

Deep Social Force

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Abstract

The Social Force model introduced by Helbing and Molnar in 1995 is a cornerstone of pedestrian simulation. This paper introduces a differentiable simulation of the Social Force model where the assumptions on the shapes of interaction potentials are relaxed with the use of universal function approximators in the form of neural networks. Classical force-based pedestrian simulations suffer from unnatural locking behavior on head-on collision paths. In addition, they cannot model the bias of pedestrians to avoid each other on the right or left depending on the geographic region. My experiments with more general interaction potentials show that potentials with a sharp tip in the front avoid locking. In addition, asymmetric interaction potentials lead to a left or right bias when pedestrians avoid each other.

Keywords: Social Force, Coordinate MLP, Fourier Features

1. Introduction

The goal is to model pedestrian behavior with a higher fidelity. So far, pedestrian simulations do not even model people avoiding each other on the right in continental, western Europe which limits any application of pedestrian simulations to counter flows: people passing by each other in corridors. In fact, most force-based pedestrian simulation models display unnatural behavior when two people need to avoid each other in head-on paths.

This project brings together classical pedestrian simulations, deep neural networks and differentiable simulations. However, this project will not do a sweeping black-box replacement of the entire model. The goal is to generalize the Social Force model (Helbing and Molnar, 1995) with deep learning while keeping its interpretable nature of forces derived from gradients of scalar potentials.

Social Force in Transportation, Robotics and Crowd Modeling. The Social Force model is a cornerstone of pedestrian modeling. It is a micro-simulation of pedestrian behavior based on simple interaction potentials that exhibit complex behaviors at scale. Social Forces are the gradients of interpretable potentials.

While the interpretability of the interaction potentials is a feature of the Social Force model, their shape constraint to a family of falling exponential functions, or any other parametric form, is unnatural. The model would preserve its interpretability if the interaction potentials were replaced by arbitrary, continuous and differentiable functions.

Universal Functions for Interaction Potentials. This paper describes interaction potentials by universal function approximators in the form of deep neural networks. In this form, Social Forces are derived from gradients of the neural network using automatic differ-

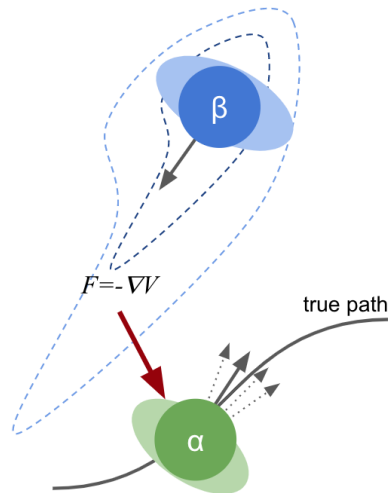




Figure 1: Illustration of a pedestrian interaction with the Deep Social Force model. In this interaction, pedestrian α feels a force F that is created by the gradient of the repulsive potential V that surrounds pedestrian β (illustrated by blue, dashed equipotential lines). From the predicted path of pedestrian α , an optimizer can modify the generic shape of V so that the prediction will match the true path.

entiation with efficient BackProp (LeCun et al., 2012). The same automatic differentiation implementation is used to compute the gradients of these potentials.

The ultimate goal is to use a more expressive model combined with a stochastic learning technique to infer model parameters from larger amounts of data. It will still be possible to plot these potentials and to interpret their shapes as before.

This paper proposes a method that retains the structure and interpretability of the Social Force model and removes assumptions on the shapes of the interaction potentials. The interaction potentials underlying the Social Force model are of particular parametric forms. While the parameters of these potentials can be adjusted to better approximate real-world data, the shape is constrained by the choice of the parametric function. Here, these functions are learned with neural networks without constraining them to particular shapes. Figure 1, illustrates how different choices of parameters for the interaction potential influence the predicted path of a pedestrian. Social interactions among pedestrians are complex and therefore should not rely on a particular parametric form.

The software is available at <https://github.com/svenkreiss/socialforce>. All results are produced with an executable book (Community, 2020) hosted at <https://www.svenkreiss.com/socialforce/>. Figures contain deep links to animated visualizations with this  Animation symbol or to executable Jupyter notebooks with the  Notebook symbol.

