

# Model Sketching by Abstraction Refinement for Lifted Model Checking (Extended Version)\*

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## ABSTRACT

In this work, we show how the use of verification and analysis techniques for model families (software product lines) with numerical features provides an interesting technique to synthesize complete models from sketches (i.e. partial models with holes). In particular, we present an approach for synthesizing PROMELA model sketches using variability-specific abstraction refinement for *lifted (family-based) model checking*.

## CCS CONCEPTS

• **Theory of computation** → **Logic**; Models of computation; • **Software and its engineering** → *Software notations and tools*.

## KEYWORDS

Model sketching, Product-line (lifted) model checking

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## 1 INTRODUCTION

This paper presents a novel synthesis framework for reactive models that adhere to a given set of properties. The input is a *sketch* [18], i.e. a partial model with holes, where each hole is a placeholder that can be replaced with one of finitely many options; and a *set of properties* that the model needs to fulfill. Model sketches are represented in the PROMELA modelling language [13] and properties are expressed in LTL [2]. The synthesizer aims to generate as output a *sketch realization*, i.e. a complete model instantiation, which satisfies the given properties by suitably filling the holes.

In this work, we frame the model sketching problem as a verification/analysis problem for model families (a.k.a. Software Product Lines – SPLs) [3], and then formulate an abstraction refinement algorithm that operates on model families to efficiently solve it. SPL methods and architectures allow building a family of similar models, known as *variants (family members)*, from a common code base. A custom variant is specified in terms of suitable *features* selected for that particular variant at compile-time.

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All possible model sketch realizations constitute a model family, where each hole is represented by a numerical feature with the same domain. In contrast to Boolean features that have only two values, numerical features can have a range of numbers as explicit values. Hence, the model sketching problem reduces to selecting correct variants (family members) from the resulting model family [8]. The automated analysis of such families for finding a correct variant is challenging since in addition to the state-space explosion affecting each family member, the family size (i.e., the number of variants) typically grows exponentially in the number of features. A naive *brute force enumerative* solution is to check each individual variant of the model family by applying an off-the-shelf model checker. This is shown to be very inefficient for large families [3, 16].

This paper applies an abstraction refinement procedure over the compact, all-in-one, representation of model families, called featured transition system (FTS) [3, 5, 6], to solve the model sketching problem. More specifically, we first devise variability abstractions tailored for model families that contain numerical features. Variability abstractions represent a configuration-space reduction technique that compresses the entire model family (with many configurations and variants) into an abstract model (with a single abstract configuration and variant), so that the result of model checking a set of LTL properties in the abstract model is preserved in all variants of the model family. The procedure is first applied on an abstract model that represents the entire model family, and then is repeated on refined abstract models that represent suitable sub-families of the original model family. Hence, the abstraction refinement approach [4–6, 10, 11] starts from considering all possible variants, and successively splits the entire family into indecisive and incorrect sub-families with respect to the given set of properties. The approach is sound and complete: either a correct complete model (variant) does exist and it is computed, or no such model exists and the procedure reports this. Because of its special structure and possibilities for sharing of equivalent execution behaviours and model checking results for many variants, this algorithm is often able to converge to a solution very fast after a handful of iterations even for sketches with large search spaces.

We have implemented our prototype model synthesizer, called PROMELASKETCHER. It uses variability-specific abstraction refinement for lifted model checking of model families with numerical features, and calls the SPIN model checker [13] to verify the generated abstract models. The abstraction and refinement are done in an efficient manner as source-to-source transformations of PROMELA code, which makes our procedure easy to implement/maintain as

a simple meta-algorithm script. We illustrate this approach for automatic completion of various PROMELA model sketches. We also compare its performance with the brute-force approach.

## 2 MODEL FAMILIES

*Featured transition system.* Let  $\mathbb{F} = \{A_1, \dots, A_k\}$  be a finite and totally ordered set of *numerical features* available in a model family. Let  $\text{dom}(A) \subseteq \mathbb{Z}$  denote the set of possible values that can be assigned to feature  $A$ . A valid combination of feature's values represents a *configuration*  $k$ , which specifies one *variant* of a model family. It is given as a *valuation function*  $k : \mathbb{F} \rightarrow \mathbb{Z}$ , which assigns a value from  $\text{dom}(A)$  to each feature  $A$ . We assume that only a subset  $\mathbb{K}$  of all possible configurations are *valid*. Each configuration  $k \in \mathbb{K}$  can be given by a formula:  $(A_1 = k(A_1)) \wedge \dots \wedge (A_k = k(A_k))$ .

