs 	Optimal design with peak-power constraint.
	[50]	Instantaneous MSE	<ul style="list-style-type: none"> • Equivalent convex reformulation 	Optimal design with sum-power constraint.
	[56]	Instantaneous MSE	<ul style="list-style-type: none"> • $\mathbf{T}\mathbf{x}^\dagger$: KKT condition • $\mathbf{R}\mathbf{x}^\ddagger$: First-order optimality condition 	Wide-band transmission.
	[58]	Confidence level with given MSE requirement	<ul style="list-style-type: none"> • $\mathbf{T}\mathbf{x}$: Normalized transmit power with random phase shift • $\mathbf{R}\mathbf{x}$: Energy detection of received signals across multiple channel uses while subtracting the noise energy based on the statistical information 	Computation over correlated MACs.
	[59]	Instantaneous MSE	<ul style="list-style-type: none"> • $\mathbf{T}\mathbf{x}$: Equivalent convex reformulation • $\mathbf{R}\mathbf{x}$: Kalman filter / Linear filter obtained via the first-order optimality condition 	Computation of spatial-and-temporal correlated signals.
	[135]	MSE	<ul style="list-style-type: none"> • $\mathbf{T}\mathbf{x}$: Channel-inversion policy • $\mathbf{R}\mathbf{x}$: First-order optimality condition 	Multi-slot aggregation with only once transmission per device.
SIMO	[37]	Instantaneous MSE	<ul style="list-style-type: none"> • Threshold-based structure developed in SISO for obtaining transmit scalars and the magnitude of receive beamforming vector • Random generation for approximating the direction of receive beamforming vector 	Near-optimal design.
	[51]	Instantaneous MSE	<ul style="list-style-type: none"> • $\mathbf{T}\mathbf{x}$: Uniform-forcing design • $\mathbf{R}\mathbf{x}$: Semidefinite relaxation (SDR) / Successive convex approximation (SCA) 	Proposal of uniform-forcing transceiver design.
	[113]	Instantaneous MSE	<ul style="list-style-type: none"> • $\mathbf{T}\mathbf{x}$: Uniform-forcing design • $\mathbf{R}\mathbf{x}$: Difference-of-convex (DC) programming 	Computation assisted by RIS.
	[115]	Instantaneous MSE	<ul style="list-style-type: none"> • $\mathbf{T}\mathbf{x}$: Uniform-forcing design. • $\mathbf{R}\mathbf{x}$: SCA and Mirror-Prox algorithm. 	Computation assisted by RIS.
	[136]	Instantaneous MSE	<ul style="list-style-type: none"> • $\mathbf{T}\mathbf{x}$: Uniform-forcing design • $\mathbf{R}\mathbf{x}$: Branch-and-bound (BnB) algorithm 	Optimal design for receive beamforming under the uniform-forcing optimization framework.
MIMO	[31]	Instantaneous MSE	<ul style="list-style-type: none"> • $\mathbf{T}\mathbf{x}$: Uniform-forcing design • $\mathbf{R}\mathbf{x}$: Differential geometry 	Channel feedback via AirComp.
	[52]	Instantaneous MSE	<ul style="list-style-type: none"> • $\mathbf{T}\mathbf{x}$: Uniform-forcing design • $\mathbf{R}\mathbf{x}$: Scaled unitary matrix 	Computation with antenna selection.
	[53]	Instantaneous MSE	<ul style="list-style-type: none"> • $\mathbf{T}\mathbf{x}$: Uniform-forcing design • $\mathbf{R}\mathbf{x}$: Decomposed architecture consisting of inner and outer components 	Computation over clustered IoT networks.
	[137]	Instantaneous MSE	<ul style="list-style-type: none"> • $\mathbf{T}\mathbf{x}$: KKT condition • $\mathbf{R}\mathbf{x}$: SCA or block coordinate descent (BCD) for receive analog beamforming, and the first-order optimality condition for receive digital beamforming 	Hybrid receive beamforming design.
	[138]	Instantaneous MSE	<ul style="list-style-type: none"> • $\mathbf{T}\mathbf{x}$: KKT condition • $\mathbf{R}\mathbf{x}$: Riemannian conjugate gradient (RCG) algorithm for receive analog beamforming and the first-order optimality condition for receive digital beamforming 	Computation assisted by RIS.
	[139]	Time-average MSE	<ul style="list-style-type: none"> • $\mathbf{T}\mathbf{x}$: SCA • $\mathbf{R}\mathbf{x}$: SCA for receive analog beamforming and the first-order optimality condition for receive digital beamforming 	Two-timescale hybrid beamforming design.

[†] Transmitter design method.

[‡] Receiver design method.

[51], [52]. Specifically, an AP with multiple antennas is able to increase the diversity gain for reducing the distortion of aggregated signals [51]. Besides, equipping multiple antennas at both the AP and the devices makes it possible to simultaneously compute multiple functions of multimodal sensing data, where various data from multiple devices can be aggregated at different antennas [31], [52]. In such cases, the receive/transmit scaling factor becomes a multidimensional complex beamforming vector, which complicates the formulation of MSE and leaves the optimal transceiver design to be solved for SIMO/MIMO AirComp. Even though,

a sub-optimal method, called uniform-forcing transceiver design, is developed in [51] to facilitate the optimization on SIMO/MIMO AirComp. The uniform-forcing design allows all devices to perfectly compensate the equivalent fading channels during the transmission while satisfying their transmit power constraints, where the equivalent fading channel is a combination of the receive beamforming vector/matrix and the fading channel vector/matrix [31], [51], [52]. Under this scheme, the received signal at the AP becomes an unbiased estimation of the sum of transmitted symbols [37], which forces the error caused by the signal-magnitude misalignment

to be zero but at the cost of elevating the noise-induced aggregation error [49]. The uniform-forcing transceiver design makes the MSE minimization problem more tractable in multi-antenna scenarios, which stimulates a growing body of research to explore the possible enhancement in the functional computation via SIMO/MIMO AirComp.

