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Robust Stochastic Observer-based Attack-Tolerant Missile Guidance Control Design Under Malicious Actuator And Sensor Attacks

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ABSTRACT In this study, the robust H_∞ stochastic observer-based attack-tolerant guidance control strategy is designed for the nonlinear stochastic missile guidance control system under the external disturbance and measurement noise as well as actuator attack signal and sensor attack signal. To simplify the attack signal estimation, a novel nonsingular smoothed dynamic model is introduced to efficiently describe the actuator and sensor attack signals. Consequently, the state/attack signal estimation can be easily achieved by using conventional Luenberger observer. Next, to attenuate the effect of external disturbance, measurement noise and approximation errors of attack signal on the missile guidance control system, the robust H_∞ attack-tolerant guidance control performance is considered and the design condition is derived in terms of nonlinear Hamilton-Jacobi inequality (HJI) constrained problem. Since HJI can not be easily solved analytically or numerically, the Takagi-Sugeno (T-S) fuzzy modelling method is utilized to facilitate the robust H_∞ attack-tolerant guidance control strategy design. Thereafter, the H_∞ observer-based attack-tolerant control design problem is transformed into linear matrix inequalities problem (LMIP) which can be solved very efficiently by using the convex optimization techniques. Simulation example, with the comparison between the proposed method and conventional robust missile guidance strategy, is given to illustrate the design procedures and validate the performance of the proposed method.

INDEX TERMS Missile guidance control system, Hamilton- Jacobi inequality, Robust H_∞ observer-based control problem, Stochastic control, Attack-tolerant control

I. INTRODUTCTION

WITH the development of national military, missile guidance control to hit a target precisely becomes more and more important. In many application aspects, especially in the aerospace field and military studies, missile guidance has always been a popular issue [1], [2], [3]. A missile is a precision-guided system, and it can propel itself. It is different from an unguided self-propelled munition system. When it comes to missile types, there are different missiles for different purposes; for example, air-to-air missiles, air-to-surface missiles, surface-to-surface missiles, surface-to-air missiles, and anti-satellite weapons [4]. In addition, there are some important system components about missiles, i.e., flight system, engine, warhead, and targeting and/or missile guidance. The guidance control system in a missile can almost be regarded as the human pilot of an airplane. Therefore, the control principles of missile guidance are vital to the control engineers. Many guidance controls have been discussed in [4], [5], and many guidance technologies have been developed to improve guidance control performance and to attenuate the effect of environmental disturbances. These guidance techniques are mainly based on classical control theory.

For years, various guidance laws have been utilized with different control concepts. Currently, most popular terminal guidance laws defined by Locke involve line-of-sight (LOS) guidance [6], [7], LOS rate guidance [8], [9], command-to-line-of-sight (CLOS) guidance [10], [11], and other advanced guidances such as proportional navigation guidance (PNG) [12]– [14], command to optimal interception point (COIP) guidance [15], augmented proportional navigation guidance (APNG) [16], optimal guidance law [17]– [19], linear quadratic Gaussian (LQG) theory [20], [21], sliding mode theory [22], H_∞ robust theory, impact angle control [23]– [25] and fuzzy logic control theory [26]– [28], etc. For the above conventional guidance strategy designs, it assumed the information of target and missile can be obtained perfectly by the seeker of missile for guidance control design, namely it neglect the effect of measurement noise in seeker. Hence, the conventional design methods will be more conservative for practical applications.

Of these current guidance technologies, guidance control commands proportional to the LOS angle rate are generally used by most high-speed missiles recently to correct missile course [12]– [14]. This guidance method is referred to as PNG, and is quite successful against non-maneuvering targets. However, since PNG exhibits the optimal performance with constant-velocity targets, it is not effective to guide for uncertain target maneuvers, and often leads to unacceptable miss distance [29]. Furthermore, in particular, relative motion between missile and target is highly nonlinear and uncertain due to unmodeled dynamics and parameter perturbations resulting from the missile modelling. Therefore, in a well-considered missile guidance system, the robustness of guidance performance w.r.t the hostile interferences, which are viewed as environmental disturbances, must be considered in the missile guidance control procedure. As a consequence, using a nonlinear stochastic dynamic model to describe a missile system would be more appealing. Generally, the modelling uncertainty of missile and the accumulated angle error of gyroscope could be modelled by continuous Wiener processes [30]; the inaccurate measurement of sensor on the seeker in the missile due to the target suddenly maneuvering could be modelled by discontinuous Poisson jump processes [31].

In the LOS guidance control of tactical missile, the target such as ballistic missile or fighter plane will perform rolling or swaggering side drift by its two-side jets to avoid the precise targeting by the seeker of tactical missile. Further, the target will also send malicious attack signal to interfere the sensor on the seeker of tactical missile. The side-step maneuvering of target by its two-side jets can be considered as an equivalent attack signal on actuator of the LOS guidance control system of tactical missile. The malicious interference signal emitted by the target on the sensor (i.e., the seeker) of tactical missile would be considered as sensor attack signal. To address this issue, the attack-tolerant control (ATC) scheme have been put forward to eliminate the effect of unknown attack signals on system. In general, the ATC scheme includes two parts with (i) attack signal estimation and (ii) attack signal compensation [32], [33], [34]. In the first part, a specific observer is constructed to simultaneously estimate the state variable and attack signal. Then, by utilizing the estimated malicious attack signals, the control strategy can be implemented to cancel the effect of real attack signal on missile guidance system. Besides, without the direct attack signal estimation, the hybrid fault tree analysis method is provided to identify all possible attack signals in the missile system in [35]. In [36], an adaptive attack tolerant control strategy is used to achieve the formation tracking control of missile system and the effect of attack signal is passively attenuated by the proposed adaptive control strategy. For the previous researches of missile control system, there have few studies to address the attack signal estimation and the corresponding compensation.

To the best of authors' knowledge, the stochastic effects, which are inevitable in the missile system, are not considered in the previous studies. Also, instead of direct attack signal estimation/compensation, the current studies of missile guidance control system aim to passively eliminate the effect of attack signal. In this situation, the designed guidance strategies in previous articles may be conservative for real application. Moreover, for the conventional ATC scheme which based on descriptor model of attack signal [32]– [40], the resulting design conditions involve matrix equations which are not solvable in general. Consequently, it is highly desirable to apply advanced control techniques to develop an effective observer-based attack-tolerant control law to improve the engagement performance of tactical missiles under external disturbance, intrinsic stochastic fluctuation and actuator and sensor attack signals.

In this study, a robust observer-based attack-tolerant guidance control design is proposed for tactical missile to precisely hit the target under malicious actuator and sensor attack signals as well as external disturbance. It is worth to point out that the attack signals on actuator and sensor from target can be also considered as equivalent attack signals on actuator and sensor of missile guidance systems. To estimate the attack signals, two nonsingular smoothed signal models are proposed to describe the unavailable actuator and sensor attack signals. Then, to avoid the corruption of attack signals, two smoothed signal models are embedded in the missile guidance model as an augmented missile guidance system so that the missile state and attack signals on sensor and actuator can be estimated by the conventional Luenberger observer for the attack-tolerant guidance control design. Due to strongly nonlinear behavior between the missile and target, it becomes very difficult to solve the robust H_∞ observer-based attack-tolerant guidance control problem because we need to solve a nonlinear Hamilton-Jacobin inequalities (HJIs) for Luenberger observer and controller to achieve the H_∞ observer-based attack-tolerant guidance control of tactical missile system under malicious actuator and sensor attack as well as external disturbance. Unfortunately, at current, there still exists no good method to solve such HJI for H_∞ observer-based attack-tolerant guidance control problem. By the T-S fuzzy approximation method [41], [42], the stochastic nonlinear guidance control system can be approximated by

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interpolating a set of local linear systems. Then, the fuzzy observer-based attack-tolerant guidance controllers are introduced to efficiently estimate system state and attack signal for guidance control of the stochastic nonlinear missile guidance system, which at the same time can eliminate the effects of external disturbance, approximation error of attack signals, and measurement noises on the estimation and guidance of the augmented guidance system below a prescribed attenuation level, so that a desired robust H_∞ observer-based attack-tolerant guidance control performance can be guaranteed. With the help of T-S fuzzy interpolation scheme, the difficult HJI for robust H_∞ observer-based attack-tolerant guidance control design of missile guidance control system is transformed to a set of linear matrix inequalities (LMIs), which could be easily solved via the LMI TOOLBOX in MATLAB. Finally, the simulation results show that the optimal H_∞ observer-based attack-tolerant guidance control performance under actuator and sensor attack can be achieved by the proposed H_∞ observer-based attack-tolerant guidance control method.

