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Robust Finite-Time H_∞ Control for the Uncertain Singular System

YANG DING,HANG ZHANG, TIANBO WANG
School of Mathematics, Physics and Statistics, Shanghai University of Engineering Science, Shanghai 201620, China

Corresponding author: YANG DING. (e-mail: dingyangsues@163.com).

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ABSTRACT This paper addresses the robust finite-time H_∞ control problem for the uncertain singular system by using the stability theory of dynamical systems. Firstly, a lemma is provided to show that the singular system is finite-time stability by using the state space decomposition approach. Similar with the proof method of this Lemma, the singular system is divided into a differential system and an algebra one. Then, some conditions are derived to ensure the singular system being finite-time H_∞ stability based on the obtained lemma, and the state feedback control law is designed. These conditions are provided in the form of the linear matrix inequalities and can be easily solved. Finally, a numerical example is given to illustrate the effectiveness of the obtained results.

INDEX TERMS Finite-time stability, H_∞ control, singular system, uncertainty.

I. INTRODUCTION

Singular systems can usually describe the behavior of numerous physical systems such as biological systems, mechanical engineering systems, economical systems and so on [1]-[4]. Because singular systems are composed of algebraic equations and differential equations, the dynamical character of singular systems is more complex than some common dynamical systems. Thus the stability problems related to singular systems are more difficult to be deal with and become an important topic for the researchers in the past decades. For example, the authors studied the stability of continuous singular switched systems by using the state-space decomposition approach and provided some sufficient conditions to guarantee the stability of these systems in [5]. By using the sliding mode control, the authors investigated the admissibility and state estimation of singular stochastic Markovian jump systems with uncertainties in [6] and [7], respectively. By using the fuzzy control, the authors studied the stability of singular system and proposed a kind of fuzzy controller in [8]. For the discrete singular systems, the authors investigated the issue of robust observer based on H_∞ control for uncertain discrete singular systems with time varying delays via sliding mode control in [9].

Among all kinds of the control issues of the dynamical systems with the external disturbances, H_∞ control is a powerful tool [10]- [14], which can describe the numerical relationship between the measured output and the external disturbances. For example, by constructing a delay-product-type augmented Lyapunov-Krasovskii functional, the authors investigated the robust H_∞ control for a class of uncertain nonlinear time-delay systems in [10].The authors investigated H_∞ control for a time-varying delay system by using the T-S fuzzy control and provided a fuzzy controller in [12].

Uncertain systems are usually used to describe some practical systems which part of parameters or structures are unknown. For these kinds of systems, there exist lots of related research methods and results [15]- [19]. For example, [16] investigated the robust stability of uncertain linear neutral systems with discrete and distributed delays and proposed some stability criteria. By using the free weighting matrices and a sliding mode control, the authors studied the stability of the uncertain discrete singular systems with external disturbances and time-varying delays in [18]. For its application, the authors investigated the problem of designing a non-fragile state estimator for a class of uncertain discrete-time neural networks with time-delays by using the Lyapunov stability theory and the explicit expression of the desired estimators in [19].

Recently, it can be found some papers related to the finite-time stability of the dynamical systems in [20]- [26]. Finite-time stability means that the quadratic function related with the system states is bounded in finite time interval when the initial values lie in a given bound domain. In fact, finite-time stability is much more accord with the practical systems. For example, by constructing appropriate Lyapunov-like functionals and using the average dwell time technique, the authors investigated the finite-time synchronization problem for a class of uncertain coupled switched neural networks under asynchronous switching condition in [24]. The authors investigates the problem of decentralized adaptive fuzzy finite-time control for switched nonlinear large-scale systems with actuator and sensor faults by using constructed observer and fuzzy logic method in [25]. The finite-time non-fragile passivity control for neural networks with time-varying delay is studied by constructing a new Lyapunov-Krasovskii function and utilizing Wirtinger-type inequality in [26].

Motivated by the above discussion, we will focus on investigating the robust finite-time H_∞ control for a class of continuous uncertain singular systems in the paper. As far as we know, there are few relevant results. The main contributions of this paper are summarized as follows: (1) Consider the finite-time stability and H_∞ control for an uncertain singular system by using the state space decomposition approach; (2) This paper provides some stable conditions depending on the system parameters and the design method of state feedback controller, and these results reveals directly the connection of the system parameters and its performance; (3) The results are given in the form of linear matrix inequalities and are easy to solve.

The rest of this paper is organized as follows. In section 2, the dynamical system and some preliminaries are given. In section 3, the finite-time H_∞ stability criteria are derived from the state feedback controller. In section 4, a numerical example is provided to illustrate the effectiveness of the proposed method. In the last section, conclusions are presented.

