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A Qualitative Report on Diffusion based Image Inpainting Models

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Abstract: Diffusion equations have been successfully applied in the field of digital image processing for the past twenty years, describing the random motion of the particles in physics. Image inpainting is a significant research problem in the image processing. Its main intent is to complete the unknown parts of the image from the knowledge of known parts of the image. This research problem can be used to restore damaged photograph, random loss of wavelet coefficients during transmission, superimposed text, noise, and/or blur. According to available models on digital image inpainting, this paper attempts to make an outline of state-of-the-art diffusion based image inpainting models with corresponding mathematical representation. We also compared the state-of-the-art diffusion based inpainting techniques in terms of its main idea, type of distortion, strengths, and weaknesses.

Keywords: Inpainting, Diffusion, Variational methods, Partial Differential Equations, Fractional Calculus

1. Introduction

Digital image inpainting is a progressive and fascinating research topic in past few years where retouching and restoration of damaged regions is done in an indistinguishable form for anyone having no knowledge of the reference image. Inpainting is executed by professional artists in the fine art museums. They propagated the colors from the boundary into the damaged parts and filled in the gap [1], [2].

The professional artists are carried out this retouching work, which is exhaustive and subjective also consume more time. To replace the manual work, the computer graphics community is inspired to deal the work using graphics algorithms to recover the small damages and cracks in the digital images of ancient paintings and old photos. The examples of damaged images are presented in Figure 1.

Image inpainting is regarded as a branch of image restoration where image inpainting and the traditional restoration problems are different [1], [2]. In traditional restoration problems, such as haze removal and motion deblurring target region is damaged but not totally unknown. On the other hand, in the inpainting issues, information can only be inferred from the outside of the target region.

Inpainting has been developed throughout the past two decades. There are diverse applications of image like covering the scratch removal in the restoration of historical images [1], occlusions removal such as text, logos, and subtitles [2], lost blocks recovery in the transmission of wireless images [3], objects removal in image editing [4]. Other applications comprise of eliminating illustrations like location and orientation from medical, aerial, and military images.

Image inpainting approaches depend upon the source regions in the image used to complete the missing or unknown regions. These can be classified into four groups. These are diffusion based (generally called image inpainting) [1], [2], [3], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37] texture-based (generally called texture synthesis) [38], [39], hybrid- based [40], [41], [42], [43], and learning based image inpainting models [44], [45], [46], [47] (generally called image completion).

There is wide distinction between image inpainting, texture synthesis, and image completion, however all these are allied techniques. Many researchers handled these terms with the similar interpretation and for all the cases the inpainting term is used in general way. The main variations between these allied methods are the size of the missing part or unknown part to be recovered and the type of information to be filled in the missing part. The typical prerequisite of all the allied methods is the missing regions are to be known in advance.

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Figure 1. Damaged Images due to: First Row: Mask; Second Row: cracks; Third Row: Text / Scratches

Image inpainting techniques are also called diffusion based inpainting models and these very useful for the filling the small missing regions, i.e., scratches, text, cracks, and noise of structure based images. Texture synthesis based models are helpful for expanding the texture size and fill the texture regions. Image completion models are helpful for the large missing regions (object removal) of structure and texture based images. However, these are not work well when the images are with many small damaged regions. The detailed comparison is mentioned in Table I.

In this article, diffusion-based image inpainting models and the review of the available literature are given in section 2. The evaluation methods and comparison of state-of-the-art diffusion-based image inpainting models are presented in section 3. Section 4 comprises concluding remarks.

2. Diffusion-based Image Inpainting Models

In this section, we briefly present an overview of the existing survey papers on image inpainting. Image inpainting models are mainly based on the diffusion process. These methods are grouped into PDE & variational based, convolution based, and Wavelet based methods.

PDE & variational based inpainting models are mainly used to remove scratches, text, and noise. Convolution based models are useful for the removal of scratches. Wavelet based inpainting models are used to fill the missing coefficients. In the next subsection, each one is illustrated categorically.

Diffusion based image inpainting models introduce smoothness priors which use Partial Differential Equations (PDEs) to propagate the local structures from outward to inward of the missing part as presented in the Figure 2. Here, Ω indicates the missing, damaged, or unknown regions to be inpainted, and Ω^c the source or known region of the image. There are number of variants based on distinct methods (linear, isotropic, non-linear, or anisotropic) to favor the diffusion in appropriate directions or according to structure curvature available in a local neighborhood.

TABLE I. Comparison of diffusion-, texture-, and hybrid-, learning-based image inpainting models

S.No	Feature	Diffusion-based Models	Texture-based Models	Hybrid-based Models	Learning-based Models
1	Size of the hole	Small	Large	Large	Large
2	Data to be filled	Geometrical	Texture	Combination of structure and texture	Combination of structure and texture
3	Images required	One	One/Two	One	Image database
4	Computational time	Mostly high	Mostly medium	Mostly low	Very high
5	Applications	Removal of text, scratches, noise, blur, and reconstruction of lost blocks	Large texture generation, removal of objects, and covering of surface	Removal of objects, recovery of large degraded region, and Editing of an image	Removal of object, restoration of large damaged area, and image editing

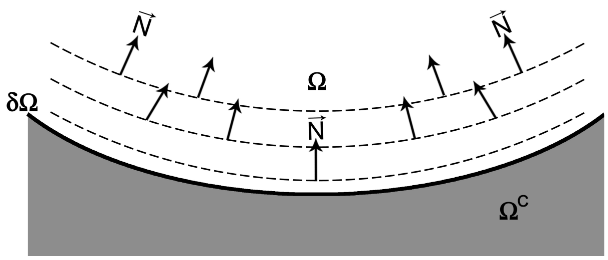

Figure 2. Diffusion direction is perpendicular to the signed distance to that of the unknown region boundary

The major significance of the diffusion-based techniques provides the continuum of geometrical structure information. The main objective of these models is to smoothly join the lines of equal gray values (isophotes) and level curves within the damaged region in an appropriate process.

A. *PDE & variational based models*

The PDE models follow the directions of isophotes in the image to implement the image restoration. These are further divided into integer calculus based models and fractional calculus-based models.

1) *Integer calculus based models*

Masnou and Morel [1] proposed the first article on image inpainting that retains the image structures. It explains the inpainting concept as a dis-occlusion concept. This model used geodesic curve to combine the isophotes traced at the boundary region. The geodesic curve joins the curves with the same orientation and color using possible path which is the shortest between the two points. This model infers that the intersection of isophotes that are connected never happens. Many drawbacks exist including quite insignificant approach for connecting the isophotes, unsustained angles of connection, and it is working only with very small missing area.

