

MIMO Radar Transmit Beampattern Shaping for Spectrally Dense Environments

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Abstract—Designing unimodular waveforms with a desired beampattern, spectral occupancy and orthogonality level is of vital importance in the next generation Multiple-Input Multiple-Output (MIMO) radar systems. Motivated by this fact, in this paper, we propose a framework for shaping the beampattern in MIMO radar systems under the constraints simultaneously ensuring unimodularity, desired spectral occupancy and orthogonality of the designed waveform. In this manner, the proposed framework is the most comprehensive approach for MIMO radar waveform design focusing on beampattern shaping. The problem formulation leads to a non-convex quadratic fractional programming. We propose an effective iterative to solve the problem, where each iteration is composed of a Semi-Definite Programming (SDP) followed by eigenvalue decomposition. Some numerical simulations are provided to illustrate the superior performance of our proposed over the state-of-the-art.

Index Terms—MIMO Radar, Beampattern Shaping, Waveform Design, Spectral Masking, Orthogonality, SDP

I. INTRODUCTION

Transmit beampattern shaping by controlling the spatial distribution of the transmit power, can play an important role in improving the radar performance through enhanced power efficiency, better detection probability, target identification, improved interference mitigation, among others. The goal is to focus the transmit power on desired angles while minimizing it at undesired angles [1]. Recently, the beampattern shaping via waveform design in Multiple-Input Multiple-Output (MIMO) radar systems has been widely studied. From a waveform design perspective, there are two methods for beampattern shaping, indirect and direct methods [2], [3]. In indirect (two-step) method, the waveform correlation matrix is firstly designed and the waveform matrix is subsequently obtained through one of the decomposition methods [4]–[12] while in direct method, the waveform is designed in one step [2], [3], [13]–[19]. On the other hand, there are several metrics (objective functions) to obtain the optimum beampattern such as, beampattern matching, spatial-Integrated Sidelobe Level Ratio (ISLR)/Peak Sidelobe Level Ratio (PSLR) minimization, and Signal to Interference plus Noise Ratio (SINR) maximization.

a) Beampattern Matching: In beampattern matching, the aim is to minimize the difference between the desired and designed beampattern. For instance, the following papers have worked on designing the waveform covariance matrix employing beampattern matching. The authors in [4] devised a

method to address the joint beampattern shaping and the cross-correlation minimization in spatial domain through Semidefinite Quadratic Programming (SQP) technique. In [5], Cyclic Algorithm (CA) is presented to shape the beampattern under low Peak-to-Average Ratio (PAR) constraint. In [10], [11], the authors propose a covariance matrix design method based on Discrete Fourier Transform (DFT) coefficients and Toeplitz matrices. The DFT-based technique provides a well-match transmit beampattern at low complexity. However, the drawback of the DFT-based method is that, for small number of antennas, the performance of the DFT-based method is slightly poorer. On the other hand, several papers focus on designing directly the transmit waveforms for beampattern shaping. For example, in [2], two optimization algorithms based on Alternating Direction Method of Multipliers (ADMM) are proposed under constant modulus constraint for the probing waveform. In [3], a method based in ADMM is proposed to design a beampattern in wide-band systems. In [17], a method for beampattern matching is addressed based on gradient decent which they term it Projection, Descent, and Retraction (PDR). In [18], the authors propose a method based on Majorization-Minimization (MM) for beampattern matching under PAR constraint in two cases of wide- and narrow-band.

b) Spatial-ISLR and PSLR minimization: In Spatial-ISLR and PSLR minimization approach, the aim is to minimize the ratio of *summation of beampattern response on undesired over desired angles*, and to minimize the ratio of *maximum beampattern response on undesired angles over minimum beampattern response on desired angles*, respectively. In [8], a method based on Semi-definite Relaxation (SDR) under constant energy and 3 dB main beam-width constraint is proposed to minimize the spatial-ISLR. In [20], the robust designs of waveform covariance matrix through optimizing the worst case transmit beampattern are considered to minimize the spatial-ISLR and -PSLR of beampatterns, respectively. Unlike two aforementioned methods, [13], [16], [19] propose a direct waveform design solution. The authors in [13] propose the efficient UNImodular set of seQUence design (UNIQUE) algorithm based on Coordinate Descent (CD) method to minimize spatial- and range-ISLR under four different constraints, namely, limited energy, PAR, continuous and discrete phase constraints. The method proposed in [19] is similar to UNIQUE without considering range-ISLR metric and PAR and limited energy constraints. A method based on ADMM is proposed in [16] to minimize the spatial-PSLR under constant modulus constraint.

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c) *SINR maximization*: In SINR optimization approaches, the problem does not deal with the beampattern directly. However, the beampattern is implicitly shaped as a result of transmit waveform optimization. For example [6], [7] address the problem of waveform design in the presence of signal dependence clutter. In these works, an iterative approach is presented to jointly optimize the transmit waveform and receive filter to maximize the output SINR. The authors in [21] propose Majorized Iterative Algorithm (MIA) based on MM method for joint waveform and filter design under similarity, constant modulus (Majorized Iterative Algorithm - Constant Modulus Constraint (MIA-CMC)) and PAR (Majorized Iterative Algorithm - PAR Constraint (MIA-PC)) constraints. While Space-Time Transmitting Code (STTC) [22] is proposed based on CD to solve the problem under similarity, uncertain steering matrices, continuous or discrete phase constraints. In [22], a Dinkelbach based method and exhaustive search is proposed for continuous and discrete phase constraints respectively.

In order to form the virtual array and enhancing the angular resolution, the received signal in MIMO radar system should be separable (orthogonal) in receiver while a set of arbitrary waveforms are adopted in the transmit side. In order to obtain the orthogonality, the waveform should have small cross-correlation [23]. Also, small auto-correlation sidelobes is a requirement, to avoid masking weak targets within the range sidelobes of a strong target, and to mitigate the harmful effects of distributed clutter returns close to the target of interest. Recently, many optimization techniques, e.g., Multi-Cyclic Algorithm-New (CAN) [24], [25], Iterative Direct Search [26], Integrated Sidelobe Level (ISL) New [27], [28], MM-Corr [29], Binary Sequences seTs (BiST) [30], UNIQUE [13] and Weighted BSUM sEquence SeT (WeBEST) [31] are proposed to design orthogonal sets of sequences, minimizing the ISL/Peak Sidelobe Level (PSL) metrics. However, beampattern shaping in MIMO radar systems yield a correlated waveform which, is in contradiction with orthogonality [13], [32]. In this context there are few papers which addressed these two aspects in MIMO radar systems. For instance [32] proposes beampattern matching problem under particular constraints on the waveform cross-correlation matrix. In [33], the authors minimize the difference between desired and undesired beampattern responses for one sub-pulse. Then the quasi-orthogonality of other sub-pulses are obtained by random permutation. In [34], the authors combine a beampattern matching by orthogonality requirement as a penalty in the objective function and use the PDR approach for the solution. In [35], the authors propose a method based on ADMM to design a beampattern with good cross-correlation. In [13], UNIQUE is proposed as a unified framework to minimize the spatial- and range-ISLR using weighted sum technique under limited energy, PAR, continuous and discrete phase constraints. By choosing an appropriate value for the regularization parameter, UNIQUE is able to make trade-off between having a good orthogonality and beampattern shaping.