1) *SIMO AirComp*: By implementing the uniform-forcing design for SIMO AirComp, the MSE minimization problem becomes a min-max optimization problem and can be further reduced to a quadratically constrained quadratic programming (QCQP) problem, which, however, is non-convex and non-deterministic polynomial-time (NP) hard [51]. Even though, it is interesting to find that the resulting QCQP problem has the same mathematical form as the downlink multicast beamforming design [140], which establishes the AirComp-multicasting duality similar to the uplink-downlink duality for multiuser MIMO communications [31], [51], [141]. As mentioned in [31], the AirComp-multicasting duality holds because both the receive beamforming in AirComp and the multicast beamforming in multicasting aim to be aligned with multiple fading channels but with different targets, i.e., reducing MSE in AirComp and increasing minimum SINR in multicasting. Therefore, based on such a duality, the existing optimization methods for multicasting problem can be directly leveraged to tackle the MSE minimization problem in SIMO AirComp, such as SDR [140], SCA [142], and BnB algorithms [143].

In particular, the non-convex QCQP problem for the MSE minimization can be converted into an semidefinite programming (SDP) problem, which can be solved with the SDR technique by dropping the rank-one constraint [144]. Due to the weak capability of obtaining the rank-one solution of the SDR technique in high-dimensional space [145], the authors in [51] further propose an SCA algorithm by iteratively approximating the non-convex quadratic constraints to convex linear constraints, which significantly enhances the solution quality initiated by the SDR algorithm. Also to overcome the limitations of the SDR technique, the authors in [113] propose a DC programming algorithm to induce the rank-one solution of the SDP problem. The main idea of the DC algorithm is to represent the rank-one constraint as a DC function and then set it as a penalty term in the objective function. By linearizing the concave term in the DC function, the reformulated problem can converge to a rank-one solution of the original SDP problem through multiple iterations [146]. Motivated by [143], the authors in [136] propose a globally optimal design for the resulting QCQP problem based on the BnB algorithm, where the solution is induced by iteratively reducing upper bound and lifting lower bound for the objective function until they converging to the same value with judiciously designed branching strategy. In contrast to the above works that focus on solving the QCQP problem, the authors in [115] propose an SCA-based algorithm to tackle the original min-max problem. As the subproblem in each SCA iteration can be transformed into a smooth convex-concave saddle point problem, the authors in [115] further utilize the Mirror-Prox method [147] to solve each subproblem with a low computational complexity, which is much friendly to accommodate the scenarios with massive

devices. Moreover, the authors in [37] develop an algorithm to find near-optimal solutions for SIMO AirComp. Specifically, by denoting the receive beamforming vector as the product of a scaling factor and a unit vector, it is able to first find the optimal receive and transmit scaling factors by utilizing the optimal design developed for SISO AirComp [37], [49], and then find the suboptimal unit vector that induces the minimum MSE in a sequence of randomly generated unit vectors.

2) *MIMO AirComp*: The uniform-forcing design for MIMO AirComp enables the transmit beamforming matrices at devices to not only compensate for non-uniform channel fading, but also eliminate the interference caused by data for computing different functions [52]. Besides, the receive beamforming matrix can be set as a scaled unitary matrix, which is a sub-optimal setting but effectively simplifies the receiver design with marginal performance loss in MIMO systems [31], [148]. To reduce the overhead for additional CSI acquisition at the AP, the authors in [52] propose an antenna selection scheme when the number of receive antennas is larger than the number of functions to be computed. Instead of selecting a subset of receive antennas at the AP, the authors in [31] formulate an approximate problem for optimizing the receive beamforming matrix on a Grassmann manifold by tightening the power constraints, and then exploit the differential geometry method to induce a closed-form solution.

Although large-antenna arrays provide rich spatial degrees of freedom and tremendous beamforming gain, they inevitably lead to high complexity in the transceiver design for MIMO AirComp. To tackle this issue, the authors in [53] propose a reduced-dimension receive beamforming design in clustered IoT networks, where the channel model is characterized by the clustered transmitters and rich local scattering [149]. By exploiting the structure of clustered MIMO channels, the optimal aggregation beamformer is decomposable with inner component for channel-dimension reduction and outer component for equalization of the low-dimensional small-scale fading channels [53]. Furthermore, the authors in [137] propose a hybrid beamforming design for MIMO AirComp. To minimize the MSE, an alternating-optimization-based algorithm is considered in [137] by jointly optimizing the transmit digital beamforming at devices and the receive hybrid beamforming at the AP, where the transmit, receive analog, and receive digital beamformings are alternately obtained via the Lagrangian duality method, SCA or BCD method, and the first-order optimality condition, respectively. With such a hybrid beamforming design, the required radio-frequency (RF) chains at the AP as well as the complexity of digital processing can be significantly reduced while achieving a close performance to the MIMO AirComp with a fully-digital receiver. Besides, the authors in [138] propose a low-complexity RCG algorithm to obtain the receive analog beamforming in hybrid beamforming design for MIMO AirComp. Moreover, the authors in [139] propose a mixed-timescale hybrid combining scheme to minimize the average MSE for multi-modal sensing via MIMO AirComp, where the baseband combiner at the AP is designed according to the real-time effective CSI, and the RF combiner at the AP and the transmit beamforming of devices are adapted to the long-term statistical CSI.