A *transition system* [2] is a tuple  $\mathcal{T} = (S, I, \text{trans}, AP, L)$ , which is used to describe behaviours of single systems. We write  $s_1 \rightarrow s_2$  whenever  $(s_1, s_2) \in \text{trans}$ . A *path* of a TS  $\mathcal{T}$  is an *infinite* sequence  $\rho = s_0 s_1 s_2 \dots$  with  $s_0 \in I$  s.t.  $s_i \rightarrow s_{i+1}$  for all  $i \geq 0$ . The *semantics* of a TS  $\mathcal{T}$ , denoted  $\llbracket \mathcal{T} \rrbracket_{TS}$ , is the set of its paths.

A *featured transition system* (FTS) represents a compact model, which describes the behaviour of a whole family of systems in a single monolithic description. The set of feature expressions,  $\text{FeatExp}(\mathbb{F})$ , are propositional logic formulae over constraints of  $\mathbb{F}$ :  $\psi ::= \text{true} \mid A \bowtie n \mid \neg\psi \mid \psi \wedge \psi$ , where  $A \in \mathbb{F}$ ,  $n \in \mathbb{Z}$ , and  $\bowtie \in \{=, <\}$ . We write  $\llbracket \psi \rrbracket$  for the set of configurations that satisfy  $\psi$ , i.e.  $k \in \llbracket \psi \rrbracket$  iff  $k \models \psi$ . A *featured transition system* (FTS) is  $\mathcal{F} = (S, I, \text{trans}, AP, L, \mathbb{F}, \mathbb{K}, \delta)$ , where  $(S, I, \text{trans}, AP, L)$  form a TS;  $\mathbb{F}$  is a set of available features;  $\mathbb{K}$  is a set of valid configurations; and  $\delta : \text{trans} \rightarrow \text{FeatExp}(\mathbb{F})$  is a total function decorating transitions with presence conditions (feature expressions). The *projection* of an FTS  $\mathcal{F}$  to a configuration  $k \in \mathbb{K}$ , denoted as  $\pi_k(\mathcal{F})$ , is the TS  $(S, I, \text{trans}', AP, L)$ , where  $\text{trans}' = \{t \in \text{trans} \mid k \models \delta(t)\}$ . We lift the definition of *projection* to sets of configurations  $\mathbb{K}' \subseteq \mathbb{K}$ , denoted as  $\pi_{\mathbb{K}'}(\mathcal{F})$ , by keeping transitions admitted by at least one of configurations in  $\mathbb{K}'$ . That is,  $\pi_{\mathbb{K}'}(\mathcal{F})$ , is the FTS  $(S, I, \text{trans}', AP, L, \mathbb{F}, \mathbb{K}', \delta')$ , where  $\text{trans}' = \{t \in \text{trans} \mid \exists k \in \mathbb{K}'.k \models \delta(t)\}$  and  $\delta'$  is the restriction of  $\delta$  to  $\text{trans}'$ . The *semantics* of an FTS  $\mathcal{F}$ , denoted as  $\llbracket \mathcal{F} \rrbracket_{FTS}$ , is the union of paths of the projections on all valid variants  $k \in \mathbb{K}$ , i.e.  $\llbracket \mathcal{F} \rrbracket_{FTS} = \cup_{k \in \mathbb{K}} \llbracket \pi_k(\mathcal{F}) \rrbracket_{TS}$ .

*Abstraction.* We start working with Galois connections between Boolean complete lattices of feature expressions, and then induce a notion of abstraction of FTSs. The Boolean complete lattice of feature expressions is:  $(\text{FeatExp}(\mathbb{F})_{\equiv}, \models, \vee, \wedge, \text{true}, \text{false}, \neg)$ , where the elements of  $\text{FeatExp}(\mathbb{F})_{\equiv}$  are equivalence classes of formulae  $\psi$  obtained by quotienting by the semantic equivalence  $\equiv$ .

The *join abstraction*,  $\alpha_{\mathbb{K}}^{\text{join}} : \text{FeatExp}(\mathbb{F}) \rightarrow \text{FeatExp}(\emptyset)$ , replaces each feature expression  $\psi$  in an FTS with true if there exists at least one configuration from  $\mathbb{K}$  that satisfies  $\psi$ . The abstract sets of features and configurations are:  $\alpha_{\mathbb{K}}^{\text{join}}(\mathbb{F}) = \emptyset$  and  $\alpha_{\mathbb{K}}^{\text{join}}(\mathbb{K}) = \{\text{true}\}$ . The abstraction and concretization functions between  $\text{FeatExp}(\mathbb{F})$  and  $\text{FeatExp}(\emptyset)$ , which form a Galois connection [6], are:

$$\alpha_{\mathbb{K}}^{\text{join}}(\psi) = \begin{cases} \text{true} & \text{if } \exists k \in \mathbb{K}.k \models \psi \\ \text{false} & \text{otherwise} \end{cases}, \quad \gamma_{\mathbb{K}}^{\text{join}}(\text{true}) = \text{true}, \gamma_{\mathbb{K}}^{\text{join}}(\text{false}) = \bigvee_{k \in \mathbb{K}} k$$