The contributions of this study are described as follows:

(I) Compared with the previous studies which focused on the deterministic missile guidance control system [1], [10], [11], the continuous Wiener process and discontinuous Poisson process are introduced to missile guidance control system to model intrinsic random fluctuations during the guidance control process of missile. Moreover, the attack signals on the missile guidance control system and the sensor of seeker on missile are considered, too. As a result, the proposed stochastic nonlinear missile guidance control system is very close to real missile guidance control system and thus the proposed robust H_∞ observer-based attack tolerant control is more practical than previous studies.

(II) Unlike the conventional singular descriptor model for the estimation of attack (fault) signal, a novel nonsingular smoothed dynamic model is proposed to efficiently describe the actuator and sensor attack signals so that the conventional Luenberger observer could be employed to precisely estimate state variables and actuator and sensor attack signals simultaneously for the robust H_∞ observer-based attack-tolerant guidance control design. As a result, different than the conventional singular descriptor models in the field of attack signal estimation, which have to solve strict algebraic equations, the design of proposed fuzzy Luenberger-type observer for state/attack signal estimation can be simply implemented without solving any algebraic equation.

(III) The H_∞ observer-based attack-tolerant guidance control problem is transformed to an equivalent H_∞ stabilization problem of the augmented system of missile guidance control system and estimation error system to significantly simplify the design procedure of H_∞ observer-based attack-tolerant guidance controller. Since the effect of attack signal approximation error as well as external disturbance and measurement noise is considered in the proposed H_∞ observer-based attack-tolerant guidance performance, the proposed observer-based guidance control can effectively attenuate these undesirable effects during the missile guidance control process. Moreover, a two-step design procedure is proposed to transform the optimal H_∞ fuzzy observer-based attack-tolerant guidance control design problem to an LMIs-constrained optimization problem, which could be easily solved with the help of LMI TOOLBOX in MATLAB.

This study is organized as follows: In Section II, the missile system and problem formulation are introduced. The robust H_∞ observer-based attack-tolerant guidance control design problem for nonlinear stochastic missile system is investigated in Section III. In Section IV, the robust H_∞ observer-based attack-tolerant guidance control design via fuzzy method is discussed. In Section V, a simulation example, with the comparison between the proposed method and conventional robust missile guidance strategy, is provided to illustrate the design procedure of H_∞ observer-based attack-tolerant guidance control design and to confirm the robust guidance performance of missile guidance control system. Conclusions are given in Section VI.

Notation: A^T: the transpose of matrix A; $A \geq 0 (A > 0)$: symmetric positive semi-definite (symmetric positive definite) matrix A; I_n: the n-by-n identity matrix; $\|x\|$: the Euclidean norm for the given vector $x \in \mathbb{R}^n$; $\mathcal{L}_2(\mathbb{R}^+;\mathbb{R}^n) = \{v(t) : \mathbb{R}^+ \to \mathbb{R}^n \parallel (\int_0^\infty v^T(t) v(t) dt)^{\frac{1}{2}} < \infty\}$; C^2: the class of functional $V(x)$: $\mathbb{R}^n \to \mathbb{R}^1$ which are twice continuously differentiable with respect to x; V_x: the gradient column vector of function $V(x) : \mathbb{R}^n \to \mathbb{R}^1$ which is continuously differentiable, (i.e., $\frac{\partial V(x)}{\partial x}$); V_{xx}: the Hessian-matrix with elements of second partial derivatives of function $V(x) : \mathbb{R}^n \to \mathbb{R}^1$ which is twice continuously differentiable, (i.e., $\frac{\partial^2 V(x)}{\partial x^2}$); $0_{a\times b}$ expresses the zero matrix with dimension $a \times b$; $\lambda_{\max}(A)$: the maximum eigenvalue of real-value symmetric matrix A; $eig(A)$ denotes the set of the eigenvalues of matrix A. S denotes the set of one-dimensional complex number. $\mathrm{col}[D]$ denotes the column space of matrix D. Matrices, if their dimension are not particularly defined, are assumed to be with appropriate dimension for algebraic operation.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

A. SYSTEM DESCRIPTION

Consider the 3-D missile guidance control system in the spherical coordinate (r, θ, ϕ) with the origin fixed at the missile. The pursuit-evasion geometry between the tactical missile at the origin and the target such as ballistic missile is shown in Fig. 1. Let $(\overrightarrow{e_r}, \overrightarrow{e_\theta}, \overrightarrow{e_\phi})$ be the unit vector along the coordinate axis. The 3-D relative velocity is obtained through the differentiation of the relative distance vector $\overrightarrow{r}$ along with the line of sight (LOS) as follows [43]:

$$\dot{\overrightarrow{r}} = \dot{r}\overrightarrow{e_r} + r\dot{\theta}\cos\phi\overrightarrow{e_\theta} + r\dot{\phi}\overrightarrow{e_\phi} \quad (1)$$

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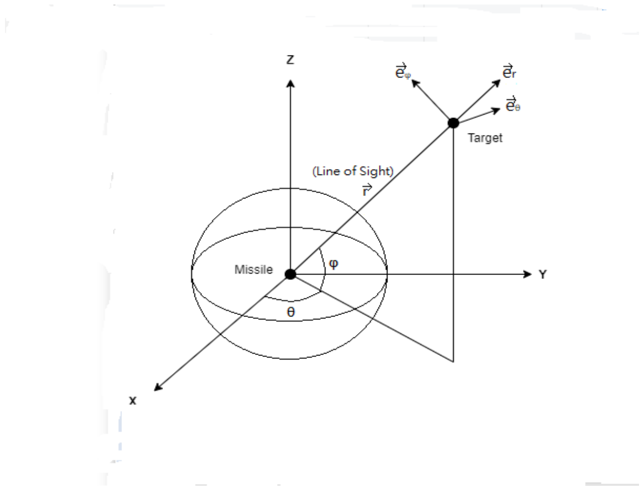