2. Related Work

This paper focuses on predicting human navigation patterns in crowded scenes where human-space and more importantly human-human interactions are key to model. As briefly introduced in the previous section, there are two general streams of methods: interpretable low capacity ones and data-driven high capacity ones with low interpretability.

Interpretable models. Most of interpretable models are based on hand-crafted energy potentials that require some domain expertise. Helbing and Molnar (Helbing and Molnar, 1995) have highly impacted the field by introducing a pedestrian motion model based on attractive and repulsive forces referred to as the *Social Force* model. This has been shown to achieve competitive results even on modern pedestrian datasets (Lerner et al., 2007b; Pellegrini et al., 2009). This method was used extensively to study panic behavior in dense crowds (Helbing et al., 2005; Helbing and Johansson, 2009; Helbing et al., 2000), applied in robotics (Luber et al., 2010) and activity understanding (Mehran et al., 2009; Yamaguchi et al., 2011; Pellegrini et al., 2010; Leal-Taixé et al., 2011; Leal-Taixé et al., 2014; Choi and Savarese, 2012, 2014). Variants of the model have been proposed by extending it to a multi-class case (Robicquet et al., 2016) or by including rotational inertia (Farina et al., 2017). Using a dataset of dense crowds, Seer (Seer et al., 2012) have calibrated the Social Force parameters to produce a model with better predictive power. Recently, Chen (Chen et al., 2018) presented a nice review on all extensions of the Social Force model.

Related approaches have been used to model human-human interactions. Treuille *et al.* (Treuille et al., 2006) use continuum dynamics, Antonini *et al.* (Antonini et al., 2006) propose a Discrete Choice framework and Wang *et al.* (Wang et al., 2008), Tay *et al.* (Tay and Laugier, 2008) use Gaussian processes. Such functions have also been used to study stationary groups (Yi et al., 2015; Park and Shi, 2015).

Analyzing the potentials behind all these hand-crafted models is in fact the backbone for interpretability. However, the manually chosen functions behind these potentials limit the predictability power of these models.

Non-interpretable high capacity models. High capacity models learn human-human and human-space interactions in a more generic, data-driven fashion. They are data hungry with respect to hand-crafted methods. Yet, with enough data, they outperform previous works in terms of prediction accuracy. Since 2016, a collection of methods have been proposed based on Recurrent Neural Networks (RNN) (Pfeiffer et al., 2017; Alahi et al., 2016; Gupta et al., 2018). The challenge remains on how to design the RNN architectures, or how to represent interactions. Alahi *et al.* (Alahi et al., 2016) suggest a social tensor to represent human-human interactions. Gupta *et al.* (Gupta et al., 2018) present an adversarial training to learn the distribution of trajectories. While these methods have the capacity to learn complex interactions, it is difficult to understand the “why” or simply the impact of surrounding humans on each other. Hence, we propose to take the benefits of both streams of methods and introduce a non-parametric, Deep Social Force model.

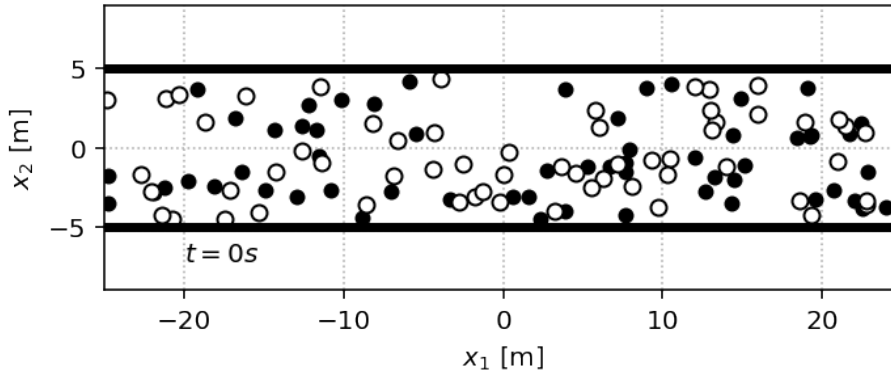


Figure 2: Corridor example. Pedestrians represented by filled circles walk towards a goal on the right and pedestrians represented by empty circles walk towards a goal on the left. The size of the circle represents the walking speed of a pedestrian as in Helbing and Molnar (1995). [Animation](#).