Given the FTS  $\mathcal{F} = (S, I, \text{trans}, AP, L, \mathbb{F}, \mathbb{K}, \delta)$ , we will define a TS  $\alpha_{\mathbb{K}}^{\text{join}}(\mathcal{F}) = (S, I, \text{trans}', AP, L)$  to be its *abstraction*, where

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### Algorithm 1: ARP( $\mathcal{F}, \mathbb{K}, \phi$ )

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**Input:** An FTS  $\mathcal{F}$ , a configuration set  $\mathbb{K}$ , and an LTL formula  $\phi$   
**Output:** Correct variants  $k \in \mathbb{K}$ , s.t.  $\pi_k(\mathcal{F}) \models \phi$   
**Global:** end := false

- 1  $c = (\alpha_{\mathbb{K}}^{\text{join}}(\mathcal{F}) \models \phi)$  ;
- 2 **if** ( $c = \text{null}$ ) **then** {end := true; **return**  $\mathbb{K}$  } ;
- 3  $\psi := \text{FeatExp}(c)$  ;
- 4 **if** ( $\text{sat}(\psi \wedge (\bigvee_{k \in \mathbb{K}} k))$ ) **then**
- 5      $(\psi_1, \dots, \psi_n) := \text{Split}(\llbracket \neg\psi \rrbracket \cap \mathbb{K})$  ;
- 6     **if** (end) **then return**  $\emptyset$  ;
- 7      $\text{ARP}(\pi_{\llbracket \psi_1 \rrbracket}(\mathcal{F}), \llbracket \psi_1 \rrbracket, \phi); \dots; \text{ARP}(\pi_{\llbracket \psi_n \rrbracket}(\mathcal{F}), \llbracket \psi_n \rrbracket, \phi)$
- 8 **else**
- 9      $\psi' = \text{CraigInterpolation}(\psi, \mathbb{K})$  ;
- 10    **if** (end) **then return**  $\emptyset$  ;
- 11     $\text{ARP}(\pi_{\llbracket \psi' \rrbracket}(\mathcal{F}), \llbracket \psi' \rrbracket, \phi); \text{ARP}(\pi_{\llbracket \neg\psi' \rrbracket}(\mathcal{F}), \llbracket \neg\psi' \rrbracket, \phi)$

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$\text{trans}' = \{t \in \text{trans} \mid \alpha_{\mathbb{K}}^{\text{join}}(\delta(t)) = \text{true}\}$ . Note that transitions in the abstract TS  $\alpha_{\mathbb{K}}^{\text{join}}(\mathcal{F})$  describe the behaviour that is possible in some variants of the concrete FTS  $\mathcal{F}$ , but not need be realized in the other variants. The information about which transitions are associated with which variants is lost, thus causing a precision loss in the abstract model. This way,  $\llbracket \alpha_{\mathbb{K}}^{\text{join}}(\mathcal{F}) \rrbracket_{TS} \supseteq \cup_{k \in \mathbb{K}} \llbracket \pi_k(\mathcal{F}) \rrbracket_{TS}$ . We say that a TS  $\mathcal{T}$  satisfies a LTL formula  $\phi$ , written  $\mathcal{T} \models \phi$ , iff all paths of  $\mathcal{T}$  satisfy formula  $\phi$  [2]. We say that an FTS  $\mathcal{F}$  satisfies  $\phi$ , written  $\mathcal{F} \models \phi$ , iff all its variants satisfy  $\phi$ , i.e.  $\forall k \in \mathbb{K}. \pi_k(\mathcal{F}) \models \phi$ .

**THEOREM 2.1 (PRESERVATION RESULTS, [6]).** *For every  $\phi \in \text{LTL}$  [2],  $\alpha_{\mathbb{K}}^{\text{join}}(\mathcal{F}) \models \phi \implies \mathcal{F} \models \phi$ .*

The problem of evaluating  $\mathcal{F} \models \phi$  can be reduced to a number of smaller problems by partitioning the configuration space  $\mathbb{K}$ . Let the subsets  $\mathbb{K}_1, \mathbb{K}_2, \dots, \mathbb{K}_n$  form a *partition* of  $\mathbb{K}$ . Then,  $\mathcal{F} \models \phi$  iff  $\pi_{\mathbb{K}_i}(\mathcal{F}) \models \phi$  for all  $i = 1, \dots, n$ .