VI. PRACTICAL IMPLEMENTATIONS

Various techniques have been developed to promote the performance of AirComp. Unfortunately, they generally assume some ideal communication conditions and ignore practical implementation issues. In this section, we will review the literature that considers system design for AirComp while taking into account the practical implementation issues.

A. Efficient Channel Feedback

In most of the existing studies on AirComp, perfect CSI is generally assumed to be available at the AP for implementing various advanced techniques, such as power control and beamforming design. Conventionally, the uplink and downlink communications between the AP and devices can work in two modes, i.e., frequency-division duplexing (FDD) mode and time-division duplexing (TDD) mode, which incurs different channel estimation methods. In FDD systems, the downlink and uplink transmissions are carried out in different frequency bands, and thus it is required to separately send pilot signals to estimate downlink and uplink channels. On the other hand, in TDD systems, the downlink and uplink transmissions are implemented in the same frequency channel but in different time slots. This enables the AP to first broadcast pilot signals to devices for estimating the local CSI, and then obtain the global CSI according to the feedback from devices by assuming the channel reciprocity. Since AirComp focuses on uplink transmission, TDD is commonly considered in the existing studies to achieve local CSI estimation at each device based on the pilot signal broadcast by the AP. However, the signaling overhead of the above training procedure is linearly scaling with the number of devices for CSI feedback, which may lead to high communication latency in ultra-dense network.

To address this issue, various channel feedback approaches have been developed to realize efficient AirComp design while avoiding massive overhead for CSI gathering. Specifically, the authors in [52], [150] exploit the “OR” property [151] of wireless channels to determine the optimal receive scaling factor at the AP, which is equal to the minimum ratio of the channel gain to the effective signal power among different devices. Instead of obtaining global CSI at the AP, each device quantizes the corresponding ratio into a binary sequence based on its local CSI, and then the AP can obtain the binary representation of the minimum ratio by determining one significant bit per feedback [52], [150]. As the overhead of this method is mainly determined by the length of the quantized binary sequence, it is able to significantly reduce the time duration for CSI gathering as compared with the conventional approach that collects the global CSI [52]. Besides, ARQ strategy is exploited in [48] to avoid massive CSI aggregation in AirComp systems.

In addition to using AirComp for achieving efficient WDA, the authors in [31], [133], [152] also leverage the AirComp to accelerate the system optimization. Since the receive beamformer design derived in [31], [133], [152] can be represented as a nomographic function of signals transmitted from dispersed devices, the AP is able to directly obtain the optimal receive beamforming design by aggregating the

properly designed feedback signals based on the local CSI via AirComp. Furthermore, similar to the quantization method considered in [52], [150], the authors in [31] propose an AirComp-based feedback for optimizing the receive scaling factor with several feedback rounds. Specifically, by predefining a quantizer codebook containing the values generated by uniformly partitioning the initial feedback-quantization range, each device is able to transmit a one-hot vector that comprises a single “1” located at the corresponding codebook index and “0”s at rest of indices, which indicates the value range of its local feedback signal. After aggregating those one-hot vectors via AirComp, it is able to obtain the largest index of non-zero elements to refresh the quantization range, thereby approaching the optimal value round by round until meeting the demands on the solution accuracy [31]. By assuming the channel reciprocity, the above methods enable each of devices to obtain the local CSI and transmit beamforming design via downlink broadcasting, and enable the AP to optimize the receive beamforming via AirComp, which requires only a single symbol duration in each transmission phase. Therefore, it is able to achieve efficient system design with a time complexity independent of the number of devices. To relieve the burden on estimating the individual channel of each device, the authors in [153] propose a random orthogonalization method for MIMO AirComp, where only a summation channel is required to be estimated at the BS via a common pilot transmission from devices. A channel echo mechanism is further developed in [153] by letting the BS feed back the summation channel to each of the devices, which is able to alleviate the effect of channel fading and make the random orthogonalization much applicable in the scenarios where the BS with small-scale antennas.

B. Robust Design

In practice, the transmitters and receivers can only obtain imperfect CSI due to various reasons, e.g., inaccurate channel estimation and finite-rate feedback [154]. Hence, simply implementing the system design derived from the assumption of perfect CSI may inevitably cause performance degradation in practical wireless networks with imperfect CSI. To address this issue, robust design is required to provide a certain level of the system performance, which is generally characterized by two classes of models, namely stochastic (expectation-based) and deterministic (worst-case) models. In the stochastic model, the uncertain CSI error follows a Gaussian distribution with known statistics (e.g., the mean and the covariance), based on which the system is designed to optimize the average or outage performance [155], [156]. In the deterministic model, the norm of uncertain CSI errors is bounded by a known spherical uncertainty region. This enables the robust design to optimize the worst-case performance, which generally induces a min-max (or max-min) optimization problem for achieving a guaranteed performance level for any channel realizations within the uncertainty region [157], [158].