Notation. R^n and $R^{n\times m}$, respectively, denote the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, $||a|| = \sqrt{a_1^2+a_2^2+...+a_n^2}$ represents the norm of vector $a=(a_1,a_2,...,a_n)^T\in R^n$. I_n is an $n\times n$ identical matrix, $\lambda_{max}(A)$ and $\lambda_{min}(A)$ stand for the maximum and the minimum eigenvalues of matrix A, respectively. $*$ denotes the symmetric parts on the matrix principal diagonal. In this paper, all matrices are assumed to have appropriate dimensions.

II. PROBLEM STATEMENT AND PRELIMINARIES

In this paper, we consider the following singular system

$$\begin{cases} E\dot{x} = (A+\triangle A)x(t) + (B+\triangle B)u(t) + D_1w(t), \\ \tilde{z}(t) = (C+\triangle C)x(t) + D_2w(t), \end{cases} \tag{1}$$

where $x(t)=(x_1(t),x_2(t),...,x_n(t))^T\in R^n$ is the system state, $\tilde{z}(t)=(\tilde{z}_1(t),\tilde{z}_2(t),...,\tilde{z}_q(t))^T\in R^q$ is the controlled output, $u(t)\in R^m$ is the control input. $A\in R^{n\times n}, B\in R^{n\times m}, C\in R^{q\times n}, D_1\in R^{n\times p}, D_2\in R^{q\times p}$ are some known matrices. $\Delta A, \Delta B$ and ΔC are some unknown matrices and express the uncertainty of system parameters and satisfy

$$[\Delta A, \Delta B] = G_1H(t)[F_1,F_2], \Delta C = G_2H(t)F_3,$$

where $H(t)$ is an unknown time-varying matrix and satisfies $H^T(t)H(t)\le I$ for $t\ge 0$. G_1,G_2,F_1, F_2,F_3 are some known matrices with appropriate dimensions. The disturbance $w(t)\in R^p$ satisfies $||w(t)||\le\sqrt{d}$ for a given scalar $d\ge 0$. E is an n-order singular matrix and satisfies $rank(E)=r<n$.

Definition 1. [27] For the singular system

$$E\dot{x}(t) = Ax(t). \tag{2}$$

(i) Singular system (2) is called regular if $det(sE-A)$ is not identical zero.

(ii) Singular system (2) is called impulse-free if $deg(det(sE-A)) = rank(E)$.

Definition 2. For given scalars $c_2>c_1>0$ and $T>0$, system (1) with $u(t)=0$ is said to be the finite-time stability on (c_1,c_2,R,T) if

$$x^T(0)Rx(0)\le c_1 \Rightarrow x^T(t)Rx(t)\le c_2, \forall t\in[0,T],$$

where $R\in R^{n\times n}$ is a positive define symmetric matrix.

Definition 3. [28] Define the H_∞ norm of system (1) as

$$||T_{zw}||_\infty = sup\{\frac{||\tilde{z}(t)||}{||w(t)||}, w(t)\in L_2[0,+\infty), ||w(t)||\ne 0\}.$$

Under zero initial condition, if $||T_{zw}||_\infty<\gamma$, then system (1) is said to possess H_∞ performance with attenuation index γ.

In this paper, we intend to design the following state feedback controller

$$u(t) = Kx(t) \tag{3}$$

such that system (1) is finite-time stable and possesses H_∞ performance with attenuation index γ, where $K\in R^{m\times n}$ is the control gain matrix to be determined in the later.

Substituting (3) into system (1), one gets the closed-loop system

$$\begin{cases} E\dot{x} = \bar{A}x(t) + D_1w(t), \\ \tilde{z}(t) = (C+\triangle C)x(t) + D_2w(t), \end{cases} \tag{4}$$

where $\bar{A} = A+BK+\triangle A+\triangle BK$.

In what follows, we give some necessary lemmas to be used in the later.

Lemma 1. For any a, $b\in R^n$ and $\varepsilon_1>0$, there is

$$2a^Tb\le\varepsilon_1a^Ta+\varepsilon_1^{-1}b^Tb.$$

Lemma 2. For given scalars $c_2>c_1>0, T>0$ and a positive define symmetric matrix R, singular system

$$E\dot{x}(t) = Ax(t) + Dw(t), \tag{5}$$

is finite-time stability if there exist a positive define symmetric matrix $P\in R^{n\times n}$, nonsingular matrices $M\in R^{n\times n}$ and $N\in R^{n\times n}$, scalars $\delta>0$, $\mu>0$ and $\alpha>0$ such that