3. Social Force Examples

Several toy examples that demonstrate the properties of Social Force were introduced in the original paper (Helbing and Molnar, 1995). These examples are re-implemented here with modern software tools and open sourced.

Corridor. Figure 2 shows a reproduction of the corridor example. Pedestrians represented by filled circles walk towards a goal on the right and pedestrians represented by empty circles walk towards a goal on the left. The size of the circle represents the walking speed of a pedestrian as in the original paper. The original paper included a study on lane-forming behavior that is not reproduced here. However, once we have generalized interaction potentials, an extension of this example will show the effect of left-right asymmetries in interaction potentials in Section 5.

Gate. Figure 3 shows the gate example. Pedestrians are trying to pass through a gate to get to the opposite side of the barrier. In the original paper, the emergent behavior of grouping was observed: groups of a few people pass the gate at a time. It was surprising that pedestrians did not pass individually or that not all of pedestrians of one side pass before the other side. While I was able to reproduce that behavior, I want to point out that this behavior is only present when the pedestrians have a preferred walking speed distribution, *i.e.* pedestrians move at different preferred speeds.

Baseline Implementation. All operations are vectorized. The equations of motion are integrated with the LeapFrog algorithm (Birdsall and Langdon, 2004) with 25 integration steps per second.

4. Method

The goal is to create a more expressive Social Force model, the *Deep Social Force* model. The ultimate aim is to calibrate models on large pedestrian datasets. The expressive detail

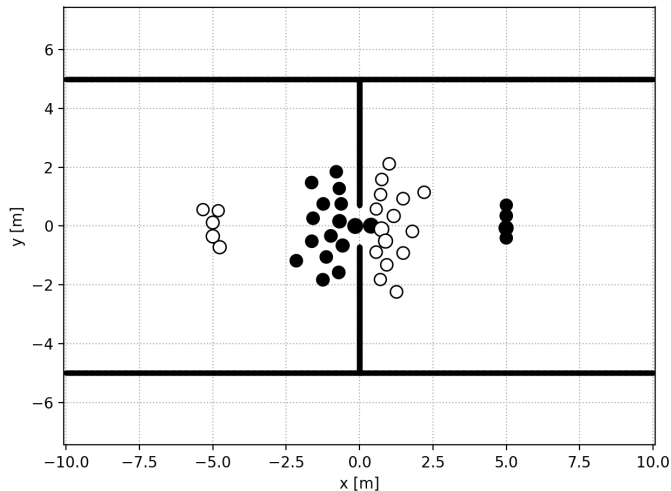


Figure 3: Gate example. Pedestrians represented by filled circles walk towards a goal on the right and pedestrians represented by empty circles walk towards a goal on the left. Five pedestrians have already reached the left goal and four have reached the right goal. At this moment, a group of two pedestrians is passing towards the right. The size of the circle represents the walking speed of a pedestrian as in Helbing and Molnar (1995).

of the model is adjustable by the number of hidden units used for the deep neural networks and can increase as more data becomes available in the future. We have shown in our own work how a Social Force model can help a robot navigate a crowded space (Chen et al., 2019). We have also observed the limits of the Social Force model where two robots that need to pass each other did not effectively negotiate the situation because they had no preference to pass each other on the right. The aim is to discover these intuitive behavioral patterns from data with this Deep Social Force model.

The technical implementation requires two key ingredients. First, we use automatic differentiation in PyTorch (Paszke et al., 2017) to calculate the gradients of the interaction potentials to obtain the forces. Second, we want to avoid imposing assumptions on the potentials from their parametric form so we use non-parametric potentials in the form of neural networks.

Automatic Differentiation of Potentials. With traditional methods, the dimensionality of the parameter space is crucial for numerical optimizers and gradient approximation. In addition, second-order optimizers like L-BFGS (Byrd et al., 1995; Broyden, 1970; Fletcher, 1970; Goldfarb, 1970; Shanno, 1970) need to store an approximation of the function curvature. Our proposed method provides exact gradients without costly numerical gradient estimation and uses first-order optimization methods – Stochastic Gradient Descent (Bottou, 2010) – that are suitable for the stochastic setting. This is faster and more stable than numeric differentiation and more flexible than hand-coded derivatives of particular potentials.

Universal Potentials. Deep neural networks are universal function approximators (Cybenko, 1989; Hornik et al., 1989) in certain limits.