As the CSI plays an important role in achieving the magnitude alignment at the AP, robust design is also critical for AirComp to achieve a reliable functional computation

under imperfect CSI. To this end, robust AirComp design under different imperfect CSI models has been investigated in [119], [120], [152], [159]. Specifically, the authors in [159] propose a robust parallel analog function computation design for MIMO AirComp with imperfect CSI modeled by the worst-case model. The corresponding robust transceiver design is formulated as an optimization problem to minimize the worst-case MSE, which can be solved by an alternating optimization algorithm after converting the semi-infinite constraints into linear matrix inequalities (LMIs). To reduce the channel training complexity, the authors in [152] present an imperfect CSI model under over-the-air signaling procedure, where the channel uncertainty vector/matrix is directly added on the superimposed feedback of effective CSI, rather than the individual local CSI as in conventional signaling procedure that collects the global CSI at the AP. The corresponding robust transceiver design is then proposed in [152] by considering both the stochastic and deterministic models, which achieves smaller MSE with a low training complexity than methods assuming perfect CSI. Furthermore, the authors in [119] and [120] propose the robust design for RIS-assisted AirComp, where the CSI of reflective links is imperfect due to the erroneous channel estimation and weak signal processing capability of the passive reflecting elements at the RIS. Moreover, transceiver design based on the statistics of channel estimation errors is developed in [160] for SISO and SIMO AirComp with imperfect CSI.

On the other hand, perfect synchronization is another ideal assumption in most of the existing studies on AirComp, which, however, is difficult to realize in large-scale networks. If there are asynchronous concurrent signals arriving at the AP, the accuracy of estimated function may be severely deteriorated. To address this issue, the authors in [134] propose a robust design with coarse synchronization by letting devices transmit random sequences at a transmit power being proportional to the real-valued sensor information. Besides, the authors in [161] propose a maximum-likelihood estimation design for misaligned AirComp with residual channel-gain variation and symbol-timing asynchrony among devices. As the maximum-likelihood estimation proposed in [161] is susceptible to noise, a Bayesian approach is further proposed in [162] by exploiting two pieces of statistical information of transmitted data, i.e., the first and the second sample moments, which addresses the error propagation and noise enhancement problems caused by the maximum-likelihood estimator.

C. Blind Data Fusion

As discussed above, most of the current design for AirComp relies on the instantaneous CSI at the AP and/or the devices. Nonetheless, the cumbersome CSI acquisition inevitably introduces additional communication delay and signaling overhead, which stimulates the CSI-free design, also known as blind design, for AirComp systems. Initially, to explore the impact of CSI on function computation in AirComp systems, the authors in [163] conduct the performance analysis for the scenarios with different local CSI available at devices. It shows that the channel magnitude information at devices is sufficient to

achieve the same performance as the case with perfect local CSI, and no CSI is needed at devices if a multi-antenna AP has access to the statistical channel knowledge [163]. The authors in [164] develop a blind AirComp to achieve desired function computation by leveraging the blind demixing, which is a powerful tool for recovering a sequence of signals without accessing any CSI [165], and further propose a randomly initialized Wirtinger flow method to solve the resulting blind demixing problem with provable optimality guarantees. To mitigate the possible destructive signal superposition due to the phase difference when CSI is not available, the authors in [58] apply random phase shifts to the transmitted signals at different devices, and then average them over multiple channel uses to compensate for the phase difference and reduce the impact of additive noise. In addition, the authors in [166], [167] leverage the one-bit quantization to achieve blind AirComp, where the receiver obtains the sign of aggregated signals based on majority vote by detecting the energy accumulated on different OFDM subcarriers [166] and superposed pulse-position modulation symbols [167], which eliminate the reliance on the CSI and relaxing the requirement of synchronization. Furthermore, the balanced number system is employed in [168] to enable continuous-valued computations in the digital AirComp system without the need of CSI acquisition.

D. Energy-Efficient Design

The energy consumption for data transmission is a critical issue to be considered in sustainable IoT networks. In order to prolong the network lifetime, the energy-efficient design is required for enhancing the applicability of AirComp in various scenarios. For instance, the authors in [169] propose an MMSE estimation scheme that achieves an energy-efficient AirComp by utilizing the spatial correlations among transmitted signals over AWGN channels. Due to the spatial correlation, only a small number of observations are required to be sampled for the functional computation while achieving an acceptable MSE level, which effectively reduces the energy consumption for data transmission and prolong the network life with densely deployed devices [169]. In addition, the authors in [170] propose a power minimization scheme for MIMO AirComp, where the transmit power is minimized under an MSE constraint.

On the other hand, since IoT devices are usually energy-constrained and battery-powered, how to efficiently recharge the hard-to-replace batteries is another challenge to be considered to increase the sustainability of services provided by these devices. Fortunately, the emergence of WPT technique enables the devices to harvest energy from the ambient RF signals radiated by the energy transmitter [171]. Hence, the integration of WPT and AirComp provides an attractive solution to enhance the efficiency and accuracy of functional computation. Motivated by this, the authors in [172] develop a wirelessly powered AirComp framework by jointly optimizing the energy beamforming and transceiver design, where the AP serves as not only a power beacon but also a data FC. To further enhance the spectrum- and energy-efficiency in severe propagation environment, RIS is deployed in [114] to assist

the downlink WPT and uplink AirComp between the AP and the devices.