On the other hand, spectral shaping is an important aspect

of resource management in cognitive¹ MIMO radar systems. Using this approach, the cognitive radar system is able to utilize effectively the available bandwidth. One attractive application of spectral shaping is coexistence of communications and cognitive MIMO radar systems, which the whole bandwidth is shared between these two systems based on the priorities [37]. There are several methods for spectral shaping. For instance, in [38]–[43] spectral matching approach is proposed to shape the spectral of the transmit waveform. In [44], [45], the authors consider a waveform design approach to maximize SINR, while the spectral behaviour is considered as a constraint. In [46], [47], the ratio of the maximum stop-band level to the minimum pass-band level is considered as the objective function to shape the spectrum. Spectral Integrated Level Ratio (SILR) minimization approach is consider under continuous and discrete phase constraints in [37]. The design of constant modulus waveform for beampattern matching under spectral constraint are addressed in [42], [48]. To tackle the non-convex optimization problem the authors in [42] and [48] propose Iterative Beampattern with Spectral design (IBS) and Beampattern Optimization With Spectral Interference Control (BIC) methods respectively.

A. Contribution

In this paper we consider the spatial-ISLR as design metric similar to [19]. In [19], the authors proposed CD-based method to enhance the performance of the radar in spatial domain. However, in this paper, we deal with the design of waveform considering the features in three domains: ISLR in the spatial and range domain and masking in the spectral domain. Particularly, we propose a waveform design framework to shape the beampattern with practical constraints, namely, spectral masking, 3 dB beam-width, constant modulus and similarity constraints. Spectral masking constraint plays an important role in cognitive MIMO radar systems in several scenarios, such as spectral sharing in coexistence of MIMO radar and MIMO communication. The 3 dB beam-width constraint ensures that the beampattern has a good response at the mainlobe. This constraint can be used in the emerging 4D-imaging automotive MIMO radar systems, wherein the Short-Range Radar (SRR), Mid-Range Radar (MRR) and Long-Range Radar (LRR) configurations are merged, to provide unique and high angular resolution in the entire radar detection range [13], [19]. Constant modulus waveforms are attractive for radar system designers due to efficient utilization of the limited transmitter power. Besides, since constant modulus is a kind of only phase-modulated sequence, implementing of constant modulus waveform is simple. As to the orthogonality, we incorporate the beampattern shaping optimization problem with similarity constraints to make a trade-off between having a good beampattern response and orthogonality [13]. This constraint imposes that the optimize waveform inherit some properties of a reference waveform. In fact, we consider the designed waveform to be similar to a specific waveform which

¹Cognitive MIMO radar systems are smart sensors which interact with the environment to adapt the properties of the waveform with the environment to enhance their performance [36].

have a good orthogonality to form the virtual array in receivers and enhance its angular resolution.

It is desirable to include all these properties to improve radar performance in emerging applications and in the emerging scenario of crowded environments with interference from other radars or communication systems. In this context, the contributions of the work are listed below.

- *Incorporation of metrics from multiple domains:* Radar tasks are influenced by parameters in the spatial, temporal and spectral domain. Hence it is pertinent to consider quality metrics in all these domains to improve performance. Thus, while it is highly interesting to consider all the metrics in the waveform design, the existing works consider only a selection of these performance metrics. A problem set-up involving these key performance indicators in different domains is lacking in literature. In this context the proposed framework incorporating all the metrics is the most comprehensive approach for MIMO radar waveform design focusing on beampattern shaping; it subsumes existing works as special cases. The gains obtained by incorporating these metrics over the existing works bears testimony to their impact.
- *Novel optimization framework:* The incorporation of all the aforementioned quality metrics adds further complexity to the waveform optimization and these cannot be handled by the existing frameworks. In this context, the paper also offers a novel optimization framework to solve the non-convex multi-variable and NP-hard problem. In an attempt to solve this problem, we propose an indirect method based on Semi-definite Programming (SDP). We first recast the waveform-design problem as a rank-one constrained optimization problem. Then, unlike the conventional methods which drop the rank one constraint, we propose a new iterative algorithm for efficiently solving the resulting rank-one constrained optimization problem. Each iteration of the proposed iterative algorithm is composed of an SDP followed by an Eigenvalue Decomposition (ED). In every iteration, we force the second largest eigen value towards zero to obtain the rank one solution. We prove that the proposed iterative algorithm converges to a local minima of the rank-one constrained optimization problem. Further, we compute the computational complexity of the proposed iterative algorithm. In addition the proposed framework can be extended to apply other convex constraints.

B. Organization and Notation

The rest of this research is organized as follows. In Section II, the system model and the design problem for minimizing the spatial-ISLR under constant modulus, 3 dB beam-width, similarity and spectral masking, constraints is formulated. We develop the iterative Waveform design for beampattern shaping and Spectral masking (WISE) framework based on SDP to obtain a rank-one solution in Section III. Finally we provide numerical experiments to verify the effectiveness of proposed algorithm in Section IV.

Notations: This paper uses lower-case and upper-case boldface for vectors (\mathbf{a}) and matrices (\mathbf{A}) respectively. The conjugate, transpose and the conjugate transpose operators are denoted by the $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^\dagger$ symbols respectively. Besides the Frobenius norm, ℓ_2 and ℓ_p norm, absolute value and round operator are denoted by $\|\cdot\|_F$, $\|\cdot\|_2$, $\|\cdot\|_p$, $|\cdot|$ and $\lfloor \cdot \rfloor$ respectively. For any matrix \mathbf{A} , $\text{Tr}(\mathbf{A})$, $\text{Diag}(\mathbf{A})$ and $\text{Rank}(\mathbf{A})$ stand for the trace, diagonal vector and the rank of \mathbf{A} , respectively. The letter j represents the imaginary unit (i.e., $j = \sqrt{-1}$), while the letter (i) is use as step of a procedure. Finally $\mathbf{1}$ and $\mathbf{0}$ denote a matrix/vector with proper size which all the elements are equal to one and zero respectively.

II. SYSTEM MODEL

We consider a colocated narrow-band MIMO radar system, with M transmit antennas, each transmitting a sequence of length N in the fast-time domain. Let the matrix $\mathbf{S} \in \mathbb{C}^{M \times N}$ denote the transmitted set of sequences in baseband, i.e,

$$\mathbf{S} \triangleq \begin{bmatrix} s_{1,1} & s_{1,2} & \dots & s_{1,N} \\ s_{2,1} & s_{2,2} & \dots & s_{2,N} \\ \vdots & \vdots & \vdots & \vdots \\ s_{M,1} & s_{M,2} & \dots & s_{M,N} \end{bmatrix}.$$

Let us further denote that $\mathbf{S} \triangleq [\bar{\mathbf{s}}_1, \dots, \bar{\mathbf{s}}_N] \triangleq [\bar{\mathbf{s}}_1^T, \dots, \bar{\mathbf{s}}_N^T]^T$, where the vector $\bar{\mathbf{s}}_n \triangleq [s_{1,n}, s_{2,n}, \dots, s_{M,n}]^T \in \mathbb{C}^M$ ($n = \{1, \dots, N\}$) indicates the n^{th} time-sample across the M transmitters (the n^{th} column of matrix \mathbf{S}) while the $\tilde{\mathbf{s}}_m \triangleq [s_{m,1}, s_{m,2}, \dots, s_{m,N}]^T \in \mathbb{C}^N$ ($m = \{1, \dots, M\}$) indicates the N samples of m^{th} transmitter (the m^{th} row of matrix \mathbf{S}). In this paper, we deal with the design of \mathbf{S} considering features in three domains: ISLR in the spatial and range domain and masking in the spectral domain. To this end, we now introduce the system model to describe in spatial and spectral domains. Subsequently, we also introduce similarity constraints to impose the range-ISLR features.