*Abstraction Refinement Framework.* The abstraction refinement procedure ARP for checking  $\mathcal{F} \models \phi$  is illustrated by Algorithm 1. We first construct an initial abstract model  $\alpha_{\mathbb{K}}^{\text{join}}(\mathcal{F})$ , and check  $\alpha_{\mathbb{K}}^{\text{join}}(\mathcal{F}) \models \phi$  (Line 1). If the abstract model satisfies the given property (i.e., the counterexample  $c$  is null), then all variants from  $\mathbb{K}$  satisfy it and we stop. In this case, the global variable end is also set to true making all other recursive calls to ARP to end (Lines 2, 6, 10). Otherwise, a non-null counterexample  $c$  is found. Let  $\psi$  be the feature expression computed by conjoining feature expressions labelling all transitions that belong to path  $c$  when  $c$  is simulated in  $\mathcal{F}$  (Line 3). There are two cases to consider.

First, if  $\psi \wedge (\bigvee_{k \in \mathbb{K}} k)$  is satisfiable (i.e.  $\mathbb{K} \cap \llbracket \psi \rrbracket \neq \emptyset$ ), then the found counterexample  $c$  is *genuine* for variants in  $\mathbb{K} \cap \llbracket \psi \rrbracket$ . For the other variants from  $\mathbb{K} \cap \llbracket \neg\psi \rrbracket$ , the found counterexample cannot be executed (Lines 5,6,7). We call `Split` to split the space  $\mathbb{K} \cap \llbracket \neg\psi \rrbracket$  in sub-families  $\llbracket \psi_1 \rrbracket, \dots, \llbracket \psi_n \rrbracket$ , such that all atomic constraints in  $\psi_i$  are of the form:  $A \bowtie n$ , where  $A \in \mathbb{F}$  and  $n \in \text{dom}(A)$ . In particular, the `Split` function takes as input a set of configurations and returns a list of sets of configurations. For example, assume that we have two numerical features  $\text{Min} \leq A \leq \text{Max}$  and  $\text{Min} \leq B \leq$

Max. If  $\psi = (A = 3)$ , then  $\text{Split}([\neg\psi])$  is  $(\text{Min} \leq A \leq 2) \wedge (\text{Min} \leq B \leq \text{Max})$  and  $(4 \leq A \leq \text{Max}) \wedge (\text{Min} \leq B \leq \text{Max})$ . Finally, we call ARP to verify the sub-families:  $\pi_{[\psi_1]}(\mathcal{F}), \dots, \pi_{[\psi_n]}(\mathcal{F})$ . Note that if  $\mathbb{K} \cap [\neg\psi] = \emptyset$ ,  $\text{Split}$  updates variable end to true and so no recursive ARPs are called.

Second, if  $\psi \wedge (\bigvee_{k \in \mathbb{K}} k)$  is unsatisfiable (i.e.  $\mathbb{K} \cap [\psi] = \emptyset$ ), then the found counterexample  $c$  is *spurious* for all variants in  $\mathbb{K}$  (due to incompatible feature expressions) (Lines 9,10,11). A feature expression  $\psi'$  used for constructing refined sub-families is determined by means of Craig interpolation [15] from  $\psi$  and  $\mathbb{K}$ . First, we find the minimal unsatisfiable core  $\psi^c = X \wedge Y = \text{false of } \psi \wedge (\bigvee_{k \in \mathbb{K}} k)$ . Next, the interpolant  $\psi'$  is computed, such that  $\psi'$  summarizes and translates why  $X$  is inconsistent with  $Y$  in their shared language. Finally, we call the ARP to check  $\pi_{[\psi']}(\mathcal{F}) \models \phi$  and  $\pi_{[\neg\psi']}(\mathcal{F}) \models \phi$ . By construction, it is guaranteed that the spurious counterexample  $c$  does not occur in both  $\pi_{[\psi']}(\mathcal{F})$  and  $\pi_{[\neg\psi']}(\mathcal{F})$  [12].

Note that abstract models we obtain are ordinary TSs where all feature expressions are replaced with true. Therefore, the verification step  $\alpha_{\mathbb{K}}^{\text{join}}(\mathcal{F}) \models \phi?$  (Line 1) can be performed using a single-system model checker such as SPIN. Also note that we call ARP until we find a correct variant (variable end is set to true) or the updated set of configurations  $\mathbb{K}$  becomes empty. Therefore,  $\text{ARP}(\mathcal{F}, \mathbb{K}, \phi)$  terminates and is correct.

