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# Improved extended dissipativity results for T-S fuzzy generalized neural networks with mixed interval time-varying delays

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**ABSTRACT** The asymptotic stability and extended dissipativity performance of T-S fuzzy generalized neural networks (GNNs) with mixed interval time-varying delays are investigated in this paper. It is noted that this is the first time that extended dissipativity performance in the T-S fuzzy GNNs has been studied. To obtain the improved results, we construct the Lyapunov-Krasovskii functional (LKF), which consists of single, double, triple, and quadruple integral terms containing full information of the delays and a state variable. Moreover, an improved Wirtinger inequality, a new triple integral inequality, and zero equation, along with a convex combination approach, are used to deal with the derivative of the LKF. By using Matlab's LMI toolbox and the above methods, the new asymptotic stability and extended dissipativity conditions are gained in the form of linear matrix inequalities (LMIs), which include passivity,  $L_2 - L_\infty$ ,  $H_\infty$ , and dissipativity performance. Finally, numerical examples that are less conservative than previous results are presented. Furthermore, we give numerical examples to demonstrate the correctness and efficacy of the proposed method for asymptotic stability and extended dissipativity performance of the T-S fuzzy GNNs, including a particular case of the T-S fuzzy GNNs.

**INDEX TERMS** T-S fuzzy generalized neural networks, Asymptotic stability, Extended dissipativity analysis, Mixed interval time-varying delays.

## I. INTRODUCTION

VARIOUS types of neural networks (NNs) have attracted the attention of researchers in the past few decades because neural networks have a wide range of applications in many fields such as combinatorial optimization, speed detection of moving objects, pattern classification, associative memory design, and other areas [1]–[5]. And we need to first perform a theoretical stability analysis of the equilibrium point to achieve the mentioned applications. Moreover, an essential factor affecting the model of the system to be used in the stability analysis is the time delay. Time delay is a natural phenomenon that always occurs in neural networks. Note that the latency of information processing and the limited speed of information transmission between neurons causes the discrete time delay [6], [7]. On the other hand,

since the variety of sizes and lengths of the axon, nerve impulses are distributed, which causes the distributed time delay [8]. The presence of such delays frequently leads to system instability, oscillation, and decreased performance. Therefore, time delays cannot be avoided in the analysis of stability and performance for NNs, and many researchers have studied NNs with distributed and discrete time delays [9]–[11]. Additionally, mixed interval time-varying delays can occur in many actual industrial systems, such as the reduced-order aggregate model for large-scale converters [12], a multiagent-based consensus algorithm in the energy internet [13], dual-predictive control for AC microgrids [14].

Recently, several researchers have studied the dynamical behaviors of static neural networks (SNNs) [15] or local field neural networks (LFNNs) [16] separately due to differences

in neuron states or local field states. Furthermore, these two models are not equivalent but can be combined into a compact model using reasonable assumptions. Thus, a unified system model was first created by Zhang and Han [17] that included both LFNNs and SNNs, called generalized neural networks (GNNs). Analysis of the stability and performance for GNNs with time delay has become increasingly popular in recent years. For example, Chen et al. [18] analyzed the stability of GNNs with time-varying delay by delay-partitioning method; moreover, they obtained improved criteria by using Free-Matrix-based integral inequality, Peng-Park's inequality, and the novel integral inequality. In [19], the problem of stability analysis for GNNs with time-varying delay is examined based on the new proposed LKF and the developed inequality.

It is well known that most dynamic systems in the real world are complex, ambiguous, and nonlinear that are difficult to control or manipulate. The fuzzy logic theory is an interesting and effective method for dealing with analysis and synthesis issues of complex nonlinear systems. Among the various types of fuzzy approaches, Takagi-Sugeno (T-S) fuzzy [20] approach is popular and successful for dealing with complex nonlinear systems using linear sub-systems. These linear sub-systems are combined through fuzzy membership functions. In addition, the neural networks model also has uncertainty or vagueness, so fuzzy logic has been applied to analyze the dynamical behavior of neural networks. For example, Datta et al. [21] used T-S fuzzy logic to describe Hopfield neural networks (HNNs), and novel stability conditions for fuzzy HNNs are obtained by using Wirtinger inequality. The global exponential stability for the T-S fuzzy Cohen-Grossberg neural network is discussed in [22] by considering the effect of non-singular M-matrix properties and the Lyapunov stability technique. Also, in the T-S fuzzy GNNs model, a nonlinear GNNs system can be represented as a weighted sum of some simple linear GNNs subsystems; then, it provides an excellent chance to use the well-established theory of linear GNNs systems to investigate the complex nonlinear GNNs systems. So, it is interesting to study the T-S fuzzy with the GNNs model.

On the other hand, dissipativity is a widely used and effective tool for analyzing nonlinear systems by describing the energy-related input-output relationship. The concept of dissipativity theory was investigated in 1972 by Willems [23], then attracted considerable attention from researchers as it not only combines passivity and  $H_\infty$  performance but can also be applied for control performance analysis, such as power converters [24] and chemical process control [25]. The study of dissipativity problems for NNs and T-S fuzzy NNs are contained in [26]–[28]. Meanwhile, the  $L_2 - L_\infty$  method is an effective tool for dealing with external interference or uncertainty in the system. Recently, the  $L_2 - L_\infty$  method has been applied to many filtering problems to minimize the maximum value of the estimation error. For example, Choi et al. [29] proposed an  $L_2 - L_\infty$  filtering for T-S fuzzy NNs in order to reduce the effect of external disturbances

in the state estimation error of T-S fuzzy NNs. In [30], the problems of exponential dissipative and  $L_2 - L_\infty$  filtering for the discrete-time switched NNs are investigated by using the discrete-time Wirtinger-type inequality. However, the aforementioned  $L_2 - L_\infty$  studies were not linked to dissipativity performance. To accommodate this demand, Zhang et al. [31] devised a novel scheme known as an extended dissipativity performance, which combines all of these performances. Hence, various researches of GNNs with time-varying delay have been examined on extended dissipativity performance. For example, Manivannan et al. [32] examined the exponential stability and extended dissipativity for GNNs with interval time-varying delays by using the appropriate LKFs, reciprocally convex combination (RCC) approach, the Wirtinger single integral inequality (WSII), and the Wirtinger double integral inequality (WDII). The extended dissipativity state estimation problem for GNNs with mixed time-varying delays is studied in [33] by using Jensen's inequality, RCC idea together with the WDII. Furthermore, the problem of extended dissipative for GNNs with interval time-delays via non-fragile control is investigated by using WSII, WDII, and RCC idea [34]. Unfortunately, no studies have been conducted to investigate the extended dissipativity for T-S fuzzy GNNs with interval distributed and discrete time-varying delays.

By the above discussions, the asymptotic stability and extended dissipativity performance problem is studied for the T-S fuzzy GNNs with mixed interval time-varying delays in this work. The major contributions are listed as follows:

- We construct the model via T-S fuzzy logic, where linear sub-systems are blended through fuzzy membership functions. Moreover, the model contains different continuous neuron activation functions  $f, g, h$ , which are different from [32], [35], [36]. The mixed interval time-varying delays such that do not necessitate being differentiable functions and the lower bound of the time-varying delays does not have to be 0. The output consists of the disturbance, the state vector, and the state vector with interval discrete time-varying delay terms. So, our model is more general and covers others such as [6], [7], [15], [21], [26], [32], [35], [37], [38].

- We construct the suitable Lyapunov-Krasovskii functional, which consists of single, double, triple, and quadruple integral terms containing information about the lower and upper bounds of the delays  $\eta_1, \eta_2, d_1, d_2$ , and a state  $y(t)$ . Furthermore, the LKF contains a new triple integral term  $\frac{(\eta_2^2 - \eta_1^2)}{2} \int_{-\eta_2}^{-\eta_1} \int_{\alpha}^0 \int_{t+\beta}^t y^T(\tau) R y(\tau) d\tau d\beta d\alpha$  and a new quadruple integral term  $\frac{(\eta_2^3 - \eta_1^3)}{6} \int_{-\eta_2}^{-\eta_1} \int_{\alpha}^0 \int_{\beta}^0 \int_{t+\varphi}^t \dot{y}^T(\tau) T \dot{y}(\tau) d\tau d\varphi d\beta d\alpha$  that do not appear in [6], [7], [15], [26], [32], [36]–[38].

- For the first time, an improved Wirtinger inequality, a new triple integral inequality, and zero equation together with convex combination approach are used in this work; as a result, we obtain more general results and maximum allowable delay bounds greater than in previous literature [6], [7], [15], [36]–[38].

• We gain the new extended dissipative criteria that include passivity,  $L_2 - L_\infty$ ,  $H_\infty$ , and dissipativity performance, and the optimal dissipativity performance less conservative than the performance in [26], [32].

This article is divided into five sections, which are as follows. Section 2 includes preliminaries and problem formulation. In section 3, theorems of asymptotic stability and extended dissipativity performance for T-S fuzzy GNNs and a particular case of T-S fuzzy GNNs are addressed. Numerical examples are given in Section 4 to demonstrate the effectiveness of our results. Section 5 provides conclusions and recommendations for future work.

*Notations:*  $\mathbb{R}^n$  and  $\mathbb{R}^+$  stand for, respectively, the  $n$ -dimensional Euclidean space and the set of non negative real numbers.  $\mathbb{R}^{a \times b}$  is the set of  $a \times b$  real matrix.  $\mathcal{C}([-\tau, 0], \mathbb{R}^n)$  is the space of all continuous vector functions mapping  $[-\tau, 0]$  into  $\mathbb{R}^n$ , where  $\tau \in \mathbb{R}^+$ .  $\mathcal{L}_2[0, \infty)$  represents the space of functions  $\psi : \mathbb{R}^+ \rightarrow \mathbb{R}^n$  with the norm  $\|\psi\|_{\mathcal{L}_2} = \left[ \int_0^\infty |\psi(s)|^2 ds \right]^{\frac{1}{2}}$ .  $Q^T$  means that the transpose of the matrix  $Q$ .  $\text{Sym}\{Q\}$  denotes  $Q + Q^T$ .  $Q > (\geq) 0$  represents the symmetric matrix  $Q$  is positive (semi-positive) definite.  $e_k$  stands for the unit column vector having one element on its  $k$ th row and zeros elsewhere.  $I$  represents the identity matrix with appropriate dimensions.

## II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the T-S fuzzy GNNs with mixed interval time-varying delays as follows:

**Rule i:** IF  $v_1(t)$  is  $F_{i1}$  and  $\dots$  and  $v_p(t)$  is  $F_{ip}$  THEN

$$\begin{aligned} \dot{y}(t) &= -A_i y(t) + B_{0i} f(Wy(t)) + B_{1i} g(Wy(t - \eta(t))) \\ &\quad + B_{2i} \int_{t-d_2(t)}^{t-d_1(t)} h(Wy(s)) ds + B_{3i} u(t), \quad (1) \\ z(t) &= C_{1i} y(t) + C_{2i} y(t - \eta(t)) + C_{3i} u(t), \\ y(t) &= \varphi(t), \forall t \in [-\iota, 0], \end{aligned}$$

where  $v_1(t), v_2(t), \dots, v_p(t)$  denote the premises variables;  $F_{il}, i = 1, 2, 3, \dots, m, l = 1, 2, 3, \dots, p$  represent the fuzzy membership functions,  $m$  represent the number of IF-THEN rules and  $p$  represent number of premise variables;  $y(t) = [y_1(t), y_2(t), \dots, y_n(t)]^T \in \mathbb{R}^n$  is the neuron state vector;  $A_i$  is a positive diagonal matrix;  $B_{0i}, B_{1i}, B_{2i}$ , and  $W$  are connection weight matrices;  $B_{3i}, C_{1i}, C_{2i}$ , and  $C_{3i}$  are given matrices;  $u(t) \in \mathbb{R}^n$  is the disturbance input which belongs to  $\mathcal{L}_2[0, \infty)$ ;  $z(t) \in \mathbb{R}^n$  is the output vector;  $\varphi(t) \in \mathcal{C}[-\iota, 0], \mathbb{R}^n$  represents the initial function. The variable  $\eta(t)$  is the interval discrete time-varying delay that correspond to  $0 \leq \eta_1 \leq \eta(t) \leq \eta_2$ .  $d_i(t)$  ( $i = 1, 2$ ) represent the interval distributed time-varying delays that correspond to  $0 \leq d_1 \leq d_1(t) \leq d_2(t) \leq d_2$  where  $\eta_1, \eta_2, d_1, d_2, \iota = \max\{\eta_2, d_2\}$  are real numbers.  $f(\cdot), g(\cdot), h(\cdot) \in \mathbb{R}^n$  are the neuron activation functions that correspond to the following assumptions:

**(H1)** The neuron activation function  $f_i(\cdot)$  for  $i \in \{1, 2, \dots, n\}$  is bounded and continuous such that

$$F_i^- \leq \frac{f_i(Wu_1) - f_i(Wu_2)}{Wu_1 - Wu_2} \leq F_i^+$$

for all  $u_1 \neq u_2$ ,  $F_i^-$  and  $F_i^+$  are real constants, and  $f_i(0) = 0$ .

**(H2)** The neuron activation function  $g_i(\cdot)$  for  $i \in \{1, 2, \dots, n\}$  is bounded and continuous such that

$$G_i^- \leq \frac{g_i(Wu_1) - g_i(Wu_2)}{Wu_1 - Wu_2} \leq G_i^+$$

for all  $u_1 \neq u_2$ ,  $G_i^-$  and  $G_i^+$  are real constants, and  $g_i(0) = 0$ .

**(H3)** The neuron activation function  $h_i(\cdot)$  is bounded and continuous such that

$$H_i^- \leq \frac{h_i(Wu_1) - h_i(Wu_2)}{Wu_1 - Wu_2} \leq H_i^+$$

for all  $u_1 \neq u_2$ ,  $H_i^-$  and  $H_i^+$  are real constants, and  $h_i(0) = 0$ .

Given  $v_l(t) = v_l^0$ , where  $v_l^0$  are singletons, then the truth values of  $\dot{y}(t)$  for each  $i^{\text{th}}$  rule are as follows:

$$x_i(v(t)) = (F_{i1}(v_1(t)) \wedge \dots \wedge F_{ip}(v_p(t))),$$

where  $F_{i1}(v_1(t)), \dots, F_{ip}(v_p(t))$ ,  $i = 1, 2, \dots, m$  denotes the grade of the membership of  $v_1(t), \dots, v_p(t)$  in  $F_{il}$  and  $\wedge$  denotes the 'min' operator.