A common parametric potential (Helbing and Molnar, 1995) for human-human interactions is a falling exponential:

$$V_{\alpha\beta}(b) = V_{\alpha\beta}^0 \exp(-b/\sigma) \quad (1)$$

with b being a function of the distance and velocities of the pedestrians and where the parameters to optimize θ are the peak values of the potential V^0 and width of the potential σ , *i.e.* $\theta = \{V^0, \sigma\}$. Here, a multi-layer perceptron (MLP) is used as a non-parametric¹ replacement:

$$V_{\alpha\beta}(b) = \text{Softplus } L_{1 \times 5} \text{ Softplus } L_{5 \times 1} b \quad (2)$$

where the parameters to optimize θ are the entries of the linear operators $\{L_{1 \times 5}, L_{5 \times 1}\}$ and the chosen activation function and output non-linearity here is a Softplus so that the outputs are bounded on one side only. This MLP replacement removes all hand-engineered properties of parametric potentials and discovers those and additional properties from data instead.

The potentials are not directly observed. The potential parameters θ are inferred through the observed interactions of pedestrian paths. Every loss function evaluation is a simulation of all pedestrian paths in the given scenes. Automatic differentiation is used to obtain the gradients of the potentials. It is the same technique that is used to obtain the gradients for the parameter update with Stochastic Gradient Descent (SGD) (Bottou, 2010).

Fourier Features. Recent progress in neural rendering has recaptured the research communities interest in Fourier Features (Rahimi et al., 2007; Tancik et al., 2020) as inputs to coordinate MLPs. We construct multi-dimensional MLPs with Fourier Features (FF) that take the usual b and in addition the perpendicular d_{\perp} and parallel distance d_{\parallel} as inputs. The first operation on these three inputs is a transformation to Fourier Features. In particular, we use three one-dimensional feature transformations to 64 Fourier Features each. The complete MLP is:

$$\text{MLP}(b, d_{\perp}, d_{\parallel}) = \text{Softplus } L_{1 \times 64} [\text{Softplus } L_{64 \times 64}]^3 \text{ Softplus } L_{64 \times 192} \text{ FF}_{192 \times 3} \begin{bmatrix} b \\ d_{\perp} \\ d_{\parallel} \end{bmatrix}. \quad (3)$$

5. Results

Fusing force-based models with deep neural networks is novel and unconventional. The resulting model has the benefits of deep learning while still being interpretable.

1. The term “non-parametric” refers to functions that can increase their capacity as more data becomes available; however, non-parametric functions do have parameters.

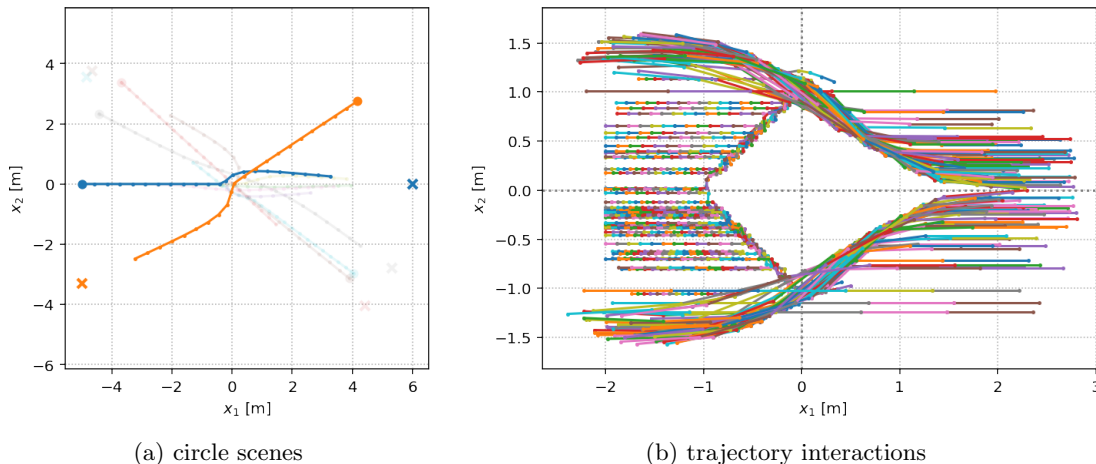


Figure 4: In (a), five “circle scenes” are shown and one is highlighted in blue and orange. The primary pedestrian (blue) always moves from left to right on the x -axis. The other pedestrian (orange) crosses the scene at a random angle. [Notebook](#). In (b), the coordinate frame is fixed to the primary pedestrian and the trajectories of the secondary pedestrian are plotted for many synthetic scenes. In this particular instance, the interaction potential is shaped like a distorted diamond. [Notebook](#).