E. Prototype Validation

So as to put theory into practice, various prototypes have been built to validate feasibility and effectiveness of AirComp in real world [150], [173]–[176]. Specifically, the authors in [173] propose to calculate mathematical functions via AirComp by encoding values into Poisson distributed burst sequences. To verify the proposed transmission scheme, a wireless sensor network platform with 15 simple nodes and a central receiver is developed in [173], where each sensor node consists of one micro-controller (e.g., the Arduino Uno micro-controller board), one sensor, and a 2.4 GHz sinusoid signal generator. By implementing an experiment of mean temperature sensing, it is verified that the computation error can be reduced by increasing the length of burst sequences [173]. The authors in [174] establish a testbed to validate the transmission scheme proposed in [134]. The implemented experiment is performed on self-developed multi-antenna software-defined radio devices, where one of these antennas is configured as the FC and others emulate separate single-antenna sensor nodes. Besides, the authors in [150] and [175] leverage the universal software radio peripheral (USRP) platform to verify the proposed transmission schemes for AirComp. Moreover, the authors in [176] design a prototype based on the Xilinx software-defined radio platform to prove the concept of AirComp-assisted FL. As the time and carrier frequency offsets in OFDM transmission induce phase noises to the transmitted signals, which may lead to the failure of coherent waveform superposition, an efficient protocol is proposed in [176] to address this issue by adding one more signaling round for phase error estimation and compensation.

VII. APPLICATIONS OF AIRCOMP

In this section, we discuss the applications of AirComp in two categories of networks, i.e., *IoT networks* and *edge intelligent networks*, where the IoT networks are mainly introduced from the perspectives of *distributed sensing* and *autonomous control*, while the edge intelligent networks are emphasized by *distributed learning*, *privacy preservation*, *Byzantine resilience*, and *federated analytics*. Some of the main application scenarios are shown in Fig. 11.

A. IoT Networks

With the advancement of IoT techniques, pervasive devices are connected to the communication network for providing ubiquitous services, especially for distributed sensing and autonomous control. In such scenarios, massive transient data needs to be aggregated in time for further processing and feedback, which causes the conventional “compute-after-communicate” multiple-access strategies poorly suited under stringent delay requirements. Fortunately, the emergence of AirComp enables us to achieve fast WDA in a “compute-when-communicate” manner for sensing and control applications.

1) *Distributed Sensing*: Distributed sensing aims to monitor and draw the digital picture of physical world with vastly and densely distributed sensors, e.g., the average temperature/humidity of an area for environmental monitoring, which are more interested in obtaining the function value of sensor readings rather than the individual raw data. As such an objective is consistent with the paradigm of AirComp, it is able to achieve efficient distributed sensing by conducting simultaneous transmission among all sensors, while the desired function can be directly computed over the air. Meanwhile, the characteristics of wireless sensor networks can be further leveraged to promote the performance of AirComp. For instance, the channel correlation can be leveraged to facilitate the distributed sensing in the systems with relatively stable radio environment and locations of sensor nodes [58]. In densely deployed IoT networks, the adjacent devices shall generate spatial-and-temporal correlated observations due to the similarity of their settled environments, which can be used to reduce the aggregation error [59] and the energy consumption [169] of AirComp. Benefiting from the WPT technique, wireless-powered AirComp can prolong the service time of devices for distributed sensing tasks [114], [172]. Moreover, UAV can be exploited to enable flexible AirComp to gather the data from high-mobility and vast-distributed sensor nodes by constructing favorable channels [31], [129]. In addition, the authors in [177] propose a radix-partition-based AirComp for remote state estimation in IoT networks, which can efficiently map the sensor measurements to the allocated resource pool without the activity detection.

2) *Autonomous Control*: Autonomous control has been highly demanded in smart industry and agriculture in recent years, where a group of agents desire to perform some actions to accomplish an overall cooperative task by interacting with each other [178]–[180]. Herein, each agent needs to iteratively gather information from others for updating its own state until convergence, which generally consists of two phases within each iteration, i.e., the communication phase for information exchanges and the computation phase for state updates. By integrating these two phases, efficient network-wide consensus can be achieved via AirComp in a decentralized manner [88]–[95]. The authors in [88] leverage coded AirComp to achieve reliable local averaging over noisy MACs through the use of linear-structured codes, while uncoded AirComp with analog transmission is exploited in [89], [91], [95] for average consensus. Furthermore, the AirComp-enabled consensus has been deployed to support efficient formation control [93] and solve linear equations [94] as well.