A. System Model in Spatial Domain

Let assume a colocated MIMO radar system with an Uniform Linear Array (ULA) structure for the transmit array characterized by the following steering vector [23],

$$\mathbf{a}(\theta) = [1, e^{j\frac{2\pi d}{\lambda} \sin(\theta)}, \dots, e^{j\frac{2\pi d(M-1)}{\lambda} \sin(\theta)}]^T \in \mathbb{C}^M, \quad (1)$$

where d is the distance between the transmitter antennas and λ is the signal wavelength. The beampattern in the direction of θ can be written as [4], [20], [23],

$$P(\mathbf{S}, \theta) = \frac{1}{N} \sum_{n=1}^N \left| \mathbf{a}^\dagger(\theta) \bar{\mathbf{s}}_n \right|^2 = \frac{1}{N} \sum_{n=1}^N \bar{\mathbf{s}}_n^\dagger \mathbf{A}(\theta) \bar{\mathbf{s}}_n$$

where, $\mathbf{A}(\theta) = \mathbf{a}(\theta) \mathbf{a}^\dagger(\theta)$. Let Θ_d and Θ_u be the sets of desired and undesired angles in the spatial domain, respectively. These two sets satisfy, $\Theta_d \cap \Theta_u = \emptyset$ and $\Theta_d \cup \Theta_u \subset [-90^\circ, 90^\circ]$. In this regard the spatial-ISLR is given by the following expression [13],

$$f(\mathbf{S}) \triangleq \frac{\sum_{\theta \in \Theta_u} P(\mathbf{S}, \theta)}{\sum_{\theta \in \Theta_d} P(\mathbf{S}, \theta)} = \frac{\sum_{n=1}^N \bar{\mathbf{s}}_n^\dagger \mathbf{A}_u \bar{\mathbf{s}}_n}{\sum_{n=1}^N \bar{\mathbf{s}}_n^\dagger \mathbf{A}_d \bar{\mathbf{s}}_n}, \quad (2)$$

where $\mathbf{A}_u \triangleq \frac{1}{N} \sum_{\theta \in \Theta_u} \mathbf{A}(\theta)$ and $\mathbf{A}_d \triangleq \frac{1}{N} \sum_{\theta \in \Theta_d} \mathbf{A}(\theta)$.

B. System Model in Spectrum Domain

Let $\mathbf{F} \triangleq [\mathbf{f}_0, \dots, \mathbf{f}_{N-1}] \in \mathbb{C}^{N \times N}$ be the DFT matrix, where, $\mathbf{f}_k \triangleq [1, e^{-j\frac{2\pi k}{N}}, \dots, e^{-j\frac{2\pi k(N-1)}{N}}]^T \in \mathbb{C}^N$, $k = \{0, \dots, N-1\}$. Let $\mathcal{U} = \cup_{k=1}^{K_u} (u_{k,1}, u_{k,2})$ be the K_u number of normalized frequency stop-bands, where $0 \leq u_{k,1} < u_{k,2} \leq 1$ and $\cap_{k=1}^{K_u} (u_{k,1}, u_{k,2}) = \emptyset$. Thus, the undesired discrete frequency bands are given by $\mathcal{V} = \cup_{k=1}^{K_u} ([Nu_{k,1}], [Nu_{k,2}])$. In this regards the absolute value of the spectrum an undesired frequency bins can be expressed as $|\mathbf{G}\tilde{\mathbf{s}}_m|$, where, $\mathbf{G} \in \mathbb{C}^{K \times N}$ contains the rows of \mathbf{F} corresponding to the frequencies in \mathcal{V} , and K is the number of undesired frequency bins [37].

C. Problem Formulation

We aim to design a set of constant modulus sequences for MIMO radar such that the transmit beampattern is steered towards desired directions and has nulls at undesired directions simultaneously, with spectrum compatibility and similarity constraints. To this end, we can formulate the following optimization problem,

$$\left\{ \begin{array}{l} \min_{\mathbf{S}} \quad f(\mathbf{S}) = \frac{\sum_{n=1}^N \tilde{\mathbf{s}}_n^\dagger \mathbf{A}_u \tilde{\mathbf{s}}_n}{\sum_{n=1}^N \tilde{\mathbf{s}}_n^\dagger \mathbf{A}_d \tilde{\mathbf{s}}_n} \quad (3a) \\ \text{s.t.} \quad 0.5 \leq \frac{\sum_{n=1}^N \tilde{\mathbf{s}}_n^\dagger \mathbf{A}(\theta_d) \tilde{\mathbf{s}}_n}{\sum_{n=1}^N \tilde{\mathbf{s}}_n^\dagger \mathbf{A}(\theta_0) \tilde{\mathbf{s}}_n} \leq 1, \quad (3b) \\ |s_{m,n}| = 1, \quad (3c) \\ \max \{ |\mathbf{G}\tilde{\mathbf{s}}_m| \} \leq \gamma, \quad m \in \{1, \dots, M\}, \quad (3d) \\ \frac{1}{\sqrt{MN}} \|\mathbf{S} - \mathbf{S}_0\|_F \leq \delta, \quad (3e) \end{array} \right.$$

where (3b) indicates the 3 dB beam-width constraint to guarantee the beampattern response at all desired angles is at least half the maximum power. In (3b), $\theta_d \in \{\theta | \forall \theta \in \Theta_d\}$ and θ_0 , denotes the the angle with maximum power, which is usually chosen to be at the center point of Θ_d . The constraint (3c) indicates the constant modulus property; this is attractive for radar system designers since its allows for the efficient utilization of the limited transmitter power. The constraint (3d) indicates the spectrum masking and guarantees the power of spectrum in undesired frequencies not to be greater than γ . Finally, the constraint (3e) has been imposed on the waveform to control properties of the optimized code (such as orthogonality) similar to the reference waveform \mathbf{S}_0 , for instance this helps controlling ISLR in range domain. If we consider \mathbf{S} and \mathbf{S}_0 to be a constant modulus waveform, the maximum admissible value of the similarity constraint parameter would be $\delta = \sqrt{2}$ ($0 \leq \delta \leq \sqrt{2}$).

In (3), the objective function (3a) is a fractional quadratic function, (3b) and (3d) are non-convex inequality constraints. The (3c) is a non-affine equality constraint while, the inequality constraint (3e) yields a convex set. Therefore, we encounter a non-convex, multi-variable and NP-hard optimization problem [13]. In the following, we propose an iterative method based on SDP to solve the problem efficiently.