The next image inpainting algorithm was popularized by Bertalmio et al. [2] which is based on PDE techniques. They utilized an anisotropic diffusion method that transports image Laplacian from the neighborhood of the surrounding known area into the inside of the damaged area. The directions of the diffusion are taken as known by the isophotes directions predicted by the normal to the image gradient in each pixel. This model is mathematically represented as given in Eq. (1).

$$\frac{\partial u}{\partial t} = \nabla(\Delta u)\nabla^{\perp} u \tag{1}$$

Here $\nabla^{\perp} u$ denotes the isophotes direction. The term in Eq. (1) is the gradient of Laplacian smoothness in the isophotes direction, pointing to a smooth transition of known data (the Laplacian), i.e., the image information is diffused interior of the damaged area in such a way that it aims at retaining the isophotes direction. It works well for small and structure missing area. Because of the smoothness process, this model produces blurring results.

Marcelo et al. [5] enhanced their model by proposing Navier-Stokes for fluid dynamic equations. The image intensity u is analogous to the stream function of fluid; direction of isophote $\upsilon = \nabla^{\perp} u$ is analogous to fluid velocity; and smoothness of image $\omega = \Delta u$ is analogous to vorticity.

$$\frac{\partial \omega}{\partial t} + \upsilon.\nabla\omega = \nu\nabla.(g(|\nabla\omega|)\nabla\omega) \tag{2}$$

Here, ν is a scalar quantity. The anisotropic diffusion of the smoothness ω is defined in terms of the parameter g. This development improves the speed and stability of the preceding model. However, this model introduces blurred results and handles only small missing areas with structure components. Further, in this model, the tuning of the parameters is done with the help user.

Inspired by Marcelo et al. [2] PDE approach, a Total Variation (TV) method to image inpainting using Bayesian approach and variational methods was proposed by Shen and Chan [6]. This TV model was first implemented for image deblurring and denoising applications by Rudin et al. [48] in 1992.

As the energy function depends on the TV-norm, the name TV model is accorded. The TV image inpainting approach is denoted as

$$\min_{u \in BV(\Omega)} J_{TV}(u) = \int_{E \cup \Omega} |\nabla u| dxdy + \frac{\lambda_\Omega}{2} \int_\Omega |u - u_0|^2 dxdy \quad (3)$$

Here, Ω is the inpainting (open) domain with its boundary $\delta\Omega$, and E is an extended domain around the $\delta\Omega$ as shown in the Figure 3. The TV model utilizes an Euler-

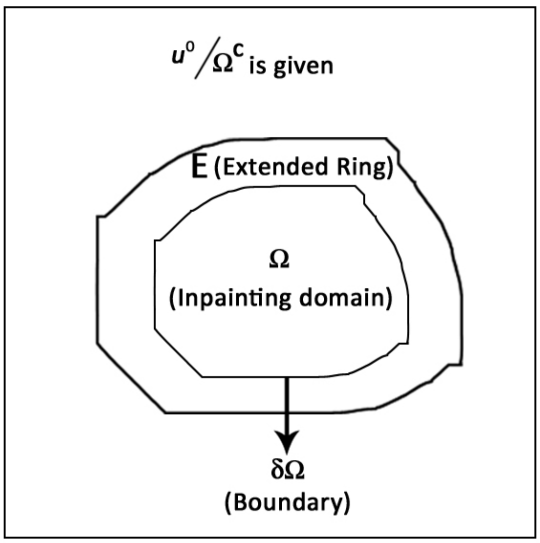

Figure 3. For a regular inpainting concept, the image is damaged on an inpainting domain Ω and the known region is usually noisy

Lagrange equation and anisotropic diffusion to propagate the information from the outward to the inward of the missing part via isophotes contrast as

$$\frac{\partial u}{\partial t} = -\nabla . \left(\frac{\nabla u}{|\nabla u|} \right) + \lambda_\Omega (u - u_0) \quad (4)$$

with Neumann's boundary condition $\partial u / \partial \vec{N} = 0$ on $\delta\Omega$, where $\lambda_\Omega = \begin{cases} \lambda, & (x, y) \in \Omega \\ 0, & \text{otherwise} \end{cases}$. The primary term denotes the regularization, where as, the subsequent term represents a data fidelity term to reconstruct the image when the noise is present. λ represents the Lagrange multiplier which tunes the weight of two terms. The numerical implementation of this model is

$$\nabla \cdot \left(\frac{\nabla u}{|\nabla u|} \right) = \nabla_{x-} \left(\frac{\nabla_{x+} u}{(\nabla_{x+} u + \nabla_{y+} u + \epsilon)^{\frac{1}{2}}} \right) + \nabla_{y-} \left(\frac{\nabla_{y+} u}{(\nabla_{x+} u + \nabla_{y+} u + \epsilon)^{\frac{1}{2}}} \right) \quad (5)$$

where, $\nabla_{x-} u, \nabla_{y-} u$ are backward differences of image u in the directions of x and y respectively, $\nabla_{x+} u, \nabla_{y+} u$ are forward differences of image u in the directions of x and $y$respectively and ϵ is a small constant for the elimination of 'divide by zero'error.

This second order image inpainting method (here the method's order is calculated by the differentials with highest order in the corresponding E-L equation) fails in connectivity principle, (connecting the edges over long distances) and staircasing effect (the smooth diffusion of isophotes into the missing domain). In order to solve these two problems, the authors further proposed a new model.

In the new model [7], the geometric information of isophotes is utilized and it is based on Curvature Driven Diffusion (CDD). It remodels the coefficient of diffusion to be stronger when the isophotes have curvature with large value as given in Eq. (6).

$$\frac{\partial u}{\partial t} = \nabla \cdot \left(g(|k|) \frac{\nabla u}{|\nabla u|} \right) \quad (6)$$

where, g is a continuous function which penalizes large curvatures, and encourages diffusion when the curvature is small. Typical example for the conductivity coefficient g is $g(s) = s^p$, where $s > 0$ and $p \geq 1$. This model permits the filling process to proceed over larger areas. Inpainted results usually look blurred, despite some broken edges by CDD method. Linear interpolation is still reflected in the level lines while solving the connectivity principle in visual perception.