B. Edge Intelligent Networks

Edge intelligence aims to distill the intelligence at network edge by exploiting the geographically dispersed data and computing power, which saves the radio resources for centralizing data, and reduces the response time and energy consumption [1], [5], [181]. We shall discuss the applications in edge intelligent networks from four aspects, i.e., distributed learning, differential privacy, Byzantine resilience, and federated analytics, where AirComp can be exploited to facilitate

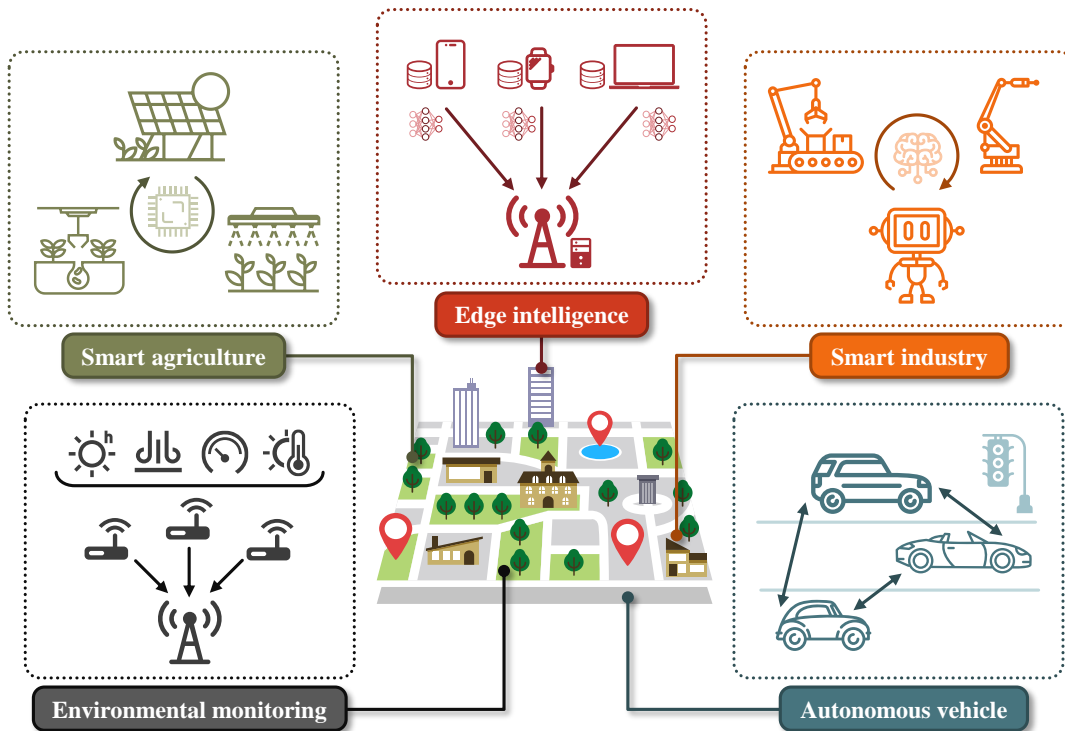


Fig. 11. Application scenarios of AirComp.

the implementation of corresponding techniques in wireless networks.

1) *Distributed Learning*: Due to the data explosion in modern society, AI has been regarded as a revolutionary strategy to efficiently extract knowledge from enormous data. In general, AI models are trained in a centralized server by collecting the local datasets from dispersed devices, which, however, induces several drawbacks. Firstly, the enormous data upload and re-storage in the server consume vast amounts of time, energy, communication, and storage resources. Secondly, data generated at local devices are usually privacy sensitive, which is violated when the raw data is gathered to the central server for a global model training. To overcome these challenges, distributed learning techniques have been developed to enable dispersed edge devices to collaboratively train a shared global model efficiently. Chief among them is the renowned FL framework [36], [182]–[187], where only the model/gradient parameters are exchanged for ensuring the privacy of the local data.

Despite avoiding the transmission of large volumes of data, limited radio resources is still a bottleneck for implementing FL in wireless networks, due to the high dimensional model/gradient parameters required to be periodically exchanged between the server and edge devices for complicated learning tasks. Therefore, conventional OMA schemes become inefficient for exchanging high-dimensional model parameters in terms of the communication delay by allocating orthogonal radio resources to participating devices. To resolve such concerns, AirComp enables the server to directly obtain the weighted-sum local models during the uplink aggregation, which achieves much lower communi-

cation latency as compared with the OMA schemes [188]–[191]. However, by adopting uncoded AirComp to assist FL, the uplink aggregation error is inevitable due to the channel fading and receiver noise, sometimes as well as the inter-cell interference, which stimulates the research to achieve high-accuracy model aggregation with several advanced techniques, such as power control [192]–[196] and beamforming design [189], [197], [198]. Besides, scheduling policy design is also considered in AirComp-assisted FL due to the concerns of aggregation error rather than the communication delay as in OMA strategies [188], [189], [199]. Moreover, auxiliary equipments like half-duplex relay and RIS are leveraged in [83], [138], [200]–[203] to further increase the aggregation accuracy in severe propagation environment. On the other hand, one-bit quantization based digital AirComp is applied in [41], [70], [166], [167], [204] to against the channel fading and receiver noise, which achieves reliable model aggregation but at the cost of the model precision.

2) *Privacy Preservation*: Although the learning tasks in edge intelligent networks is conducted distributively without exposing the raw data, the risk of privacy leakage still exists in such scenarios, where the confidential data can be potentially inferred from the publicly shared model/gradient parameters [205]–[207]. To quantify the information leakage, differential privacy (DP) is a well-established criterion to measure the sensitivity of the statistical change of the dataset with a fresh input [208]. A certain level of DP can be guaranteed by introducing perturbations into the aggregated model to maintain the disclosed statistics, thereby masking the contribution of arbitrary individual data. This leads to a trade-off between the privacy and learning accuracy due to additional artificial local

perturbation introduced to transmitted signals for camouflaging private information [209]. Meanwhile, additional power consumption is needed to provide such an artificial noise mask, which may cause insufficient power at devices for subsequent computation and communication.