$$PE = E^TP^T\ge 0, \tag{6}$$

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$$PA + A^T P^T + \alpha PE + \delta^{-1} PDD^T P^T < 0, \quad (7)$$

$$(1 + 2||A_{22}^{-1}A_{21}||^2) \cdot \frac{\lambda_{max}(R_{11})}{\lambda_{min}(P_{11})\cdot\lambda_{min}(R_{11})} [\frac{\lambda_{max}(P_{11})}{\lambda_{min}(N^T RN)} c_1 + \frac{\delta d}{\alpha}(e^{\alpha T} - 1)] + 2d||A_{22}^{-1}D_{21}||^2 \le \mu^{-1} c_2, \quad (8)$$

and

$$\begin{bmatrix} -\mu I & N^T R \\ * & -R \end{bmatrix} < 0 \quad (9)$$

hold, where P_{11}, R_{11}, A_{21}, A_{22}, D_{21} will be defined in the later.

Proof. Because of $Rank(E) = r < n$, there exist two non-singular matrices M and N such that $MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$. Firstly, we will show that system (5) is regular and impulse-free if (6) and (7) hold.

Writing $MAN = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ and $MD = \begin{bmatrix} D_{11} \\ D_{21} \end{bmatrix}$, $z(t) = N^{-1}x(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix}$, $z_1(t) \in R^r$ and $z_2(t) \in R^{n-r}$, then system (5) is

$$\begin{cases} \dot{z}_1(t) = A_{11}z_1(t) + A_{12}z_2(t) + D_{11}w(t), \\ 0 = A_{21}z_1(t) + A_{22}z_2(t) + D_{21}w(t). \end{cases} \quad (10)$$

From (6), and letting $N^T PM^{-1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$, one has

$$N^T PEN = N^T PM^{-1} \cdot MEN = \begin{bmatrix} P_{11} & 0 \\ P_{21} & 0 \end{bmatrix}$$

and

$$N^T E^T P^T N = N^T E^T M^T \cdot M^{-T} P^T N = \begin{bmatrix} P_{11}^T & P_{21}^T \\ 0 & 0 \end{bmatrix},$$

which shows $P_{11} = P_{11}^T$ and $P_{21} = 0$. Moreover,

$$N^T E^T REN = N^T E^T M^T \cdot M^{-T} RM^{-1} \cdot MEN = \begin{bmatrix} R_{11} & 0 \\ 0 & 0 \end{bmatrix},$$

where $M^{-T}RM^{-1} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}$. Since $M^{-T}RM^{-1}$ is a positive define symmetric matrix, which gives $R_{11} > 0$. For any $a = (a_1, a_2, ..., a_n)^T \in R^n$, there are

$$\begin{aligned} \lambda_{min}(R_{11}) \sum_{i=1}^{r} a_i^2 &\le a^T N^T E^T RENa \\ &= a^T \begin{bmatrix} R_{11} & 0 \\ 0 & 0 \end{bmatrix} a \\ &\le \lambda_{max}(R_{11}) a^T a, \end{aligned} \quad (11)$$

$$\begin{aligned} \lambda_{min}(P_{11}) \sum_{i=1}^{r} a_i^2 &\le a^T N^T PENa \\ &= a^T \begin{bmatrix} P_{11} & 0 \\ 0 & 0 \end{bmatrix} a \\ &\le \lambda_{max}(P_{11}) \sum_{i=1}^{r} a_i^2 \\ &\le \lambda_{max}(P_{11}) a^T a \end{aligned} \quad (12)$$

and

$$\lambda_{min}(N^T RN) a^T a \le a^T N^T RNa \le \lambda_{max}(N^T RN) a^T a.$$

From the above inequalities, we get

$$PE \le \frac{\lambda_{max}(P_{11})}{\lambda_{min}(N^T RN)} R.$$

Next, we will show the finite-time stability of system (5) under the conditions (6) $\sim$ (9). Choosing a Lyapunov function as

$$V(t) = x^T(t) PEx(t),$$

then its derivative along with the trajectory of (5) is

$$\begin{aligned} \dot{V}(t) + \alpha V(t) &= 2x^T(t) PE\dot{x}(t) + \alpha x^T(t) PEx(t) \\ &= 2x^T(t) P[Ax(t) + Dw(t)] + \alpha x^T(t) PEx(t) \\ &= x^T(t)[PA + A^T P^T + \alpha PE]x(t) \\ &+ 2x^T(t) PDw(t) \\ &\le x^T(t)[PA + A^T P^T + \alpha PE \\ &+ \delta^{-1} PDD^T P^T]x(t) + \delta w^T(t) w(t), \end{aligned} \quad (13)$$