Shen et al. [8] extended the earlier idea of Masnou and Morel [1] using Euler's elastica energy. In this concept, the researchers presented the elastica PDE model (fourth order) which joins the curvature driven diffusion and the transport process of the method [2]. Therefore, it does not allow kinks at the inpainting region boundary because the curvature of the kinks is infinite. Moreover, curved edges, as well as straight edges, are extended exactly across the inpainting region. Euler's elastica repairs the missing region by the following energy

$$\min_{u \in BV(\Omega)} J_{Elas}(u) = \int (a + bk^2) |\nabla u| dxdy \quad (7)$$

In this functional, a and b are positive real numbers and k represents the curvature of u. Minimizing this energy is equivalent to connecting sharp edges according to the curvature of the level lines in the known region.

Two non-texture image inpainting methods are proposed by Esedoglu and Shen [9] to attain more natural visual effect. In the first concept, the authors utilized Mumford-Shah image model which is the extension of TV technique [6] to decrease the computation time and order of approximation. In order to quicken the numerical convergence and make it simple the Γ-convergence approximation is adopted. This method produces artificial corners since it adopts straight lines. Further, this model offends the concept of the connectivity principle. In the second method, the authors extended the first one based on Euler's elastic curve's high-order correction instead of straight-line curve.

Telea [10] is described a Fast Marching Technique (FMT) that estimates the missing pixels in single iteration via weighted means of already calculated pixels. This method recovers the missing pixels as a level sets by diffusing image estimator along first order derivative of the image. Hence, the computational time is decreased. This method utilized the surrounding of the known pixels to calculate the image smoothness as a weighted average to fill the unknown pixels. Firstly, the FMT completes the neighborhood pixels to the source region then manages a narrow band of pixels which isolates source region from the missing region.

$$u(q) = \frac{\sum_{r \in B_\epsilon(q)} w(q,r)[u_0(r) + \nabla u_0(r)(q-r)]}{\sum_{r \in B_\epsilon(q)} w(q,r)} \quad (8)$$

where, q and r represent the unknown and known pixels respectively. The term $B_\epsilon(q)$ is a known neighborhood of the pixel q and $w(q,r)$ is the normalized weight function. The drawback of this technique is in introducing blur when the missing area is wider than ten pixels.

Xu et al. [11] presented a faster technique based on PDEs. This technique is called as Quick Curvature-Driven Diffusion (QCDD) and produces better results with lesser computation time. QCDD model is introduced based on the CDD model (6) as given in Eq. (9)

$$\frac{\partial u}{\partial t} = \nabla \cdot (g(|k|)\nabla u) \quad (9)$$

Both, CDD and QCDD models are supported by "connectivity principle". These techniques connect a few broken edges, but produce a blurry look after inpainting.

Li and Wang [13] presented curvature driven diffusion image inpainting method using p-Laplace operator. This model mainly reverts scratches and removes texts from images. The p-Laplace non-linear anisotropic diffusion is utilized in this model to repair the damaged regions.

$$\frac{\partial u}{\partial t} = \nabla \cdot \left(|\Delta_p u| \frac{\nabla u}{|\nabla u|} \right) \quad (10)$$

where,$|\Delta_p u| = div(|\nabla u|^{(p-2)} \nabla u), 1 < p < \infty$

An image inpainting model with non-iterative process via coherence transport equation is introduced by Bornemann and Marz [11]. The authors re-investigated the anisotropic diffusion steps that are ignored in the method [2] to interleave the stabilization. Further, they reframed the transport equation in the method [1] to eliminate the iteration process and keeps quick processing. The coherence direction of the image is estimated using structure tensor. According to the coherence value strength, the model modifies the diffusion from normal diffusion or directional transport. This model works well in the vicinity of the missing regions and structure region, however, fails in the texture region. This model introduces visual artifacts when the filling region is large. Further, user interaction is required to tune the parameters.

In [12], Barcelos and Batista have presented a new technique for simultaneous image inpainting and noise removal. This method is the improved version of Marcelo et al. [2] approach. It simultaneously completed the missing regions and eliminated the noise with two different models. The denoising is performed in two different ways, i.e., within the inpainting domain, the Mean Curvature Flow (MCF) is applied for the smoothing, while outside the inpainting domain the removal of noise is implemented in such a way that encourage smoothning inside the region and discourage smoothning across boundaries. In addition to smoothing, it allows the transportation of available information from outward to inward of the missing area. This approach allows the concurrent use of inpainting and noise removal of distinct regions of an image.

A fast image inpainting approach using TV approach was proposed by Lu et al. [15]. They proposed priority TV model depends on the analysis of local characteristics of the pixels around the missing area. The filling process is done layer by layer according to the pixel's priority at edges and the priority is computed via the source boundary pixels in the vicinity of the unknown pixel. Higher priority layers are filled first. The computational time of this model is lesser than the TV model proposed in [7]. However, this method is still slow iterative process and unsuccessful in producing quality output.

Biradar and Kohir [16] re-investigated the PDE approach [2] and introduced a non-linear median filter to propagate median value from outside to inside missing region. This model handles well only for thin missing area like text or scratches removal. When the unknown region is large, this method introduces blur and artifacts.

A third-order diffusion-based image inpainting model driven by Gauss curvature for the reconstruction of images was proposed by Jidesh and George [17]. In this model, level curve's Gauss curvature value is considered for the diffusion of information into the missing region. It is the multiplication of the two principal curvatures. Whenever one of the principal curvatures is zero its value becomes zero. Therefore, the evolution preserved few structures which are meaningful with non-zero value of mean curvature. So, the diffusion process effectively eliminates the noise, since these features possesses non-zero value of Gaussian curvature.

$$\frac{\partial u}{\partial t} = -\nabla.\left(\frac{\chi(G)\nabla u}{|\nabla u}\right) + \lambda\kappa(\kappa u - u_0) \tag{11}$$

$$\chi(G) = \begin{cases} 1, & (x,y) \notin \Omega \\ f(G), & (x,y) \in \Omega \end{cases} \tag{12}$$

where $f(G) = |G|/max(G(x,y))), \forall(x,y)$ and $|.|$ denotes the absolute function.

$$G = \frac{u_{xx}u_{yy} - u_{xy}^2}{(1 + u_x^2 + u_y^2)^2} \tag{13}$$

Gopinath et al. [18] proposed fourth-order complex diffusion-based image inpainting. This model improves the filling capacity of anisotropic diffusion process. The authors considered the complex diffusion coefficient as a conductivity parameter.

$$\frac{\partial u}{\partial t} = -\nabla^2(c(Imag(u))\nabla^2 u) \tag{14}$$

where,

$$c(s) = \frac{e^{j\theta}}{1 + \left(\frac{s}{k_c\theta}\right)^2} \tag{15}$$

Barbu [20] proposed a robust second order PDE model with new conduction coefficient. A numerical approximation based on a consistent and stable finite-difference based approach is utilized to discretize this non-linear diffusion model. This is a fast inpainting model and performs well in noisy conditions. However, this method requires proper tuning of several parameters.