Applying the center-average defuzzifier approach, the system (1) can be expressed as follows:

$$\begin{aligned} \dot{y}(t) &= \sum_{i=1}^m \omega_i(v(t)) \{ -A_i y(t) + B_{0i} f(Wy(t)) \\ &\quad + B_{1i} g(Wy(t - \eta(t))) \\ &\quad + B_{2i} \int_{t-d_2(t)}^{t-d_1(t)} h(Wy(s)) ds + B_{3i} u(t) \} \quad (2) \\ z(t) &= \sum_{i=1}^m \omega_i(v(t)) \{ C_{1i} y(t) + C_{2i} y(t - \eta(t)) + C_{3i} u(t) \}, \end{aligned}$$

where  $\omega_i(v(t)) = \frac{x_i(v(t))}{\sum_{i=1}^m x_i(v(t))}$ ,  $\forall t$  and  $i = 1, 2, \dots, m$ , is called the fuzzy weighting function which satisfies

$$\omega_i(v(t)) \geq 0, \quad \sum_{i=1}^m \omega_i(v(t)) = 1. \quad (3)$$

The T-S fuzzy GNNs (2) can be expressed compactly as

$$\begin{aligned} \dot{y}(t) &= -\tilde{A} y(t) + \tilde{B}_0 f(Wy(t)) + \tilde{B}_1 g(Wy(t - \eta(t))) \\ &\quad + \tilde{B}_2 \int_{t-d_2(t)}^{t-d_1(t)} h(Wy(s)) ds + \tilde{B}_3 u(t) \quad (4) \\ z(t) &= \tilde{C}_1 y(t) + \tilde{C}_2 y(t - \eta(t)) + \tilde{C}_3 u(t), \end{aligned}$$

where  $\tilde{A} = \sum_{i=1}^m \omega_i(v(t)) A_i$ ,  $\tilde{B}_0 = \sum_{i=1}^m \omega_i(v(t)) B_{0i}$ ,  $\tilde{B}_1 = \sum_{i=1}^m \omega_i(v(t)) B_{1i}$ ,  $\tilde{B}_2 = \sum_{i=1}^m \omega_i(v(t)) B_{2i}$ ,

$$\tilde{B}_3 = \sum_{i=1}^m \omega_i(v(t))B_{3i}, \tilde{C}_1 = \sum_{i=1}^m \omega_i(v(t))C_{1i}, \tilde{C}_2 = \sum_{i=1}^m \omega_i(v(t))C_{2i}, \tilde{C}_3 = \sum_{i=1}^m \omega_i(v(t))C_{3i}.$$

**Remark 1.** The T-S fuzzy GNNs (4) is a general type of delay T-S fuzzy GNNs model that includes both T-S fuzzy LFNNs and T-S fuzzy SNNs, and it can be easily modified to each of them by changing the values of  $\tilde{B}_0, \tilde{B}_1, \tilde{B}_2$ , and  $W$ , i.e.,

- When  $W = I$ , the T-S fuzzy GNNs (4) becomes the following model, namely **T-S fuzzy LFNNs**:

$$\begin{aligned} \dot{y}(t) &= -\tilde{A}y(t) + \tilde{B}_0f(y(t)) + \tilde{B}_1g(y(t - \eta(t))) \\ &\quad + \tilde{B}_2 \int_{t-d_2(t)}^{t-d_1(t)} h(y(s))ds + \tilde{B}_3u(t), \\ z(t) &= C_1y(t) + \tilde{C}_2y(t - \eta(t)) + \tilde{C}_3u(t). \end{aligned}$$

- When  $\tilde{B}_0 = \tilde{B}_1 = \tilde{B}_2 = I$ , the T-S fuzzy GNNs (4) converts to the following model, namely **T-S fuzzy SNNs**:

$$\begin{aligned} \dot{y}(t) &= -\tilde{A}y(t) + f(Wy(t)) + g(Wy(t - \eta(t))) \\ &\quad + \int_{t-d_2(t)}^{t-d_1(t)} h(Wy(s))ds + \tilde{B}_3u(t), \\ z(t) &= \tilde{C}_1y(t) + \tilde{C}_2y(t - \eta(t)) + \tilde{C}_3u(t). \end{aligned}$$

To achieve the main results in the next section, we need to introduce the definition, lemmas, and assumptions.

**Assumption (H4)** [39] Given  $\tilde{\Gamma}_1 \leq 0, \tilde{\Gamma}_3, \tilde{\Gamma}_4 \geq 0$  be real symmetric matrices, and  $\tilde{\Gamma}_2$  be a real matrix, the conditions hold as follows:

- (1)  $\|\tilde{C}_3\| \cdot \|\tilde{\Gamma}_4\| = 0$ ;
- (2)  $(\|\tilde{\Gamma}_1\| + \|\tilde{\Gamma}_2\|) \cdot \|\tilde{\Gamma}_4\| = 0$ ;
- (3)  $\tilde{C}_3^T \tilde{\Gamma}_1 \tilde{C}_3 + \tilde{C}_3^T \tilde{\Gamma}_2 + \tilde{\Gamma}_2^T \tilde{C}_3 + \tilde{\Gamma}_3 > 0$ .

**Definition 1.** [39] For given real matrices  $\tilde{\Gamma}_1, \tilde{\Gamma}_2, \tilde{\Gamma}_3$ , and  $\tilde{\Gamma}_4$  corresponding Assumption (H4), the T-S fuzzy GNNs (4) is said to be extended dissipative if for any  $t_f \geq 0$  and all  $u(t) \in \mathcal{L}_2[0, \infty)$  with the zero initial state, there exists a scalar  $\beta$  such that

$$\int_0^{t_f} J(\tau) d\tau \geq \sup_{0 \leq t \leq t_f} z^T(t) \tilde{\Gamma}_4 z(t) + \beta \quad (5)$$

where

$$\begin{aligned} J(\tau) &= z^T(\tau) \tilde{\Gamma}_1 z(\tau) + 2z^T(\tau) \tilde{\Gamma}_2 u(\tau) \\ &\quad + u^T(\tau) \tilde{\Gamma}_3 u(\tau). \end{aligned} \quad (6)$$

**Remark 2.** The relation (5) expresses a new general performance that covers other performances, i.e.

- When  $\tilde{\Gamma}_1 = 0, \tilde{\Gamma}_2 = 0, \tilde{\Gamma}_3 = \gamma^2 I, \tilde{\Gamma}_4 = I$ , and  $\beta = 0$  then the relation (5) yields the  $L_2 - L_\infty$  performance;
- When  $\tilde{\Gamma}_1 = -I, \tilde{\Gamma}_2 = 0, \tilde{\Gamma}_3 = \gamma^2 I, \tilde{\Gamma}_4 = 0$ , and  $\beta = 0$  then the relation (5) becomes the  $H_\infty$  performance;
- When  $\tilde{\Gamma}_1 = 0, \tilde{\Gamma}_2 = I, \tilde{\Gamma}_3 = \gamma I, \tilde{\Gamma}_4 = 0$ , and  $\beta = 0$  then the relation (5) degenerates the passivity performance;

- When  $\tilde{\Gamma}_1 = Q, \tilde{\Gamma}_2 = S, \tilde{\Gamma}_3 = R - \gamma I, \tilde{\Gamma}_4 = 0$ , and  $\beta = 0$  then the relation (5) determines the  $(Q, S, R) - \gamma$ -dissipativity performance.

**Lemma 1.** [40] Given  $Q \in \mathbb{R}^{n \times n}, Q = Q^T > 0$ , scalars  $t, a_1$  and  $a_2$  correspond to  $a_2 \geq a_1 \geq 0$ , and a vector function  $y : [t - a_2, t] \rightarrow \mathbb{R}^n$  such that the integrations concerned are well defined, the following inequality holds:

$$\begin{aligned} &\frac{1}{2}(a_2^2 - a_1^2) \int_{-a_2}^{-a_1} \int_{t+\alpha}^t y^T(\tau) Q y(\tau) d\tau d\alpha \\ &\geq \int_{-a_2}^{-a_1} \int_{t+\alpha}^t y^T(\tau) d\tau d\alpha Q \int_{-a_2}^{-a_1} \int_{t+\alpha}^t y(\tau) d\tau d\alpha. \end{aligned}$$

**Lemma 2.** [41] Given  $Q \in \mathbb{R}^{n \times n}, K \in \mathbb{R}^{2n \times 2n}$  and  $L \in \mathbb{R}^{2n \times n}$  with  $\begin{bmatrix} K & L \\ * & Q \end{bmatrix} \geq 0$ , and such that the following inequality is well defined, then it satisfies that:

$$\begin{aligned} &-\int_{-a_2}^{-a_1} \int_{\alpha}^0 \int_{t+\beta}^t \dot{y}^T(\tau) Q \dot{y}(\tau) d\tau d\beta d\alpha \\ &\leq \Theta_1^T(t) \left[ (a_2^2 - a_1^2) \text{Sym}\{L\Theta\} + \frac{a_2^3 - a_1^3}{6} K \right] \Theta_1(t), \end{aligned}$$

where  $\Theta = [I, -I]$  and

$$\Theta_1 = \left[ y^T(t), \int_{-a_2}^{-a_1} \int_{t+\alpha}^t \frac{2}{a_2^2 - a_1^2} y^T(\tau) d\tau d\alpha \right]^T.$$

**Lemma 3.** [42] Given matrix  $Q > 0$ , the following inequality holds for all continuous differentiable function  $y(t)$  in  $[a_1, a_2] \in \mathbb{R}^n$ :

$$\begin{aligned} &-\int_{a_1}^{a_2} \dot{y}^T(\tau) Q \dot{y}(\tau) d\tau \leq -\frac{1}{a_2 - a_1} \left( \Theta_2^T Q \Theta_2 \right. \\ &\quad \left. + 3\Theta_3^T Q \Theta_3 + 5\Theta_4^T Q \Theta_4 + 7\Theta_5^T Q \Theta_5 \right), \end{aligned}$$

where

$$\begin{aligned} \Theta_2 &= y(a_2) - y(a_1), \\ \Theta_3 &= y(a_2) + y(a_1) - \frac{2}{a_2 - a_1} \int_{a_1}^{a_2} y(\tau) d\tau, \\ \Theta_4 &= y(a_2) - y(a_1) + \frac{6}{a_2 - a_1} \int_{a_1}^{a_2} y(\tau) d\tau \\ &\quad - \frac{12}{(a_2 - a_1)^2} \int_{a_1}^{a_2} \int_u^{a_2} y(\tau) d\tau du, \\ \Theta_5 &= y(a_2) + y(a_1) - \frac{12}{a_2 - a_1} \int_{a_1}^{a_2} y(\tau) d\tau \\ &\quad + \frac{60}{(a_2 - a_1)^2} \int_{a_1}^{a_2} \int_u^{a_2} y(\tau) d\tau du \\ &\quad - \frac{120}{(a_2 - a_1)^3} \int_{a_1}^{a_2} \int_u^{a_2} \int_\alpha^{a_2} y(\tau) d\tau d\alpha du. \end{aligned}$$

### III. MAIN RESULTS

For the convenience of consideration, we use the following denotations in the rest of this article:

$$\begin{aligned}
 J_1 &= e_1 - e_3, J_2 = e_1 + e_3 - 2e_7, \\
 J_3 &= e_1 - e_3 + 6e_7 - 12e_{13}, \\
 J_4 &= e_1 + e_3 - 12e_7 + 60e_{13} - 120e_{17}, \\
 J_5 &= e_1 - e_4, J_6 = e_1 + e_4 - 2e_8, \\
 J_7 &= e_1 - e_4 + 6e_8 - 12e_{14}, \\
 J_8 &= e_1 + e_4 - 12e_8 + 60e_{14} - 120e_{18}, \\
 J_9 &= e_5 - e_4, J_{10} = e_5 + e_4 - 2e_{10}, \\
 J_{11} &= e_5 - e_4 + 6e_{10} - 12e_{15}, \\
 J_{12} &= e_5 + e_4 - 12e_{10} + 60e_{15} - 120e_{19}, \\
 J_{13} &= e_3 - e_5, J_{14} = e_3 + e_5 - 2e_9, \\
 J_{15} &= e_3 - e_5 + 6e_9 - 12e_{16}, \\
 J_{16} &= e_3 + e_5 - 12e_9 + 60e_{16} - 120e_{20}, \\
 J_{17} &= [e_1 \quad 2e_{12} + 2e_{11}], J_{18} = e_5 W^T G_p^T - e_6, \\
 J_{19} &= e_6 - G_m W e_5, J_{20} = e_1 W^T F_p^T - e_{21}, \\
 J_{21} &= e_{21} - F_m W e_1, J_{22} = e_1 W^T H_p^T - e_{22}, \\
 J_{23} &= e_{22} - H_m W e_1, \\
 \bar{L} &= \begin{bmatrix} L_1 + L_1^T & -L_1 + L_2^T \\ * & -L_2 - L_2^T \end{bmatrix}, \\
 \bar{K} &= \begin{bmatrix} K_1 + K_1^T & K_2 + K_3^T \\ * & K_4 + K_4^T \end{bmatrix}, \\
 F_p &= \text{diag}\{F_1^+, F_2^+, \dots, F_n^+\}, \\
 F_m &= \text{diag}\{F_1^-, F_2^-, \dots, F_n^-\}, \\
 G_p &= \text{diag}\{G_1^+, G_2^+, \dots, G_n^+\}, \\
 G_m &= \text{diag}\{G_1^-, G_2^-, \dots, G_n^-\}, \\
 H_p &= \text{diag}\{H_1^+, H_2^+, \dots, H_n^+\}, \\
 H_m &= \text{diag}\{H_1^-, H_2^-, \dots, H_n^-\},
 \end{aligned}$$