where we use the fact that there exists $\delta > 0$ such that

$$2x^T(t) PDw(t) \le \delta^{-1} x^T(t) PDD^T P^T x(t) + \delta w^T(t) w(t).$$

By (7) and integrating (13) in interval $[0, t] \subseteq [0, T]$, one has

$$\begin{aligned} V(t) &\le e^{-\alpha t}[V(0) + \delta \int_0^T e^{\alpha\tau} w^T(\tau) w(\tau) d\tau] \\ &\le \frac{\lambda_{max}(P_{11})}{\lambda_{min}(N^T RN)} x^T(0) Rx(0) + \frac{\delta d}{\alpha}(e^{\alpha T} - 1) \\ &\le \frac{\lambda_{max}(P_{11})}{\lambda_{min}(N^T RN)} c_1 + \frac{\delta d}{\alpha}(e^{\alpha T} - 1). \end{aligned}$$

On the other hand, there is

$$\begin{aligned} V(t) &\ge \frac{\lambda_{min}(P_{11})}{\lambda_{max}(R_{11})} x^T(t) E^T REx(t) \\ &= \frac{\lambda_{min}(P_{11})}{\lambda_{max}(R_{11})} x^T(t) N^{-T} \cdot N^T E^T REN \cdot N^{-1} x(t) \\ &\ge \frac{\lambda_{min}(P_{11})}{\lambda_{max}(R_{11})} z_1^T(t) R_{11} z_1(t). \end{aligned}$$

Thus

$$||z_1(t)||^2 \le \frac{\lambda_{max}(R_{11})}{\lambda_{min}(P_{11})\cdot\lambda_{min}(R_{11})} [\frac{\lambda_{max}(P_{11})}{\lambda_{min}(N^T RN)} c_1 + \frac{\delta d}{\alpha}(e^{\alpha T} - 1)]. \quad (14)$$

Furthermore, by (7) there is

$$PA + A^T P^T < 0,$$

so

$$N^T PM^{-1} \cdot MAN + N^T A^T M^T \cdot M^{-T} P^T N < 0,$$

that is

$$\begin{bmatrix} P_{11}A_{11} + A_{11}^T P_{11}^T & P_{11}A_{12} + P_{12}A_{22} \\ +P_{12}A_{21} + A_{21}^T P_{12}^T & +A_{21}^T P_{22}^T \\ * & P_{22}A_{22} + A_{22}^T P_{22}^T \end{bmatrix} < 0,$$

thus $P_{22}A_{22} + A_{22}^T P_{22}^T < 0$. Because of $P_{22} > 0$, A_{22} is an invertible matrix. From Definition 1, system (5) is singular and impulse-free.

According to (10), there is

$$\begin{aligned} ||z_2(t)|| &= ||-A_{22}^{-1}A_{21}z_1(t) - A_{22}^{-1}D_{21}w(t)|| \\ &\le ||A_{22}^{-1}A_{21}|| \cdot ||z_1(t)|| + ||A_{22}^{-1}D_{21}|| \cdot ||w(t)||. \end{aligned} \quad (15)$$

By (8), one gets

$$\begin{aligned} x^T(t)Rx(t) &= z^T(t)N^TRNz(t) \\ &\leq \lambda_{max}(N^TRN)(||z_1(t)||^2+||z_2(t)||^2) \\ &\leq \lambda_{max}(N^TRN)(||z_1(t)||^2+2||A_{22}^{-1}A_{21}||^2 \\ &\quad \cdot||z_1(t)||^2+2||A_{22}^{-1}D_{21}||^2\cdot||w(t)||^2) \\ &\leq \lambda_{max}(N^TRN)\{(1+2||A_{22}^{-1}A_{21}||^2) \\ &\quad \cdot\frac{\lambda_{max}(R_{11})}{\lambda_{min}(P_{11})\cdot\lambda_{min}(R_{11})} \\ &\quad \cdot[\frac{\lambda_{max}(P_{11})}{\lambda_{min}(N^TRN)}c_1+\frac{\delta d}{\alpha}(e^{\alpha T}-1)] \\ &\quad +2d||A_{22}^{-1}D_{21}||^2\} \\ &\leq c_2, \end{aligned} \tag{16}$$

which shows that system (5) is finite-time stability. The proof of Lemma 2 is completed.

Remark 1. The finite-time stability for the singular dynamical systems is an interesting topic, there exist some existing papers [29], [30]. Viewed from the used methods, there mainly include two kinds of methods. One is the state space decomposition approach, the other is to construct some special Lyapunov functions as [29]. In fact, compared with some dynamical systems, singular systems are composed of algebraic equations and differential equations. Thus, the difficult is how to decompose them into two subsystems and finds out the connection of the states of the two subsystems by using suitable method.