Training / Calibration. The aim is to train deep neural networks – aka to calibrate the parameters of interaction potentials – with a method that scales to large datasets. The deep learning community uses stochastic optimization methods that operate on batches of data and never on the whole dataset at once. While traditional Social Force models are calibrated with second order methods (*e.g.* Yamaguchi et al. (2011); Seer et al. (2012)), we aim to use the first order Stochastic Gradient Descent (SGD) (Bottou, 2010). SGD can make use of the gradients that our differentiable simulation provides and allows to operate on small batches of data.

Synthetic Scenes. A “Scene” is comprised of a few seconds of observed pedestrian trajectories, see Figure 4. Each trajectory is given as a list of (x, y) coordinates similar to latitudes and longitudes of GPS tracks. We can use the interaction potentials given by Helbing and Molnar (1995) or create our own to generate synthetic scenes. The goal is to have a diverse set of these scenes that probe all aspects of the interaction potential.

Automatic Differentiation of Potentials. With traditional methods, the dimensionality of the parameter space is crucial for numerical optimizers and the first-order gradient approximation. Automatic differentiation is faster and more stable than numeric differentiation and more flexible than hand-coded derivatives of particular potentials. For a small multi-layer perceptron (MLP) with only 10 parameters (1 input, 1 output, 5 hidden units), it is possible to compare the numeric finite difference method against back-propagation. Because of the enormous reduction in function evaluations when back-propagated gradients are available, the finite difference method is more than an order of magnitude slower even on this small model. Finite difference methods are infeasible for larger MLPs. Below, we

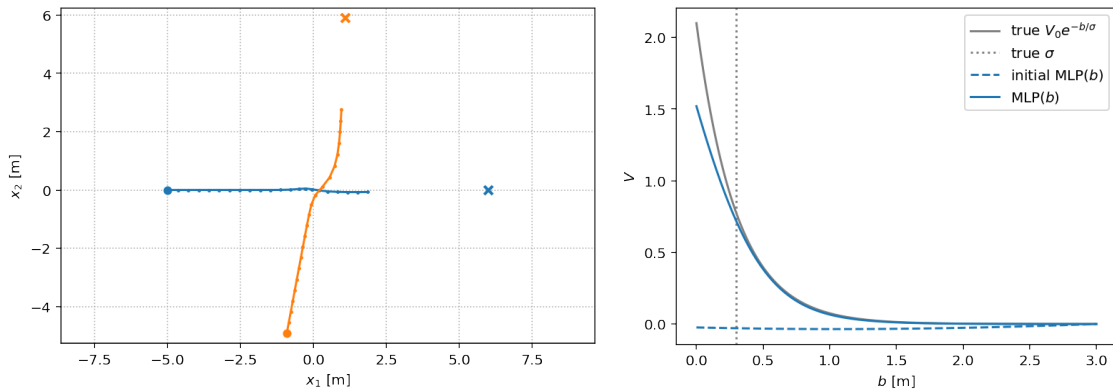
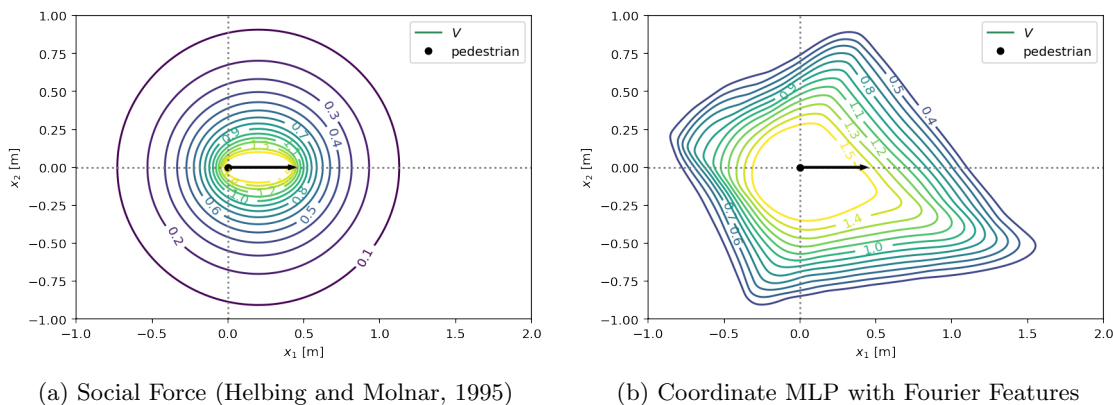