III. PROPOSED METHOD

The maximum of $P(\mathbf{S}, \theta)$ is clearly M^2 , and occurs when $\tilde{\mathbf{s}}_n = \mathbf{a}(\theta)$ $n = \{1, \dots, N\}$. Therefore, the denominator of (3a) satisfies, $\sum_{n=1}^N \tilde{\mathbf{s}}_n^\dagger \mathbf{A}_d \tilde{\mathbf{s}}_n \leq K_d M^2$, where K_d is the number of desired angles. Thus, the problem (3) can be equivalently written as [49],

$$\left\{ \begin{array}{l} \min_{\mathbf{S}} \quad \sum_{n=1}^N \tilde{\mathbf{s}}_n^\dagger \mathbf{A}_u \tilde{\mathbf{s}}_n \quad (4a) \\ \text{s.t.} \quad \sum_{n=1}^N \tilde{\mathbf{s}}_n^\dagger \mathbf{A}_d \tilde{\mathbf{s}}_n \leq K_d M^2, \quad (4b) \\ \sum_{n=1}^N \tilde{\mathbf{s}}_n^\dagger \mathbf{A}(\theta_d) \tilde{\mathbf{s}}_n \leq \sum_{n=1}^N \tilde{\mathbf{s}}_n^\dagger \mathbf{A}(\theta_0) \tilde{\mathbf{s}}_n, \quad (4c) \\ \sum_{n=1}^N \tilde{\mathbf{s}}_n^\dagger \mathbf{A}(\theta_0) \tilde{\mathbf{s}}_n \leq 2 \sum_{n=1}^N \tilde{\mathbf{s}}_n^\dagger \mathbf{A}(\theta_d) \tilde{\mathbf{s}}_n, \quad (4d) \\ |s_{m,n}| = 1, \quad (4e) \\ \|\mathbf{G}\tilde{\mathbf{s}}_m\|_{p \rightarrow \infty} \leq \gamma, \quad m \in \{1, \dots, M\}, \quad (4f) \\ \frac{1}{\sqrt{MN}} \|\mathbf{S} - \mathbf{S}_0\|_F \leq \delta \quad (4g) \end{array} \right.$$

In (4), constraints (4c) and (4d) are obtained by expanding (3b) constraint. Besides, we replace the non-convex constraint $\max\{|\mathbf{G}\tilde{\mathbf{s}}_m|\}$ (3d) with $\|\mathbf{G}\tilde{\mathbf{s}}_m\|_{p \rightarrow \infty}$ (4f), which is a convex constraint for each finite p .

Problem (4) is still non-convex with respect to \mathbf{S} due to (4e). To cope with this problem, defining $\mathbf{X}_n \triangleq \tilde{\mathbf{s}}_n \tilde{\mathbf{s}}_n^\dagger$, we recast (4) as follows:

$$\left\{ \begin{array}{l} \min_{\mathbf{S}, \mathbf{X}_n} \quad \sum_{n=1}^N \text{Tr}(\mathbf{A}_u \mathbf{X}_n) \quad (5a) \\ \text{s.t.} \quad \sum_{n=1}^N \text{Tr}(\mathbf{A}_d \mathbf{X}_n) \leq K_d M^2, \quad (5b) \\ \sum_{n=1}^N \text{Tr}(\mathbf{A}(\theta_d) \mathbf{X}_n) \leq \sum_{n=1}^N \text{Tr}(\mathbf{A}(\theta_0) \mathbf{X}_n), \quad (5c) \\ \sum_{n=1}^N \text{Tr}(\mathbf{A}(\theta_0) \mathbf{X}_n) \leq 2 \sum_{n=1}^N \text{Tr}(\mathbf{A}(\theta_d) \mathbf{X}_n), \quad (5d) \\ \text{Diag}(\mathbf{X}_n) = \mathbf{1}_M, \quad (5e) \\ (4f), (4g), \quad (5f) \\ \mathbf{X}_n \succcurlyeq \mathbf{0}, \quad (5g) \\ \mathbf{X}_n = \tilde{\mathbf{s}}_n \tilde{\mathbf{s}}_n^\dagger, \quad (5h) \end{array} \right.$$

It is readily observed that, in (5), the objective function and all the constraints but (5h) are convex in \mathbf{X}_n and \mathbf{S} . In the following, we first present an equivalent reformulation for (5), which paves the way for iteratively solving this non-convex optimization problem .

Theorem 3.1: Defining $\mathbf{Q}_n \triangleq \begin{bmatrix} 1 & \tilde{\mathbf{s}}_n^\dagger \\ \tilde{\mathbf{s}}_n & \mathbf{X}_n \end{bmatrix} \in \mathbb{C}^{(M+1) \times (M+1)}$

and considering slack variables $\mathbf{V}_n \in \mathbb{C}^{(M+1) \times M}$ and $b_n \in \mathbb{R}$. The optimization problem (5) is equivalent to,

$$\left\{ \begin{array}{l} \min_{\mathbf{S}, \mathbf{X}_n, b_n} \sum_{n=1}^N \text{Tr}(\mathbf{A}_u \mathbf{X}_n) + \eta \sum_{n=1}^N b_n \\ \text{s.t.} \quad (5b), (5c), (5d), (5e), (5f), (5g) \\ \mathbf{Q}_n \succcurlyeq \mathbf{0}, \\ b_n \mathbf{I}_M - \mathbf{V}_n^\dagger \mathbf{Q}_n \mathbf{V}_n \succcurlyeq \mathbf{0}, \\ b_n \geq 0, \end{array} \right. \quad (6a) \text{---} (6e)$$

where η is a regularization parameter.

proof 3.2: See Appendix A.

The problem (6) can be solved iteratively by alternating between the parameters. Let, $\mathbf{V}_n^{(i)}$, $\mathbf{Q}_n^{(i)}$, $\mathbf{S}^{(i)}$, $\mathbf{X}_n^{(i)}$ and $b_n^{(i)}$ be the values of \mathbf{V}_n , \mathbf{Q}_n , \mathbf{S} , \mathbf{X}_n and b_n at i^{th} iteration, respectively. Given $\mathbf{V}^{(i-1)}$ and $b_n^{(i-1)}$, the optimization problem with respect to $\mathbf{S}^{(i)}$, $\mathbf{X}_n^{(i)}$ and $b_n^{(i)}$ at the i^{th} iteration becomes,

$$\left\{ \begin{array}{l} \min_{\mathbf{S}^{(i)}, \mathbf{X}_n^{(i)}, b_n^{(i)}} \sum_{n=1}^N \text{Tr}(\mathbf{A}_u \mathbf{X}_n^{(i)}) + \eta \sum_{n=1}^N b_n^{(i)} \\ \text{s.t.} \quad \sum_{n=1}^N \text{Tr}(\mathbf{A}_d \mathbf{X}_n^{(i)}) \leq K_d M^2, \\ \sum_{n=1}^N \text{Tr}(\mathbf{A}(\theta) \mathbf{X}_n^{(i)}) \leq \sum_{n=1}^N \text{Tr}(\mathbf{A}(\theta_0) \mathbf{X}_n^{(i)}), \\ \sum_{n=1}^N \text{Tr}(\mathbf{A}(\theta_0) \mathbf{X}_n^{(i)}) \leq 2 \sum_{n=1}^N \text{Tr}(\mathbf{A}(\theta) \mathbf{X}_n^{(i)}), \\ \text{Diag}(\mathbf{X}_n^{(i)}) = \mathbf{1}_M, \\ \|\mathbf{G} \bar{\mathbf{s}}_m^{(i)}\|_{p \rightarrow \infty} \leq \gamma, \quad m \in \{1, \dots, M\}, \\ \frac{1}{\sqrt{MN}} \|\mathbf{S}^{(i)} - \mathbf{S}_0\|_F \leq \delta, \\ \mathbf{X}_n^{(i)} \succcurlyeq \mathbf{0}, \\ \mathbf{Q}_n^{(i)} \succcurlyeq \mathbf{0}, \\ b_n^{(i)} \mathbf{I}_M - \mathbf{V}_n^{(i-1)\dagger} \mathbf{Q}_n^{(i)} \mathbf{V}_n^{(i-1)} \succcurlyeq \mathbf{0}, \\ b_n^{(i-1)} \geq b_n^{(i)} \geq 0, \end{array} \right. \quad (7a) \text{---} (7k)$$