Fortunately, AirComp allows the individual data to be hidden in the superimposed signal, which prevents the attacks from accessing a specific user's model information. Besides, the inherent non-eliminated channel noise during analog transmissions can be regarded as a random perturbation conducted on the received data. The authors in [210] demonstrate that the channel noise in AirComp-assisted FL contributes to DP requirements, which reduces or even completely saves the energy for adding local artificial perturbations. The analyses in [211], [212] show that the privacy leakage per user scales as $\mathcal{O}(1/\sqrt{K})$ in AirComp-assisted FL, but scales as a constant and does not decay with the number of devices, K , in the counterpart OMA schemes. Besides, due to the inherent channel noise is uncontrollable during the model aggregation, the adaptive power control is further investigated in [210], [213], [214] to ensure that the privacy level can be achieved in different training rounds. Moreover, the authors in [215] present a scalable FL framework based on uncoded AirComp and alternating direction method of multipliers (ADMM), which reveals that the wireless channel interference and perturbations can be harnessed to increase the spectral efficiency while ensuring the privacy requirements.

3) *Byzantine Resilience*: The distributed nature of edge intelligence makes it vulnerable to adversarial attacks, namely Byzantine attacks, from malicious devices, which cause unpredictable errors to deteriorate the learning performance. It has been shown that no updates in a linear combination manner, e.g., model averaging in FL, can tolerate even a single Byzantine attack [216], which leads to non-convergence of the learning algorithm. The primary method for achieving Byzantine resilience during the training process is to remove the adversarial attacks from valuable local updates by information comparison [216]–[219]. However, as the true value of each local update is required for such comparison, it inadvertently increases the risk of privacy leakage.

As stated in the above subsection, AirComp is able to utilize the waveform superposition to mask individual local information into the crowd without additional encryption, while the random channel noise can further enhance the privacy. In conjunction with such superiority, several Byzantine-resilient methods for AirComp-assisted FL are proposed to achieve secure and robust learning procedures [220]–[222]. Specifically, the authors in [220] employ the Weiszfeld algorithm [223] to against Byzantine attack by introducing a smoothed geometric median aggregation rule for numerical stability, where AirComp expedites the inherent additive operation of the Weiszfeld algorithm. The authors in [221] develop a power control policy, named best effort voting (BEV), integrated with SGD to enhance the robustness of AirComp-assisted FL to Byzantine attacks. Moreover, a Byzantine-resilient approach is proposed in [222] by dividing the participating devices into random groups and aggregating updates from only one group per time slot, which effectively reduces the impact

of Byzantine attacks verified by the theoretical analysis and simulation results.

4) *Federated Analytics*: Although FL has flourished in many fields recently, in many applications we are only interested in comprehending the characteristic of local data, e.g., the most frequently played songs and the operation status on edge devices, where the ML technique for model training is not necessary. Therefore, federated analytics (FA) is aroused to obtain descriptive statistics of local data in a private fashion [224], [225]. In particular, FA aims to apply data science methods to locally analyze the raw data, and then the server aggregates local results to draw conclusions, which differs from the FL paradigm emphasizing the collaborative model training instead of measuring the data quality. Various applications operated via FA have been presented, such as discovering and adding out-of-dictionary words to the smart keyboard [226], constructing self-regulating FL systems [227], and anomaly analytics for local poisoning attacks [228].

Additionally, two standard techniques in data analytics, i.e., principle component analysis (PCA) and matrix factorization (MF), are exploited in FA for distributed data analysis with privacy concerns [229], [230]. Herein, PCA is a classic technique for extracting the linear structure of a dataset, which is useful for feature extraction and data compression. Besides, MF aims to factorize one matrix into the product of two lower-rank matrices to simplify subsequent operations, which is widely employed in various signal processing and machine learning applications including data clustering and item recommendations. As the objective of federated PCA and federated MF can be formulated as minimizing the approximation error [230], [231], it is able to leverage modified distributed gradient descent algorithm to realize efficient estimation, where the global gradient aggregation can be efficiently obtained via AirComp in wireless networks. Furthermore, the authors in [231] demonstrate that the inherent channel noise of AirComp can help the descent path to escape from saddle points, thereby accelerating the convergence speed of the federated PCA.

VIII. FUTURE RESEARCH DIRECTIONS

Heretofore, a growing body of research on AirComp has been developed to enhance the performance of wireless communication systems. With the emergence of new scenarios and technologies, we believe that there are more interesting issues to be explored in the future. In this section, we present some future research directions yet to be further investigated on AirComp.

A. Learning for AirComp

The existing literature on AirComp mainly adopts convex optimization based methods to implement system design, which generally suffers from a high computational complexity, especially when the design variables are high dimensional. This ruthlessly creates a gap between the theoretical analysis and the practical demand for real-time signal processing. Besides, the transmit and receive scalars/matrices are usually coupled in the optimization problem of AirComp, which can be solved by optimizing the transceiver in an alternative

manner with multiple iterations until convergence. However, the obtained solutions are usually sub-optimal in this case. Meanwhile, a set of real-time input parameters, e.g., instantaneous CSI, need to keep invariant in practice during the entire optimization procedures to ensure that the optimized results are still feasible in the current transmission environment. The above issues make the convex optimization based methods somewhat inefficient and difficult to apply in practice.