Figure 5: Fit of an MLP-based radial potential for human-human interaction to a single synthetic path of two pedestrians crossing and avoiding each other (left). The value V is shown (right) as a function of reduced distance b . [Notebook](#).



(a) Social Force (Helbing and Molnar, 1995)

(b) Coordinate MLP with Fourier Features

Figure 6: A 2D visualization of the standard interaction potential in the original Social Force model (Helbing and Molnar, 1995) is shown in (a). In (b), the result of an inference of a high-capacity MLP with about 10,000 parameters on a synthetic simulation that was generated with an asymmetric interaction potential is shown. [Notebook](#).

will see a parameter inference on large MLPs for 2D asymmetric potentials with over 10,000 parameters using back-propagated gradients through a differentiable simulation.

Baseline: Parameter Inference for 1D MLP. The result of an inference of MLP parameters on synthetic data is shown in Figure 5. This demonstrates that the differentiable simulation can recover the exact potential as the generative potential with SGD. Note that the inferred potential is a universal function in the form of an MLP whereas the generating potential was a falling exponential function.

Asymmetric 2D Potentials Above, the interaction potential in the one dimension of reduced distance b (Helbing and Molnar, 1995) was generalized with a neural network. Now, we move to more generic 2D potentials. Figure 6a shows the interaction potential in 2D for

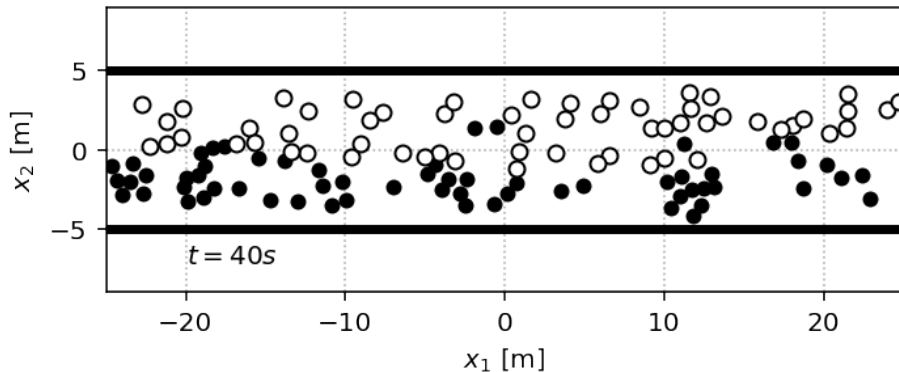


Figure 7: Filled circles represent pedestrians that move to the right and empty circles pedestrians that move to the left. Because of the asymmetry of the new potential, pedestrians prefer to stay on the right side of oncoming pedestrians which implies that all pedestrians prefer the right side of the corridor in their direction of movement. [Animation](#). The corridor example is based on a scenario in Helbing and Molnar (1995) which is reproduced for reference in Figure 2.

the classical Social Force model. It is an exact ellipse where the pedestrian is at one of the focal points and the distance of the focal points of the ellipse is related to the velocity of the pedestrian. The variable b is the semi-minor axis of this ellipse.

To move to 2D, parallel d_{\parallel} and perpendicular d_{\perp} distances are added to the set of input variables. However, I was not able to achieve any reasonable inference results with this configuration. Inspired by recent successes in neural rendering, I added a Fourier Feature layer (FF) (Tancik et al., 2020; Rahimi et al., 2007). The FF layer takes the three input variables and converts them to 192 Fourier Features. The FF layer itself does not contain any trainable parameters. The number of MLP parameters have nevertheless increased to about 10,000 due to the increased input space and deeper architecture.

This 2D potential with its many parameters has a high capacity to model many interaction types. With the differentiable simulation and gradient back-propagation, we can optimize this large dimensional parameter space. On a simulation that was generated with an asymmetric potential, we can infer that asymmetry for this 2D MLP and obtain the potential that is shown in Figure 6b.