Once $\mathbf{X}_n^{(i)}$, $\mathbf{S}_n^{(i)}$ and $b_n^{(i)}$ are found by solving (7), $\mathbf{V}_n^{(i)}$ can be obtained by seeking an $(M+1) \times M$ matrix with orthonormal columns such that $b_n^{(i)} \mathbf{I}_M \succcurlyeq \mathbf{V}_n^{(i)\dagger} \mathbf{Q}_n^{(i)} \mathbf{V}_n^{(i)}$. Choosing $\mathbf{V}_n^{(i)}$ to be equal to the matrix composed of the eigenvectors of $\mathbf{Q}_n^{(i)}$ corresponding to its M smallest eigenvalues, and following similar arguments provided after (10) in the Appendix, we have [50, Corollary 4.3.16],

$$\begin{aligned} \mathbf{V}_n^{(i)\dagger} \mathbf{Q}_n^{(i)} \mathbf{V}_n^{(i)} &= \text{Diag}([\rho_1^{(i)}, \rho_2^{(i)}, \dots, \rho_M^{(i)}]^T) \\ &\preccurlyeq \text{Diag}([\nu_1^{(i-1)}, \nu_2^{(i-1)}, \dots, \nu_M^{(i-1)}]^T) \preccurlyeq b_n^{(i)} \mathbf{I}_M, \end{aligned} \quad (8)$$

where, $\rho_1^{(i)} \leq \rho_2^{(i)} \leq \dots \leq \rho_{M+N}^{(i)}$ and $\nu_1^{(i-1)} \leq \nu_2^{(i-1)} \leq \dots \leq \nu_M^{(i-1)}$ denote the eigenvalues of $\mathbf{Q}_n^{(i)}$ and $\mathbf{V}_n^{(i-1)\dagger} \mathbf{Q}_n^{(i)} \mathbf{V}_n^{(i-1)}$, respectively. It follows from (8) that the matrix composed of the eigenvectors of $\mathbf{Q}_n^{(i)}$ corresponding to its M smallest eigenvalues is the appropriate choice for $\mathbf{V}_n^{(i)}$.

Accordingly, at each iteration of the proposed algorithm which we term as WISE, we need to solve a SDP followed

Algorithm 1 :WISE in MIMO Radar Systems

Input: $\gamma, \delta, \mathbf{S}_0, \mathcal{U}$.

Initialization:

- 1) $i := 0$;
- 2) Obtain $\mathbf{Q}_n^{(0)}$ by dropping (7j) and (7k) then solving (7);
- 3) $\mathbf{V}_n^{(0)}$ is the M eigenvectors of $\mathbf{Q}_n^{(0)}$, corresponding to the M lowest eigenvalues;
- 4) $b_n^{(0)}$ is the second largest eigenvalue of $\mathbf{Q}_n^{(0)}$;

Optimization:

- 1) **while**, $\xi \geq e_1$ and $\max \left\{ \|\bar{\mathbf{s}}_n \bar{\mathbf{s}}_n^\dagger - \mathbf{X}_n\|_F \right\} \geq e_2$ **do**
- 2) $i := i + 1$;
- 3) Obtain $\mathbf{S}^{(i)}$, $\mathbf{X}_n^{(i)}$ and $b_n^{(i)}$ by solving (7);
- 4) $\mathbf{V}_n^{(i)}$ is the M eigenvectors of $\mathbf{Q}_n^{(i)}$, by dropping the eigenvector correspond to the largest eigenvalue.
- 5) $b_n^{(i)}$ is the second largest eigenvalue of $\mathbf{Q}_n^{(i)}$.
- 6) **end while**

Output: $\mathbf{S}^* = \mathbf{S}^{(i)}$.

by an Eigenvalue Decomposition (EVD). **Algorithm 1** summarizes the steps of the WISE approach for solving (3). In order to initialize the algorithm, $\mathbf{V}_n^{(0)}$ can be found through the eigenvalue decomposition of $\mathbf{Q}_n^{(0)}$ obtained from solving (7) without considering (7j) and (7k) constraints. Further, we terminate the algorithm when $\bar{\mathbf{s}}_n \bar{\mathbf{s}}_n^\dagger$ converges to \mathbf{X}_n . In this regards, let us assume that,

$$\xi_{n,1} \geq \xi_{n,2} \geq \dots \geq \xi_{n,m} \geq \dots \geq \xi_{n,M} \geq 0,$$

be the eigenvalues of \mathbf{X}_n , we consider $\xi \triangleq \frac{\max\{\xi_{n,2}\}}{\min\{\xi_{n,1}\}} < e_1$ ($e_1 > 0$) as the termination condition. In this case the *second* largest eigenvalue of \mathbf{X}_n is negligible comparing to its largest eigenvalue and can be concluded that the solution is rank one. In addition, we consider $\max \left\{ \|\bar{\mathbf{s}}_n \bar{\mathbf{s}}_n^\dagger - \mathbf{X}_n\|_F \right\} < e_2$ ($e_2 > 0$) as the second termination condition.

We note that the proposed algorithm, which is based on alternating optimization method, is guaranteed that the objective function converges to at least a local minimum of (6) [51].

1) *Convergence:* It readily follows from (7k) that $\lim_{k \rightarrow \infty} \frac{|b_n^{(k)}|}{|b_n^{(k-1)}|} \leq 1$. This implies that $b_n^{(i)}$ converges at least sub-linearly to zero [52]. Hence, there exists some \mathcal{I} such that $b_n^{(i)} \leq \epsilon$ ($\epsilon \rightarrow 0$) for $i \geq \mathcal{I}$. Making use of this fact, we can deduce from (7j) that,

$$\mathbf{V}_n^{(i-1)\dagger} \mathbf{Q}_n^{(i)} \mathbf{V}_n^{(i-1)} \preccurlyeq \epsilon \mathbf{I}_M, \quad \epsilon \rightarrow 0, \quad (9)$$

for $i \geq \mathcal{I}$. Then, it follows from (9) and (8) that $\text{Rank}(\mathbf{Q}_n^{(i)}) \simeq 1$ for $i \geq \mathcal{I}$, thereby $\mathbf{X}_n^{(i)} = \bar{\mathbf{s}}_n^{(i)} \bar{\mathbf{s}}_n^{(i)\dagger}$ for $i \geq \mathcal{I}$. This implies that $\mathbf{X}_n^{(i)}$, for any $i \geq \mathcal{I}$, is a feasible point for the optimization problem (6). On the other hand, considering the fact that $b_n^{(i)} \leq \epsilon$ for $i \geq \mathcal{I}$, we conclude that $\mathbf{X}_n^{(i)}$ for $i \geq \mathcal{I}$ is also a minimizer of the function $\sum_{n=1}^N \text{Tr}(\mathbf{A}_u \mathbf{X}_n)$. These imply that $\mathbf{X}_n^{(i)}$ for $i \geq \mathcal{I}$ is at least a local minimizer of the optimization problem (6). The proves the convergence of the proposed iterative algorithm.

2) *Computational Complexity:* In each iteration, **Algorithm 1** needs to perform the following steps:

- *Solving (7)*: Needs the solution of a SDP, whose computational complexity is $\mathcal{O}(M^{3.5})$ [53].
- *Obtaining $\mathbf{V}_n^{(i)}$ and $b_n^{(i)}$* : Needs the implementation of a Single Value Decomposition (SVD), whose computational complexity is $\mathcal{O}(M^3)$ [54].