FIGURE 1: 3-D pursuit-evasion geometry in the missile guidance control system

Thus, the relative acceleration at the direction of $\overrightarrow{e_r}$, $\overrightarrow{e_\theta}$, and $\overrightarrow{e_\phi}$ can be obtained by differentiating the above equation in the following [44]:

$$\begin{aligned}
&\ddot{r} - r\dot{\phi}^2 - r\dot{\theta}^2\cos^2\phi = w_r \\
&r\ddot{\theta}\cos\phi + 2\dot{r}\dot{\theta}\cos\phi - 2r\dot{\phi}\dot{\theta}\sin\phi = w_\theta - u_\theta \\
&r\ddot{\phi} + 2\dot{r}\dot{\phi} + r\dot{\theta}^2\cos\phi\sin\phi = w_\phi - u_\phi
\end{aligned} \tag{2}$$

where u_θ and u_ϕ are the control input of missile; w_r, w_θ, and w_ϕ are the target acceleration vectors. Therefore, the kinematic between the missile and the target in (2) can be represented by the following state space system [1], [2]:

$$\begin{aligned}
&\dot{x}(t) = F(x(t)) + Bu(t) + Dw(t) \\
&y(t) = C(x(t)) + n(t)
\end{aligned} \tag{3}$$

where $x(t) = [r(t)\ \theta(t)\ \phi(t)\ V_r(t)\ V_\theta(t)\ V_\phi(t)]^T$ is the state variable with relative distance $r(t)$, relative yaw angle $\theta(t)$, relative pitch angle $\phi(t)$, relative velocity $V_r(t)$, relative angular velocity of yaw angle $V_\theta(t)$ and relative angular velocity of pitch angle $V_\phi(t)$; $u(t) = [u_\theta\ u_\phi]^T$ is the control input; $w(t)$ denotes the target acceleration vector, which is unavailable for missile and is considered as the external disturbance to the missile guidance control system; $y(t)$ denotes the measurement output by laser sensor of the seeker in missile with measurement noise $n(t)$, $C(x(t))$ is the nonlinear output matrix. The matrices in (3) are defined as follows:

$$F(x(t)) = \begin{bmatrix} V_r(t) \\ \frac{V_\theta}{r\cos\phi} \\ \frac{V_\phi}{r} \\ \frac{V_\theta^2 + V_\phi^2}{r} \\ -\frac{V_rV_\theta}{r} + \frac{V_\theta V_\phi \tan\phi}{r} \\ -\frac{V_rV_\phi}{r} + \frac{V_\theta^2\tan\phi}{r} \end{bmatrix}$$

$$B = \begin{bmatrix} 0_{4\times 2} \\ -I_2 \end{bmatrix}, D = \begin{bmatrix} 0_{3\times 3} \\ I_3 \end{bmatrix},$$

In addition, to avoid the attack of the tactical missile, the target will generate jamming signal to interfere with the laser sensor on the seeker of missile through wireless channel, which will lead to an equivalent sensor attack signal. On the other hand, the target will perform rolling or sudden side-step maneuvering through its two-side jets, which will lead to an equivalent actuator attack on the missile guidance control system. By considering the effect of sensor and actuator attack signals by hostile jamming from target as well as rolling and side-step maneuvering of two-side jets of target on missile guidance, respectively, the missile guidance control system in (3) with actuator and sensor attack should be revised as:

$$\begin{aligned}
&\dot{x}(t) = F(x(t)) + Bu(t) + Dw(t) + D_a\gamma_a(t) \\
&y(t) = C(x(t)) + n(t) + D_s\gamma_s(t)
\end{aligned} \tag{4}$$

where $\gamma_a(t) \in \mathbb{R}$ denotes the equivalent actuator attack due to sudden rolling and side-step maneuvering through two-side jets in the target; $\gamma_s(t) \in \mathbb{R}$ denotes the sensor attack on of missile due to hostile jamming from target; D_a is the actuator attack matrix; D_s is the sensor attack matrix.

Assumption 1. *The actuator attack signal $\gamma_a(t)$ and sensor attack signal $\gamma_s(t)$ are differentiable functions..*

Despite the malicious signal from the attacker, the missile guidance control system in (4) always suffers from intrinsic stochastic continuous Wiener fluctuations due to the modelling uncertainty of the missile and the accumulated angle error of the gyroscope as well as the stochastic discontinuous Poisson jump noise due to the inaccurate radar measurement of the missile because of the target's suddenly maneuvering [43]. In this situation, the missile guidance control dynamic system in (4) should be further modified as:

$$\begin{aligned}
&dx(t) = [F(x(t)) + Bu(t) + Dw(t) + D_a\gamma_a(t)]dt \\
&\qquad + H(x(t))\,dW(t) + G(x(t))\,dN(t) \\
&y(t) = C(x(t)) + n(t) + D_s\gamma_s(t)
\end{aligned} \tag{5}$$

where $H(x(t))$, and $G(x(t)) : \mathbb{R}^{n_x} \to \mathbb{R}^{n_x}$ are nonlinear Borel measurable continuous functions, which are satisfied with local Lipschitz continuity. The 1-D Wiener noise $W(t)$ is a continuous but non-differentiable stochastic process and $H(x(t))\,dW(t)$ denotes the effect of continuous stochastic intrinsic noise. $N(t)$ is a Poisson counting process with jump intensity $\lambda > 0$ and $G(x(t))\,dN(t)$ is used to describe

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the discontinuous behavior in missile guidance control system. The Wiener process $W(t)$ and the Poisson counting process $N(t)$ are defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ with the sample space Ω, filtration $\mathcal{F}_t$ generated by $W(s)$ and $N(s)$ for $s \leq t$, $\mathcal{F} = \{\mathcal{F}_t\}_{t\geq 0}$ and probability measure $\mathcal{P}$. These two process are assumed to be independent.

Remark 1. *Some important properties of Wiener process $W(t)$ and Poisson jump process $N(t)$ in this study are given as follows [45]:*
1) $E\{W(t)\} = E\{dW(t)\} = 0$.
2) $E\{dW(t)\, dW(t)\} = dt$.
3) $E\{dN(t)\} = \lambda dt$ with the Poisson jump intensity $\lambda > 0$.

In order to estimate malicious attack signal $\gamma_a(t)$ and $\gamma_s(t)$ by the conventional Luenberger observer for the ATC design, a novel dynamic smoothed model is proposed for malicious attack signals $\gamma_a(t)$ and $\gamma_s(t)$. Based on the right derivative definition of $\dot{\gamma_a}(t) = \lim_{\tau\to 0} \frac{\gamma_a(t+\tau)-\gamma_a(t)}{\tau}$, we immediately have the following approximation:

$$\begin{aligned} \dot{\gamma_a}(t) &= \tfrac{1}{\tau}(\gamma_a(t+\tau) - \gamma_a(t)) + \epsilon_{1,a}(t), \\ \dot{\gamma_a}(t-\tau) &= \tfrac{1}{\tau}(\gamma_a(t) - \gamma_a(t-\tau)) + \epsilon_{2,a}(t), \\ &\vdots \\ \dot{\gamma_a}(t-k\tau) &= \tfrac{1}{\tau}(\gamma_a(t-(k-1)\tau) - \gamma_a(t-k\tau)) + \epsilon_{k,a}(t) \end{aligned} \tag{6}$$

where $\epsilon_{1,a}(t), ..., \epsilon_{k,a}(t)$ are the corresponding approximation errors of derivative at different smoothed time points for actuator attack signal $\gamma_a(t)$, $\tau > 0$ is a small enough time interval and $k \in \mathbb{N}$ denotes the step of attack signal estimation. In addition, the future attack signal $\gamma_a(t+\tau)$ could be also represented by extrapolation as follows:

$$\gamma_a(t+\tau) = \sum_{i=0}^{k} a_i \gamma_a(t-i\tau) + \delta_a(t), \tag{7}$$

where $a_i, i = 0, ..., k$ are the extrapolation coefficients with $\sum_{i=0}^{k} a_i = 1$, $\delta_a(t)$ is the extrapolation error of $\gamma_a(t+\tau)$. Then we could obtain the following dynamic smoothed model of actuator attack signal $\gamma_a(t)$:

$$d\Gamma_a(t) = (A_{\gamma_a}\Gamma_a(t) + \epsilon_a(t))dt \tag{8}$$

where $\Gamma_a(t) = \left[\gamma_a^T(t), \gamma_a^T(t-\tau), ..., \gamma_a^T(t-k\tau)\right]^T$, $\epsilon_a(t) = \left[(\epsilon_{1,a}(t) + \delta_a(t)/\tau)^T, \epsilon_{2,a}^T(t), \cdots, \epsilon_{k,a}^T(t)\right]^T$ denotes the approximation error vector of actuator attack signal, and