2) Fractional Calculus Based Models

Recently, researchers applied fractional calculus for various image processing applications. It is a powerful tool to find the fine edges and ramps of an image. The main advantage of fractional order derivative over integer order derivative is its non-local property, i.e., derivative at a pixel not only depends on the pixel's vicinity but also on the entire pixel intensities.

The definition of fractional derivative is not unique which is explained by many mathematicians in the literature. The popular definitions are Grunwald-Letnikov (GL), Riemann-Liouville (RL), Caputo, and Riesz [27] also using Discrete Fourier Transform (DFT) [30].

Zhang et al. [23] introduced fractional calculus to the image inpainting in 2011. The authors considered the fractional order derivative in place of integer order derivative of total variational model proposed by Chan and Shen [7] with a constant p-Laplacian operator. The non-local property of fractional derivative gives the improved quality of the image. The fractional derivative is implemented based on Riemann- Liouville definition. The mathematical representation of this model is given in Eq. (16).

$$\min_{u\in BV(\Omega)} J_{FTV}(u) = \frac{1}{p}\int_{E\cup\Omega} |\nabla^\alpha u|^p dxdy \quad +\lambda_\Omega \int_\Omega |u - u_0|^2, \quad \alpha \in \mathbf{R}^+, p \in [1,2] \tag{16}$$

where α is the fractional-order. The Euler-Lagrange energy minimization of this model is represented as

$$\frac{\partial u}{\partial t} = \overline{(-1)^\alpha} div^\alpha \left(\frac{\nabla^\alpha u}{|\nabla^\alpha u|^{(2-p)}}\right) = \overline{(-1)^\alpha} curv^\alpha u \tag{17}$$

$$curv^\alpha u = \nabla^\alpha_{x-}\left(\frac{\nabla^\alpha_{x+}u}{(\nabla^\alpha_{x+}u + \nabla^\alpha_{y+}u + \epsilon)^{\frac{2-p}{2}}}\right) + \nabla^\alpha_{y-}\left(\frac{\nabla^\alpha_{y+}u}{(\nabla^\alpha_{x+}u + \nabla^\alpha_{y+}u + \epsilon)^{\frac{2-p}{2}}}\right) \tag{18}$$

Later the same authors proposed another model to remove the metal artifacts from the X-ray Computed Tomography (CT) images based on curvature driven diffusion with fractional calculus [24]. The fractional order is developed based on Grunwald - Letnikov (GL) definition. This method utilized projection data with metal regions as a missing region to restore the lost data due to the metal region. The mathematical representation of this model is

$$\frac{\partial u}{\partial t} = \nabla \cdot \left(|k^\alpha||\nabla^\alpha u|^{-1}\nabla u\right) \tag{19}$$

where, k^α is fractional curvature and it is given in Eq. (18).

In [25], The authors extended the work of [23] by using adaptive p-Laplace operator. The p-Laplacian value in model (16) is adaptively varied using the local geometrical characteristics of the image.

$$p = 1 + \frac{curv^\alpha u}{curv^\alpha u + |\nabla^\alpha u|} \tag{20}$$

The authors utilized Grunwald-Letnikov fractional derivative for filling the unknown parts and denoise the other parts of the image.

A vector valued Cahn-Hilliard equation based fractional image inpainting is developed by Bosch and Stoll [26]. This model is the generalized version of binary Cahn-Hilliard image inpainting method to gray-scale images [49]. It is further generalized method to a version of fractional-in-space. It is achieved by substituting the integer order differential with fractional order differential. The authors utilized the fractional Laplace operator based on the spectral decomposition. Fourier spectral approaches furnish efficient solvers since they return a fully diagonal approach.

Sridevi and Kumar [28] proposed fractional order non-linear complex diffusion based on Caputo definition. In this model, the coefficient of diffusion depends on the non-linear complex function for the diffusion of the information of pixel from the source regions to the missing regions. This coefficient is more powerful, which doesn't depend on the first or second order derivative. Another important observation is that when this coefficient reaches the real axis the imaginary part can able to detect the edges. The mathematical representation of this model is depicted as

$$\frac{\partial u}{\partial t} = \nabla^{\alpha}(c(Imag(u))\nabla^{\alpha}u) + \lambda_{\Omega}(u - u_0) \tag{21}$$

where,

$$c(s) = \frac{e^{j\theta}}{1 + \left(\frac{s}{k_c\theta}\right)^2} \tag{22}$$

Where, $c(.)$ is a complex diffusion coefficient. It is assumed to consider the diffusion coefficient for smaller θ values. When θ is large, the imaginary part can be fed-back to the real part and creats an unacceptable effect called ringing effect. Hence, θ value is selected as $\pi/30$ to test the inpainting performance.

The Fractional Tensor Regularization (FTR) inpainting model was proposed by Yang and Guo [29]. This method combines fractional calculus with tensor regularizer. The characteristics of fractional calculus mainly address the fine-scale features of the image and this model achieves anisotropism of tensor diffusion. This inpainting approach was determined as a technique that minimizes a functional proportional to the fractional structure tensor which comprises of the fractional differential inner product and its transposition. The fractional differential in four directions was represented based on the shifted Grunwald-Letnikov definition.

Sridevi and Kumar [30] proposed an image inpainting model based on fractional order non-linear diffusion driven by difference curvature (DC) for image reconstruction. This model simultaneously completes the inpainting regions and eliminates the noise and blur without introducing any impulsive effects. The fractional derivative is implemented based on discrete Fourier transform.

$$\frac{\partial u}{\partial t} = -D^{\alpha}\psi_{\Omega}(D^{\alpha}u) + \lambda_{\Omega}h * (h * u - u_0) \tag{23}$$

The first one represents regularization, the second one represents data fidelity and Ψ_{Ω} is the coefficient of diffusion and it is defined as $\partial u/\partial\vec{N} = 0$ on $\delta\Omega$,
where $\Psi_{\Omega} = \begin{cases} f(DC), & (x, y) \in \Omega \\ \frac{1}{|D^{\alpha}u|}, & \text{otherwise} \end{cases}$.