Let us assume that \mathcal{I} iterations are required for convergence of the **Algorithm 1**. Therefore, the overall computational complexity of **Algorithm 1** is, $\mathcal{O}(\mathcal{I}(M^{3.5} + M^3))$.

IV. NUMERICAL RESULTS

In this section, numerical results are provided for assessing the performance of the proposed algorithm for beampattern shaping and spectral matching under constant modulus constraint. Towards this end, unless otherwise explicitly stated, we consider the following set-up. For transmit parameters, we consider ULA configuration with $M = 8$ transmitters, with the spacing of $d = \lambda/2$ and each antenna transmits $N = 64$ samples. We consider an uniform sampling of regions $\theta = [-90^\circ, 90^\circ]$ with a grid size of 5° and the desired and undesired angels for beampattern shaping are $\Theta_d = [-55^\circ, -35^\circ]$ ($\theta_0 = -45^\circ$) and $\Theta_u = [-90^\circ, -60^\circ] \cup [-30^\circ, 90^\circ]$, respectively. The normalized frequency stop-band is set at $\mathcal{U} = [0.3, 0.35] \cup [0.4, 0.45] \cup [0.7, 0.8]$ and the absolute spectral mask level is set as $\gamma = 0.01\sqrt{N}$. As to the reference signal for similarity constraint, we consider \mathbf{S}_0 be a set of sequences with a good range-ISLR property, which is obtained by the method in [13]. For the optimization problem we set $\eta = 0.1$ and $p = 1000$ to approximate the (4f) constraint. The convex optimization problems are solved via the CVX toolbox [55] and the stop condition for **Algorithm 1** are set at $e_1 = 10^{-5}$ and $e_2 = 10^{-4}$, respectively.

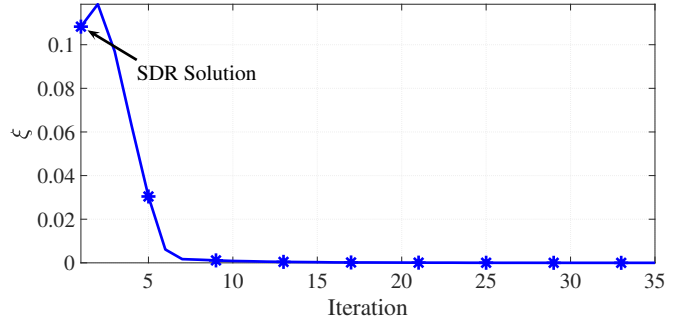
A. Convergence

Fig. 1 depicts the convergence behavior of the proposed method in different aspects. In this figure, we consider the maximum admissible value for similarity parameter, i.e., $\delta = \sqrt{2}$. Fig. 1a shows the convergence of ξ to zero. This indicates that the second largest eigenvalue of \mathbf{X}_n is negligible in comparison with the largest eigenvalue, therefore resulting in a rank one solution for $\bar{\mathbf{s}}_n$. Fig. 1b shows that the solution of \mathbf{X}_n converges to $\bar{\mathbf{s}}_n$, which confirms our claim about obtaining a rank one solution. Fig. 1c shows the convergence of the $\max\{|\mathbf{S}|\}$ to $\min\{|\mathbf{S}|\}$, which indicates the constant modulus solution.

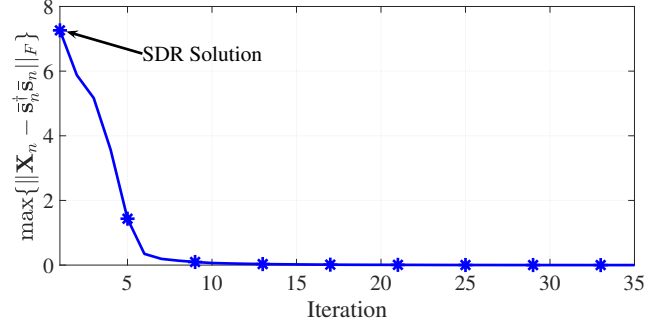
Please note that, the first iteration in Fig. 1a, Fig. 1b and Fig. 1c shows the SDR solution of (7) by dropping (7j) and (7k). As can be seen the SDR method offers neither rank one nor constant modulus solution.

B. Performance

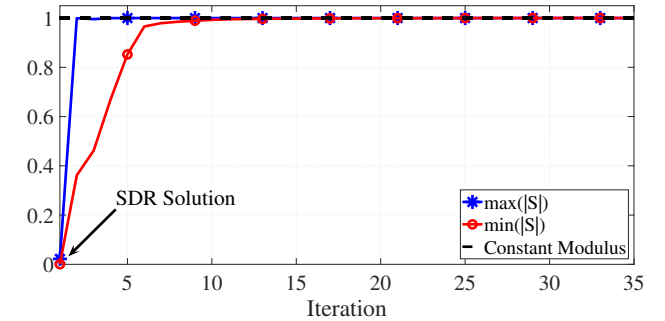
Fig. 2 compares the performance of the proposed method in terms of beampattern shaping and spectral masking with UNIQUE [13] method as a benchmark. Fig. 2a shows the beampattern response of the proposed method and UNIQUE. In this figure, for fair comparison we drop the spectral masking (7f) and 3 dB main beam-width (7c) and (7d) constraints. As



(a) Convergence of eigenvalue



(b) Convergence of arguments



(c) Convergence of constant modulus

Fig. 1: The convergence behavior of proposed method in different aspects, (a) $\xi = \frac{\max\{\xi_{n,2}\}}{\min\{\xi_{n,1}\}}$, (b) $\max\{\|\mathbf{X}_n - \mathbf{s}_n^\dagger \mathbf{s}_n\|_F\}$ and (c) Constant modulus ($M = 8$, $N = 64$, $\delta = \sqrt{2}$, $\Theta_d = [-55^\circ, -35^\circ]$, $\Theta_u = [-90^\circ, -60^\circ] \cup [-30^\circ, 90^\circ]$, $\mathcal{U} = [0.3, 0.4]$ and $\gamma = 0.01\sqrt{N}$)

can be seen, in this case the proposed method offers almost similar performance (in some undesired angles deeper nulls) as compared to UNIQUE method. However, considering the spectral masking (4f) and 3 dB main beam-width (7c) and (7d) constraints, the proposed method is able to steer the beam towards the desired and steer the nulls at undesired angles simultaneously.

The beampattern response of WISE at desired angles region and the spectrum response of the proposed method has better performance comparing to UNIQUE method. Fig. 2b shows the main beam-width response of the proposed method and UNIQUE. Since UNIQUE does not have the 3 dB main beam-width constraint, it method does not have a good main beam-width response. However, the 3 dB main beam-width

constraint incorporated in our framework improves the main beam-width response. Besides, the maximum beampattern response is located at -45° in the proposed method while there is a deviation in UNIQUE method. On the other hand Fig. 2c shows the spectrum response of the proposed method. Observe that the waveform obtained by WISE masks the spectral power in the stop-bands region (\mathcal{U}) below the γ value. However, since UNIQUE method is not spectral compatible, is unable to put notches on the stop-bands.

C. The impact of similarity parameter

In this subsection, we evaluate the impact of choosing the similarity parameter δ on performance of the proposed method. When we consider the maximum admissible value for similarity parameter, i.e., $\delta = \sqrt{2}$, we do not include similarity constraint and by decreasing δ we have the degree of freedom to enforce properties similar to the reference waveform on the optimal waveform. As mentioned in section IV, we consider \mathbf{S}_0 be a set of sequences with a good range-ISLR property as the reference signal for similarity constraint, which is obtained by UNIQUE method [13]. Therefore, by decreasing the δ we obtain a waveform with good orthogonality, which leads to omni directional beampattern.