III. MAIN RESULTS

Theorem 1. For given scalars $c_2 > c_1 > 0, T > 0$ and a positive define symmetric matrix R, if there exist a positive define symmetric matrix $P \in R^{n\times n}$, nonsingular matrices $M \in R^{n\times n}$ and $N \in R^{n\times n}$, scalars $\delta > 0$, $\mu > 0$ and $\alpha > 0$ such that

$$PE = E^TP^T \geq 0, \tag{17}$$

$$P\bar{A}+\bar{A}^TP^T+\alpha PE+\delta^{-1}PD_1D_1^TP^T<0, \tag{18}$$

$$\begin{aligned} &(1+2||\bar{A}_{22}^{-1}\bar{A}_{21}||^2)\cdot\frac{\lambda_{max}(R_{11})}{\lambda_{min}(P_{11})\cdot\lambda_{min}(R_{11})} \\ &\cdot[\frac{\lambda_{max}(P_{11})}{\lambda_{min}(N^TRN)}c_1+\frac{\delta d}{\alpha}(e^{\alpha T}-1)]+2d||\bar{A}_{22}^{-1}D_{1,21}||^2 \\ &\leq \mu^{-1}c_2, \end{aligned} \tag{19}$$

$$\begin{bmatrix} -\mu I & N^TR \\ * & -R \end{bmatrix} < 0, \tag{20}$$

$$\Phi_1 = \begin{bmatrix} \Theta_{11} & PD_1+(C+\triangle C)^TD_2 \\ * & D_2^TD_2-\gamma^2I \end{bmatrix} < 0, \tag{21}$$

where $\Theta_{11} = P\bar{A}+\bar{A}^TP^T+(C+\triangle C)^T(C+\triangle C)$, then system (4) is finite-time stability on (c_1, c_2, R, T) and $||T_{zw}||_\infty \leq \gamma$. Where $M\bar{A}N = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}$ and $MD_1 = \begin{bmatrix} D_{1,11} \\ D_{1,21} \end{bmatrix}$.

Proof. From lemma 2, we know that if inequalities (17) $\sim$ (20) hold, system (4) is the finite-time stability.

On the other hand, since

$$\begin{aligned} &\dot{V}(t)+\tilde{z}^T(t)\tilde{z}(t)-\gamma^2w^T(t)w(t) \\ &=2x^T(t)P[\bar{A}x(t)+D_1w(t)]-\gamma^2w^T(t)w(t) \\ &+[(C+\triangle C)x(t)+D_2w(t)]^T[(C+\triangle C)x(t)+D_2w(t)] \\ &=\eta^T(t)\Phi_1\eta(t), \end{aligned}$$

where $V(t)$ is the same with Lemma 2 and $\eta(t) = [x^T(t), w^T(t)]^T$. Thus, from (17) and (21), one gets

$$||\tilde{z}(t)||^2 \leq \gamma^2||w(t)||^2,$$

which shows that system (4) possesses H_∞ performance with attenuation index γ.

The inequalities in Theorem 1 are nonlinear on the unknown variables, in order to solve them easily, we can transform them into the linear matrix inequalities. Thus we provide the following results.

Theorem 2. For given scalars $c_2 > c_1 > 0, T > 0$ and a positive define symmetric matrix R, if there exist a positive define symmetric matrix $X \in R^{n\times n}$, nonsingular matrices $M \in R^{n\times n}$ and $N \in R^{n\times n}$, matrix $W \in R^{m\times n}$, positive scalars δ, μ, ε_2, ε_3, ε_4, such that (23) $\sim$ (27) hold, where $\Phi_{2,11} = AX+BW+XA^T+W^TB^T+\varepsilon_3G_1G_1^T$. Then system (4) is the finite-time stability on (c_1, c_2, R, T) and $||T_{zw}||_\infty \leq \gamma$.

Proof. Since

$$\begin{aligned} &\begin{bmatrix} P(A+BK)+(A+BK)^TP^T+\alpha PE & PD_1 \\ * & -\delta I \end{bmatrix} \\ &+\begin{bmatrix} PG_1 \\ 0 \end{bmatrix} H(t) \begin{bmatrix} F_1+F_2K & 0 \end{bmatrix} \\ &+\begin{bmatrix} (F_1+F_2K)^T \\ 0 \end{bmatrix} H^T(t) \begin{bmatrix} (PG_1)^T & 0 \end{bmatrix} \\ &\leq \begin{bmatrix} P(A+BK)+(A+BK)^TP^T+\alpha PE & PD_1 \\ * & -\delta I \end{bmatrix} \\ &+\varepsilon_2 \begin{bmatrix} PG_1 \\ 0 \end{bmatrix} \begin{bmatrix} (PG_1)^T & 0 \end{bmatrix} \\ &+\varepsilon_2^{-1} \begin{bmatrix} (F_1+F_2K)^T \\ 0 \end{bmatrix} \begin{bmatrix} F_1+F_2K & 0 \end{bmatrix}, \end{aligned}$$