Here, α is a non-integer value and is more flexible in inpainting domain and non-inpainting domain, it is denoted as $\alpha = \begin{cases} \alpha_1, & (x, y) \in \Omega \\ \alpha_2, & \text{otherwise} \end{cases}$. and $f(DC)$ is adaptively varied according to the image characteristics.

Finally, PDE & variational based models are extensively applied for recovering small damaged parts. These methods also work well in the presence of noise and work well to remove the noise in the non-inpainting regions. These methods are very useful for removing the text, scratches, and noise. The common limitations in most of these models include the processing time for the diffusion process, monotonous realization, and fail to fill the fine texture details.

B. *Convolution Based Models*

In addition to the approaches based on PDE/variational techniques, the other diffusion techniques using the operation of convolution are presented in this section.

The first convolution-based image inpainting method was initiated by Richard et al. [31]. This process is introduced mainly to minimize the computational time. It begins by removing the color features of the degraded area. The diffusion process into the unknown area is executed iteratively by the convolution of the known information and the predefined diffusion mask. The diffusion masks defined in this method look identical to weighted average of the neighborhood pixels but the center weight is with zero as shown in the Figure 4.

Diffusion limit is applied to break the iteration process using the modifications in contrast of the image. This method is simple and rapid in comparison with PDE-based approaches. When the missing area is very small and do not have any contrast edges then this model works well.

$$\begin{bmatrix} 0.125 & 0.125 & 0.125 \\ 0.125 & 0 & 0.125 \\ 0.125 & 0.125 & 0.125 \end{bmatrix} \begin{bmatrix} 0.073 & 0.176 & 0.073 \\ 0.176 & 0 & 0.176 \\ 0.073 & 0.176 & 0.073 \end{bmatrix}$$

Figure 4. The diffusion kernels used in the method [31].

A non-iterative convolution approach is introduced by Hadhoud et al. [32]. This approach addressed the drawbacks of earlier convolution-based method [28]. In this method, the diffusion kernels are modified with the zero weight from center to the bottom right corner as shown in the Figure 4. This alteration eliminates the need to repeat the convolution a greater number of times, which produces more blur in the output. To diffuse the information, this model utilizes these masks only once by convolving it with the missing area.

An adaptive convolution-based approach is introduced by Noori et al. [33]. This approach utilizes adaptive mask in place of fixed mask as shown in the Figure 4. The mask coefficients to be convolved with the damaged image are calculated from the first order derivative of source boundary pixels as depicted in the Eq. (24).

$$L(X) = \begin{cases} 1 - \left(\frac{X}{\beta}\right)^2, & |X| \le \frac{\beta}{2} \\ \left(1 - \frac{X}{\beta}\right)^2, & \frac{\beta}{2} \le |X| \le \beta \\ 0, & |X| \ge \beta \end{cases} \tag{24}$$

Here, $L(X)$ is the coefficient function, X is a first order derivative of the pixel, and the propagation softness is controlled by β. This model is better than the model in [31]. However, due to the iterative convolution process, this model still produced blurred results.

The authors in [34] improved their work with bilateral filter to retain edges. The space and range domain vicinity based mask coefficients are used. To adjust the number of iterations local variance is considered. This enhancement is efficient for eliminating the noise. However, to retain the edges, it requires large number of iterations.

Rana et al. [35] proposed dynamic masking-based convolution method for image inpainting. An edge conserving approach is considered in the dynamic masking. It is an alternate to the 2-pixel width barriers defined according to Richard's model. Without reducing the enhancement, the inpainted result is produced in a less amount of time.

Finally, convolution-based approaches are treated as the fastest models in comparison with PDE and variational-based models. These models effectively handle for the removal of noise. However, due to the convolution iteration process these models suffer from the blurred results. Further, the inpainted images' quality relies on the number of iterations.

C. Wavelet Based Models

After the release of JPEG 2000, wavelet transform is extensively applied to process the signals and images. The reason is, with the help of a smaller number of wavelet coefficients, this transform collects and maintains sharp features to produce efficient results. Especially during error transmission, compression, and storage there is a problem of missing information of structures and lost wavelet coefficients. These problems are addressed by various wavelet based image inpainting methods.

A first wavelet image inpainting based on variational methods is introduced by Tony et al. [3]. This approach completes the lost coefficients in wavelet domain based on a total variation technique to retain the geometrical features of the image and remove the noise. In their work, the authors presented the models for both noise free and noisy channels.

$$\min_{\beta_{j,k}:(j,k)\in\Omega} F(u,u_0) = \int_{\mathbf{R}^2} |\nabla_x u(\beta,x)| dx \tag{25}$$

$$\min_{\beta_{j,k}} F_\lambda(u,u_0) = \int_{\mathbf{R}^2} |\nabla_x u(\beta,x)| dx + \sum_{(j,k)} \lambda_{j,k} \left(\beta_{j,k} - \alpha_{j,k}\right)^2 \tag{26}$$

In the above equations, $u(\beta, x)$ serves as the wavelet transform, Ω is the damaged area, $\beta_{j,k}$ and $\alpha_{j,k}$ are the wavelet coefficients, and $\beta_{j,k} = \alpha_{j,k} \forall (j,k) \notin \Omega$, also

$$\lambda_{j,k} = \begin{cases} \lambda, (j,k) \notin \Omega \\ 0, (j,k) \in \Omega \end{cases} \tag{27}$$

Zhang and Chan [50] enhanced the Total variation wavelet image inpainting model to the corresponding non-local (NL) form. This model is solved by using Bregmanized operator splitting (BOS) algorithm.

Zhang et al. [51] proposed wavelet total variation image inpainting with p-Laplace operator. The main advantage of total variation approach is that it can preserve the edges effectively, however this approach suffers from the staircase effect. To reduce this issue, the authors are analyzed the physical characteristics of total variation approach and p-Laplace operator in local coordinates.