$$A_{\gamma_a} = \begin{bmatrix} \frac{\overline{a}_0}{\tau} I_{n_a} & \frac{a_1}{\tau} I_{n_a} & \frac{a_2}{\tau} I_{n_a} & \dots & \frac{a_k}{\tau} I_{n_a} \\ \frac{1}{\tau} I_{n_a} & -\frac{1}{\tau} I_{n_a} & 0 & \dots & 0 \\ 0 & \frac{1}{\tau} I_{n_a} & -\frac{1}{\tau} I_{n_a} & \dots & 0 \\ . & & . & & . \\ . & & & . & . \\ . & & & & . \\ 0 & \dots & 0 & \frac{1}{\tau} I_{n_a} & -\frac{1}{\tau} I_{n_a} \end{bmatrix},$$

with $\overline{a}_0 = -1 + a_0$. Similar to the smoothed model of actuator attack signal $\gamma_a(t)$ in (8), the dynamic smoothed model for the sensor attack signal $\gamma_s(t)$ can be written as follows:

$$d\Gamma_s(t) = (A_{\gamma_s}\Gamma_s(t) + \epsilon_s(t))dt, \tag{9}$$

where $\Gamma_s(t) = \left[\gamma_s^T(t), \gamma_s^T(t-\tau), ..., \gamma_s^T(t-k\tau)\right]^T$, $\epsilon_s(t) = \left[(\epsilon_{1,s}(t) + \delta_s(t)/\tau)^T, \epsilon_{2,s}^T(t), ..., \epsilon_{k,s}^T(t)\right]^T$,

$$A_{\gamma_s} = \begin{bmatrix} \frac{\overline{b}_0}{\tau} I_{ns} & \frac{b_1}{\tau} I_{n_s} & \frac{b_2}{\tau} I_{ns} & \dots & \frac{b_k}{\tau} I_{ns} \\ \frac{1}{\tau} I_{ns} & -\frac{1}{\tau} I_{ns} & 0 & \dots & 0 \\ 0 & \frac{1}{\tau} I_{n_s} & -\frac{1}{\tau} I_{n_s} & \dots & 0 \\ . & & . & & . \\ . & & & . & . \\ . & & & & . \\ 0 & \dots & 0 & \frac{1}{\tau} I_{n_s} & -\frac{1}{\tau} I_{n_s} \end{bmatrix},$$

,$\overline{b}_0 = -1 + b_0$ and $b_i, i = 0, ..., k$ are the extrapolation coefficients with $\sum_{i=0}^{k} b_i = 1$.

Remark 2. *In general, due to the continuity property of attack signal, it is expected that the attack signal at future sample point will more close to attack signal at current sample point. Under such thought, the non-negative extrapolation parameters $\{\alpha_i, \beta_i\}_{i=1}^k$ are chosen as decreasing series, i.e., $\alpha_i \geq \alpha_j$ and $\beta_i \geq \beta_j, \forall i, j \in \{1, \cdots, k\}, j \geq i$. Also, to avoid the over extrapolation, the sums of extrapolation parameters $\{\alpha_i\}_{i=1}^k$ and $\{\beta_i\}_{i=1}^k$ are normalized to 1, respectively, i.e., $\sum_{i=1}^k \alpha_i = 1$ and $\sum_{i=1}^k \beta_i = 1$. Besides, to have a better extrapolation performance for attack signal modeling, the designer may increase the number k of delay sample. In this case, the dimension of corresponding system matrices of dynamic smoothed model will be enlarged and it will lead to a more computational complexity for the implementation of the fuzzy Luenberger-type observer in the sequel. Obiviously, there exists a trade-off bettwen the extrapolation performance and a computation complexity in the number k of dealy sample in (8) and (9)*

Then, to estimate $x(t)$ $\gamma_a(t)$ and $\gamma_s(t)$ simultaneously, we can augment these states as $\overline{x}(t) = \left[\Gamma_a^T(t), \Gamma_s^T(t), x^T(t)\right]^T$, and the corresponding augmented missile guidance control system is given as follows:

$$\begin{aligned} d\overline{x}(t) &= [\overline{F}(\overline{x}(t)) + \overline{B}u(t) + \overline{D}\overline{w}(t)]dt + \overline{H}(\overline{x}(t))dW(t) \\ &\quad + \overline{G}(\overline{x}(t))dN(t) \\ y(t) &= \overline{C}(\overline{x}(t)) + \overline{E}\overline{w}(t) \end{aligned} \tag{10}$$

where $\overline{F}(\overline{x}(t)) = \begin{bmatrix} A_{\gamma_a}\Gamma_a(t) \\ A_{\gamma_s}\Gamma_s(t) \\ F(x(t)) + D_a S_a \Gamma_a(t) \end{bmatrix}$, $\overline{B} = \begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix}$, $\overline{D} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & D & 0 \end{bmatrix}$, $\overline{w}(t) = \begin{bmatrix} \epsilon_a(t) \\ \epsilon_s(t) \\ w(t) \\ n(t) \end{bmatrix}$,

$S_a = [I_{n_a}, 0, \cdots, 0]$, $\overline{H}(\overline{x}(t)) = \begin{bmatrix} 0 \\ 0 \\ H(x(t)) \end{bmatrix}$, $\overline{G}(\overline{x}(t)) = \begin{bmatrix} 0 \\ 0 \\ G(x(t)) \end{bmatrix}$, $\overline{C}(\overline{x}(t)) = C(x(t)) + D_s S_s \Gamma_s(t)$, $\overline{E} = [0,0,0,I]$, $S_s = [I_{n_s}, 0, \cdots, 0]$.

Remark 3. *In general, to estimate the unknown attack signal, the designer has to construct the dynamic model of attack signal. Then, the dynamic model of attack signal is augmented with the control system for state/attack signal estimation. In this situation, it is assumed the attack signal is differentiable for model construction. Moreover, for the most of researches of attack signal estimation, two common assumptions, i.e., (I) the attack signal is bounded and (II) the differential of attack signal is bounded, are made for the simplicity of design [53]. In this study, by using the proposed smoothed signal model to describe the fault signal, these two assumptions can be dropped.*

For the missile guidance control system design, since the state $\overline{x}(t)$ of stochastic nonlinear augmented missile guidance control system in (10) can not be measured directly, the following nonlinear observer-based guidance controller is proposed to estimate the states and attack signals of the augmented missile guidance system in (10) for the attack-tolerant guidance control design:

$$\begin{aligned} &d\widehat{\overline{x}}(t) = \{\overline{F}(\widehat{\overline{x}}(t)) + \overline{B}u(t) + L(\widehat{\overline{x}}(t))[y(t) - \widehat{y}(t)]\}dt \\ &u(t) = K(\widehat{\overline{x}}(t)) \end{aligned} \tag{11}$$

where $L(\widehat{\overline{x}}(t))$ denotes the nonlinear observer gain and $K(\widehat{\overline{x}}(t))$ is the nonlinear observer-based controller gain.