and using the Schur complement lemma, we know that if (28) holds, then inequality (18) holds. Pre- and postmultiplying with $diag\{P^{-1}, I, I\}$ and its transpose on the both sides of (28), respectively, using the Schur complement lemma again and letting $P^{-1} = X, KP^{-1} = W$, one gets (24).

On the other hand, according to the Schur complement lemma, inequality (21) is equivalent with

$$\begin{bmatrix} P\bar{A}+\bar{A}^TP^T & PD_1+(C+\Delta C)^TD_2 & (C+\Delta C)^T \\ * & D_2^TD_2-\gamma^2I & 0 \\ * & * & -I \end{bmatrix} < 0. \tag{22}$$

Substituting $\bar{A}$ into (22) and using Lemma 1, one gets

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$$EX^T = XE^T \geq 0, \tag{23}$$

$$\begin{bmatrix} AX + BW + X^TA^T + W^TB^T + \alpha EX + \varepsilon_2 G_1 G_1^T & D_1 & XF_1^T + W^TF_2^T \\ * & -\delta I & 0 \\ * & * & -\varepsilon_2 I \end{bmatrix} < 0, \tag{24}$$

$$\begin{aligned} &(1 + 2||\bar{A}_{22}^{-1}\bar{A}_{21}||^2) \cdot \frac{\lambda_{max}(R_{11})}{\lambda_{min}(P_{11})\cdot\lambda_{min}(R_{11})}[\frac{\lambda_{max}(P_{11})}{\lambda_{min}(N^TRN)}c_1 + \frac{\delta d}{\alpha}(e^{\alpha T} - 1)] \\ &+2d||\bar{A}_{22}^{-1}D_{1,21}||^2 \leq \mu^{-1}c_2, \end{aligned} \tag{25}$$

$$\begin{bmatrix} -\mu I & N^TR \\ * & -R \end{bmatrix} < 0, \tag{26}$$

$$\Phi_2 = \begin{bmatrix} \Phi_{2,11} & D_1 + XC^TD_2 & XC^T & XF_1^T + W^TF_2^T & XF_3^T \\ * & D_2^TD_2 - \gamma^2 I + \varepsilon_4 D_2^TG_2G_2^TD_2 & \varepsilon_4 D_2^TG_2G_2^T & 0 & 0 \\ * & * & -I + \varepsilon_4 G_2G_2^T & 0 & 0 \\ * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & -\varepsilon_4 I \end{bmatrix} < 0, \tag{27}$$

$$\begin{aligned} &\begin{bmatrix} P(A+BK) + (A+BK)^TP^T & PD_1 + C^TD_2 & C^T \\ * & D_2^TD_2 - \gamma^2 I & 0 \\ * & * & -I \end{bmatrix} \\ &+ \begin{bmatrix} PG_1 \\ 0 \\ 0 \end{bmatrix} H(t) \begin{bmatrix} F_1 + F_2K & 0 & 0 \end{bmatrix} \\ &+ \begin{bmatrix} (F_1 + F_2K)^T \\ 0 \\ 0 \end{bmatrix} H^T(t) \begin{bmatrix} (PG_1)^T & 0 & 0 \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ D_2^TG_2 \\ G_2 \end{bmatrix} H(t) \begin{bmatrix} F_3 & 0 & 0 \end{bmatrix} \\ &+ \begin{bmatrix} F_3^T \\ 0 \\ 0 \end{bmatrix} H^T(t) \begin{bmatrix} 0 & G_2^TD_2 & G_2^T \end{bmatrix} \\ &\leq \begin{bmatrix} P(A+BK) + (A+BK)^TP^T & PD_1 + C^TD_2 & C^T \\ * & D_2^TD_2 - \gamma^2 I & 0 \\ * & * & -I \end{bmatrix} \\ &+\varepsilon_3 \begin{bmatrix} PG_1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} (PG_1)^T & 0 & 0 \end{bmatrix} \\ &+\varepsilon_3^{-1} \begin{bmatrix} (F_1 + F_2K)^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} F_1 + F_2K & 0 & 0 \end{bmatrix} \\ &+\varepsilon_4 \begin{bmatrix} 0 \\ D_2^TG_2 \\ G_2 \end{bmatrix} \begin{bmatrix} 0 & G_2^TD_2 & G_2^T \end{bmatrix} \\ &+\varepsilon_4^{-1} \begin{bmatrix} F_3^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} F_3 & 0 & 0 \end{bmatrix}. \end{aligned}$$

Thus, inequality (22) holds only if inequality (29) holds.