### 3 SYNTACTIC TRANSFORMATIONS

We now present the high-level modelling language PROMELA for writing sketches and model families. Then, we describe several transformations of PROMELA sketches and model families.

*Syntax of PROMELA.* PROMELA [13] is a non-deterministic modelling language designed for describing systems composed of concurrent processes that communicate asynchronously. The basic statements of processes are given by:

```
stm ::= skip | break | x := expr | c?x | c!expr | stm1; stm2 |
if :: g1 → stm1 ··· gn → stmn fi | do :: g1 → stm1 ··· gn → stmn od
```

where  $x$  is a variable,  $expr$  is an expression,  $c$  is a channel, and  $g_i$  are conditions over variables and contents of channels.

*Sketches.* To encode sketches, a single sketching construct of type expression is included: a basic integer hole denoted by  $??$ . Each hole occurrence is assumed to be uniquely labelled as  $??_i$ , and it has a bounded integer domain  $[n_i, n'_i]$ .

*Model Families.* To encode multiple variants, a new compile-time guarded-by-features statement is included:

```
stm ::= ... | #if :: ψ1 → stm1 ... :: ψn → stmn #endif
```

where  $\psi_1, \dots, \psi_n$  are feature expressions defined over  $\mathbb{F}$ . The “#if” statement contains feature expressions  $\psi_i \in \text{FeatExp}(\mathbb{F})$  as presence conditions (guards). If presence condition  $\psi_i$  is satisfied by a configuration  $k \in \mathbb{K}$  the statement  $stm_i$  will be included in the variant corresponding to  $k$ . Hence, “#if” plays the same role as “#if” directives in C preprocessor CPP [7, 9, 14]. The semantics of PROMELA models and PROMELA model families are given in [6, 13].

*Syntactic Transformations.* Our aim is to transform an input sketch  $\hat{P}$  with a set of  $m$  holes  $??_1^{[n_1, n'_1]}, \dots, ??_m^{[n_m, n'_m]}$ , into an output

model family  $\bar{P}$  with a set of numerical features  $A_1, \dots, A_m$  with domains  $[n_1, n'_1], \dots, [n_m, n'_m]$ . The set of configurations  $\mathbb{K}$  includes all possible combinations of feature’s values. The rewrite rule for eliminating holes  $??$  from a model sketch is of the form:

```
stm[??[n,n']] ~> #if :: (A=n) → stm[n] ... :: (A=n') → stm[n'] #endif (R-1)
```

where  $stm[-]$  is a (non-compound) basic statement with a single expression  $-$  in it,  $??^{[n,n']}$  is an occurrence of a hole with domain  $[n, n']$ , and  $A$  is a fresh numerical feature with domain  $[n, n']$ . The meaning of the rule (R-1) is that if the current sketch being transformed matches the abstract syntax tree node of the shape  $stm[??^{[n,n']}$ ] then replace  $stm[??^{[n,n']}$ ] according to the rule (R-1).

We write  $\text{Rewrite}(\hat{P})$  to be the final model family obtained by repeatedly applying the rule (R-1) on sketch  $\hat{P}$  and on its transformed versions until we reach a point where it can not be applied.

We now present the syntactic transformations of model families  $\bar{P} = \text{Rewrite}(\hat{P})$  obtained from PROMELA sketches  $\hat{P}$ . We consider two transformations: projection  $\pi_{[\psi]}(\bar{P})$  and variability abstraction  $\alpha_{\mathbb{K}}^{\text{join}}(\bar{P})$ . Let  $\bar{P}$  represent a model family.

The projection  $\pi_{[\psi]}(\bar{P})$  is obtained by defining a translation recursively over the structure of  $\psi$ . Let  $\psi$  be of the form  $(A < m)$ . The rewrite rule is of the form:

```
#if :: (A=n) → stm[n] ... :: (A=m) → stm[m] ... :: (A=n') → stm[n'] #endif ~>
#if :: (A=n) → stm[n] ... :: false → stm[m] ... :: false → stm[n'] #endif (R-2)
```

where  $n \leq m \leq n'$ . That is, all guards that do not satisfy  $(A < m)$  are replaced with false. Let  $\psi$  be a feature expression of the form  $\neg\psi'$ . We first transform  $\bar{P}$  by applying the projection  $\psi'$ , then in all #if-s obtained from the projection  $\psi'$  we change the guards: those guards of the form  $(A = m')$  become false, and false guards are returned to the form  $(A = m')$  by looking at a special memo list where we keep record of them. Let  $\psi$  be of the form  $\psi_1 \wedge \psi_2$ . Then, we apply projections  $\psi_1$  and  $\psi_2$  one after the other.