Remark 4. *If we estimate $x(t)$ from $y(t)$ in (4), the malicious attack signals $\gamma_a(t)$ and $\gamma_s(t)$ will corrupt the observer directly. In the augmented missile guidance control system (10), the malicious attack signals $\gamma_a(t)$ and $\gamma_s(t)$ are embedded in the $\overline{x}(t)$. Therefore, the observer in (11) not only estimate $x(t)$, $\gamma_a(t)$ and $\gamma_s(t)$ directly, but also avoid the corruption of attack signals in the estimation.*

B. PROBLEM FORMULATION

In practical, a good guidance law must keep the relative pitch and yaw angular velocities as small as possible, i.e., the head-on condition [44]. So our design objective is to specify the observer-based guidance observer gain $L(\widehat{\overline{x}}(t))$ and corresponding controller $K(\widehat{\overline{x}}(t))$ in (11) so that $V_\theta(t)$, $V_\phi(t)$, and the estimation error $e(t) = \overline{x}(t) - \widehat{\overline{x}}(t)$ will approach to zero. Because $\overline{w}(t)$ is generally uncertain but bounded, it can be viewed as an external disturbance to the missile guidance system. Therefore, the H_∞ guidance control law has been shown to be an effective control to attenuate the effect of uncertain external disturbances on the guidance control performance below a desired level. To begin with, let us denote the state variable $\xi(t) = \begin{bmatrix} V_\theta(t) \\ V_\phi(t) \end{bmatrix}$ to be controlled as [1], [11]:

$$\xi(t) = U' x(t) \tag{12}$$

where

$$U' = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Remark 5. *When $V_\theta(t)$, $V_\phi(t) \to 0$, it means the missile and the target in the head-on condition [43], [44]. Theoretically, in the head-on condition, the missile will approach the target along with LOS because $V_r \neq 0$ in Fig. 1. However, due to attack signal, intrinsic random fluctuation and external disturbance, it is not easy to control the missile in head-on condition.*

For the stochastic augmented missile system in (10), the effects of the approximation errors of sensor and actuator attack signals, the external disturbance and the measurement noise in $\overline{w}(t)$ will deteriorate the control and estimation performance of the missile guidance system and even lead to the instability of the control system. In this situation, how to attenuate these effects to guarantee the robust guidance control performance will be an important design purpose of the robust attack-tolerant guidance control system under malicious attacks and external disturbance. Since H_∞ control is the most important robust control design to efficiently eliminate the effect of uncertain $\overline{w}(t)$ on the attack-tolerant guidance control system, it will be employed to deal with the robust H_∞ observer-based attack-tolerant guidance control design problem in (10) and (11). The following H_∞ observer-based attack-tolerant guidance control performance index is considered as the design objective,

$$\begin{aligned} &J_\infty\left(L(\widehat{\overline{x}}(t)), u(t)\right) \\ &= \frac{\begin{array}{c} E\{\int_0^{t_f}[\xi^T(t)Q_1\xi(t)dt + e^T(t)Q_2e(t) \\ +u^T(t)Ru(t)]dt - V(x(0))\} \end{array}}{E\left\{\int_0^{t_f}\overline{w}(t)^T\overline{w}(t)dt\right\}} \leq \rho^2 \\ &\forall \overline{w}(t) \in \mathcal{L}_2[0, t_f] \end{aligned} \tag{13}$$

where t_f denotes the terminal time, $Q_1, Q_2 \geq 0$ and $R = R^T > 0$ are the weighting matrices, $V(x(0))$ denotes the effect of initial condition to be deducted and ρ^2 denotes a prescribed disturbance attenuation level. For the the H_∞ observer-based attack-tolerant guidance control performance in (13), the designer aims to design the nonlinear observer $L(\widehat{\overline{x}}(t))$ and nonlinear observer-based controller $K(\widehat{\overline{x}}(t))$ in (11) to attenuate the effect of all possible disturbance $\overline{w}(t)$ on the controlled output $\xi(t)$, estimation error $e(t)$ and control input $u(t)$ under a prescribed disturbance attenuation level ρ^2 at the same time from the view point of energy.

III. H_∞ OBSERVER-BASED ATTACK-TOLERANT GUIDANCE CONTROL DESIGN FOR NONLINEAR STOCHASTIC MISSILE SYSTEM UNDER MALICIOUS ATTACKS

In this section, the robust H_∞ observer-based attack-tolerant guidance control design will be developed. By using the Itô-Lévy lemma, the design condition is transformed to an equivalent Hamilton-Jacobi inequality (HJI) problem. At first, for the convenience of the design, according to (10) and (11), the following augmented missle guidance control system is constructed as follows:

$$d\tilde{x}(t) = [\tilde{F}(\tilde{x}(t)) + \tilde{B}u(t) + \tilde{D}(\tilde{x}(t))\bar{w}(t)]dt + \tilde{H}(\tilde{x}(t))dW(t) + \tilde{G}(\tilde{x}(t))dN(t), \quad (14)$$

where $\tilde{x}(t) = [\bar{x}^T(t) \;\; e^T(t)]^T$ is the augmented system states with the system matrices $\tilde{F}(\tilde{x}(t)) = [\bar{F}^T(\bar{x}(t)), \bar{F}^T(\bar{x}(t)) - \bar{F}^T(\hat{\bar{x}}(t)) - [L^T(\hat{\bar{x}}(t))[\bar{C}((\bar{x}(t)) - \bar{C}((\hat{\bar{x}}(t)))]^T]^T$, $\tilde{B} = [\bar{B}^T, 0]^T$, $\tilde{H}(\tilde{x}(t)) = [\bar{H}^T(\bar{x}(t)), \bar{H}^T(\bar{x}(t))]^T$, $\tilde{G}(\tilde{x}(t)) = [\bar{G}^T(\bar{x}(t)), \bar{G}^T(\bar{x}(t))]^T$, $\tilde{D}(\tilde{x}(t)) = \begin{bmatrix} \bar{D} \\ \bar{D} - L(\hat{\bar{x}}(t))\bar{E} \end{bmatrix}$

Furthermore, the H_∞ observer-based attack-tolerant guidance control performance in (13) can be rewritten as:

$$J_\infty\left(L(\hat{\bar{x}}(t)), u(t)\right) = \frac{E\left\{\int_0^{t_f}[\tilde{x}^T(t)Q\tilde{x}(t) + u^T(t)Ru(t)]dt - V(\tilde{x}_0)\right\}}{E\left\{\int_0^{t_f}\bar{w}^T(t)\bar{w}(t)dt\right\}} \le \rho^2, \quad \forall \bar{w}(t) \in L_2[0, t_f] \quad (15)$$

where $Q = diag\{Q_1', Q_2\}$, $Q_1' = U'^T Q_1 U'$. The term $V(\tilde{x}_0)$ is to deduct the effect of initial condition $\tilde{x}(0)$ on the H_∞ observer-based guidance control performance. The main purpose of H_∞ observer-based attack-tolerant guidance control performance is to eliminate the effects of external disturbance, measurement noise and approximation error of actuator and sensor attack signals on the H_∞ guidance estimation and control performance in (15) of the augmented missile guidance control system (10) and observer (11). Before the main results, the following Lemmas are proposed to facilitate the control design:

Lemma 1. *[46] Define the Lyapunov function $V(\cdot) \in C^2$, $V(\cdot) \ge 0$, and $V(0) = 0$. Then , for the nonlinear stochastic augmented missile system in (14), the Itô-Lévy formula of $V(\tilde{x}(t))$ is given as :*

$$dV(\tilde{x}(t)) = \{V_{\tilde{x}}^T[\tilde{F}(\tilde{x}(t)) + \tilde{B}u(t) + \tilde{D}(\tilde{x}(t))\bar{w}(t)] + \tfrac{1}{2}\tilde{H}^T(\tilde{x}(t))V_{\tilde{x}\tilde{x}}\tilde{H}(\tilde{x}(t))\}dt + V_{\tilde{x}}^T\tilde{H}(\tilde{x}(t))dW(t) + [V(\tilde{x}(t) + \tilde{G}(\tilde{x}(t))) - V(\tilde{x}(t))]dN(t) \quad (16)$$

where the term $\frac{1}{2}\tilde{H}^T(\tilde{x}(t))V_{\tilde{x}\tilde{x}}\tilde{H}(\tilde{x}(t))$ is used to compensate the effect of Wiener process in the increment of Lyapunov function $V(\tilde{x}(t))$, and the term $[V(\tilde{x}(t) + \tilde{G}(\tilde{x}(t))) - V(\tilde{x}(t))]dN(t)$ is used to compensate the effect of Poisson process in the increment of Lyapunov function $V(\tilde{x}(t))$.