$$\begin{aligned}
 \xi^T(t) &= \left[ y^T(t), \dot{y}^T(t), y^T(t - \eta_1), y^T(t - \eta_2), \right. \\
 & y^T(t - \eta(t)), g^T(Wy(t - \eta(t))), \frac{1}{\eta_1} \int_{t-\eta_1}^t y^T(\tau) d\tau, \\
 & \frac{1}{\eta_2} \int_{t-\eta_2}^t y^T(\tau) d\tau, \frac{1}{\eta(t) - \eta_1} \int_{t-\eta(t)}^{t-\eta_1} y^T(\tau) d\tau, \\
 & \frac{1}{\eta_2 - \eta(t)} \int_{t-\eta_2}^{t-\eta(t)} y^T(\tau) d\tau, \\
 & \frac{1}{\eta_2^2 - \eta_1^2} \int_{-\eta(t)}^{-\eta_1} \int_{t+\alpha}^t y^T(\tau) d\tau d\alpha, \\
 & \frac{1}{\eta_2^2 - \eta_1^2} \int_{-\eta_2}^{-\eta(t)} \int_{t+\alpha}^t y^T(\tau) d\tau d\alpha, \\
 & \frac{1}{\eta_1^2} \int_{t-\eta_1}^t \int_{\alpha}^t y^T(\tau) d\tau d\alpha, \frac{1}{\eta_2^2} \int_{t-\eta_2}^t \int_{\alpha}^t y^T(\tau) d\tau d\alpha, \\
 & \left. \frac{1}{(\eta_2 - \eta(t))^2} \int_{t-\eta_2}^{t-\eta(t)} \int_{\alpha}^{t-\eta(t)} y^T(\tau) d\tau d\alpha, \right]
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{(\eta(t) - \eta_1)^2} \int_{t-\eta(t)}^{t-\eta_1} \int_{\alpha}^{t-\eta_1} y^T(\tau) d\tau d\alpha, \\
 & \frac{1}{\eta_1^3} \int_{t-\eta_1}^t \int_{\alpha}^t \int_{\beta}^t y^T(\tau) d\tau d\beta d\alpha, \\
 & \frac{1}{\eta_2^3} \int_{t-\eta_2}^t \int_{\alpha}^t \int_{\beta}^t y^T(\tau) d\tau d\beta d\alpha, \\
 & \frac{1}{(\eta_2 - \eta(t))^3} \int_{t-\eta_2}^{t-\eta(t)} \int_{\alpha}^{t-\eta(t)} \int_{\beta}^{t-\eta(t)} y^T(\tau) d\tau d\beta d\alpha, \\
 & \frac{1}{(\eta(t) - \eta_1)^3} \int_{t-\eta(t)}^{t-\eta_1} \int_{\alpha}^{t-\eta_1} \int_{\beta}^{t-\eta_1} y^T(\tau) d\tau d\beta d\alpha \Big], \\
 \bar{\xi}^T(t) &= [\xi^T(t), u^T(t)], \\
 \zeta^T(t) &= \left[ \xi^T(t), f^T(Wy(t)), h^T(Wy(t)), \right. \\
 & \left. \int_{t-d_2(t)}^{t-d_1(t)} h^T(Wy(\tau)) d\tau \right], \\
 \bar{\zeta}^T(t) &= [\zeta^T(t), u^T(t)].
 \end{aligned}$$

#### A. STABILITY ANALYSIS

In this subsection, we achieve the new sufficient conditions of asymptotic stability for the T-S fuzzy GNNs (4) and a particular case of the T-S fuzzy GNNs (4).

**Theorem 1.** For given scalars  $\eta_1, \eta_2, d_1, d_2, \alpha_1$ , and  $\alpha_2$ , if there exist symmetric positive definite matrices  $P, Q_1, Q_2, U_1, U_2, U_3, R, T, S \in \mathbb{R}^{n \times n}$ , positive definite matrices  $M_1, M_2 \in \mathbb{R}^{n \times n}$ , positive diagonal matrices  $Y_1, Y_2, Y_3 \in \mathbb{R}^{n \times n}$ , any matrices  $K_1, K_2, K_3, K_4, L_1, L_2 \in \mathbb{R}^{n \times n}$ , and a positive scalar  $c_1$  such that the following linear matrix inequalities hold for  $i = 1, 2, \dots, m$ :

$$\Sigma_i + \Sigma_1 + c_1 I < 0, \quad (7)$$

$$\Sigma_i + \Sigma_2 + c_1 I < 0, \quad (8)$$

$$\begin{bmatrix} K_1 + K_1^T & K_2 + K_3^T & L_1 \\ * & K_4 + K_4^T & L_2 \\ * & * & T \end{bmatrix} \geq 0, \quad (9)$$

where

$$\begin{aligned}
 \Sigma_i &= 2e_1 P e_2^T + e_1 Q_1 e_1^T - e_3 Q_1 e_3^T + e_1 Q_2 e_1^T \\
 & - e_4 Q_2 e_4^T + \eta_1^2 e_2 U_1 e_2^T - J_1 U_1 J_1^T \\
 & - 3J_2 U_1 J_2^T - 5J_3 U_1 J_3^T - 7J_4 U_1 J_4^T \\
 & + \eta_2^2 e_2 U_2 e_2^T - J_5 U_2 J_5^T - 3J_6 U_2 J_6^T \\
 & - 5J_7 U_2 J_7^T - 7J_8 U_2 J_8^T + (\eta_2 - \eta_1)^2 e_2 U_3 e_2^T \\
 & - J_9 U_3 J_9^T - 3J_{10} U_3 J_{10}^T - 5J_{11} U_3 J_{11}^T \\
 & - 7J_{12} U_3 J_{12}^T - J_{13} U_3 J_{13}^T - 3J_{14} U_3 J_{14}^T \\
 & - 5J_{15} U_3 J_{15}^T - 7J_{16} U_3 J_{16}^T \\
 & + \frac{(\eta_2^2 - \eta_1^2)^2}{4} e_1 R e_1^T - (\eta_2^2 - \eta_1^2)^2 e_{12} R e_{12}^T
 \end{aligned}$$

$$\begin{aligned}
 & -(\eta_2^2 - \eta_1^2)^2 e_{11} R e_{11}^T + \frac{(\eta_2^3 - \eta_1^3)^2}{36} e_2 T e_2^T \\
 & + J_{17} \left( (\eta_2^2 - \eta_1^2) \bar{L} + \frac{\eta_2^3 - \eta_1^3}{6} \bar{K} \right) J_{17}^T \\
 & + 2J_{18} Y_1 J_{19}^T - 2e_1 \alpha_1 M_1^T e_2^T - 2e_1 \alpha_1 M_1^T A_i e_1^T \\
 & + 2e_1 \alpha_1 M_1^T B_{1i} e_6^T - 2e_2 \alpha_2 M_2^T e_2^T \\
 & - 2e_2 \alpha_2 M_2^T A_i e_1^T + 2e_2 \alpha_2 M_2^T B_{1i} e_6^T \\
 & + (d_2 - d_1)^2 e_{22} S e_{22}^T - e_{23} S e_{23}^T + 2J_{20} Y_2 J_{21} \\
 & + 2J_{22} Y_3 J_{23} + 2e_1 \alpha_1 M_1^T B_{0i} e_{21}^T \\
 & + 2e_1 \alpha_1 M_1^T B_{2i} e_{23}^T + 2e_2 \alpha_2 M_2^T B_{0i} e_{21}^T \\
 & + 2e_2 \alpha_2 M_2^T B_{2i} e_{23}^T, \\
 \Sigma_1 & = -(\eta_2^2 - \eta_1^2)^2 e_{12} R e_{12}^T, \\
 \Sigma_2 & = -(\eta_2^2 - \eta_1^2)^2 e_{11} R e_{11}^T,
 \end{aligned}$$

then, the T-S fuzzy generalized neural networks (4) is asymptotically stable.

*Proof.* See Appendix A.  $\square$

Next, we study the particular case of the system (4) as follows:

$$\begin{aligned}
 \dot{y}(t) & = -\tilde{A}y(t) + \tilde{B}_1 g(Wy(t - \eta(t))) + \tilde{B}_3 u(t), \\
 z(t) & = \tilde{C}_1 y(t),
 \end{aligned} \tag{10}$$

where  $\tilde{A} = \sum_{i=1}^m \omega_i(v(t)) A_i$ ,  $\tilde{B}_1 = \sum_{i=1}^m \omega_i(v(t)) B_{1i}$ ,  $\tilde{B}_3 = \sum_{i=1}^m \omega_i(v(t)) B_{3i}$ ,  $\tilde{C}_1 = \sum_{i=1}^m \omega_i(v(t)) C_{1i}$ .

**Theorem 2.** For given scalars  $\eta_1, \eta_2, \alpha_1$ , and  $\alpha_2$ , if there exist symmetric positive definite matrices  $P, Q_1, Q_2, U_1, U_2, U_3, R, T \in \mathbb{R}^{n \times n}$ , positive definite matrices  $M_1, M_2 \in \mathbb{R}^{n \times n}$ , positive diagonal matrix  $Y_1 \in \mathbb{R}^{n \times n}$ , any matrices  $K_1, K_2, K_3, K_4, L_1, L_2 \in \mathbb{R}^{n \times n}$ , and a positive scalar  $a_1$  such that the following linear matrix inequalities hold for  $i = 1, 2, \dots, m$ :

$$\Omega_i + \Omega_1 + a_1 I < 0, \tag{11}$$

$$\Omega_i + \Omega_2 + a_1 I < 0, \tag{12}$$

$$\begin{bmatrix} K_1 + K_1^T & K_2 + K_3^T & L_1 \\ * & K_4 + K_4^T & L_2 \\ * & * & T \end{bmatrix} \geq 0, \tag{13}$$

where

$$\begin{aligned}
 \Omega_i & = 2e_1 P e_2^T + e_1 Q_1 e_1^T - e_3 Q_1 e_3^T + e_1 Q_2 e_1^T \\
 & - e_4 Q_2 e_4^T + \eta_1^2 e_2 U_1 e_2^T - J_1 U_1 J_1^T - 3J_2 U_1 J_2^T \\
 & - 5J_3 U_1 J_3^T - 7J_4 U_1 J_4^T + \eta_2^2 e_2 U_2 e_2^T - J_5 U_2 J_5^T \\
 & - 3J_6 U_2 J_6^T - 5J_7 U_2 J_7^T - 7J_8 U_2 J_8^T \\
 & + (\eta_2 - \eta_1)^2 e_2 U_3 e_2^T - J_9 U_3 J_9^T - 3J_{10} U_3 J_{10}^T \\
 & - 5J_{11} U_3 J_{11}^T - 7J_{12} U_3 J_{12}^T - J_{13} U_3 J_{13}^T \\
 & - 3J_{14} U_3 J_{14}^T - 5J_{15} U_3 J_{15}^T - 7J_{16} U_3 J_{16}^T \\
 & + \frac{(\eta_2^2 - \eta_1^2)^2}{4} e_1 R e_1^T - (\eta_2^2 - \eta_1^2)^2 e_{12} R e_{12}^T \\
 & - (\eta_2^2 - \eta_1^2)^2 e_{11} R e_{11}^T + \frac{(\eta_2^3 - \eta_1^3)^2}{36} e_2 T e_2^T \\
 & + J_{17} \left( (\eta_2^2 - \eta_1^2) \bar{L} + \frac{\eta_2^3 - \eta_1^3}{6} \bar{K} \right) J_{17}^T \\
 & + 2J_{18} Y_1 J_{19}^T - 2e_1 \alpha_1 M_1^T e_2^T - 2e_1 \alpha_1 M_1^T A_i e_1^T \\
 & + 2e_1 \alpha_1 M_1^T B_{1i} e_6^T - 2e_2 \alpha_2 M_2^T e_2^T \\
 & - 2e_2 \alpha_2 M_2^T A_i e_1^T + 2e_2 \alpha_2 M_2^T B_{1i} e_6^T, \\
 \Omega_1 & = -(\eta_2^2 - \eta_1^2)^2 e_{12} R e_{12}^T, \\
 \Omega_2 & = -(\eta_2^2 - \eta_1^2)^2 e_{11} R e_{11}^T,
 \end{aligned}$$

then, the T-S fuzzy GNNs (10) is asymptotically stable.

*Proof.* See Appendix B.  $\square$

## B. EXTENDED DISSIPATIVE ANALYSIS

In this subsection, based on the criteria that were developed in Theorem 1 and 2, we achieve the new sufficient conditions of extended dissipativity for the T-S fuzzy GNNs (4) and a particular case of the T-S fuzzy GNNs (4).

**Theorem 3.** For given scalars  $\eta_1, \eta_2, d_1, d_2, \alpha_1, \alpha_2$ , and a positive scalar  $b < 1$ , if there exist symmetric positive definite matrices  $P, Q_1, Q_2, U_1, U_2, U_3, R, T, S \in \mathbb{R}^{n \times n}$ , positive definite matrices  $M_1, M_2 \in \mathbb{R}^{n \times n}$ , positive diagonal matrices  $Y_1, Y_2, Y_3 \in \mathbb{R}^{n \times n}$ , any matrices  $K_1, K_2, K_3, K_4, L_1, L_2 \in \mathbb{R}^{n \times n}$ , and a positive scalar  $c_1$  such that the following linear matrix inequalities hold for  $i = 1, 2, \dots, m$ :

$$\tilde{\Sigma}_i + \Sigma_1 + c_1 I < 0, \tag{14}$$

$$\tilde{\Sigma}_i + \Sigma_2 + c_1 I < 0, \tag{15}$$

$$\begin{bmatrix} K_1 + K_1^T & K_2 + K_3^T & L_1 \\ * & K_4 + K_4^T & L_2 \\ * & * & T \end{bmatrix} \geq 0, \tag{16}$$

$$\begin{bmatrix} bP - C_{1i}^T \tilde{\Gamma}_4 C_{1i} & -C_{1i}^T \tilde{\Gamma}_4 C_{2i} \\ * & (1-b)P - C_{2i}^T \tilde{\Gamma}_4 C_{2i} \end{bmatrix} \geq 0, \tag{17}$$

where

$$\begin{aligned} \tilde{\Sigma}_i &= \bar{\Sigma}_i - e_1 C_{1i}^T \tilde{\Gamma}_1 C_{1i} e_1^T - e_1 C_{1i}^T \tilde{\Gamma}_1 C_{2i} e_5^T \\ &\quad - e_1 C_{1i}^T \tilde{\Gamma}_1 C_{3i} e_{24}^T - e_5 C_{2i}^T \tilde{\Gamma}_1 C_{1i} e_1^T \\ &\quad - e_5 C_{2i}^T \tilde{\Gamma}_1 C_{2i} e_5^T - e_5 C_{2i}^T \tilde{\Gamma}_1 C_{3i} e_{24}^T \\ &\quad - e_{24} C_{3i}^T \tilde{\Gamma}_1 C_{1i} e_1^T - e_{24} C_{3i}^T \tilde{\Gamma}_1 C_{2i} e_5^T \\ &\quad - 2e_1 C_{1i}^T \tilde{\Gamma}_2 e_{24}^T - 2e_5 C_{2i}^T \tilde{\Gamma}_2 e_{24}^T \\ &\quad - e_{24} (C_{3i}^T \tilde{\Gamma}_1 C_{3i} + 2C_{3i}^T \tilde{\Gamma}_2 + \tilde{\Gamma}_3) e_{24}^T, \\ \bar{\Sigma}_i &= \Sigma_i + 2e_{24} \alpha_1 B_{3i}^T M_1 e_1^T + 2e_{24} \alpha_2 B_{3i}^T M_2 e_2^T, \end{aligned}$$

then, the T-S fuzzy generalized neural networks (4) is extended dissipative.