Fig. 3 shows the beampattern response of the proposed method with different values of δ . As can be seen, with $\delta = \sqrt{2}$, yields an optimized beampattern and by decreasing δ the beampattern gradually tends to be omnidirectional.

On the other hand, Fig. 4a, Fig. 4c and Fig. 4e show the correlation level of the proposed method with different values of δ . Observe that with $\delta = \sqrt{2}$ we obtain fully correlated waveform and by decreasing δ the waveform gradually becomes uncorrelated. Therefore, having simultaneous beampattern shaping and orthogonality are contradictory, and the choice of δ effects a trade-off between the two and enhance the performance of radar system [13]. Besides Fig. 4b, Fig. 4d and Fig. 4f show the spectrum of the proposed method with different values of δ . As can be seen in all cases the proposed method is able to perform the spectral masking.

V. CONCLUSION

In this paper we discuss about the problem of beampattern shaping with practical constraints in MIMO radar systems namely, spectral masking, 3 dB beam-width, constant modulus and similarity constraints. Solving this problem, not considered hitherto, enables us to control the performance of MIMO radar in three domains namely, spatial, spectral and orthogonality (by similarity constraints). Accordingly, we consider a waveform design approach for beampattern shaping optimization problem under. The aforementioned problem leads to a non-convex and NP-hard optimization problem. In order to solve the problem, first by introducing a slack variable we convert the optimization problem to a linear problem with a rank one constraint. Then to tackle the the we proposed an iterative method to obtain the rank one solution. Numerical results shows that the proposed method is able to manage the resources efficiently to obtain the best performance.

APPENDIX A

It is readily confirmed that the constraint $\mathbf{X}_n = \bar{\mathbf{s}}_n \bar{\mathbf{s}}_n^\dagger$ is equivalent to $\text{Rank}(\mathbf{X}_n - \bar{\mathbf{s}}_n \bar{\mathbf{s}}_n^\dagger) = 0$. Further, it can be equivalently expressed as $1 + \text{Rank}(\mathbf{X}_n - \bar{\mathbf{s}}_n \bar{\mathbf{s}}_n^\dagger) = 1$. Since 1 is positive definite, it follows from the Guttman rank additivity formula [56] that $1 + \text{Rank}(\mathbf{X}_n - \bar{\mathbf{s}}_n \bar{\mathbf{s}}_n^\dagger) = \text{Rank}(\mathbf{Q}_n)$. Moreover, it follows from $\mathbf{X}_n = \bar{\mathbf{s}}_n \bar{\mathbf{s}}_n^\dagger$ and $1 \succ 0$ that \mathbf{Q}_n has to be positive semi-definite. These imply that the constraint $\mathbf{X}_n = \bar{\mathbf{s}}_n \bar{\mathbf{s}}_n^\dagger$ in (5) can be replaced with a rank and semi-definite constraints on matrix \mathbf{Q}_n . Hence, the optimization problem (5) can be recast as follows,

$$\left\{ \begin{array}{l} \min_{\mathbf{S}, \mathbf{X}_n} \sum_{n=1}^N \text{Tr}(\mathbf{A}_u \mathbf{X}_n) \quad (10a) \\ s.t. \quad (5b), (5c), (5d), (5e), (5f), (5g) \quad (10b) \\ \mathbf{Q}_n \succcurlyeq \mathbf{0}, \quad (10c) \\ \text{Rank}(\mathbf{Q}_n) = 1, \quad (10d) \end{array} \right.$$

Now, we show that the optimization problem (6) is equivalent to (10). Let $\rho_{n,1} \leq \rho_{n,2} \leq \dots \leq \rho_{n,M+1}$ and $\nu_{n,1} \leq \nu_{n,2} \leq \dots \leq \nu_{n,M}$ denote the eigenvalues of \mathbf{Q}_n and $\mathbf{V}_n^\dagger \mathbf{Q}_n \mathbf{V}_n$, respectively. From the constraint $b_n \mathbf{I}_M - \mathbf{V}_n^\dagger \mathbf{Q}_n \mathbf{V}_n \succcurlyeq \mathbf{0}$, we have $\nu_{n,i} \leq b_n, i = 1, 2, \dots, M$ for any \mathbf{V}_n and \mathbf{Q}_n in the feasible set of (6). Additionally, it follows from [50, Corollary 4.3.16] that $0 \leq \rho_{n,i} \leq \nu_{n,i}, i = 1, 2, \dots, M$ for any \mathbf{V}_n and \mathbf{Q}_n in the feasible set of (6). Hence, we observe that,

$$\begin{aligned} \mathbf{0} &\preccurlyeq \text{Diag}([\rho_{n,1}, \dots, \rho_{n,M}]^T) \\ &\preccurlyeq \text{Diag}([\nu_{n,1}, \dots, \nu_{n,M}]^T) \preccurlyeq b_n \mathbf{I}_M, \end{aligned} \quad (11)$$

for any \mathbf{V}_n and \mathbf{Q}_n in the feasible set of (6). It is easily observed from (6) and (11) that, by properly selecting η , the optimum value of \mathbf{V}_n will be equal to the eigenvectors of \mathbf{Q}_n corresponding to its M smallest eigenvalues and the optimum values of $b_n, \rho_{n,1}, \dots, \rho_{n,M}, \nu_{n,1}, \dots, \nu_{n,M}$ will be equal to zero. This implies that the optimum value of \mathbf{Q}_n in (11) possesses one nonzero and M zero eigenvalues. This completes the proof.

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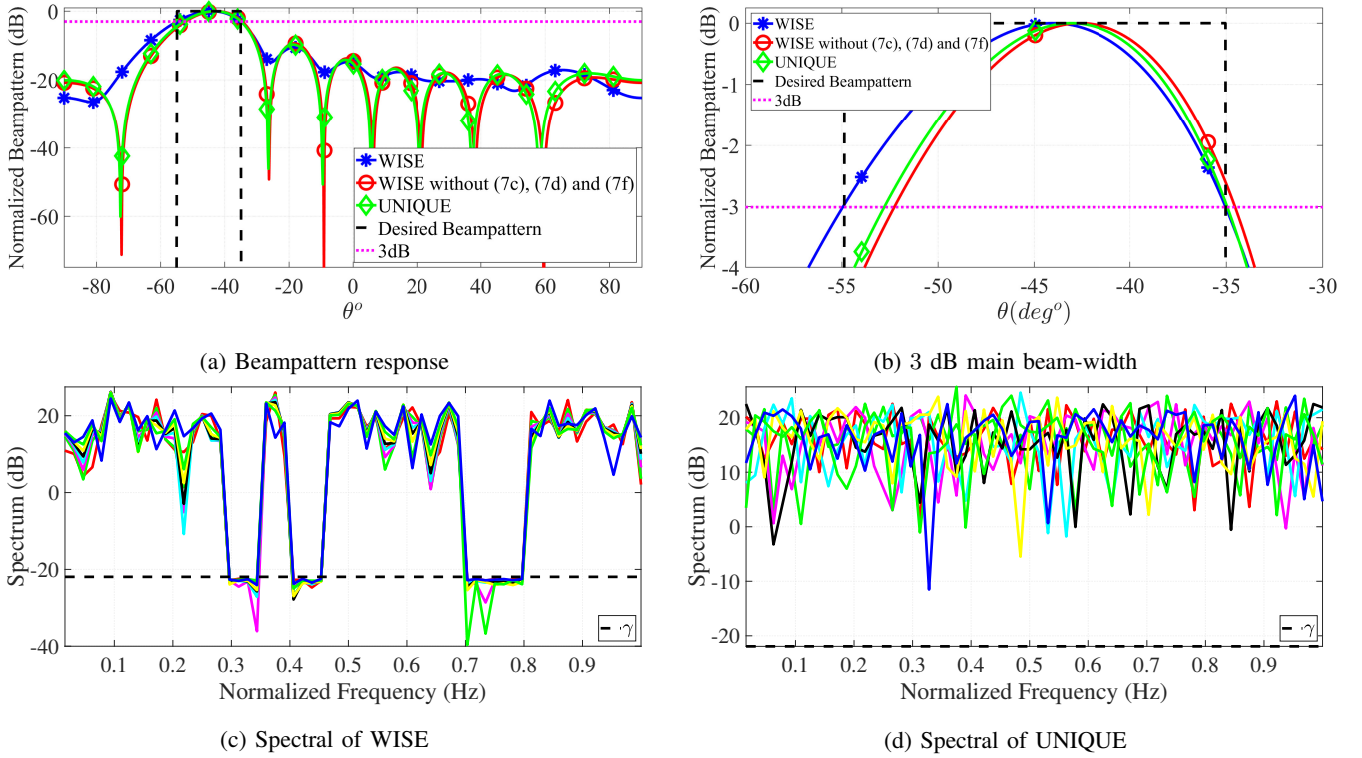