Lemma 2. *[48] Given two matrices A and B with a positive number α, the following inequality holds:*

$$A^TB + B^TA \le \alpha A^TA + \alpha^{-1}B^TB \quad (17)$$

With the help of Itô-Lévy formula in Lemma 1 and Lemma 2, we immediately have the following theorem:

Theorem 1. *If there exists a function $V(\tilde{x}(t))$ which satisfies $V(\cdot) \in C^2$, $V(\cdot) \ge 0$, and $V(0) = 0$, the nonlinear observer gain $L(\hat{\bar{x}}(t))$ and controller gain $K(\hat{\bar{x}}(t))$ such that the following HJI holds:*

$$E\{\tilde{x}^T(t)Q\tilde{x}(t) + u^T(t)Ru(t) + V_{\tilde{x}}^T[\tilde{F}(\tilde{x}(t)) + \tilde{B}u(t)] + \tfrac{1}{4\rho^2}V_{\tilde{x}}^T\tilde{D}(\tilde{x}(t))\tilde{D}^T(\tilde{x}(t))V_{\tilde{x}} + \tfrac{1}{2}\tilde{H}^T(\tilde{x}(t))V_{\tilde{x}\tilde{x}} \times \tilde{H}(\tilde{x}(t)) + \lambda[V(\tilde{x}(t) + \tilde{G}(\tilde{x}(t))) - V(\tilde{x}(t))]\} \le 0 \quad (18)$$

then the H_∞ observer-based attack-tolerant guidance control performance in (15) can be achieved with a prescribed disturbance attenuation level ρ^2.

Proof. See Appendix A. □

From the results in Theorem 1, the H_∞ observer-based attack-tolerant guidance control design problem is transformed to a HJI problem in (18).

IV. ROBUST H_∞ OBSERVER-BASED ATTACK-TOLERANT GUIDANCE CONTROL DESIGN VIA FUZZY MODEL METHOD

Since the construction for the H_∞ observer-based attack-tolerant guidance law of the stochastic missile guidance system with external disturbance, actuator and sensor attack in (5) needs to solve HJI in (18), which is very difficult to be solved even with numerical methods. To knock out these difficulties, the T-S fuzzy dynamic model proposed by Tagaki and Sugeno is applied to represent the nonlinear stochastic missile guidance system in (5) by interpolating several locally linearized systems [47]. This T-S fuzzy model is described by a group of IF-THEN rules and is used to deal with the H_∞ observer-based attack-tolerant guidance control design problem. The ith rule of T-S fuzzy model for the nonlinear missile guidance system with actuator and sensor attack signals in (5) could be described as follows [41], [42]:

Plant Rule i :
If $z_1(t)$ is F_{i1}, and ..., and $z_g(t)$ is F_{ig}
Then

$$\begin{aligned} dx(t) &= (A_ix(t) + B_iu(t) + D_iw(t) + D_a\gamma_a(t))dt + H_ix(t)dW(t) + G_ix(t)dN(t) \\ y(t) &= C_ix(t) + n(t) + D_s\gamma_s(t), \; i = 1, ..., l \end{aligned} \quad (19)$$

where $z_1(t), ..., z_g(t)$ are the premise variables, $F_{ij}$is the ith fuzzy set of the jth premise variable, for $i = 1, ..., l$, and l is the number of fuzzy rules. Therefore, the overall T-S fuzzy missile guidance system is inferred as follows [41], [42]:

$$\begin{aligned} dx(t) &= \textstyle\sum_{i=1}^{l} h_i(z(t))[(A_i x(t) + B_i u(t) + D_i w(t) \\ &\quad + D_a \gamma_a(t))dt + H_i x(t) dW(t) + G_i x(t) dN(t)] \\ y(t) &= \textstyle\sum_{i=1}^{l} h_i(z(t))(C_i x(t) + n(t) + D_s \gamma_s(t)) \end{aligned} \tag{20}$$

where $z(t) = [z_1(t), ..., z_g(t)]^T$, $\mu_i(z(t)) = \prod_{j=1}^{g} F_{ij}(z_j(t))$, and $h_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{j=1}^{l} \mu_j(z(t))}$, which satisfies $0 \le h_i(z(t)) \le 1$, and $\sum_{i=1}^{l} h_i(z(t)) = 1$. The physical meaning of the T-S fuzzy model in (20) is that the locally linearized missile guidance systems in (19) at different operation points are interpolated piecewisely via the fuzzy interpolation functions $h_i(z(t))$ to approximate the original nonlinear missile guidance system in (5).

Remark 6. *In this study, unlike the conventional descriptor model [32]– [40], the nonsingular dynamic models of attack signals $\gamma_a(t)$ and $\gamma_s(t)$ in (8) and (9) are to be embedded in the augmented system with T-S fuzzy system (19). In this situation, the conventional T-S fuzzy observer could be employed to precisely estimate the state variables and actuator and sensor attack signals to efficiently compensate the effect of attack signals and external disturbance for the observer-based attack-tolerant guidance control design.*

Now, the nonlinear missile guidance system in (5) can be rewritten as

$$\begin{aligned} dx(t) &= \textstyle\sum_{i=1}^{l} h_i(z(t))[(A_i x(t) + B_i u(t) + D_i w(t) \\ &\quad + D_a \gamma_a(t) + \Delta f(x(t)))dt + (H_i x(t) + \Delta H(x(t))) \\ &\quad \times dW(t) + (G_i x(t) + \Delta G(x(t)))dN(t)] \\ y(t) &= \textstyle\sum_{i=1}^{l} h_i(z(t))(C_i x(t) + n(t) + D_s \gamma_s(t)) \\ &\quad + \Delta C(x(t)) \end{aligned} \tag{21}$$

where $\Delta f(x(t)) = F(x(t)) - \sum_{i=1}^{l} h_i(z(t)) A_i x(t)$, $\Delta H(x(t)) = H(x(t)) - \sum_{i=1}^{l} h_i(z(t)) H_i x(t)$, $\Delta C(x(t)) = C(x(t)) - \sum_{i=1}^{l} h_i(z(t)) C_i x(t)$ and $\Delta G(x(t)) = G(x(t)) - \sum_{i=1}^{l} h_i(z(t)) G_i x(t)$ denote the fuzzy approximation error between the nonlinear missile guidance control system in (5) and the fuzzified missile guidance control system in (20).