The abstract model  $\alpha_{\mathbb{K}}^{\text{join}}(\bar{P})$  is obtained by appropriately resolving all “#if”-s. The rewrite rule is:

```
#if :: (ψ1) → stm1 ... :: (ψn) → stmn #endif ~>
if :: αℕjoin(ψ1) → stm1 ... :: αℕjoin(ψn) → stmn fi (R-3)
```

where all guards in the new if are set to *true* or *false* depending whether there is some valid configurations that satisfies that guard.

The correctness of these transformations are formally proved by structural induction on  $\hat{P}$  and  $\bar{P}$  (see Theorems A.1 and A.2 in App. A).

### 4 SYNTHESIS ALGORITHM

We can now encode the sketch synthesis problem as a lifted model checking problem. In particular, we delegate the effort of conducting an effective search of all possible sketch realizations to an efficient abstraction refinement for lifted model checking. Once the lifted model checking of the corresponding model family is performed, we can see for which variants the given property is correct. Those variants represent the correct sketch realizations.

The synthesis algorithm  $\text{SYNTHESIZE}(\hat{P}, \phi)$  for solving a sketch  $\hat{P}$  is the following. The sketch  $\hat{P}$  is first encoded as a model family

$\bar{P} = \text{Rewrite}(\hat{P})$ . Then, we call function  $\text{ARP}(\bar{P}, \mathbb{K}, \phi)$ , which takes as input the model family  $\bar{P}$ , its configuration set  $\mathbb{K}$ , and the property to verify  $\phi$ , and returns as solution a set of variants  $\mathbb{K}' \subseteq \mathbb{K}$  that satisfy  $\phi$  obtained after performing the ARP. The correctness and termination of  $\text{SYNTHESIZE}(\hat{P}, \phi)$  are shown in Theorem A.3 in App. A.

## 5 EVALUATION

*Implementation.* We have developed a prototype model synthesizer, called `PROMELASKETCHER`, for resolving `PROMELA` sketches. It uses the ANTLR parser generator [17] for processing `PROMELA` code, while projections and abstractions of `#if`-enriched `PROMELA` code are implemented using source-to-source transformations. It calls the `SPIN` [13] to verify the generated `PROMELA` models. If a counterexample trace is returned, the tool inspects the trace by using `SPIN`'s simulation mode, and generates refined abstractions. Our tool is written in `JAVA` and consists of around 2K LOC.

*Experiment setup and Benchmarks.* All experiments are executed on a 64-bit Intel®Core™ i5 CPU, Ubuntu VM, with 8 GB memory. The implementation, benchmarks, and all results are available from: [https://github.com/aleksdimovski/Promela\\_sketcher](https://github.com/aleksdimovski/Promela_sketcher). We compare our approach with the `BRUTE FORCE` enumeration approach that generates all possible sketch realizations and verifies them using `SPIN` one by one. For each experiment, we report: `TIME` which is the total time to resolve a sketch in seconds; and `CALLS` which is the number of times `SPIN` is called. We show performances for three different sizes of holes: 3-, 4-, and 8-bits. We only measure the model checking `SPIN` times to generate a process analyser (`pan`) and to execute it. We do not count the time for compiling `pan`, as it is due to a design decision in `SPIN` rather than its verification algorithm. The evaluation is performed on several suitably adjusted `PROMELA` sketches collected from the `SKETCH` project [18], `SyGuS-Comp` [1], and `SPIN` [13] (see benchmarks in App. B).

*Performance Results.* Table 1 shows the results of synthesizing our benchmarks.

`PROMELASKETCHER` needs two iterations and two calls to `SPIN` to resolve the `SIMPLE` sketch given in Fig. 1 by reporting that the hole `??` can be replaced with an integer value from  $[Min, 2]$ .

Hence, it significantly outperforms the `BRUTE FORCE` approach.

The `LOOP` sketch [18] in Fig 2 contains one hole `??` represented by feature `A`. The coarsest abstract model has an `if` statement with one optional sequence `'do :: (x>n) → ...od'` for each possible value `n` of feature `A`. `PROMELASKETCHER` reports counterexamples for the cases  $(A = Min), \dots, (A = 4)$ , and then we obtain the correct solution for the abstraction  $(5 \leq A \leq Max)$ . We have slightly changed `Loop` by replacing `x:=10` with `x:=??`, thus obtaining a sketch `Loop'` with two holes represented by two features `A1` and `A2`. All reported counterexamples have specific values for `A1` and `A2`, which are used to define refined abstract models.

`PROMELASKETCHER` needs two iterations to resolve the `LOOP-COND` sketch [18] in Fig 3, where we use feature `A` for the hole `??`. In the first iteration, `SPIN` reports an error trace that corresponds to the case when  $(A = 2)$ . In the next iteration, we obtain the correct answer for the abstraction  $(A \leq 1)$ .