Zhang et al. [25] introduced p-Laplace wavelet image inpainting with fractional calculus to improve the limitations of the method [23].

$$\min_{\beta_{j,k}:(j,k)\in\Omega} F(u,u_0) = \frac{1}{p} \int_{\mathbf{R}^2} |\nabla_x^\alpha u(\beta,x)|^p dx, \alpha \in \mathbf{R}^+, p \in [1,2] \tag{28}$$

$$\min_{\beta_{j,k}} F_\lambda(u,u_0) = \frac{1}{p} \int_{\mathbf{R}^2} |\nabla_x^\alpha u(\beta,x)| dx + \sum_{(j,k)} \lambda_{j,k} \left(\beta_{j,k} - \alpha_{j,k}\right)^2, \quad \alpha \in \mathbf{R}^+, p \in [1,2] \tag{29}$$

The parameters given in the above equations are defined as in Eqs. (25),(26).

Fractional order differentiation and p-Laplace are considered as an alternative of integer order total variation to improve the restoration ability.

Jiang and Yin [37] proposed wavelet based inpainting with fractional order TV regularization technique. In this model, primal-dual algorithm is applied to implement the TV regularization. This method is computationally efficient than the previous method.

Finally, inpainting models in wavelet domain are extensively used to restore the incomplete wavelet coefficients due to compression, transmission, and storage. These models are very effective to remove the noise from the images. The general limitations of these models are the processing time as these cannot represent diagonal curves well, and these are shift variant. Furthermore, these are not suitable with fine details.

3. Discussion and Evaluation

The digital image inpainting researchers find difficulty to evaluate their models due to the unavailability of a large database of degraded images and due to the novelty of this concept. For that, most researchers use the available databases like USC-SIPI [52], Foreground Aware [53], Places [54], ImageNet [55] and others. The available models in literature develop their own image inpainting databases by including some artificial distortion such as noise, text, scratches, objects, and masks.

A. Qualitative Analysis

The performance of the various PDE & variational image inpainting models are applied for the removal of text and noise about peppers image. All simulations are carried out using MATLAB R2016a on an Intel i3 processor with 4G RAM. The corresponding simulation results are shown in the Figure 5. Chan and Shen model [7], and Barcelos and Batista model [14] are integer order models, whereas Zhang et al. [25], Sridevi and Kumar [30], and Yang and Guo [29] are fractional order models. These models are mainly used to complete the small missing regions damaged by scratches and text.

Chan and Shen model [7] works well to complete small missing regions. However, it produces staircase effect. Barcelos & Batista model [14] works well in the presence of noise. However, this model produces more blur. Zhang et al. model [25] works well when the image with texture regions also. The selection of the fractional order is not adaptive. In Sridevi & Kumar model [30], the curvy edges and ramps are restored effectively than other models since the diffusion coefficient is adaptively changed based on the image characteristics. Yang & Guo [29] model handles subtle details and complex structures because of the characteristics of fractional calculus. Fractional order models work well in the presence of noise also. The corresponding performance analysis of these models are presented in Table II.

TABLE II. Comparison of PDE & Variational models for noise (20%) and text removal about peppers image (Input PSNR=17.6 dB)

Model	Output PSNR (dB)
Chan and Shen [7]	28.23
Barcelos and Batista [14]	31.12
Zhang et al. [25]	32.12
Sridevi and Kumar [30]	32.89
Yang and Guo [29]	33.42

Convolution based image inpainting models are mainly used to remove scratches. The performance of these models are presented in the Table III. The original image is damaged with several cross lines, the Richard et al. [31], Hadhoud et al. [32] and Noori et al. [33] methods produce edge blur. In Rana et al. model [35], for detecting the damaged or lost piece of image's region is very important where dynamic masking method points at the automatic detection target area to be inpainted by automatically generates mask image without user cooperation that contains only target areas to be inpainted. Wavelet image inpainting models are used to recover the lost wavelet coefficients. Authors used the biorthogonal wavelets (Daubechies7 – 9) with symmetric extension for these algorithms. Forward and backward biorthogonal wavelet transforms are implemented using Wavelab. Wavelet is applied up to 3 scales for the decomposition to apply the algorithm. The performance of the models in terms of PSNR for the lost wavelet coefficients is presented the Table IV.

TABLE III. Performance of Convolution based image inpainting models

Model	Output PSNR (dB)
Richard et al.[31]	12.01
Hadhoud et al. [32]	12.99
Noori et al. [33]	7.02
Rana et al. [35]	13.22

The images are collected from USC-SIPI image database [52] is used to test these algorithms. The two cases of wavelet coefficients missing: the random and whole LH loss are considered. Tony et al. [3] model can able to complete the random loss, but it produces blocky effects. Zhang and Chan model [50] utilized non-local means total variation model and PSNR is more when compared to Tony et al. model. Zhang et al. [23] used fractional order p-Laplace model produces better results when fractional order is 1.6 in noiseless environment and when order is 1.4 for noisy environment. Zhang et al. model [25] utilized adaptive p-Laplace model and produces better results in terms of PSNR. In Jiang and Yin model [37], the proper selection of the fraction depends on the image to inpaint and order is selected 1.4 for better results. This model works well for the texture images. Fractional order models work well for the texture images and in the noisy environment.

The evaluation methods of the restored images for image inpainting models, researchers utilize some performance metrics, viz., Mean Square Error (MSE), Peak Signal to Noise Ratio (PSNR), Mean Structural Similarity Index (MSSIM) [56], and Figure of Merit (FoM) [57]. In addition to these metrics some researchers utilize subjective analysis for the inpainting process. In addition to the evaluation measures, the image database used for the evaluation for the inpainting models can be different from one approach to another. For example, some approaches handle gray scale images while other approaches evaluate in RGB images or historical images. Hence, we target to summarize the diffusion-based methods in terms of its main idea of implementation, type of distortion, source of images, performance metrics, strengths, and weaknesses. These are presented in the Table V.

Finally, diffusion-based image inpainting models are mainly useful for the removal of small scratches, text, random loss of coefficients and noise to complete the image.