Remark 7. *In the fuzzy set theory [42], there have several types of membership functions (e.g., triangular, singleton and Gaussian) and the selection depends on the application fields. In the field of fuzzy-model-based control, the trapezoidal function is a common membership function to be utilized for the design due to it's simplicity. By setting the membership function as trapezoidal functions, without the adoption of any normalization method to this membership function, the choice of such membership function can directly ensure the completeness of fuzzy controller/fuzzy observer. That is, no matter the system is at any position, the entire membership functions and interpolation functions after defuzzification process will not be zero and thus the fuzzy controller and fuzzy observer will not vanish.*

Similarly, based on T-S fuzzy system in (21), the augmented missile guidance control system in (10) can be represented by the T-S fuzzy augmented missile guidance control system:

$$\begin{aligned} d\bar{x}(t) &= \textstyle\sum_{i=1}^{l} h_i(z(t)) [(\bar{A}_i \bar{x}(t) + \bar{B}_i u(t) + \bar{D}_i \bar{w}(t)) \\ &\quad + \Delta\bar{f}(\bar{x}(t)) dt + (\bar{H}_i \bar{x}(t) + \Delta\bar{H}(\bar{x}(t))) dW(t) \\ &\quad + (\bar{G}_i \bar{x}(t) + \Delta\bar{G}(\bar{x}(t))) dN(t)] \\ y(t) &= \textstyle\sum_{i=1}^{l} h_i(z(t)) (\bar{C}_i \bar{x}(t) + \bar{E}_i \bar{w}(t)) + \Delta\bar{C}(\bar{x}(t)) \end{aligned} \tag{22}$$

where the augmented state $\bar{x}(t) = [\Gamma_a^T(t), \Gamma_s^T(t), x^T(t)]^T$, the vector $\bar{w}(t) = [\epsilon_a^T(t), \epsilon_s^T(t), w^T(t), n^T(t)]^T$, $\bar{A}_i = \begin{bmatrix} A_{\gamma_a} & 0 & 0 \\ 0 & A_{\gamma_s} & 0 \\ D_a S_a & 0 & A_i \end{bmatrix}$, $\bar{B}_i = \begin{bmatrix} 0 \\ 0 \\ B_i \end{bmatrix}$, $\bar{D}_i = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & D_i & 0 \end{bmatrix}$, $\bar{C}_i = \begin{bmatrix} 0 & D_s S_s & C_i \end{bmatrix}$, $\bar{E}_i = \begin{bmatrix} 0 & 0 & 0 & I \end{bmatrix}$, $\Delta\bar{f}(\bar{x}(t)) = \begin{bmatrix} 0 \\ 0 \\ \Delta f(x(t)) \end{bmatrix}$, $\bar{H}_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & H_i \end{bmatrix}$, $\Delta\bar{C}(\bar{x}(t)) = \Delta C(x(t))$, $\Delta\bar{H}(\bar{x}(t)) = \begin{bmatrix} 0 \\ 0 \\ \Delta H(x(t)) \end{bmatrix}$, $\bar{G}_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & G_i \end{bmatrix}$, $\Delta\bar{G}(\bar{x}(t)) = \begin{bmatrix} 0 \\ 0 \\ \Delta G(x(t)) \end{bmatrix}$, the mapping matrix $S_a = [I_{n_a}, 0, ..., 0]$, and $S_s = [I_{n_s}, 0, ..., 0]$. Then, we will estimate the state, actuator attack signal, and sensor attack signal on missile guidance system in (5) from T-S fuzzy augmented system in (22). Before the further discussion, the following theorem is proposed to address the observability of T-S fuzzy local linearized system:

Theorem 2. *For the T-S fuzzy augmented system in (21), if the local matrices (A_i, C_i) for $i = 1, ..., l$ are observable, i.e.,*

$$rank \begin{bmatrix} sI_n - A_i \\ C_i \end{bmatrix} = n, \ for\ s \in S, \tag{23}$$

and the following conditions hold

$$\begin{aligned} &eig(A_i) \cap eig(A_{\gamma_a}) = \phi, eig(A_i) \cap eig(A_{\gamma_s}) = \phi \\ &eig(A_{\gamma_a}) \cap eig(A_{\gamma_s}) = \phi \end{aligned} \tag{24}$$

$$\begin{aligned} &\text{col} \begin{bmatrix} -D_a C_{\gamma_a} \\ 0 \end{bmatrix} \cap \text{col} \begin{bmatrix} sI_n - A_i \\ C_i \end{bmatrix} = \phi \\ &for\ s \in eig(A_{\gamma_a}) \end{aligned} \tag{25}$$

$$rank \begin{bmatrix} sI_{n_a(k+1)} - A_{\gamma_a} \\ -D_a C_{\gamma_a} \end{bmatrix} = n_a(k+1), \ for\ s \in eig(A_{\gamma_a}), \tag{26}$$

$$rank\left[\begin{array}{c} sI_{n_s(k+1)}-A_{\gamma_s} \\ -D_sC_{\gamma_s} \end{array}\right]=n_s(k+1),\ for\ s\in eig(A_{\gamma_s}), \tag{27}$$

then the ith T-S fuzzy local linearized system $(\overline{A}_i, \overline{C}_i)$ in the augmented T-S fuzzy system in (22) are also observable.

Proof. See Appendix B. □

Remark 8. *The physical meaning of the conditions in (26) and (27) is that the actuator attack state $\Gamma_a(t)$ of dynamic smoothed model in (8) and sensor attack state $\Gamma_s(t)$ of dynamic smoothed model in (9) are all observable.*

Suppose the following conventional T-S fuzzy Luenberger observer is proposed to deal with the estimation of the state variables and actuator and sensor attack signals of nonlinear missile guidance system in (21) or the state of the augmented missile guidance control system in (22):

Observer rule i :
If $z_1(t)$ is F_{i1}, and ..., and $z_g(t)$ is F_{ig}
then

$$\begin{aligned} &\dot{\widehat{\overline{x}}}(t)=\overline{A}_i\widehat{\overline{x}}(t)+\overline{B}_iu(t)+L_i(y(t)-\widehat{y}(t)),\\ &\widehat{y}(t)=\overline{C}_i\widehat{\overline{x}}(t), \end{aligned} \tag{28}$$

where L_i is the observer parameters to be specified for $i = 1, ..., l$. The vector $\widehat{\overline{x}}(t)$ and $\widehat{y}(t)$ are the estimated state and the estimated measurement output for the T-S fuzzy system in (22), respectively. Then, the overall T-S fuzzy observer can be designed as :

$$\begin{aligned} \dot{\widehat{\overline{x}}}(t) &= \textstyle\sum_{i=1}^{l} h_i(z(t))(\overline{A}_i\widehat{\overline{x}}(t)+\overline{B}_iu(t)+L_i(y(t)-\widehat{y}(t)))\\ &= \textstyle\sum_{i,j=1}^{l} h_i(z(t))h_j(z(t))(\overline{A}_i\widehat{\overline{x}}(t)+\overline{B}_iu(t)\\ &\quad + L_i(\overline{C}_j(\overline{x}(t)-\widehat{\overline{x}}(t))+\Delta\bar{C}(\overline{x}(t))+\overline{E}_j\overline{w}(t))\\ \widehat{y}(t) &= \textstyle\sum_{i=1}^{l} h_i(z(t))\overline{C}_i\widehat{\overline{x}}(t) \end{aligned} \tag{29}$$

Remark 9. *For the T-S fuzzy observer in (29), the estimated state can be specified as the premise variables, i.e., $z(t) = \widehat{\overline{x}}(t)$.*

Remark 10. *In this study, we utilized the fuzzy Luenberger-type observer to simultaneously estimate the augmented state of missile guidance control system and attack signals. It is worth to point out that the fuzzy Luenberger-type observer in (29) is a nonlinear observer. In fact, by applying the parallel distributed compensation (PDC) scheme [42], the local linear Luenberger-type observer is constructed for each local system. Then, the fuzzy Luenberger-type observer in (29) can be constructed by the interpolation of these local linear Luenberger-type observers.*

Then, by choosing the estimation error as $e(t) = \overline{x}(t) - \widehat{\overline{x}}(t)$ and subtracting (29) from (22), we immediately have the corresponding estimation error dynamic of T-S fuzzy observer:

$$\begin{aligned} de(t) = \textstyle\sum_{i,j=1}^{l} & h_i(z(t))h_j(z(t))[((\overline{A}_i-L_i\overline{C}_j)e(t)\\ &+(\overline{D}_i-L_i\overline{E}_j)\overline{w}(t)+\Delta\overline{f}(\overline{x}(t))-L_i\Delta\bar{C}(\overline{x}(t)))dt\\ &+(\overline{H}_i\overline{x}(t)+\Delta\overline{H}(\overline{x}(t)))dW(t)\\ &+(\overline{G}_i\overline{x}(t)+\Delta\overline{G}(\overline{x}(t)))dN(t)] \end{aligned} \tag{30}$$

By using the estimated states from the T-S fuzzy observer in (29), the following jth rule of T-S fuzzy attack-tolerant observer-based guidance control is employed to deal with the augmented missile guidance control problem.