The `WELFARE` sketch [13] in Fig 4 is a problem due to Feijen. There are three ordered lists of integers `a`, `b`, and `c`. At least one element appears in all three lists. Find the smallest indices `i`, `j`, and `k`, such that `a[i]=b[j]=c[k]`. That is, we want to find the first element that appears in all three lists. The list `c` is initialized in such a way that concrete values assigned to the first  $n - 1$  elements do not appear in lists `a` and `b`, and the last  $n$ -th element of `c` is assigned to the hole `??`. Hence, the hole `??` should be replaced with the smallest value that appear also in lists `a` and `b`. `PROMELASKETCHER` successfully partitions the configuration space and finds the correct solutions for various values assigned to lists `a`, `b`, and `c`. The number of iterations needed depends on the content of `a`, `b`, and `c`.

The `SALESMAN` sketch [13] is a well-known optimisation problem, whose `PROMELA` solution is given in Fig. 5. Given a list of `N` cities and `distances` between each pair of cities, it asks to find the shortest possible tour that visits each city and returns to the origin city. We now use our approach to find the shortest tour through the cities. We initialize variable `MAX` to an integer hole `??`. Whenever there exists a shorter tour than the one assigned to `MAX`, the given LTL property `p` fails and a counterexample is reported. Therefore, the LTL property `p` will be correct only when `MAX` is initialized to the value less or equal to the shortest possible tour. `PROMELASKETCHER` successfully finds this value for `??` in only two iterations. In the first iteration, it reports a counterexample with a tour of length  $(n + 1)$  that is greater than the shortest possible tour that is of length  $n$ . Then, in the second iteration, the abstraction  $(A \leq n)$  satisfies the property `p`.

We can see from Table 1 that `PROMELASKETCHER` significantly outperforms `BRUTE FORCE`. On our benchmarks, it translates to speed ups that range from 1.2× to 3.5× for 4-bits holes, and from 11.5× to 51.4× for 8-bits holes. This is due to the fact that the number of calls to `SPIN` and the number of partitionings of  $\mathbb{K}$  that share the same counterexamples or correct traces in `PROMELASKETCHER` are much less than the configuration space  $\mathbb{K}$  that is inspected one by one using `SPIN` in the `BRUTE FORCE`.

## 6 CONCLUSION

In this paper, we employ techniques from product-line lifted model checking for automatically resolving of model sketches. By means of an implementation and a number of experiments, we confirm that our technique is effective and works well on a variety of `PROMELA` benchmarks and LTL properties.

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**Table 1: Performance results. All times in sec.**

Bench.	3 bits				4 bits				8 bits			
	PROMELASKETCHER		BRUTE-FORCE		PROMELASKETCHER		BRUTE-FORCE		PROMELASKETCHER		BRUTE-FORCE	
	CALLS	TIME	CALLS	TIME	CALLS	TIME	CALLS	TIME	CALLS	TIME	CALLS	TIME
SIMPLE	2	0.319	8	0.648	2	0.351	16	1.250	2	0.373	256	19.24
Loop	4	0.638	8	0.614	4	0.658	16	1.228	4	1.667	256	18.95
LoopCond	2	0.392	8	0.639	2	0.448	16	1.251	2	0.778	256	19.64
Welfare	4	0.660	8	0.650	5	0.923	16	1.205	10	1.476	256	19.69
Salesman	2	0.406	8	0.689	2	0.417	16	1.359	2	0.424	256	19.41

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## A PROOFS

Let  $H$  be a set of holes in the sketch  $\hat{P}$ . We define a *control function*  $\varphi : \Phi = H \rightarrow \mathbb{Z}$  to describe the value of each hole in  $\hat{P}$ . We write  $[[\hat{P}]]_{TS}^\varphi$  for TS obtained by replacing holes in  $\hat{P}$  according to  $\varphi$ .