4. Conclusions

For the past few years image inpainting has been an active and interesting concept. There are multiple methods that have been discussed to tackle various applications such as dis-occlusion, text removal, restoration, and recovery

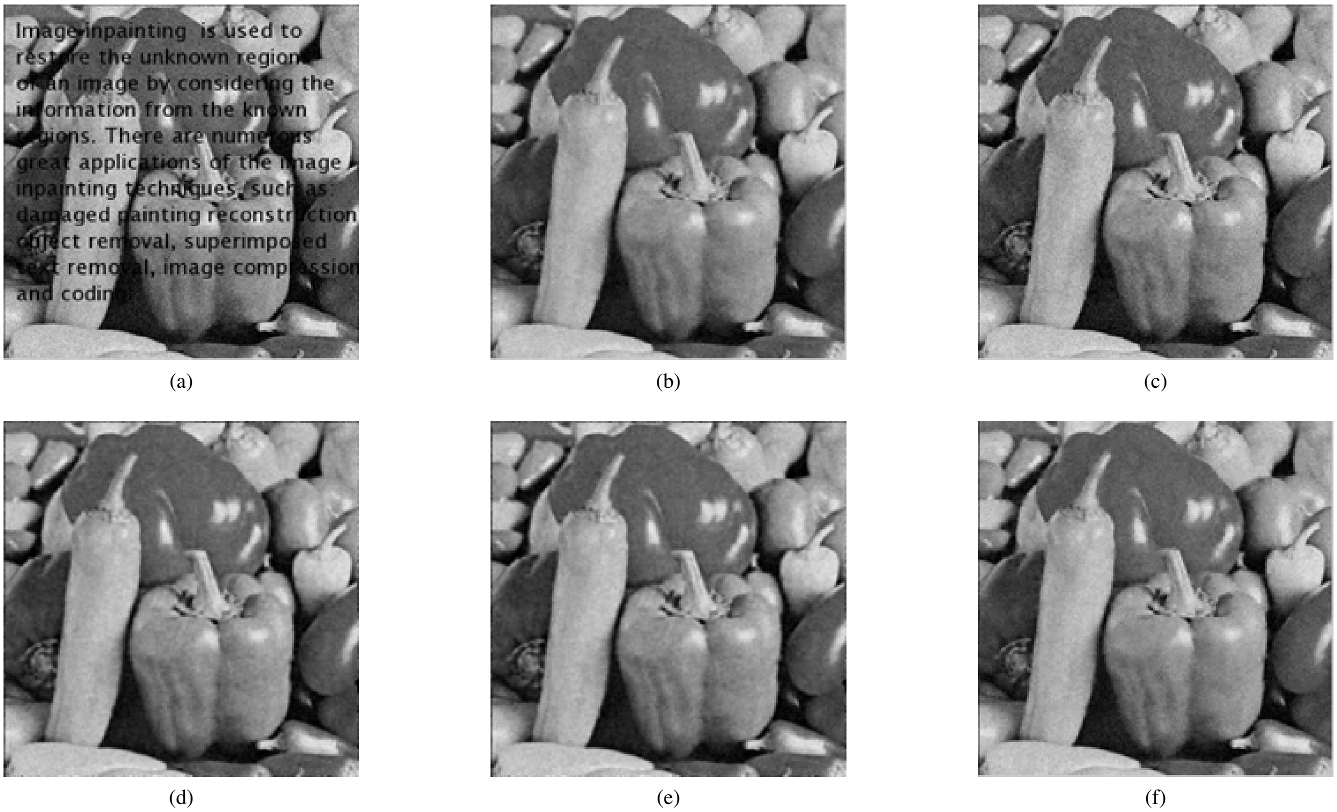

Figure 5. PDE & Variational models for noise and text removal First Row: Damaged image, Chan and Shen[7], Barcelos and Batista [14] Second Row: Zhang et al. [25], Sridevi and Kumar [27], Yang and Guo [29]

TABLE IV. Performance of Wavelet image inpainting models

Model	Name of the image	Type of distortion	Output PSNR (dB)
Tony et al. [3]	Lena	10% Random loss	24.20
Tony et al. [3]	Boat	LH loss (Gaussian noise, 15% Standard deviation)	24.32
Zhang and Chan [50]	Peppers	50% Random loss and (Gaussian noise, 10% Standard deviation)	19.65
Zhang and Chan [50]	Boat	LH loss (Gaussian noise, 15% Standard deviation)	25.48
Zhang et al. [51]	Woman	50% random loss	29.24
Zhang et al. [23]	Lena	10% Random loss	25.12
Jiang and Yin [37]	Lena	10% Random loss	34.86
Jiang and Yin [37]	Boat	LH loss (Gaussian noise, 15% Standard deviation)	26.75

of random loss during transmission. This article reviewed and presented various image inpainting models elaborately which have contributed much to the progress. Technical details of the methods in each group are given to exemplify the strengths and weaknesses.

Diffusion-based image inpainting can be used to repair damaged regions or remove the undesirable regions in an image. However, some traces are left in the inpainted image and making it easy for the forensic algorithms to locate the inpainted regions. The inpainting algorithms may be implemented for the anti-forensics.

The problems that are still open in this field are damaged complex structures, depth ambiguity, computational complexity, and quality assessment.

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TABLE V: PERFORMANCE COMPARISON OF VARIOUS IMAGE INPAINTING MODELS

S.No	Reference	Main idea of the method	Type of distortion	Source of the Images	Performance metrics	Strengths	Weaknesses
1	Masnou and Morel [1]	It utilizes geodesic curve by joining the lines of same direction and color relying on shortest path	Noise, disocclusion	Self Collected Images and Lena Image	Subjective Analysis	• Simple to realize	• Trivial method • No preservation of angles • Works only for very small region
2	Marcelo et al. [2]	It improves the method [1] of maintaining the connection angles by employing anisotropic diffusion and discretized gradient	Text and Scratch	Self Collected Images	Subjective Analysis	• Handles small region	• Time consuming • Blurred results • Numerically not stable
3	Bertalmio et al. [5]	It enhances the method [2] by utilizing the Navier–Stokes equation	Text and Scratches	Self Collected Images	Subjective Analysis	• Improves speed and stability	• Needs user interactions • Produces blurred results
4	Chan and Shen [6]	It utilizes TV method employing E-L equation and anisotropic diffusion to spread the information into unknown regions	Text, Scratch, Noise	Self Collected Images	Subjective Analysis	• Works well in noisy image	• Fails connectivity principle • Produces staircasing effect
5	Shen and Chan [7]	It improves the process [6] by CDD which handles the geometric information	Text, Scratch, Noise	Self Collected Images	Subjective Analysis	• Connects some broken edges • Supports larger area	• Blurred results • Consumes more time
6	Esedoglu and Shen [9]	It enhances the technique [6] by applying Mumford–Shah method and Γ-convergence	Scratches	Self Collected Images	Subjective Analysis	• Reduces the approximation and computation	• Generates artificial corners • Violates connectivity principle • Unstable
7	Telea [10]	It depends on gradient of the image and neighborhood of the pixel	Scratches	Self Collected Images	Subjective Analysis	• Simple and fast	• Blurred results when the region wider than 10 pixels