*Proof.* See Appendix C.  $\square$

**Theorem 4.** For given scalars  $\eta_1, \eta_2, \alpha_1$ , and  $\alpha_2$ , if there exist symmetric positive definite matrices  $P, Q_1, Q_2, U_1, U_2, U_3, R, T \in \mathbb{R}^{n \times n}$ , positive definite matrices  $M_1, M_2 \in \mathbb{R}^{n \times n}$ , positive diagonal matrix  $Y_1 \in \mathbb{R}^{n \times n}$ , any matrices  $K_1, K_2, K_3, K_4, L_1, L_2 \in \mathbb{R}^{n \times n}$ , and positive scalars  $a_1$  such that the following linear matrix inequalities hold for  $i = 1, 2, \dots, m$ :

$$\tilde{\Omega}_i + \Omega_1 + a_1 I < 0, \quad (18)$$

$$\tilde{\Omega}_i + \Omega_2 + a_1 I < 0, \quad (19)$$

$$\begin{bmatrix} K_1 + K_1^T & K_2 + K_3^T & L_1 \\ * & K_4 + K_4^T & L_2 \\ * & * & T \end{bmatrix} \geq 0, \quad (20)$$

$$P - C_{1i}^T \tilde{\Gamma}_4 C_{1i} \geq 0, \quad (21)$$

where

$$\begin{aligned} \tilde{\Omega}_i &= \bar{\Omega}_i - e_1 C_{1i}^T \tilde{\Gamma}_1 C_{1i} e_1^T - 2e_{21} \tilde{\Gamma}_2^T C_{1i} e_1^T - e_{21} \tilde{\Gamma}_3 e_{21}^T, \\ \bar{\Omega}_i &= \Omega_i + 2e_{21} \alpha_1 B_{3i}^T M_1 e_1^T + 2e_{21} \alpha_2 B_{3i}^T M_2 e_2^T, \end{aligned}$$

then, the T-S fuzzy GNNs (10) is extended dissipative.

*Proof.* See Appendix D.  $\square$

**Remark 3.** Recently, extended dissipativity for NNs and GNNs has received a lot of attention [32], [35], [39] because it not only covers the efficiency of passivity,  $H_\infty$ ,  $L_2 - L_\infty$ , and dissipativity, but it can also be applied in science and engineering fields. In 2015, Choi et al. [29] investigated an  $L_2 - L_\infty$  filtering for the T-S fuzzy NNs in order to reduce the effect of external disturbances on the state estimation error of the T-S fuzzy NNs. Furthermore, Datta et al. [21] investigated the asymptotic stability of the fuzzy HNNs with interval discrete time-varying delay. It is well known that the above model is a particular case of the T-S fuzzy GNNs, and distributed delay is unavoidable in the analysis of the delayed T-S fuzzy GNNs systems. Thus, the study of extended dissipativity for the T-S fuzzy GNNs with both interval discrete and interval distributed time-varying delays is a fascinating and challenging problem that we have explored and analyzed in this paper.

**Remark 4.** Since the NNs consist of a large number of neurons that connect to one another in a variety of axon sizes and lengths, the time delay is a normal phenomenon that occurs. In practice, the time delay can occur in an irregular manner, such that time-varying delays are not always differentiable. As a result, interval distributed and discrete time-varying delays are not required to be differentiable functions in this work.

**Remark 5.** The suitable Lyapunov-Krasovskii functional is used in this work, and it consists of single, double, triple, and quadruple integral terms that contain information about the lower and upper bounds of the delays  $\eta_1, \eta_2, d_1, d_2$ , and a state  $y(t)$ . In addition, more information on activation functions has been fully incorporated into the stability and performance analysis, which is  $F_i^- \leq \frac{f_i(W_i y(t))}{W_i y(t)} \leq F_i^+$ ,  $G_i^- \leq \frac{g_i(W_i y(t - \eta(t)))}{W_i y(t - \eta(t))} \leq G_i^+$ , and  $H_i^- \leq \frac{h_i(W_i y(t))}{W_i y(t)} \leq H_i^+$ . Furthermore, to bound the derivative of the LKF, improved Wirtinger inequality [42], a new triple integral inequality [41], zero equation, and convex combination approach are used. So, the construction of the LKF together with the assistance of the above technique is the main key to improving the results of this work.

**Remark 6.** In the proof, we use the Lyapunov-Krasovskii functional that is suitable and sufficiently informative. To estimate the derivative of LKF, we use improved Wirtinger inequality [42], a new triple integral inequality [41] with the contribution of zero equation and convex combination approach. These technique are applied to get better results than the others [6], [7], [15], [26], [32], [36]–[38]. However, such complex calculations lead to large LMIs and may be difficult to practical applications. Therefore, in the future, it will be interesting to study and develop methods to achieve results that are easier to use in practical applications.

#### IV. NUMERICAL EXAMPLES

This section includes seven numerical examples to demonstrate the efficacy of the improved results.

**Example 1.** Consider the T-S fuzzy generalized neural networks (10) with the following parameters:

$$\begin{aligned} A_1 &= \text{diag}\{7.0214, 7.4367\}, \\ B_{11} &= \begin{bmatrix} -6.4993 & -12.0275 \\ -0.6867 & 5.6614 \end{bmatrix}, \\ W &= I, G_m = 0, G_p = I, \alpha_1 = 2, \text{ and } \alpha_2 = 3. \end{aligned}$$

In this example, our objective is to estimate the upper bounds of  $\eta(t)$  so that we can compare them to other literature where the T-S fuzzy GNNs (10) is asymptotically stable. By solving Example 1 with LMIs in Theorem 2 for different values of  $\eta_1$  without the upper bound of differentiable delay ( $\mu$ ), we gain the maximum allowable upper bounds (MAUBs) of  $\eta_2$ , as shown in Table 1. Table 1 indicates that the stability criteria

in this work give less conservative results when compared to other studies [6], [7], [15], [36]–[38].

TABLE 1. MAUBs of  $\eta_2$  for various  $\eta_1$  and  $\mu$  in Example 1

$\eta_1$	Methods	$\mu = 0.5$	$\mu = 0.9$	Unknown $\mu$
0.1	[6]	0.2669	0.2668	-
	[36]	0.2678	0.2677	-
	Theorem 2	-	-	0.2955
0.3	[6]	0.3996	0.3996	-
	[36]	0.4007	0.4007	-
	[15]	0.4134	0.4134	-
	[7]	0.4229	0.4228	-
	[37]	0.4372	0.4370	-
	[38](Theorem 1)	0.4400	0.4377	-
	[38](Theorem 2)	0.4524	0.4489	-
	Theorem 2	-	-	0.4955
0.5	[6]	0.5640	0.5640	-
	[36]	0.5643	0.5643	-
	[15]	0.5743	0.5743	-
	[7]	0.5782	0.5782	-
	[37]	0.5904	0.5895	-
	[38](Theorem 1)	0.5912	0.5898	-
	[38](Theorem 2)	0.6356	0.6356	-
		Theorem 2	-	-

**Example 2.** Consider the T-S fuzzy generalized neural networks (10) with the following parameters:

$$A_1 = \text{diag}\{7.3458, 6.9987, 5.5949\},$$

$$B_{11} = \begin{bmatrix} 13.6014 & -2.9616 & -0.6936 \\ 7.4736 & 21.6810 & 3.2100 \\ 0.7290 & -2.6334 & -20.1300 \end{bmatrix},$$

$$G_m = 0, G_p = \text{diag}\{0.3680, 0.1795, 0.2876\},$$

$$W = I, \alpha_1 = 2, \text{ and } \alpha_2 = 3.$$

The goal of this example is to estimate the upper bounds of  $\eta(t)$  so that we can compare them to other literature where the T-S fuzzy GNNs (10) is asymptotically stable. By solving Example 2 with LMIs in Theorem 2 for  $\eta_1 = 0.5$  without the upper bound of differentiable delay ( $\mu$ ), we gain the MAUBs of  $\eta_2$ , as shown in Table 2. The letter NA in Table 2 shows that the maximum delay upper bounds for the relevant cases are not documented. Table 2 indicates that the stability criteria in this work give less conservative results when compared to other studies [6], [15], [36], [38].

TABLE 2. MAUBs of  $\eta_2$  for  $\eta_1 = 0.5$  and various  $\mu$  in Example 2

$\mu$	0.3	0.9	Unknown $\mu$
[6]	0.5880	0.5880	-
[36]	0.5886	0.5885	-
[15]	0.6021	NA	-
[38](Theorem 1)	0.6700	NA	-
[38](Theorem 2)	0.6867	NA	-
	Theorem 2	-	0.7472

**Example 3.** Consider the T-S fuzzy generalized neural networks (10) such that

**Rule 1:** IF  $v_1(t)$  is  $\frac{1}{e^{-2v_1(t)}}$ , THEN

$$\dot{y}(t) = -A_1 y(t) + B_{11} g(y(t - \eta(t)))$$

**Rule 2:** IF  $v_2(t)$  is  $1 - \frac{1}{e^{-2v_1(t)}}$ , THEN

$$\dot{y}(t) = -A_2 y(t) + B_{12} g(y(t - \eta(t)))$$

where

$$A_1 = A_2 = I, B_{11} = \text{diag}\{0.1, 0.3\},$$

$$B_{12} = \begin{bmatrix} 0.88 & 0.30 \\ 0.26 & -0.25 \end{bmatrix},$$

$$W = I, G_m = 0, \text{ and } G_p = I.$$

By taking parameters  $\eta_1 = 1.0, \eta_2 = 1.4, \alpha_1 = 2, \alpha_2 = 3$  and solving Example 3 with LMIs in Theorem 2, the feasible solution are gained

$$P = \begin{bmatrix} 0.0209 & -0.0160 \\ -0.0160 & 0.0739 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 8.7112 & 0 \\ 0 & 8.7111 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 8.7112 & 0 \\ 0 & 8.7111 \end{bmatrix},$$

$$U_1 = 10^{-3} \times \begin{bmatrix} 0.1831 & -0.0091 \\ -0.0091 & 0.2155 \end{bmatrix},$$

$$U_2 = 10^{-3} \times \begin{bmatrix} 0.1572 & -0.0151 \\ -0.0151 & 0.2107 \end{bmatrix},$$

$$U_3 = 10^{-3} \times \begin{bmatrix} 0.3963 & 0.0081 \\ 0.0081 & 0.3720 \end{bmatrix},$$

$$R = \begin{bmatrix} 6.0967 & 0 \\ 0 & 6.0967 \end{bmatrix},$$

$$T = \begin{bmatrix} 0.0031 & -0.0056 \\ -0.0056 & 0.0231 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 0.0175 & -0.0084 \\ -0.0075 & 0.0450 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 0.0004 & -0.0005 \\ -0.0004 & 0.0021 \end{bmatrix},$$

$$Y_1 = 10^{-3} \times \begin{bmatrix} 0.7775 & 0 \\ 0 & 0.7775 \end{bmatrix},$$

$$K_1 = 10^5 \times \begin{bmatrix} -0.0005 & 6.8500 \\ -6.8500 & -0.0005 \end{bmatrix},$$

$$K_2 = 10^{-11} \times \begin{bmatrix} 0.7045 & -0.0123 \\ -0.0487 & -0.9146 \end{bmatrix},$$

$$K_3 = 10^{-11} \times \begin{bmatrix} -0.7049 & 0.0498 \\ 0.0136 & 0.9102 \end{bmatrix},$$

$$K_4 = \begin{bmatrix} -0.2621 & 8.5711 \\ -8.5711 & -0.2621 \end{bmatrix},$$

$$L_1 = 10^{-13} \times \begin{bmatrix} -0.0108 & 0.0369 \\ 0.0318 & -0.1198 \end{bmatrix},$$

$$L_2 = 10^{-14} \times \begin{bmatrix} -0.0004 & -0.0076 \\ -0.0325 & 0.1320 \end{bmatrix},$$

$$a_1 = 8.1584 \times 10^{-6}.$$

In addition, we achieve the MAUBs of  $\eta_2$  for various values of  $\eta_1$ , as shown in Table 3. The state response solution  $y(t)$  is



depicted in Figure 1 where  $u(t) = 0$  and the initial function  $\varphi(t) = [-0.1 \cos(t) \ 0.1 \cos(t)]^T$ .