Fig. 2: Comparing the performance of WISE and UNIQUE methods in terms of (a) 3 dB main beam-width constraint, (b) spectral masking of WISE and (c) spectral masking of UNIQUE ($M = 4$, $N = 64$, $\delta = \sqrt{2}$, $\Theta_d = [-55^\circ, -35^\circ]$, $\Theta_u = [-90^\circ, -60^\circ] \cup [-30^\circ, 90^\circ]$, $\mathcal{U} = [0.3, 0.35] \cup [0.4, 0.55] \cup [0.7, 0.85]$ and $\gamma = 0.01\sqrt{N}$).

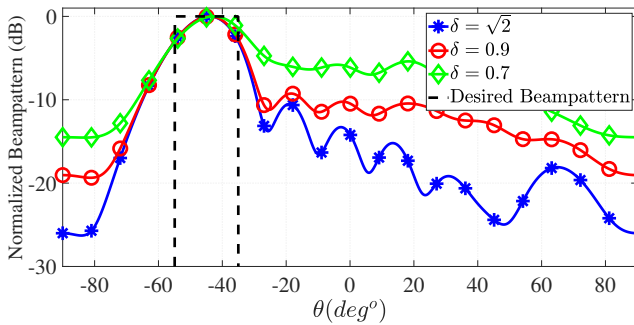


Fig. 3: The impact of choosing δ in the proposed method on Beampattern response ($M = 8$, $N = 64$, $\Theta_d = [-55^\circ, -35^\circ]$ and $\Theta_u = [-90^\circ, -60^\circ] \cup [-30^\circ, 90^\circ]$, $\mathcal{U} = [0.3, 0.35] \cup [0.5, 0.55]$ and $\gamma = 0.01\sqrt{N}$).

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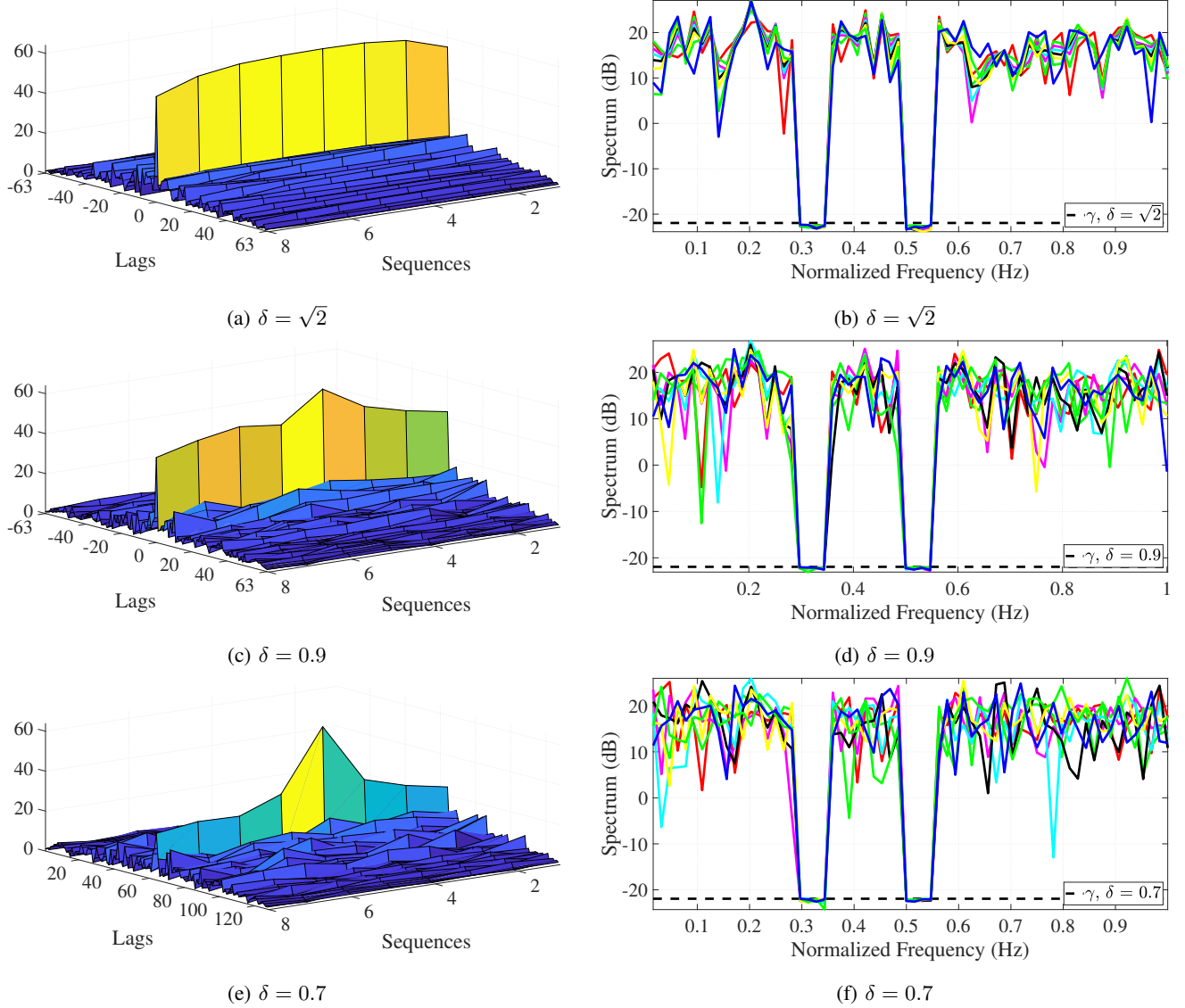


Fig. 4: The impact of choosing δ on correlation level and spectral masking ($M = 8$, $N = 64$, $\Theta_d = [-55^\circ, -35^\circ]$, $\Theta_u = [-90^\circ, -60^\circ] \cup [-30^\circ, 90^\circ]$, $\mathcal{U} = [0.3, 0.35] \cup [0.5, 0.55]$ and $\gamma = 0.01\sqrt{N}$).

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