We now have all ingredients to investigate implications from asymmetric potentials in the “corridor” scenario. Figure 7 shows a simulation of an asymmetric potential. It shows that asymmetric potentials impose a left-right bias and can model people avoiding each other on the right which ultimately leads to people preferring the right side of the corridor. In this simulation, from random initializations, after 40s all pedestrians have moved to the right in their walking direction as to avoid other pedestrians.

Inference from real data. Figure 8 shows a real world scenario on the left. Inferring an interaction potential from that scene is difficult. On the right side of this figure, I share the result of a first attempt. The MLP is initialized with a fit to synthetic data and then fine-tuned on the real world example from the “Crowd by Example” dataset (Lerner et al.,

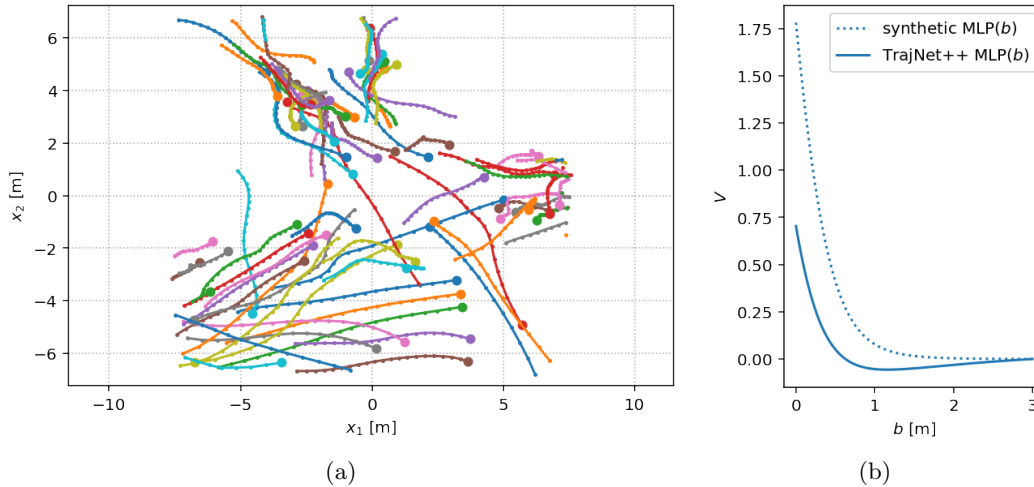


Figure 8: In (a), a scene from the “Crowd by Example” dataset (Lerner et al., 2007a) as provided in the TrajNet++ challenge (Kothari et al., 2021) is shown. In (b), a first attempt at inferring the shape of a 1D MLP potential on this real world dataset is shown. The MLP is initialized with synthetic data and then fine-tuned on the real world scene. [Notebook](#).

2007a) as provided in the TrajNet++ challenge (Kothari et al., 2021). The optimization does not converge and does not work without the initialization from synthetic data. If one wanted to force an interpretation of this result, one would highlight the two distinct regimes below and above $b = 1$ m. For $b < 1$ m, the potential is repulsive as in the classical Social Force model. For $b > 1$ m, the potential is attractive. This type of behavior would have been impossible to discover with a functional constraint to a falling exponential as in the classical Social Force model (Helbing and Molnar, 1995). The reason for the attractive nature of the potential is likely due to the large number of social groups (groups of friends or classmates) in this dataset. The classical Social Force model actually does have a separate interaction term for the attractive forces but here this behavior is discovered from data.

6. Conclusion

This paper and the associated open source software provide an entry point to reproducible research. The open source implementation allows to critically investigate previous claims.

Most of this paper has focused on toy problems. It is important to understand what works and what does not in simplified scenarios. However, when applying these methods to real data, they generally disappoint. Our modeling might still not be advanced enough and we might need probabilistic path prediction and evaluation methods. However, it might also be that the premise of forecasting pedestrian paths from a few seconds of observations of their (x, y) coordinates is too optimistic for general situations. This modeling assumption might be more valid for pedestrians in crowded train station corridors where the main short-term motion is driven by collision avoidance and less valid for the motion of students across

a sparsely populated open space on a university campus where motion is driven by other factors not represented in trajectory coordinates.

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