TABLE 3. MAUBs of  $\eta_2$  for various  $\eta_1$  in Example 3

$\eta_1$	0	0.1	0.5	1.0	1.5
Theorem 2	1.4961	1.5961	1.9961	2.4960	2.9960

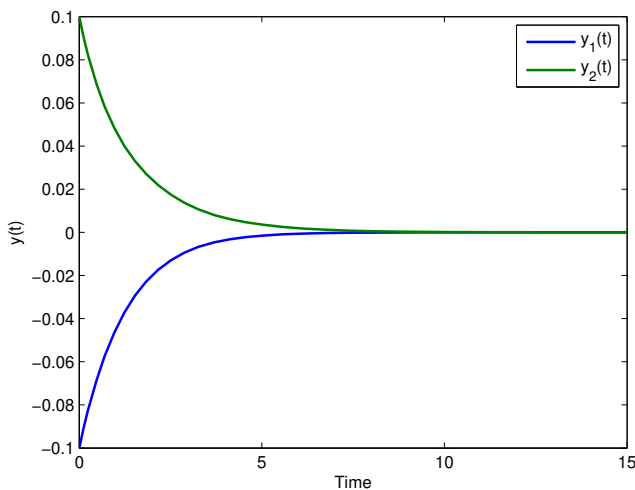


FIGURE 1. State trajectory of the T-S fuzzy GNNs (10) in Example 3

**Example 4.** Consider the T-S fuzzy generalized neural networks (4) such that

**Rule 1:** IF  $v_1(t)$  is  $\frac{1}{e^{-2v_1(t)}}$ , THEN

$$\dot{y}(t) = -A_1 y(t) + B_{01} f(Wy(t)) + B_{11} g(Wy(t - \eta(t))) + B_{21} \int_{t-d_2(t)}^{t-d_1(t)} h(Wy(s)) ds$$

**Rule 2:** IF  $v_2(t)$  is  $1 - \frac{1}{e^{-2v_1(t)}}$ , THEN

$$\dot{y}(t) = -A_2 y(t) + B_{02} f(Wy(t)) + B_{12} g(Wy(t - \eta(t))) + B_{22} \int_{t-d_2(t)}^{t-d_1(t)} h(Wy(s)) ds$$

where

$$\begin{aligned} A_1 = A_2 = 4.5I, B_{01} = \text{diag}\{1, 3\}, \\ B_{02} = \text{diag}\{1.5, 0.5\}, \\ B_{11} = \text{diag}\{0.1, 0.3\}, B_{12} = \begin{bmatrix} 0.88 & 0.30 \\ 0.26 & -0.25 \end{bmatrix}, \\ B_{21} = \text{diag}\{0.7, 0.35\}, B_{22} = \text{diag}\{1, 1.3\}, \\ W = I, G_m = F_m = H_m = 0, \text{ and} \\ G_p = F_p = H_p = I. \end{aligned}$$

By taking parameters  $d_1 = 0.8, d_2 = 2, \eta_1 = 0.2, \eta_2 = 0.45, \alpha_1 = 2, \alpha_2 = 3$  and solving Example 4 with LMIs in Theorem 1, the feasible solution are gained

$$\begin{aligned} P &= 10^{-3} \times \begin{bmatrix} 0.7909 & -0.0804 \\ -0.0804 & 0.9923 \end{bmatrix}, \\ Q_1 &= 10^{-3} \times \begin{bmatrix} 0.3917 & -0.0155 \\ -0.0155 & 0.3980 \end{bmatrix}, \\ Q_2 &= 10^{-3} \times \begin{bmatrix} 0.3916 & -0.0155 \\ -0.0155 & 0.3980 \end{bmatrix}, \\ U_1 &= 10^{-4} \times \begin{bmatrix} 0.7176 & 0.0003 \\ 0.0003 & 0.7310 \end{bmatrix}, \\ U_2 &= 10^{-4} \times \begin{bmatrix} 0.6946 & -0.0029 \\ -0.0029 & 0.7140 \end{bmatrix}, \\ U_3 &= 10^{-3} \times \begin{bmatrix} 0.2591 & 0.0295 \\ 0.0295 & 0.1549 \end{bmatrix}, \\ R &= \begin{bmatrix} 0.0387 & -0.0196 \\ -0.0196 & 0.0933 \end{bmatrix}, \\ T &= \begin{bmatrix} 0.3795 & -0.0528 \\ -0.0528 & 0.4918 \end{bmatrix}, \\ S &= 10^{-3} \times \begin{bmatrix} 0.7033 & 0.0206 \\ 0.0206 & 0.5640 \end{bmatrix}, \\ M_1 &= 10^{-3} \times \begin{bmatrix} 0.3976 & -0.0349 \\ -0.0197 & 0.4784 \end{bmatrix}, \\ M_2 &= 10^{-3} \times \begin{bmatrix} 0.0760 & -0.0117 \\ -0.0055 & 0.1003 \end{bmatrix}, \\ Y_1 &= 10^{-3} \times \begin{bmatrix} 0.2875 & 0 \\ 0 & 0.2875 \end{bmatrix}, \\ Y_2 &= \begin{bmatrix} 0.0019 & 0 \\ 0 & 0.0019 \end{bmatrix}, \\ Y_3 &= \begin{bmatrix} 0.0011 & 0 \\ 0 & 0.0011 \end{bmatrix}, \\ K_1 &= 10^7 \times \begin{bmatrix} 0 & -9.3505 \\ 9.3505 & 0 \end{bmatrix}, \\ K_2 &= 10^8 \times \begin{bmatrix} -1.5278 & -0.3030 \\ 0.3601 & 1.4057 \end{bmatrix}, \\ K_3 &= 10^8 \times \begin{bmatrix} 1.5278 & -0.3601 \\ 0.3030 & -1.4057 \end{bmatrix}, \\ K_4 &= 10^7 \times \begin{bmatrix} 0 & -3.0671 \\ 3.0671 & 0 \end{bmatrix}, \\ L_1 &= \begin{bmatrix} -0.6415 & 0.0208 \\ 0.0208 & -0.6861 \end{bmatrix}, \\ L_2 &= \begin{bmatrix} 0.6407 & -0.0204 \\ -0.0204 & 0.6842 \end{bmatrix}, \\ c_1 &= 1.1141 \times 10^{-6}. \end{aligned}$$

In addition, we achieve the MAUBs of  $\eta_2$  for various values of  $\eta_1$ , as shown in Table 4. The state response solution  $y(t)$  is depicted in Figure 2 where  $u(t) = 0$  and the initial function  $\varphi(t) = [-0.1 \cos(t) \ 0.1 \cos(t)]^T$ .

TABLE 4. MAUBs of  $\eta_2$  for various  $\eta_1$  in Example 4

$\eta_1$	0.0	0.2	0.5	0.75	1.0
Theorem 1	0.8150	0.8340	0.8700	0.9652	1.1005

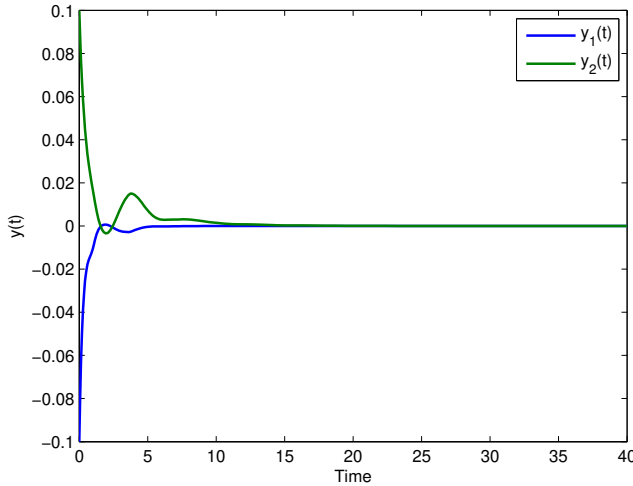


FIGURE 2. State trajectory of the T-S fuzzy GNNs (4) in Example 4

**Example 5.** Consider the T-S fuzzy generalized neural networks (10) with the following matrices:

$$A_1 = \text{diag}\{7.0214, 7.4367\}, B_{11} = I,$$

$$W = \begin{bmatrix} -6.4993 & -12.0275 \\ -0.6867 & 5.6614 \end{bmatrix},$$

$$G_m = \text{diag}\{0.3680, 0.1795\},$$

$$G_p = \text{diag}\{0.3680, 0.1795\}, \text{ and } C_{11} = 0.$$

We are letting  $\tilde{\Gamma}_1 = -I$ ,  $\tilde{\Gamma}_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ ,  $\tilde{\Gamma}_3 = 3I - \gamma I$ , and  $\tilde{\Gamma}_4 = 0$  for the purposes of comparing those results in [26], [32]. By taking parameters  $\alpha_1 = 2, \alpha_2 = 3, \eta_1 = 0.1, \eta_2 = 0.3$  and solving Example 5 with LMIs in Theorem 4, the optimal dissipativity performance  $\gamma$  without the upper bound of differentiable delay ( $\mu$ ) are listed in Table 5. Table 5 indicates that the dissipativity performance in our work is better than those in [26], [32].

TABLE 5. Maximum dissipativity performance  $\gamma$  in Example 5

$\mu$	0.3	0.5	0.7	0.9	Unknown $\mu$
[26]	1.5245	1.5104	1.5051	1.5037	-
[32]	2.2680	2.1757	2.1205	2.0092	-
Theorem 4	-	-	-	-	2.9999

**Example 6.** Consider the T-S fuzzy GNNs (10) where  $i = 2$

and

$$A_1 = 3I, B_{11} = \text{diag}\{0.1, 0.3\}, B_{31} = I, C_{11} = 0.5I,$$

$$A_2 = 3I, B_{12} = \begin{bmatrix} 0.88 & 0.3 \\ 0.26 & -0.25 \end{bmatrix}, B_{32} = 1.5I, C_{12} = I,$$

$$G_m = 0, G_p = W = I, \alpha_1 = 2, \text{ and } \alpha_2 = 3.$$

This example is used to illustrate the extended dissipativity performance of the T-S fuzzy GNNs (10), which includes the  $H_\infty$ ,  $L_2 - L_\infty$ , passivity, and dissipativity performances. By solving Example 6 with LMIs in Theorem 4 for different values of  $\tilde{\Gamma}_1, \tilde{\Gamma}_2, \tilde{\Gamma}_3$ , and  $\tilde{\Gamma}_4$ , we achieve four cases as follow:

(1)  $H_\infty$  performance: When  $\tilde{\Gamma}_1 = -I, \tilde{\Gamma}_2 = 0, \tilde{\Gamma}_3 = \gamma^2 I$ , and  $\tilde{\Gamma}_4 = 0$ , the extended dissipativity performance is reduced to the  $H_\infty$  performance. By solving the LMIs in Theorem 4 with  $\eta_1 = 0.5$  and various  $\eta_2$ , the  $H_\infty$  performance index  $\gamma$  are presented in Table 6. The plot of  $H(t) = \sqrt{\int_0^t z^T(\theta)z(\theta) d\theta / \int_0^t u^T(\theta)u(\theta) d\theta}$  versus time is shown in Figure 3, where the initial condition  $\varphi(t) = [-0.1 \ 0.1]^T$ . Obviously,  $H(t)$  converges to 2.0954.

(2) Passivity performance: When  $\tilde{\Gamma}_1 = 0, \tilde{\Gamma}_2 = I, \tilde{\Gamma}_3 = \gamma I$ , and  $\tilde{\Gamma}_4 = 0$ , the extended dissipativity performance becomes the passivity performance. By solving the LMIs in Theorem 4 with  $\eta_1 = 0.5$  and various  $\eta_2$ , the passivity performance index  $\gamma$  are listed in Table 6. The plot of  $P(t) = -2 \int_0^t z^T(\theta)u(\theta) d\theta / \int_0^t u^T(\theta)u(\theta) d\theta$  versus time is depicted in Figure 4, where the initial condition  $\varphi(t) = [-0.1 \ 0.1]^T$ . Obviously,  $P(t)$  converges to 0.0617.

(3)  $L_2 - L_\infty$  performance: If we take  $\tilde{\Gamma}_1 = 0, \tilde{\Gamma}_2 = 0, \tilde{\Gamma}_3 = \gamma^2 I$ , and  $\tilde{\Gamma}_4 = I$ , the extended dissipativity performance converted into the  $L_2 - L_\infty$  performance. By solving the LMIs in Theorem 4 with  $\eta_1 = 0.5$  and different  $\gamma$ , we gain the MAUBs of  $\eta_2$ , as shown in Table 7. The plot of  $L(t) = \sqrt{z^T(t)z(t) / \int_0^t u^T(\theta)u(\theta) d\theta}$  versus time is presented in Figure 5, where the initial condition  $\varphi(t) = [-0.1 \ 0.1]^T$ . Obviously,  $\sup_t L(t) = 0.0273$ .

(4) Dissipativity performance: If we set  $\tilde{\Gamma}_1 = -I, \tilde{\Gamma}_2 = I, \tilde{\Gamma}_3 = \mathcal{R} - \gamma I, \mathcal{R} = 4I$ , and  $\tilde{\Gamma}_4 = 0$ , the extended dissipativity performance is converted into the dissipativity performance. By solving the LMIs in Theorem 4 with  $\eta_1 = 0.5$  and different  $\gamma$ , we obtain the MAUBs of  $\eta_2$ , as shown in Table 7. The plot of  $D(t) = \left( \int_0^t -z^T(\theta)z(\theta) + 2z^T(\theta)u(\theta) + 4u^T(\theta)u(\theta) d\theta \right) / \left( \int_0^t u^T(\theta)u(\theta) d\theta \right)$  versus time is depicted in Figure 6, where the initial condition  $\varphi(t) = [-0.1 \ 0.1]^T$ . Obviously,  $D(t)$  converges to -0.4526.

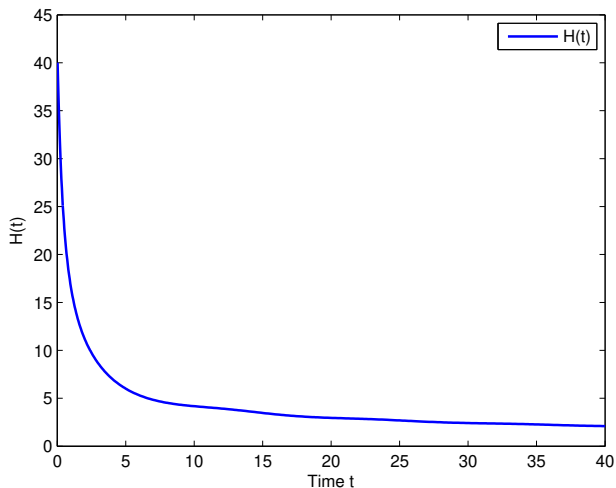
TABLE 6. Minimum  $\gamma$  for  $H_\infty$  case and passivity case in Example 6 with different  $\eta_2$

$\eta_2$	1.0	1.2	1.5	1.8
$H_\infty$ case	0.7971	0.8794	1.1667	2.7810
Passivity case	0.0459	0.1023	0.2991	1.4051

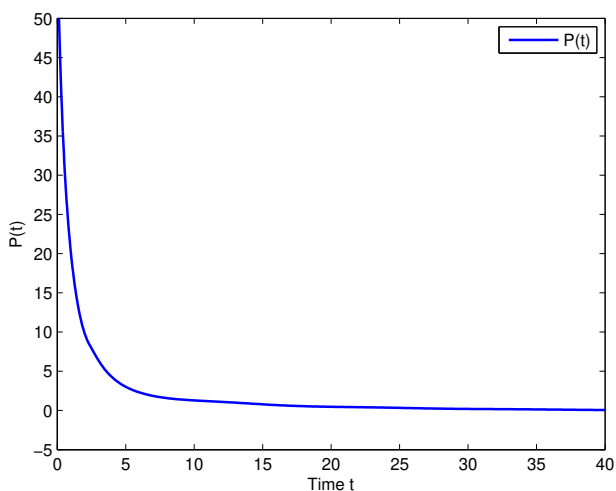
**Example 7.** In this example, we illustrate the extended dissipativity performance of the T-S fuzzy GNNs (4), including