Machine learning (ML) based optimization methods have recently been recognized as a promising solution for future wireless networks [181], [232]–[235]. Specifically, ML is capable of building a complicated functional relationship implied in the input and output data, which enables the wireless network to achieve efficient system design by modeling the transmitter, channel, and receiver as one deep neural network (DNN) [236], [237]. This stimulates future work to leverage the ML technique to implement the transceiver design of AirComp by treating it as a learning task, which makes it possible to achieve more accurate function computation in a more efficient manner as compared with conventional convex optimization based methods. ML also makes it possible to find near-optimal solutions to the transceiver design of SIMO/MIMO AirComp, which is much intractable with the conventional convex optimization based methods.

B. AirComp for Edge Inference

ML techniques have been broadly employed to the design and optimization of wireless communication networks, where the ML solutions are generally obtained from the well-trained DNNs by the inference. Herein, the inference generally requires the input data to pass through several layers of DNNs to realize a composite functional computation. Meanwhile, as the functional computation can be achieved over the air during communication via AirComp, it is natural to consider the possibility of shifting part of inference into the air. The authors in [238] develop an over-the-air convolution method, called AirNN, to shift the convolution operation of convolutional neural networks (CNNs) from devices into the ambient environment. To achieve the desired convolution operation, multiple programmable RISs are deployed to modify the reflecting signals to ensure that the superimposed signal can be combined in a deterministic manner at the AP, thereby resembling the procedure of input data passing through a convolutional layer [238]. This work is a step forward in the use of AirComp to facilitate the ML process at wireless terminals. Nonetheless, there are still several challenges to be addressed in achieving such convolution operations, such as the precise synchronization requirement, energy consumption and latency considerations, and the scalability of practical applications. Moreover, it remains an open issue to explore the feasibility of AirComp to achieve the computation of different kinds of layers in DNNs.

C. Secure AirComp

In foreseen data-driven intelligent wireless networks, data security and privacy are important issues to be considered beyond the requirement of efficiency and reliability during the

data transmission. By harnessing the intra-cell interference, it is natural for AirComp to provide inherent privacy preservation through hiding individual data in the superimposed signals masked by the receiver noise [24], which effectively prevents eavesdroppers from recovering information of a specific device. Nevertheless, there is still a risk of leakage of the functional computation results.

To avoid such eavesdropping, the authors in [239] propose to include a jammer in AirComp system, which ensures that the jamming signal can be canceled at the legitimate receiver but is indistinguishable from white noise at the eavesdropper by exploiting two information theoretical tools, i.e., coding for the compound channel and channel resolvability. In addition to the initial theoretical analysis and outlook on secure AirComp presented in [239], the authors in [240] propose a transceiver design for secure AirComp to minimize the computation error at the FC with a guaranteed error at the eavesdropper above a threshold. The above works motivate us to further explore the practical implementation and optimization under different network architectures in the future. On the other hand, adversarial devices may exist to destroy AirComp by transmitting irrelevant signals over the same radio channel, thereby leading to invalid function computation at the receiver. Therefore, it is critical to design an attack-resilient AirComp to provide secure and reliable services in the presence of adversarial devices.

D. Integrated Sensing/Communication and AirComp

Except for the investigation on co-existed services provided by dedicated devices in a cellular network, recently there has been a focus on how to enable the devices to provide integrated dual-functional services. One of the representative works is the integrated sensing and communication (ISAC) technique [241], [242], which captures the integration gain to improve the efficiency of spectrum and hardware utilization, and the coordination gain to implement mutual assistance while balancing the dual-functional performance. This motivates the exploration on how to build a multi-functional network by integrating AirComp and other services with multi-modal devices. Thereupon, the authors in [243], [244] consider a dual-functional task by integrating communication and AirComp, where the signals transmitted by each device consisting of a computation signal for AirComp and a communication signal for information upload. As channel interference has different impacts on the communication and AirComp, transmit and receive beamformings are optimized for coordinating the interference to minimize the MSE of AirComp while maximizing the SINR/sum-rate of communication services [243], [244]. In addition, the authors in [245] integrate radar sensing and AirComp to realize target detection and function computation simultaneously, which demonstrates the effectiveness of integration framework for improving the spectral efficiency in IoT networks. The study on integrated sensing/communication and AirComp is still in its infancy, and the sensing to be integrated for complex tasks, e.g., vehicular tracking and gesture recognition, warrant further investigations.

IX. CONCLUSIONS

A comprehensive overview and research outlook of AirComp systems were presented in this paper. We first introduced the basics of AirComp from the mathematical form and communication design perspectives, which laid the groundwork for subsequent developments of AirComp. Then, AirComp over different network architectures were elaborated in terms of single-cell, multi-cell, hierarchical, decentralized, RIS-aided, and UAV-aided networks, and the critical issues were discussed along with the existing works studied in these scenarios. Subsequently, a literature review on technologies from the information theory and signal processing perspectives were presented, which described the efforts that have been made to analyze and optimize AirComp systems in recent years. By considering the issues that may be encountered in the practical deployment of AirComp, we further reviewed the related works in terms of efficient channel feedback, robust design, blind data fusion, energy-efficient design, and prototype validation. In addition, we introduced the applications of AirComp in IoT and edge intelligent networks, which demonstrated the advantages of AirComp in reducing communication latency, increasing spectral efficiency, and privacy preservation. Finally, we identified several future research directions to motivate future works of AirComp.

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