Where $\Phi_{3,11} = P(A + BK) + (A + BK)^TP^T + \varepsilon_3 PG_1G_1^TP^T$. Pre- and postmultiplying both sides of (29) with $diag\{P^{-1}, I, I, I, I\}$ and its transpose, respectively, and using the Schur complement lemma, we obtain (27). The proof is completed.

Remark 2. Based on the above results, we can obtain some special cases. For example, when $E = I_n$ in (4), this system is robust H_∞ finite-time stability only if replacing E in (23) $\sim$ (27) with I_n and other necessary modification. In fact, for this case, there have been some existing results [13], [31]. Which shows that our results are more general. Certainly, the obtained results suit for system (4) without uncertainty.

IV. NUMERICAL SIMULATION

In this section, we give a numerical example to show the effectiveness of our results.

Example 1. Consider singular system (1) with the following parameters

$$E = \begin{bmatrix} 1 & 0 & 2 \\ 0.5 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}, A = \begin{bmatrix} -14 & 1 & -3 \\ 0 & -5 & 2 \\ 1 & 0 & -6 \end{bmatrix},$$

$$B = \begin{bmatrix} -1 & 1 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}, C = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \end{bmatrix}, D_2 = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix}, G_1 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \end{bmatrix}, G_2 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 0.1 & 0.1 & 0.3 \end{bmatrix}, F_2 = \begin{bmatrix} 0.2 & -0.5 \end{bmatrix}, F_3 = \begin{bmatrix} 0.1 & 0 & 0.5 \end{bmatrix},$$

$$R = \begin{bmatrix} 0.6 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix}, c_1 = 48, c_2 = 80, H(t) = sint.$$

By using the LMI toolbox in the MATLAB, we obtain the following solutions of the inequalities (23) $\sim$ (27):

$$M = \begin{bmatrix} 0.1603 & 0.0061 & 0.2220 \\ 0.7182 & 0.5368 & -0.5332 \\ -0.4286 & 0.8571 & 0.2857 \end{bmatrix},$$

$$N = \begin{bmatrix} 0.1633 & 0.9866 & 0 \\ 0 & 0 & 1 \\ 0.9866 & -0.1633 & 0 \end{bmatrix},$$

$$X = \begin{bmatrix} 1.3546 & -0.7605 & -0.8447 \\ -0.7605 & 1.6999 & 0.4489 \\ -0.8447 & 0.4489 & 2.9862 \end{bmatrix},$$

$$W = \begin{bmatrix} -6.1905 & 0.8414 & 1.8664 \\ 4.6434 & -5.0307 & 4.4245 \end{bmatrix},$$

$$K = \begin{bmatrix} -6.2602 & -2.0860 & -0.8323 \\ 4.2363 & -1.8451 & 2.9573 \end{bmatrix},$$

$\varepsilon_2 = 12.2332$, $\varepsilon_3 = 13.1024$, $\varepsilon_4 = 3.3855$, $\delta = 5.8727$, $\mu = 14.7735$,

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$$\begin{bmatrix} P(A+BK)+(A+BK)^TP^T+\alpha PE+\varepsilon_2 PG_1G_1^TP & PD_1 & (F_1+F_2K)^T \\ * & -\delta I & 0 \\ * & * & -\varepsilon_2 I \end{bmatrix} < 0, \tag{28}$$

$$\Phi_3 = \begin{bmatrix} \Phi_{3,11} & PD_1+C^TD_2 & C^T & (F_1+F_2K)^T & F_3^T \\ * & D_2^TD_2-\gamma^2I+\varepsilon_4 D_2^TG_2G_2^TD_2 & \varepsilon_4 D_2^TG_2G_2^T & 0 & 0 \\ * & * & -I+\varepsilon_4 G_2G_2^T & 0 & 0 \\ * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & -\varepsilon_4 I \end{bmatrix} < 0, \tag{29}$$

$\gamma = 2.2$. For the initial values $x(0) = (5, 3, -5)^T$, Fig.1 is the state trajectories of system (4). Fig.2 is the curve of the state function $x^T(t)Rx(t)$, which shows that system (4) is finite-time stability. In fact, from these two figures, we known that finite-time stability is milder than the asymptotical stability. Fig.3 is the curve of H_∞ performance index $\gamma(t) = \frac{||z(t)||}{||w(t)||}$ for system (4), which shows that system (4) has H_∞ performance index γ.