**TABLE 7.** MAUBs of  $\eta_2$  for  $L_2 - L_\infty$  case and dissipativity case in Example 6 with various  $\gamma$

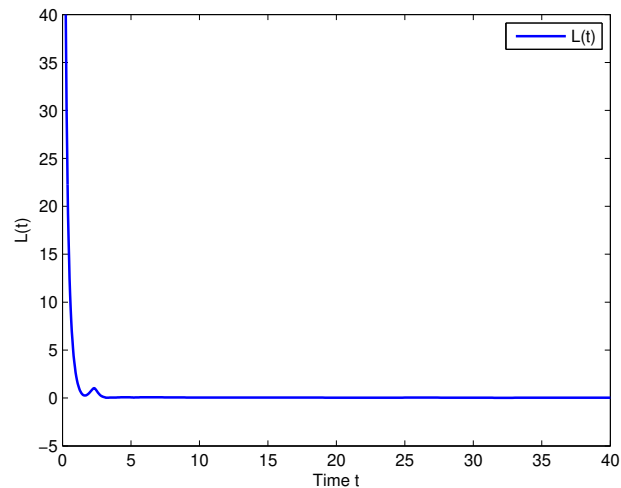
$\gamma$	1.0	1.5	2.0	2.5
$L_2 - L_\infty$ case	1.1767	1.6267	1.7633	1.8218
Dissipativity case	1.7436	1.7245	1.6984	1.6597



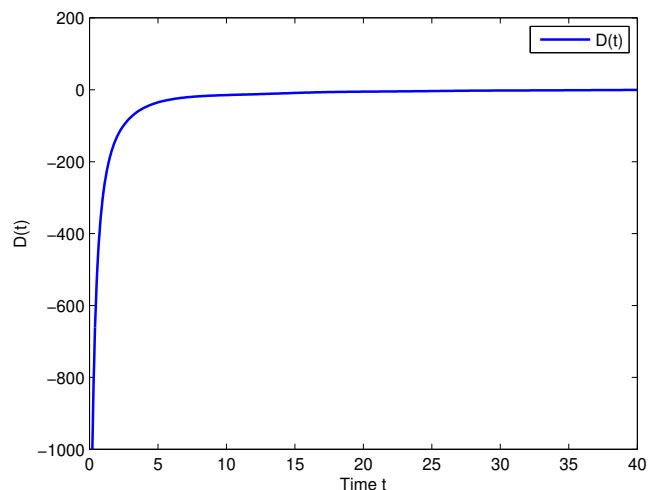
**FIGURE 3.** State trajectory of  $H(t)$  in Example 6



**FIGURE 4.** State trajectory of  $P(t)$  in Example 6



**FIGURE 5.** State trajectory of  $L(t)$  in Example 6



**FIGURE 6.** State trajectory of  $D(t)$  in Example 6

Consider the T-S fuzzy GNNs (4) where  $i = 2$  and

$$\begin{aligned}
 A_1 &= 5.5I, B_{01} = \text{diag}\{1, 3\}, B_{11} = \text{diag}\{0.1, 0.3\}, \\
 B_{21} &= \text{diag}\{0.7, 0.35\}, B_{31} = \begin{bmatrix} 0.2 & -0.01 \\ 0.01 & 0.2 \end{bmatrix}, \\
 C_{11} &= \text{diag}\{2, 1\}, C_{21} = \text{diag}\{0.1, -0.01\}, \\
 C_{31} &= \text{diag}\{1, 1.9\}, A_2 = 5.5I, B_{02} = \text{diag}\{1.5, 0.5\}, \\
 B_{12} &= \begin{bmatrix} 0.88 & 0.3 \\ 0.26 & -0.25 \end{bmatrix}, B_{22} = \text{diag}\{1, 1.3\}, \\
 B_{32} &= \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, C_{12} = \text{diag}\{1, 0.1\}, \\
 C_{22} &= \text{diag}\{0.1, 0.01\}, C_{32} = \text{diag}\{1.5, 1.9\}, \\
 F_m &= G_m = H_m = 0, F_p = G_p = H_p = I, W = I, \\
 d_1 &= 0.5, d_2 = 1.2, \alpha_1 = 2, \text{ and } \alpha_2 = 3.
 \end{aligned}$$

the  $H_\infty$ ,  $L_2 - L_\infty$ , passivity, and dissipativity performances.

By setting  $\Gamma_1 = -I$ ,  $\Gamma_2 = I$ ,  $\Gamma_3 = (5 - \gamma)I$ ,  $\Gamma_4 = I$  and solving Example 7 with LMIs (14)-(17) in Theorem 3, the MAUBs of  $\eta_2$  for different values of  $\eta_1$  are presented in Table 8. The state response solution  $y(t)$  in Example 7 is shown in Figure 7 where  $u(t) = [0.002 \cos(0.3t) \quad -0.003 \sin(0.3t)]^T$  and the initial function  $\varphi(t) = [-0.25 \cos(t) \quad 0.25 \cos(t)]^T$ .

TABLE 8. MAUBs of  $\eta_2$  for different values of  $\eta_1$  in Example 7

$\eta_1$	0.5	1.0	1.5	2.0	2.5
Theorem 3	1.6814	2.1772	2.4563	2.7360	3.0302

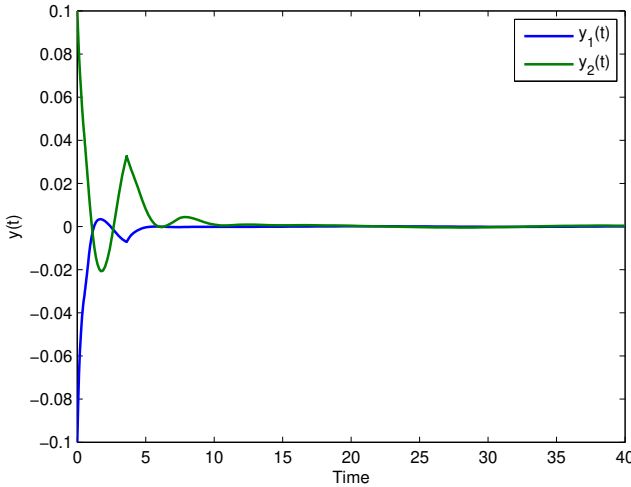


FIGURE 7. State trajectory of the T-S fuzzy GNNs (4) in Example 7

**Remark 7.** In the future topic, it is very challenging to apply some lemmas or Lyapunov-Krasovskii functional used in this paper into the memristive systems [43]–[45] to achieve improved stability criteria, which are applied in next generation computer [46], [47] and powerful brain-like “neural” computer [48], [49].

## V. CONCLUSION

In this article, we investigated the extended dissipativity problem for the T-S fuzzy GNNs with mixed interval time-varying delays. Firstly, we gain the novel asymptotic stability conditions for the T-S fuzzy GNNs and a particular case of the T-S fuzzy GNNs by using an appropriate LKF consisting of single, double, triple, and quadruple integral terms, a new triple integral inequality, an improved Wirtinger inequality, zero equation together with a convex combination approach. Next, the asymptotic stability results are developed to the analysis of extended dissipativity performance for the T-S fuzzy GNNs and a particular case of the T-S fuzzy GNNs, which covers  $L_2 - L_\infty$ ,  $H_\infty$ , passivity, and dissipativity performance. Furthermore, we obtain the less conservative results for maximum allowable delay bounds and the optimal

dissipativity performance for a particular case of the T-S fuzzy GNNs. Finally, illustrative examples are given to demonstrate the correctness and effectiveness of the proposed method. The proposed results and methods in this work are expected to be extended in the future topic to the exponential projective synchronization problem of T-S fuzzy GNNs, the extended dissipativity analysis of T-S fuzzy memristive GNNs, and so on [43], [45], [50].

## APPENDIX A.

**Proof of Theorem 1.** Let us use the Lyapunov-Krasovskii functional candidate for the T-S fuzzy GNNs (4) as follows:

$$V(y(t), t) = \sum_{i=1}^9 V_i(y(t), t),$$

where

$$V_1(y(t), t) = y^T(t) P y(t),$$

$$V_2(y(t), t) = \int_{t-\eta_1}^t y^T(\tau) Q_1 y(\tau) d\tau,$$

$$V_3(y(t), t) = \int_{t-\eta_2}^t y^T(\tau) Q_2 y(\tau) d\tau,$$

$$V_4(y(t), t) = \eta_1 \int_{-\eta_1}^0 \int_{t+\tau}^t \dot{y}^T(u) U_1 \dot{y}(u) du d\tau,$$

$$V_5(y(t), t) = \eta_2 \int_{-\eta_2}^0 \int_{t+\tau}^t \dot{y}^T(u) U_2 \dot{y}(u) du d\tau,$$

$$V_6(y(t), t) = (\eta_2 - \eta_1) \int_{-\eta_2}^{-\eta_1} \int_{t+\tau}^t \dot{y}^T(u) U_3 \dot{y}(u) du d\tau,$$

$$V_7(y(t), t) = \frac{(\eta_2^2 - \eta_1^2)}{2} \times \int_{-\eta_2}^{-\eta_1} \int_{\alpha}^0 \int_{t+\beta}^t y^T(\tau) R y(\tau) d\tau d\beta d\alpha,$$

$$V_8(y(t), t) = \frac{(\eta_2^3 - \eta_1^3)}{6} \int_{-\eta_2}^{-\eta_1} \int_{\alpha}^0 \int_{\beta}^0 \int_{t+\varphi}^t \dot{y}^T(\tau) \times T \dot{y}(\tau) d\tau d\varphi d\beta d\alpha,$$

$$V_9(y(t), t) = (d_2 - d_1) \int_{-d_2}^{-d_1} \int_{t+\tau}^t h^T(W y(s)) \times Sh(W y(s)) ds d\tau.$$

We find time derivatives of  $V_i(y(t), t)$ ,  $i = 1, 2, \dots, 9$ , along the trajectories of the T-S fuzzy GNNs (4), we gain

$$\dot{V}_1(y(t), t) = y^T(t) P \dot{y}(t) + \dot{y}^T(t) P y(t), \quad (22)$$

$$\dot{V}_2(y(t), t) = y^T(t) Q_1 y(t) - y^T(t - \eta_1) Q_1 y(t - \eta_1), \quad (23)$$

$$\dot{V}_3(y(t), t) = y^T(t) Q_2 y(t) - y^T(t - \eta_2) Q_2 y(t - \eta_2), \quad (24)$$

$$\dot{V}_4(y(t), t) = \eta_1^2 \dot{y}^T(t) U_1 \dot{y}(t) - \eta_1 \int_{t-\eta_1}^t \dot{y}^T(\beta) U_1 \dot{y}(\beta) d\beta, \quad (25)$$

$$\begin{aligned} \dot{V}_5(y(t), t) &= \eta_2^2 \dot{y}^T(t) U_2 \dot{y}(t) \\ &\quad - \eta_2 \int_{t-\eta_2}^t \dot{y}^T(\beta) U_2 \dot{y}(\beta) d\beta, \end{aligned} \quad (26)$$

$$\begin{aligned} \dot{V}_6(y(t), t) &= (\eta_2 - \eta_1)^2 \dot{y}^T(t) U_3 \dot{y}(t) \\ &\quad - (\eta_2 - \eta_1) \int_{t-\eta_2}^{t-\eta_1} \dot{y}^T(\beta) U_3 \dot{y}(\beta) d\beta, \end{aligned} \quad (27)$$

$$\begin{aligned} \dot{V}_7(y(t), t) &= \frac{(\eta_2^2 - \eta_1^2)^2}{4} y^T(t) R y(t) - \frac{(\eta_2^2 - \eta_1^2)}{2} \\ &\quad \times \int_{-\eta_2}^{-\eta_1} \int_{t+\alpha}^t y^T(\beta) R y(\beta) d\beta d\alpha, \end{aligned} \quad (28)$$

$$\begin{aligned} \dot{V}_8(y(t), t) &= \frac{(\eta_2^3 - \eta_1^3)^2}{36} \dot{y}^T(t) T \dot{y}(t) - \frac{(\eta_2^3 - \eta_1^3)}{6} \\ &\quad \times \int_{-\eta_2}^{-\eta_1} \int_{\alpha}^0 \int_{t+\beta}^t \dot{y}^T(\tau) T \dot{y}(\tau) d\tau d\beta d\alpha, \end{aligned} \quad (29)$$

$$\begin{aligned} \dot{V}_9(y(t), t) &\leq (d_2 - d_1)^2 \zeta^T(t) e_{22} S e_{22}^T \zeta(t) \\ &\quad - \zeta^T(t) e_{23} S e_{23}^T \zeta(t). \end{aligned} \quad (30)$$