**THEOREM A.1.** *Let  $\hat{P}$  be a sketch and  $\varphi$  be a control function, s.t. features  $A_1, \dots, A_n$  correspond to holes  $??_1, \dots, ??_n$ . We define a configuration  $k \in \mathbb{K}$ , s.t.  $k(A_i) = \varphi(??_i)$  for  $1 \leq i \leq n$ . Let  $\bar{P} = \text{Rewrite}(\hat{P})$ . We have:  $[[\hat{P}]]_{TS}^\varphi \equiv [[\pi_k(\bar{P})]]_{FTS}$ .*

**PROOF.** By induction on the structure of  $\hat{P}$ . The only interesting case is a basic statement  $stm[??_i]$  for rule (R-1), since in all other cases we have identity transformations.

$$\begin{aligned} & [[\pi_k(\llbracket \#if :: (A_i = n) \rightarrow stm[n] \dots :: (A_i = n') \rightarrow stm[n'] \#endif \rrbracket_{FTS})]]_{TS} \\ \stackrel{\text{def. of } \pi_k}{=} & \llbracket stm[k(A_i)] \rrbracket_{TS} \stackrel{\text{hypoth.}}{=} \llbracket stm[\varphi(??_i)] \rrbracket_{TS} \stackrel{\text{def. of } ??}{=} \llbracket stm[??_i] \rrbracket_{TS}^\varphi \quad \square \end{aligned}$$

**THEOREM A.2.** *Let  $\bar{P}$  be a PROMELA family and  $[[\bar{P}]]_{FTS}$  be its FTS. Then:  $\pi_{[[\psi]]}(\llbracket \bar{P} \rrbracket_{FTS}) \equiv [[\pi_{[[\psi]]}(\bar{P})]]_{FTS}$  and  $\alpha_{\mathbb{K}}^{\text{join}}(\llbracket \bar{P} \rrbracket_{FTS}) \equiv [[\alpha_{\mathbb{K}}^{\text{join}}(\bar{P})]]_{TS}$ .*

**PROOF.** By induction on the structure of  $\bar{P}$ . The only interesting case is “#if”, since in all other cases we have an identity translation. We can see that projection  $\pi_{[[\psi]]}$  and abstraction  $\alpha_{\mathbb{K}}^{\text{join}}$  are applied on feature expressions  $\psi$ , which can be introduced in FTSs only through “#if”-s. Thus, it is the same whether  $\pi_{[[\psi]]}$  and  $\alpha_{\mathbb{K}}^{\text{join}}$  are applied directly on FTS  $[[\bar{P}]]_{FTS}$  after the FTS is built by following the operational semantics of “#if”, or  $\pi_{[[\psi]]}$  and  $\alpha_{\mathbb{K}}^{\text{join}}$  are first applied on “#if”-s using rules (R-2), (R-3) and then FTS is built.  $\square$

**THEOREM A.3.** *SYNTHESIZE( $\hat{P}, \phi$ ) is correct and terminates.*

**PROOF.** The procedure SYNTHESIZE( $\hat{P}, \phi$ ) terminates since all steps in it are terminating. The correctness of SYNTHESIZE( $\hat{P}, \phi$ ) follows from the correctness of Rewrite (see Theorem A.1), ARP and syntactic transformations (see Theorem A.2).  $\square$

## B BENCHMARKS

---

```

init {
  byte x; int y;
  do :: break :: x++ od;
  y := ??*x;
  assert (y ≤ x+x) }

```

---

Figure 1: SIMPLE sketch.

---

```

init {
  byte x; int y:=0;
  do :: break :: x++ od;
  do :: (x>0) → x-;
    if :: (y<??) → y++
      :: else → y-fi;
    :: else → break od;
  assert (y ≤ 1) }

```

---

Figure 3: LOOPCOND sketch.

---

```

byte N:=4, MAX, distance[16];
byte city, dest, tour, seen;
bool visited[4];
#define Dist(a,b) distance[4*a+b]
inline travel2(dest) {
  (city != dest ∧ tour ≤ MAX) →
  tour := tour + Dist(city,dest)
  city := dest
  if :: (¬visited[city]) →
    visited[city]:=true;
    seen++
  :: else → break fi }

```

---



---

```

init {
  byte x:=10;
  int y:=0;
  do :: (x>??) → x-;
    y++
  :: else → break
  od;
  assert (y < 6) }

```

---

Figure 2: LOOP sketch.

---

```

int a[5], b[5], c[5];
init {
  byte i, j, k;
  a[0]:=1;... a[4]:=18;
  b[0]:=4;... a[4]:=25;
  c[0]:=5;... c[4]:=??;
  do :: a[i]<b[j] ∧ i<4 → i++;
    :: b[j]<c[k] ∧ j<4 → j++;
    :: c[k]<a[i] ∧ k<4 → k++;
    :: else → break od;
  assert(a[i]=b[j] ∧ b[j]=c[k]
    ∧ c[k]=a[i]) }

```

---

Figure 4: WELFARE sketch.

---

```

init {
  MAX:=??;
  Dist(0,1)=20;... Dist(3,2)=12;
  do :: select (dest: 0 .. (N-1)) →
    travel2(dest)
  od;
  ltl p {[] (seen<N ∧ tour>MAX) }
}

```

---

Figure 5: SALESMAN sketch.