8	Xu et al. [11]	It is a small extension to the method [7] to handle the limitation of PDE	Text, Scratches	Internet Images	Computational Complexity	• Improves processing time and connectivity problem	• Blurred results and artifacts when the missing area is large
9	Li and Wang [13]	It fully uses the nonlinear anisotropic diffusion of p-Laplace operator	Text, Scratches	Self Collected Images	PSNR	• Improves quality and processing time	• Artifacts in the smoothness of isophotes exist
10	Bornemann and Marz [12]	It enhances the anisotropic diffusion and is based on coherence transport equation	Text, Scratches, Salt & Pepper noise	Internet Images	Subjective Analysis	• Non-iterative and fast • Generates quality results around boundary	• Creates artifacts for large region as well as texture regions • Relies on tuning of parameters
11	Barcelos and Batista [14]	It is the improved version of method in [2]. Mean curvature flow diffusion is applied in the inpainting regions	Text, Scratches, Noise	Internet Images	Subjective Analysis	• Small improvement in the quality of the image	• Blurs the edges
12	Lu et al. [15]	It extends the method [6] and is based on the analysis of local characteristics around the damaged region	Scratches	Self Collected historical images	PSNR, Computational time	• Improves computational time	• Produces blurred results
13	Biradar and Kohir [16]	It is a small extension to the method [2] to address the limitation of partial differential equation	Scratches	Internet Images and Self Collected	PSNR	• Little bit enhancement in computational complexity	• Produces blurred and visual artifacts when the unknown region is large
14	Jidesh and George [17]	It is the second-order nonlinear diffusion driven by Gauss curvature. This is also used to eliminate Gaussian noise and blur in the other regions of the image	Random loss, Noise, and Blur	Internet Images	PSNR, SSIM, FoM	• Faster than CDD • Remove the noise well	• Produces small blur and staircasing effect
15	Gopinath et al. [18]	It is the fourth-order diffusion and uses the complex diffusion coefficient as conductivity parameter	Scratches	Internet Images	PSNR and SSIM	• Improves quality of the image	• Produces blurred results

16	Barbu [20]	It introduces a new edge stopping function for the diffusion process	Scratches, Noise	USC-SIPI	PSNR and SSIM	• Fast inpainting model • performs well in noisy conditions	• Requires more user interaction to tune several parameters
17	Zhang et al. [23]	It improves TV model [6] by considering fractional calculus. Riemann-Liouville fractional derivative definition is considered	Text, scratches, Noise	USC-SIPI	PSNR	• Eliminates staircase effects • Solves the connectivity principle	• Blurs the edges • Reduces contrast of the image
18	Zhang et al. [25]	It extends TV model [6] by considering fractional calculus and adaptive p-Laplace operator. Grünwald-Letnikov fractional derivative definition is considered	Text, scratches, Noise	USC-SIPI	PSNR	• Eliminates staircase effects • Solves the connectivity principle. • User interaction is reduced	• Blurs the edges • Reduces contrast of the image
19	Bosch and Stoll [26]	It extends Cahn-Hilliard model [49] by considering fractional Laplacian operator via spectral decomposition	Mask	Internet Images	PSNR and SSIM	• Increases sharpness • Improves visual quality	• Consumes more computational time
20	Sridevi and Kumar [28]	It uses fractional-order diffusion driven by nonlinear complex conduction coefficient. The fractional derivative is utilized based on Caputo definition	Text, scratches, Noise	USC-SIPI	PSNR, SSIM, and MI	• Eliminates staircase effects • Minimizes edge blurring • Enhances the contrast of the image	• Consumes more computational time
21	Yang and Guo[29]	It integrates fractional calculus with tensor regularizer. Deduction of fractional derivative in four directions conforming to the shifted GL definition	Text, Mask, Noise	USC-SIPI	PSNR	• Overcomes the shortages of the traditional tensor regularization model	• Consumes more computational time

22	Sridevi and Kumar [30]	It uses an adaptive fractional-order diffusion driven by difference curvature. The fractional derivative is utilized based on DFT	Text, scratches, Random loss, Noise, Blur	USC-SIPI	PSNR, SSIM and FoM	• Minimizes edge blurring • Minimizes blocky effect	• Consumes more computational time
23	Richard et al. [31]	It iteratively convolves predefined diffusion weighted averaging mask with the missing region	Text, Scratches	Internet images	MSE and Computational time	• Fast and simple	• Employs very small region with low contrast edge • Blurred results
24	Hadhoud et al. [32]	It expands and enhances the technique [31] by altering the diffusion masks with a weight of zero at the bottom right corner	Text, Scratches	Internet Images	PSNR	• Simple and faster than the method [31] • Non-iterative	• Uses for scratches and text removal • Does n't work well in the presence of noise & produces blurred edges
25	Noori et al. [33]	It extends the method [31] by using adaptive convolution mask which depends on gradient of known neighborhoods	Text, Scratches	Internet Images	PSNR	• Produces better results than [31] • Efficient for noise removal	• Produces blurred results • Requires more number of iterations to recover the edges
26	Rana et al. [35]	It is based on the methods [28,29] followed by edge detection and smoothing the image	Scratches	Internet Images	PSNR	• Produces better results than [31] • Edges are preserved	• • Not suitable for textured images
27	Tony et al. [3]	It combines total variation minimization and wavelet transform to recover the lost blocks during transmission	Loss of wavelet coefficients and Noise	USC-SIPI	PSNR	• Supports noisy images as well as noiseless images	• Unsuitable for fine structures • Only for lost blocks
28	Zhang and Chan [50]	It utilizes TV model with non-local means with Bregmanized operator splitting	Loss of wavelet coefficients and Noise	USC-SIPI	PSNR	• Computationally less	• Handles small corrupted data
29	Zhang et al. [51]	It combines total variation model and p-Laplace operator	Loss of wavelet coefficients and Noise	USC-SIPI	PSNR	• Produces better results than [3]	• Only for lost blocks

30	Zhang et al. [25]	It combines the method [3] with p-Laplacian operator and fractional-order derivative in wavelet domain	Loss of wavelet coefficients and Noise	USC-SIPI	PSNR	• Produces better results than [3]	• Extensive time consumption in fractional-order gradient
31	Jiang and Yin [37]	It combines total variation model and fractional calculus in wavelet domain. Primal-Dual method is considered for the energy minimization	Loss of wavelet coefficients and Noise	USC-SIPI	PSNR	• Produces better results than [25]	• Extensive time consumption in fractional-order gradient

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