By using Lemma 3, we get

$$\begin{aligned} -\eta_1 \int_{t-\eta_1}^t \dot{y}^T(\beta) U_1 \dot{y}(\beta) d\beta &\leq -\zeta^T(t) (e_1 - e_3) \\ &\quad \times U_1 (e_1 - e_3)^T \zeta(t) - 3\zeta^T(t) (e_1 + e_3 - 2e_7) \\ &\quad \times U_1 (e_1 + e_3 - 2e_7)^T \zeta(t) \\ &\quad - 5\zeta^T(t) (e_1 - e_3 + 6e_7 - 12e_{13}) \\ &\quad \times U_1 (e_1 - e_3 + 6e_7 - 12e_{13})^T \zeta(t) \\ &\quad - 7\zeta^T(t) (e_1 + e_3 - 12e_7 + 60e_{13} - 120e_{17}) \\ &\quad \times U_1 (e_1 + e_3 - 12e_7 + 60e_{13} - 120e_{17})^T \zeta(t), \end{aligned} \quad (31)$$

$$\begin{aligned} -\eta_2 \int_{t-\eta_2}^t \dot{y}^T(\beta) U_2 \dot{y}(\beta) d\beta &\leq -\zeta^T(t) (e_1 - e_4) \\ &\quad \times U_2 (e_1 - e_4)^T \zeta(t) - 3\zeta^T(t) (e_1 + e_4 - 2e_8) \\ &\quad \times U_2 (e_1 + e_4 - 2e_8)^T \zeta(t) \\ &\quad - 5\zeta^T(t) (e_1 - e_4 + 6e_8 - 12e_{14}) \\ &\quad \times U_2 (e_1 - e_4 + 6e_8 - 12e_{14})^T \zeta(t) \\ &\quad - 7\zeta^T(t) (e_1 + e_4 - 12e_8 + 60e_{14} - 120e_{18}) \\ &\quad \times U_2 (e_1 + e_4 - 12e_8 + 60e_{14} - 120e_{18})^T \zeta(t), \end{aligned} \quad (32)$$

$$\begin{aligned} &-(\eta_2 - \eta_1) \int_{t-\eta_2}^{t-\eta_1} \dot{y}^T(\beta) U_3 \dot{y}(\beta) d\beta \\ &= -(\eta_2 - \eta_1) \int_{t-\eta_2}^{t-\eta(t)} \dot{y}^T(\beta) U_3 \dot{y}(\beta) d\beta \\ &\quad - (\eta_2 - \eta_1) \int_{t-\eta(t)}^{t-\eta_1} \dot{y}^T(\beta) U_3 \dot{y}(\beta) d\beta \end{aligned}$$

$$\begin{aligned} &\leq -\zeta^T(t) (e_5 - e_4) U_3 (e_5 - e_4)^T \zeta(t) \\ &\quad - 3\zeta^T(t) (e_5 + e_4 - 2e_{10}) U_3 (e_5 + e_4 - 2e_{10})^T \zeta(t) \\ &\quad - 5\zeta^T(t) (e_5 - e_4 + 6e_{10} - 12e_{15}) \\ &\quad \times U_3 (e_5 - e_4 + 6e_{10} - 12e_{15})^T \zeta(t) \\ &\quad - 7\zeta^T(t) (e_5 + e_4 - 12e_{10} + 60e_{15} - 120e_{19}) \\ &\quad \times U_3 (e_5 + e_4 - 12e_{10} + 60e_{15} - 120e_{19})^T \zeta(t) \\ &\quad - \zeta^T(t) (e_3 - e_5) U_3 (e_3 - e_5)^T \zeta(t) \\ &\quad - 3\zeta^T(t) (e_3 + e_5 - 2e_9) U_3 (e_3 + e_5 - 2e_9)^T \zeta(t) \\ &\quad - 5\zeta^T(t) (e_3 - e_5 + 6e_9 - 12e_{16}) \\ &\quad \times U_3 (e_3 - e_5 + 6e_9 - 12e_{16})^T \zeta(t) \\ &\quad - 7\zeta^T(t) (e_3 + e_5 - 12e_9 + 60e_{16} - 120e_{20}) \\ &\quad \times U_3 (e_3 + e_5 - 12e_9 + 60e_{16} - 120e_{20})^T \zeta(t). \end{aligned} \quad (33)$$

In addition, we derive the following inequality based on Lemma 1:

$$\begin{aligned} &-\frac{(\eta_2^2 - \eta_1^2)}{2} \int_{-\eta_2}^{-\eta_1} \int_{t+\alpha}^t y^T(\beta) R y(\beta) d\beta d\alpha \\ &\leq -(\eta_2^2 - \eta_1^2)^2 \zeta^T(t) e_{12} R e_{12}^T \zeta(t) \\ &\quad - \kappa (\eta_2^2 - \eta_1^2)^2 \zeta^T(t) e_{12} R e_{12}^T \zeta(t) \\ &\quad - (\eta_2^2 - \eta_1^2)^2 \zeta^T(t) e_{11} R e_{11}^T \zeta(t) \\ &\quad - (1 - \kappa) (\eta_2^2 - \eta_1^2)^2 \zeta^T(t) e_{11} R e_{11}^T \zeta(t), \end{aligned} \quad (34)$$

where  $\kappa = \frac{\eta^2(t) - \eta_1^2}{\eta_2^2 - \eta_1^2}$ .

Based on Lemma 2 and inequality (9), we achieve

$$\begin{aligned} &-\frac{(\eta_2^3 - \eta_1^3)}{6} \int_{-\eta_2}^{-\eta_1} \int_{\alpha}^0 \int_{t+\beta}^t \dot{y}^T(\tau) T \dot{y}(\tau) d\tau d\beta d\alpha \\ &\leq \zeta^T(t) [e_1 \quad 2e_{12} + 2e_{11}] \left( (\eta_2^2 - \eta_1^2) \bar{L} + \frac{\eta_2^3 - \eta_1^3}{6} \bar{K} \right) \\ &\quad \times [e_1 \quad 2e_{12} + 2e_{11}]^T \zeta(t). \end{aligned} \quad (35)$$

From assumptions **(H1)**, **(H2)**, and **(H3)**, we gain the following relations:

$$\begin{aligned} &2 \left( F_p W y(t) - f(W y(t)) \right)^T Y_2 \\ &\quad \times \left( f(W y(t)) - F_m W y(t) \right) \geq 0, \end{aligned} \quad (36)$$

$$\begin{aligned} &2 \left( G_p W y(t - \eta(t)) - g(W y(t - \eta(t))) \right)^T Y_1 \\ &\quad \times \left( g(W y(t - \eta(t))) - G_m W y(t - \eta(t)) \right) \geq 0, \end{aligned} \quad (37)$$

$$\begin{aligned} &2 \left( H_p W y(t) - h(W y(t)) \right)^T Y_3 \\ &\quad \times \left( h(W y(t)) - H_m W y(t) \right) \geq 0. \end{aligned} \quad (38)$$

Consider the T-S fuzzy GNNs (4), we obtain

$$0 = 2 [y^T(t)\alpha_1 M_1^T + \dot{y}^T(t)\alpha_2 M_2^T] [-\dot{y}(t) - \tilde{A}y(t) + \tilde{B}_0 f(Wy(t)) + \tilde{B}_1 g(Wy(t - \eta(t))) + \tilde{B}_2 \int_{t-d_2(t)}^{t-d_1(t)} h(Wy(s))ds + \tilde{B}_3 u(t)]. \quad (39)$$

Combining from (22)-(39), we have

$$\dot{V}(y(t), t) \leq \sum_{i=1}^m \omega_i \bar{\zeta}^T(t) \{ \kappa \bar{\Sigma}_i^{(1)} + (1 - \kappa) \bar{\Sigma}_i^{(2)} \} \bar{\zeta}(t), \quad (40)$$

where  $\bar{\Sigma}_i^{(1)} = \bar{\Sigma}_i + \Sigma_1$ ,  $\bar{\Sigma}_i^{(2)} = \bar{\Sigma}_i + \Sigma_2$ , and  $\bar{\Sigma}_i = \Sigma_i + 2e_{24}\alpha_1 B_{3i}^T M_1 e_1^T + 2e_{24}\alpha_2 B_{3i}^T M_2 e_2^T$  with  $\Sigma_i$ ,  $\Sigma_1$ , and  $\Sigma_2$  are defined in (7) and (8).

Consider (40) with  $u(t) = 0$  (doesn't have disturbance), we obtain

$$\dot{V}(y(t), t) \leq \sum_{i=1}^m \omega_i \zeta^T(t) \{ \kappa \Sigma_i^{(1)} + (1 - \kappa) \Sigma_i^{(2)} \} \zeta(t),$$

where  $\Sigma_i^{(1)} = \Sigma_i + \Sigma_1$  and  $\Sigma_i^{(2)} = \Sigma_i + \Sigma_2$ .

From the condition (3), the upper bound of  $\dot{V}(y(t), t)$  is negative if the following inequality holds:

$$\kappa \Sigma_i^{(1)} + (1 - \kappa) \Sigma_i^{(2)} < -c_1 I. \quad (41)$$

The inequality (41) can be expressed as follows:

$$\kappa (\Sigma_i^{(1)} + c_1 I) + (1 - \kappa) (\Sigma_i^{(2)} + c_1 I) < 0. \quad (42)$$

Since  $0 \leq \kappa \leq 1$ , the term  $\kappa (\Sigma_i^{(1)} + c_1 I) + (1 - \kappa) (\Sigma_i^{(2)} + c_1 I)$  is a convex combination of  $\Sigma_i^{(1)} + c_1 I$  and  $\Sigma_i^{(2)} + c_1 I$ . The combination is negative definite only if each term is negative; so, (42) is equivalent to (7) and (8). Then, the T-S fuzzy generalized neural networks (4) is asymptotically stable.

## APPENDIX B.

**Proof of Theorem 2.** Let us use the Lyapunov-Krasovskii functional candidate for the T-S fuzzy GNNs (10) as follows:

$$V(y(t), t) = \sum_{i=1}^8 V_i(y(t), t),$$

where

$$\begin{aligned} V_1(y(t), t) &= y^T(t) P y(t), \\ V_2(y(t), t) &= \int_{t-\eta_1}^t y^T(\tau) Q_1 y(\tau) d\tau, \\ V_3(y(t), t) &= \int_{t-\eta_2}^t y^T(\tau) Q_2 y(\tau) d\tau, \\ V_4(y(t), t) &= \eta_1 \int_{-\eta_1}^0 \int_{t+\tau}^t \dot{y}^T(u) U_1 \dot{y}(u) du d\tau, \\ V_5(y(t), t) &= \eta_2 \int_{-\eta_2}^0 \int_{t+\tau}^t \dot{y}^T(u) U_2 \dot{y}(u) du d\tau, \\ V_6(y(t), t) &= (\eta_2 - \eta_1) \int_{-\eta_2}^{-\eta_1} \int_{t+\tau}^t \dot{y}^T(u) U_3 \dot{y}(u) du d\tau, \\ V_7(y(t), t) &= \frac{(\eta_2^2 - \eta_1^2)}{2} \\ &\quad \times \int_{-\eta_2}^{-\eta_1} \int_{\alpha}^0 \int_{t+\beta}^t y^T(\tau) R y(\tau) d\tau d\beta d\alpha, \\ V_8(y(t), t) &= \frac{(\eta_2^3 - \eta_1^3)}{6} \int_{-\eta_2}^{-\eta_1} \int_{\alpha}^0 \int_{\beta}^0 \int_{t+\varphi}^t \dot{y}^T(\tau) \\ &\quad \times T \dot{y}(\tau) d\tau d\varphi d\beta d\alpha. \end{aligned}$$

We find time derivatives of  $V_i(y(t), t)$ ,  $i = 1, 2, \dots, 8$ , along the trajectories of the T-S fuzzy GNNs (10), we gain

$$\dot{V}_1(y(t), t) = y^T(t) P \dot{y}(t) + \dot{y}^T(t) P y(t), \quad (43)$$

$$\begin{aligned} \dot{V}_2(y(t), t) &= y^T(t) Q_1 y(t) \\ &\quad - y^T(t - \eta_1) Q_1 y(t - \eta_1), \end{aligned} \quad (44)$$

$$\begin{aligned} \dot{V}_3(y(t), t) &= y^T(t) Q_2 y(t) \\ &\quad - y^T(t - \eta_2) Q_2 y(t - \eta_2), \end{aligned} \quad (45)$$

$$\begin{aligned} \dot{V}_4(y(t), t) &= \eta_1^2 \dot{y}^T(t) U_1 \dot{y}(t) \\ &\quad - \eta_1 \int_{t-\eta_1}^t \dot{y}^T(\beta) U_1 \dot{y}(\beta) d\beta, \end{aligned} \quad (46)$$

$$\begin{aligned} \dot{V}_5(y(t), t) &= \eta_2^2 \dot{y}^T(t) U_2 \dot{y}(t) \\ &\quad - \eta_2 \int_{t-\eta_2}^t \dot{y}^T(\beta) U_2 \dot{y}(\beta) d\beta, \end{aligned} \quad (47)$$

$$\begin{aligned} \dot{V}_6(y(t), t) &= (\eta_2 - \eta_1)^2 \dot{y}^T(t) U_3 \dot{y}(t) \\ &\quad - (\eta_2 - \eta_1) \int_{t-\eta_2}^{t-\eta_1} \dot{y}^T(\beta) U_3 \dot{y}(\beta) d\beta, \end{aligned} \quad (48)$$

$$\begin{aligned} \dot{V}_7(y(t), t) &= \frac{(\eta_2^2 - \eta_1^2)^2}{4} y^T(t) R y(t) - \frac{(\eta_2^2 - \eta_1^2)}{2} \\ &\quad \times \int_{-\eta_2}^{-\eta_1} \int_{t+\alpha}^t y^T(\beta) R y(\beta) d\beta d\alpha, \end{aligned} \quad (49)$$

$$\begin{aligned} \dot{V}_8(y(t), t) &= \frac{(\eta_2^3 - \eta_1^3)^2}{36} \dot{y}^T(t) T \dot{y}(t) - \frac{(\eta_2^3 - \eta_1^3)}{6} \\ &\quad \times \int_{-\eta_2}^{-\eta_1} \int_{\alpha}^0 \int_{t+\beta}^t \dot{y}^T(\tau) T \dot{y}(\tau) d\tau d\beta d\alpha. \end{aligned} \quad (50)$$