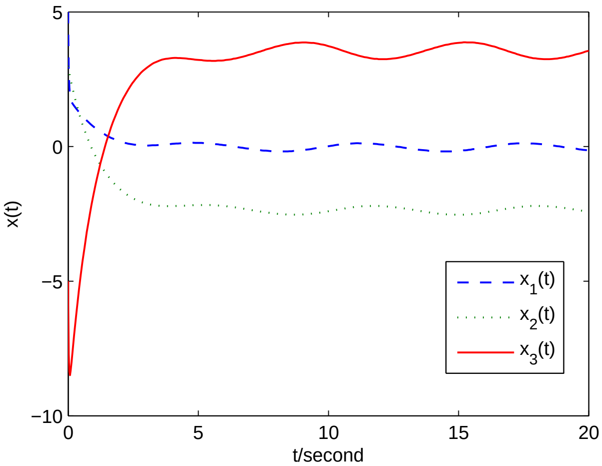

Fig.1. The state trajectories of system (4).

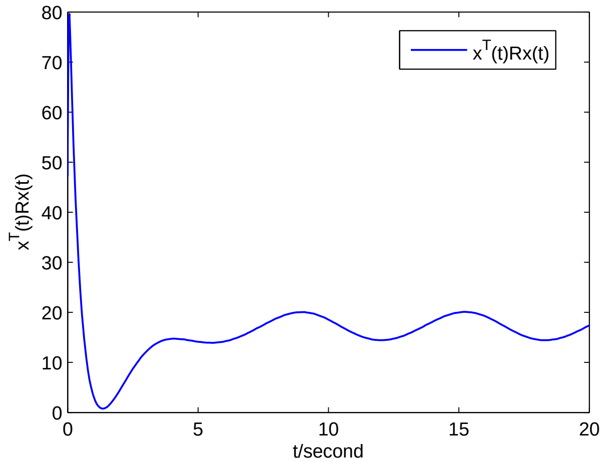

Fig.2. The curve of the state function $x^T(t)Rx(t)$.

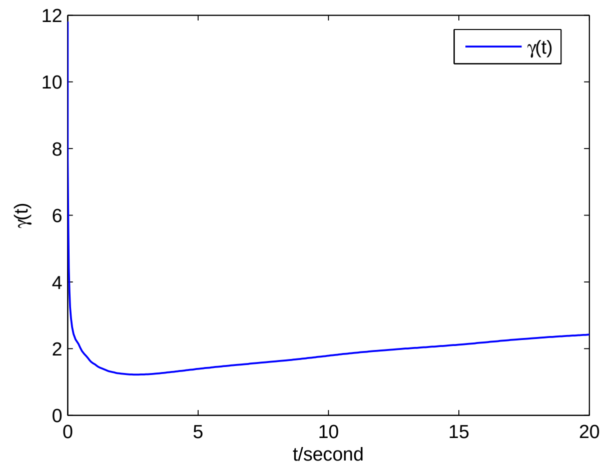

Fig.3. The curve of H_∞ performance index $\gamma(t) = \frac{||z(t)||}{||w(t)||}$ of system (4).

V. CONCLUSION

In this paper, the robust finite-time H_∞ control problem for the uncertain singular system has been investigated. Through the state space decomposition and constructing a Lyapunov functional, some conditions which guarantee the studied singular system to be finite-time stability and possess H_∞ performance with attenuation index γ have obtained. These results have preferably revealed the relationship between the finite-time stability and system parameters. Moreover, these results are provided by the form of the linear matrix inequalities and are easy to solve. A numerical example has shown that our method is effective.

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YANG DING received the B. Eng. degree from the School of Hangzhou Dianzi University, Hangzhou, China, in 2018. He is currently pursuing the M.Eng. degree with the School of Mathematics, Physics and Statistics, Shanghai University of Engineering Science, Shanghai, China.

His current research interest is finite time stability and singular system.

HANG ZHANG received the B. Eng. degree from the School of Electrical Information Engineering, Dalian Jiaotong University, Dalian, China, in 2018. He is currently pursuing the M.Eng. degree with the School of Mathematics, Physics and Statistics, Shanghai University of Engineering Science, Shanghai, China.

His current research interest is finite time stability and sliding mode control.

TIANBO WANG received his B.S. degree from Hubei University in 1998. He obtained his M.S. degree from Wuhan University in 2005. He received his Ph.D. degree in Automatic control theory and Engineering from Donghua University in 2013. Now he is an associate professor of Shanghai University of Engineering Science, Shanghai, China.

His current research interests include the stability analysis and control for the nonlinear dynamical systems, and the synchronization of complex dynamical networks.