By using Lemma 3, we get

$$\begin{aligned}
& -\eta_1 \int_{t-\eta_1}^t \dot{y}^T(\beta) U_1 \dot{y}(\beta) d\beta \leq -\xi^T(t) (e_1 - e_3) \\
& \quad \times U_1 (e_1 - e_3)^T \xi(t) - 3\xi^T(t) (e_1 + e_3 - 2e_7) \\
& \quad \times U_1 (e_1 + e_3 - 2e_7)^T \xi(t) \\
& \quad - 5\xi^T(t) (e_1 - e_3 + 6e_7 - 12e_{13}) \\
& \quad \times U_1 (e_1 - e_3 + 6e_7 - 12e_{13})^T \xi(t) \\
& \quad - 7\xi^T(t) (e_1 + e_3 - 12e_7 + 60e_{13} - 120e_{17}) \\
& \quad \times U_1 (e_1 + e_3 - 12e_7 + 60e_{13} - 120e_{17})^T \xi(t), \quad (51)
\end{aligned}$$

$$\begin{aligned}
& -\eta_2 \int_{t-\eta_2}^t \dot{y}^T(\beta) U_2 \dot{y}(\beta) d\beta \leq -\xi^T(t) (e_1 - e_4) \\
& \quad \times U_2 (e_1 - e_4)^T \xi(t) - 3\xi^T(t) (e_1 + e_4 - 2e_8) \\
& \quad \times U_2 (e_1 + e_4 - 2e_8)^T \xi(t) \\
& \quad - 5\xi^T(t) (e_1 - e_4 + 6e_8 - 12e_{14}) \\
& \quad \times U_2 (e_1 - e_4 + 6e_8 - 12e_{14})^T \xi(t) \\
& \quad - 7\xi^T(t) (e_1 + e_4 - 12e_8 + 60e_{14} - 120e_{18}) \\
& \quad \times U_2 (e_1 + e_4 - 12e_8 + 60e_{14} - 120e_{18})^T \xi(t), \quad (52)
\end{aligned}$$

$$\begin{aligned}
& -(\eta_2 - \eta_1) \int_{t-\eta_2}^{t-\eta_1} \dot{y}^T(\beta) U_3 \dot{y}(\beta) d\beta \\
& = -(\eta_2 - \eta_1) \int_{t-\eta_2}^{t-\eta(t)} \dot{y}^T(\beta) U_3 \dot{y}(\beta) d\beta \\
& \quad - (\eta_2 - \eta_1) \int_{t-\eta(t)}^{t-\eta_1} \dot{y}^T(\beta) U_3 \dot{y}(\beta) d\beta \\
& \leq -\xi^T(t) (e_5 - e_4) U_3 (e_5 - e_4)^T \xi(t) \\
& \quad - 3\xi^T(t) (e_5 + e_4 - 2e_{10}) U_3 (e_5 + e_4 - 2e_{10})^T \xi(t) \\
& \quad - 5\xi^T(t) (e_5 - e_4 + 6e_{10} - 12e_{15}) \\
& \quad \times U_3 (e_5 - e_4 + 6e_{10} - 12e_{15})^T \xi(t) \\
& \quad - 7\xi^T(t) (e_5 + e_4 - 12e_{10} + 60e_{15} - 120e_{19}) \\
& \quad \times U_3 (e_5 + e_4 - 12e_{10} + 60e_{15} - 120e_{19})^T \xi(t) \\
& \quad - \xi^T(t) (e_3 - e_5) U_3 (e_3 - e_5)^T \xi(t) \\
& \quad - 3\xi^T(t) (e_3 + e_5 - 2e_9) U_3 (e_3 + e_5 - 2e_9)^T \xi(t) \\
& \quad - 5\xi^T(t) (e_3 - e_5 + 6e_9 - 12e_{16}) \\
& \quad \times U_3 (e_3 - e_5 + 6e_9 - 12e_{16})^T \xi(t) \\
& \quad - 7\xi^T(t) (e_3 + e_5 - 12e_9 + 60e_{16} - 120e_{20}) \\
& \quad \times U_3 (e_3 + e_5 - 12e_9 + 60e_{16} - 120e_{20})^T \xi(t). \quad (53)
\end{aligned}$$

In addition, we derive the following inequality based on Lemma 1:

$$\begin{aligned}
& -\frac{(\eta_2^2 - \eta_1^2)}{2} \int_{-\eta_2}^{-\eta_1} \int_{t+\alpha}^t \dot{y}^T(\beta) R y(\beta) d\beta d\alpha \\
& \leq -(\eta_2^2 - \eta_1^2)^2 \xi^T(t) e_{12} R e_{12}^T \xi(t) \\
& \quad - \kappa(\eta_2^2 - \eta_1^2)^2 \xi^T(t) e_{12} R e_{12}^T \xi(t)
\end{aligned}$$

$$\begin{aligned}
& -(\eta_2^2 - \eta_1^2)^2 \xi^T(t) e_{11} R e_{11}^T \xi(t) \\
& - (1 - \kappa)(\eta_2^2 - \eta_1^2)^2 \xi^T(t) e_{11} R e_{11}^T \xi(t), \quad (54)
\end{aligned}$$

where  $\kappa = \frac{\eta^2(t) - \eta_1^2}{\eta_2^2 - \eta_1^2}$ .

Based on Lemma 2 and inequality (13), we achieve

$$\begin{aligned}
& -\frac{(\eta_2^3 - \eta_1^3)}{6} \int_{-\eta_2}^{-\eta_1} \int_{\alpha}^0 \int_{t+\beta}^t \dot{y}^T(\tau) T \dot{y}(\tau) d\tau d\beta d\alpha \\
& \leq \xi^T(t) [e_1 \quad 2e_{12} + 2e_{11}] \left( (\eta_2^2 - \eta_1^2) \bar{L} + \frac{\eta_2^3 - \eta_1^3}{6} \bar{K} \right) \\
& \quad \times [e_1 \quad 2e_{12} + 2e_{11}]^T \xi(t). \quad (55)
\end{aligned}$$

From assumption **(H2)**, we gain the following relation:

$$\begin{aligned}
& 2 \left( G_p W y(t - \eta(t)) - g(W y(t - \eta(t))) \right)^T Y_1 \\
& \quad \times \left( g(W y(t - \eta(t))) - G_m W y(t - \eta(t)) \right) \geq 0. \quad (56)
\end{aligned}$$

Consider the system (10), we obtain

$$\begin{aligned}
0 = & 2 \left[ y^T(t) \alpha_1 M_1^T + \dot{y}^T(t) \alpha_2 M_2^T \right] \left[ -\dot{y}(t) - \tilde{A} y(t) \right. \\
& \left. + \tilde{B}_1 g(W y(t - \eta(t))) + \tilde{B}_3 u(t) \right]. \quad (57)
\end{aligned}$$

Combining from (43)-(57), we get

$$\dot{V}(y(t), t) \leq \sum_{i=1}^m \omega_i \xi^T(t) \{ \kappa \bar{\Omega}_i^{(1)} + (1 - \kappa) \bar{\Omega}_i^{(2)} \} \bar{\xi}(t), \quad (58)$$

where  $\bar{\Omega}_i^{(1)} = \bar{\Omega}_i + \Omega_1$  and  $\bar{\Omega}_i^{(2)} = \bar{\Omega}_i + \Omega_2$ ,  $\bar{\Omega}_i = \Omega_i + 2e_{21} \alpha_1 B_{3i}^T M_1 e_1^T + 2e_{21} \alpha_2 B_{3i}^T M_2 e_2^T$  with  $\Omega_i$  and  $\Omega_1, \Omega_2$  are defined in (11) and (12).

Consider (58) with  $u(t) = 0$  (doesn't have disturbance), we obtain

$$\dot{V}(y(t), t) \leq \sum_{i=1}^m \omega_i \xi^T(t) \{ \kappa \Omega_i^{(1)} + (1 - \kappa) \Omega_i^{(2)} \} \xi(t),$$

where  $\Omega_i^{(1)} = \Omega_i + \Omega_1$  and  $\Omega_i^{(2)} = \Omega_i + \Omega_2$ .

From the condition (3), the upper bound of  $\dot{V}(y(t), t)$  is negative if the following inequality holds:

$$\kappa \Omega_i^{(1)} + (1 - \kappa) \Omega_i^{(2)} < -a_1 I. \quad (59)$$

The inequality (59) can be expressed as follows:

$$\kappa(\Omega_i^{(1)} + a_1 I) + (1 - \kappa)(\Omega_i^{(2)} + a_1 I) < 0 \quad (60)$$

Since  $0 \leq \kappa \leq 1$ , the term  $\kappa(\Omega_i^{(1)} + a_1 I) + (1 - \kappa)(\Omega_i^{(2)} + a_1 I)$  is a convex combination of  $\Omega_i^{(1)} + a_1 I$  and  $\Omega_i^{(2)} + a_1 I$ . The combination is negative definite only if the vertices are negative; so, (60) is equivalent to (11) and (12). Then, the T-S fuzzy GNNs (10) is asymptotically stable.

### APPENDIX C.

**Proof of Theorem 3.** By using inequality (40) in Theorem 1, condition (6), and LMIs (14)- (17) we achieve

$$\dot{V}(y(t), t) - J(t) \leq \sum_{i=1}^m \omega_i \bar{\zeta}^T(t) \{ \kappa \tilde{\Sigma}_i^{(1)} + (1 - \kappa) \tilde{\Sigma}_i^{(2)} \} \bar{\zeta}(t) \leq 0,$$

$$\dot{V}(y(t), t) \leq \sum_{i=1}^m \omega_i \bar{\zeta}^T(t) \{ \kappa \tilde{\Sigma}_i^{(1)} + (1 - \kappa) \tilde{\Sigma}_i^{(2)} \} \bar{\zeta}(t) + J(t) \leq J(t), \quad (61)$$

where  $\tilde{\Sigma}_i^{(1)} = \tilde{\Sigma}_i + \Sigma_1$  and  $\tilde{\Sigma}_i^{(2)} = \tilde{\Sigma}_i + \Sigma_2$ .

Then we integrate both sides of (61) from 0 to  $t \geq 0$  and setting  $\beta \leq -V(y(0), 0)$ , we gain

$$\int_0^t J(\tau) d\tau \geq V(y(t), t) - V(y(0), 0) \geq y^T(t)Py(t) + \beta. \quad (62)$$

Consider two cases of  $\tilde{\Gamma}_4 = 0$  and  $\tilde{\Gamma}_4 \neq 0$ .

First, we consider  $\tilde{\Gamma}_4 = 0$ , from inequality (62) we get

$$\int_0^{t_f} J(\tau) d\tau \geq \beta. \quad (63)$$

This implies Definition 1 with  $\tilde{\Gamma}_4 = 0$ .

The next case is  $\tilde{\Gamma}_4 \neq 0$ , as stated in assumption (H4), we acquire  $\tilde{\Gamma}_1 = 0, \tilde{\Gamma}_2 = 0, \tilde{\Gamma}_3 > 0$ , and  $C_{3i} = 0$ . So, for any  $0 \leq t \leq t_f$  and  $0 \leq t - \eta(t) \leq t_f$ , (62) goes to

$$\int_0^{t_f} J(\tau) d\tau \geq \int_0^t J(\tau) d\tau \geq y^T(t)Py(t) + \beta,$$

and

$$\int_0^{t_f} J(\tau) d\tau \geq \int_0^{t-\eta(t)} J(\tau) d\tau \geq y^T(t-\eta(t))Py(t-\eta(t)) + \beta,$$

respectively. In addition, for  $t - \eta(t) \leq 0$ , we have

$$\begin{aligned} & y^T(t-\eta(t))Py(t-\eta(t)) + \beta \\ & \leq \|P\| \|y(t-\eta(t))\|^2 + \beta \\ & \leq \|P\| \sup_{-\eta_2 \leq \theta \leq 0} |\varphi(\theta)|^2 + \beta \\ & \leq -V(y(0), 0) \\ & \leq \int_0^{t_f} J(\tau) d\tau. \end{aligned}$$

So,  $\exists b \in \mathbb{R}^+$  such that  $b < 1$ ,

$$\begin{aligned} \int_0^{t_f} J(\tau) d\tau & \geq \beta + by^T(t)Py(t) \\ & + (1-b)y^T(t-\eta(t))Py(t-\eta(t)). \end{aligned}$$

By the relationship between  $z(t)$  and inequality (17) yields the following equation:

$$\begin{aligned} z(t)^T \tilde{\Gamma}_4 z(t) & = - \begin{bmatrix} y(t) \\ y(t-\eta(t)) \end{bmatrix}^T \\ & \times \begin{bmatrix} bP - C_{1i}^T \tilde{\Gamma}_4 C_{1i} & -C_{1i}^T \tilde{\Gamma}_4 C_{2i} \\ * & (1-b)P - C_{2i}^T \tilde{\Gamma}_4 C_{2i} \end{bmatrix} \\ & \times \begin{bmatrix} y(t) \\ y(t-\eta(t)) \end{bmatrix} + by^T(t)Py(t) \\ & + (1-b)y^T(t-\eta(t))Py(t-\eta(t)). \end{aligned}$$

Therefore, for any  $t$  such that  $0 \leq t \leq t_f$ , it is obvious that

$$\int_0^{t_f} J(\tau) d\tau \geq z(t)^T \tilde{\Gamma}_4 z(t) + \beta. \quad (64)$$

Then, taking the supremum over  $t$  in (63) and (64), the T-S fuzzy generalized neural networks (4) is extended dissipative. The proof is complete.

### APPENDIX D.

**Proof of Theorem 4.** By using inequality (58) in Theorem 2, condition (6), and LMIs (18)-(21) we achieve

$$\dot{V}(y(t), t) - J(t) \leq \sum_{i=1}^m \omega_i \bar{\xi}^T(t) \{ \kappa \tilde{\Omega}_i^{(1)} + (1 - \kappa) \tilde{\Omega}_i^{(2)} \} \bar{\xi}(t) \leq 0,$$

$$\dot{V}(y(t), t) \leq \sum_{i=1}^m \omega_i \bar{\xi}^T(t) \{ \kappa \tilde{\Omega}_i^{(1)} + (1 - \kappa) \tilde{\Omega}_i^{(2)} \} \bar{\xi}(t) + J(t) \leq J(t), \quad (65)$$

where  $\tilde{\Omega}_i^{(1)} = \tilde{\Omega}_i + \Omega_1$  and  $\tilde{\Omega}_i^{(2)} = \tilde{\Omega}_i + \Omega_2$ .

Then we integrate both sides of (65) from 0 to  $t \geq 0$  and setting  $0 = \beta \leq -V(y(0), 0)$ , we gain

$$\int_0^t J(\tau) d\tau \geq V(y(t), t) - V(y(0), 0) \geq y^T(t)Py(t). \quad (66)$$

Consider two cases of  $\tilde{\Gamma}_4 = 0$  and  $\tilde{\Gamma}_4 \neq 0$ .

First, we consider  $\tilde{\Gamma}_4 = 0$ , from inequality (66) we get

$$\int_0^{t_f} J(\tau) d\tau \geq 0. \quad (67)$$

This implies Definition 1 with  $\tilde{\Gamma}_4 = 0$ .

The next case is  $\tilde{\Gamma}_4 \neq 0$ , as stated in Assumption (H4), we acquire  $\tilde{\Gamma}_1 = 0, \tilde{\Gamma}_2 = 0$ , and  $\tilde{\Gamma}_3 > 0$ . So, for any  $0 \leq t \leq t_f$ , (66) goes to

$$\int_0^{t_f} J(\tau) d\tau \geq \int_0^t J(\tau) d\tau \geq y^T(t)Py(t).$$

By the relationship between  $z(t)$  and inequality (21) yields the following equation:

$$\begin{aligned} z(t)^T \tilde{\Gamma}_4 z(t) & = -y^T(t) \left( P - C_{1i}^T \tilde{\Gamma}_4 C_{1i} \right) y(t) \\ & + y^T(t)Py(t) \\ & \leq y^T(t)Py(t). \end{aligned}$$



Therefore, for any  $t$  such that  $0 \leq t \leq t_f$ , it is clear that

$$\int_0^{t_f} J(\tau) d\tau \geq z(t)^T \tilde{\Gamma}_4 z(t). \quad (68)$$

Then, taking the supremum over  $t$  in (67) and (68), the T-S fuzzy GNNs (10) is extended dissipative. The proof is complete.

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