Guidance Control Rule j
If $z_1(t)$ is F_{j1}, and,..., $z_g(t)$ is F_{jg},
then

$$u(t)=K_j\widehat{\overline{x}}(t). \tag{31}$$

where K_j is the fuzzy controller gain, for $j = 1, ..., l$, and the overall T-S fuzzy observer-based attack-tolerant guidance control can be expressed as

$$u(t)=\sum\nolimits_{j=1}^{l} h_j(z(t))K_j\widehat{\overline{x}}(t), \tag{32}$$

Remark 11. *From the structure of fuzzy controller in (32), the controller will use the estimation of $\overline{x}(t)$ (i.e., $\widehat{\overline{x}}(t)$ in (29)) to control the missile guidance control system and eliminate the effect of attack signal. Thus, once the attack signal on actuator is estimated, the proposed fuzzy controller $u(t) = \sum_{j=1}^{l} h_j(z(t))K_j\widehat{\overline{x}}(t)$ will use information of estimated attack signal $\hat{\Gamma}_a(t)$ in $\widehat{\overline{x}}(t)$ to compensate the effect of attack signal on actuator. On the other hand, once the attack signal on sensor is estimated, the estimated attack signal on sensor $\hat{\Gamma}_s(t)$ in $\widehat{\overline{x}}(t)$ is directly used to compensate the effect of real attack signal on sensor. Further, since the H_∞ observer-based attack-tolerant guidance control performance in (15) is considered in the design, the effect of $\overline{w}(t)$ on guidance control performance and estimation performance is efficiently attenuated by the proposed fuzzy controller and observer simultaneously.*

To simplify the design, the augmented T-S fuzzy missile guidance control system in (22) and the estimation error dynamic of T-S fuzzy observer in (30) are augmented as one new state variables $\widetilde{x}(t) = [\overline{x}^T(t)\ e^T(t)]^T$. Then, we immediately obtain the following state space model of $\widetilde{x}(t)$:

$$\begin{aligned} d\widetilde{x}(t) = \textstyle\sum_{i,j=1}^{l} & h_i(z(t))h_j(z(t))[(\widetilde{A}_{ij}\widetilde{x}(t)+\widetilde{D}_i\bar{w}(t)\\ &+\Delta\widetilde{f}(\widetilde{x}(t))+\widetilde{I}_i\Delta\widetilde{C}(\widetilde{x}(t)))dt\\ &+(\widetilde{H}_i\widetilde{x}(t)+\Delta\widetilde{H}(\widetilde{x}(t)))dW(t)\\ &+(\widetilde{G}_i\widetilde{x}(t)+\Delta\widetilde{G}(\widetilde{x}(t)))dN(t)] \end{aligned} \tag{33}$$

where $\widetilde{A}_{ij} = \begin{bmatrix} \overline{A}_i+\overline{B}_iK_j & -\overline{B}_iK_j \\ 0 & \overline{A}_i-L_i\overline{C}_j \end{bmatrix}$, $\widetilde{G}_i = \begin{bmatrix} \overline{G}_i & 0 \\ \overline{G}_i & 0 \end{bmatrix}$, $\widetilde{H}_i = \begin{bmatrix} \overline{H}_i & 0 \\ \overline{H}_i & 0 \end{bmatrix}$ $\Delta\widetilde{f}(\widetilde{x}(t)) = [\Delta\overline{f}^T(\overline{x}(t))\ \Delta\overline{f}^T(\overline{x}(t))]^T$, $\Delta\widetilde{H}(\widetilde{x}(t)) = [\Delta\overline{H}^T(\overline{x}(t))\ \Delta\overline{H}^T(\overline{x}(t))]^T$,

$\Delta\widetilde{G}(\widetilde{x}(t)) = [\Delta\overline{G}^T(\overline{x}(t))\ \Delta\overline{G}^T(\overline{x}(t))]^T$, $\widetilde{D}_i = \left[\overline{D}_i^T\ (\overline{D}_i - L_i\overline{E}_j)^T\right]^T$, $\widetilde{I}_i = \left[0\ -L_i^T\right]^T$, $\Delta\widetilde{C}(\widetilde{x}(t))) = \Delta\bar{C}(\overline{x}(t))$.

Remark 12. *If the designers want to achieve a great control/estimation performance, it needs a large number of operation points to cover all the nonlinearities in nonlinear stochastic missile system to obtain a better model approximation and therefore a better control/estimation performance. However, in this case, the computational complexity of fuzzy observer and fuzzy observer-based controller will increase and it may lead to the infeasibility of the design conditions. To address this issue, recent researchers have developed a new scheme called "Mismatched Premise Membership Functions (MPMF) [52]." Under the scheme of MPMF, the number of IF-THEN rules of the controller/observer with the corresponding operation points can be freely chosen and is different than the system model. In this situation, it can reduce the number of operation points to save the computation time.*

Since the augmented disturbance $\bar{w}(t)$ which includes external disturbance, approximation errors of attack signals, and measurement noises in (33) will significantly influence observer-based state estimation and guidance control performance, H_∞ fuzzy observer-based attack-tolerant guidance control performance in (15) is employed as the design objective to efficiently attenuate the effect of $\bar{w}(t)$ and fuzzy approximation errors on the fuzzy observer-based attack-tolerant guidance control. Before the further discussion, the following assumption is made to deal with the external disturbance and fuzzy approximation error in (33)

Assumption 2. *There exists a set of scalars $\{r_i \geq 0\}_{i=1}^4$ such that the following inequalities hold*

$$\begin{array}{c}\Delta\widetilde{f}^T(\widetilde{x}(t))\Delta\widetilde{f}(\widetilde{x}(t)) \leq r_1\widetilde{x}^T(t)\widetilde{x}(t)\\ \Delta\widetilde{C}^T(\widetilde{x}(t))\Delta\widetilde{C}(\widetilde{x}(t))) \leq r_2\widetilde{x}^T(t)\widetilde{x}(t)\\ \Delta\widetilde{H}^T(\widetilde{x}(t))\Delta\widetilde{H}(\widetilde{x}(t)) \leq r_3\widetilde{x}^T(t)\widetilde{x}(t)\\ \Delta\widetilde{G}^T(\widetilde{x}(t))\Delta\widetilde{G}(\widetilde{x}(t)) \leq r_4\widetilde{x}^T(t)\widetilde{x}(t)\end{array} \quad (34)$$

By the above assumption, we immediately have the following result:

Theorem 3. *If there exists fuzzy observer gains $\{L_i\}_{i=1}^l$ and fuzzy controller gains $\{K_i\}_{i=1}^l$ in (29) and (32), respectively, and a positive-definite matrix $P = P^T > 0$ such that the following matrix inequalities hold:*

$$\Pi_{ij} < 0,\ i,j = 1,\cdots,l \quad (35)$$

$$P < \alpha I \quad (36)$$

where $\Pi_{ij} = Q + \bar{K}_j^T R\bar{K}_j + P\widetilde{A}_{ij} + \widetilde{A}_{ij}^T P + r_1 I + \alpha^2 I + r_2 I + P\widetilde{I}_i\widetilde{I}_i^T P + \frac{1}{\rho^2}P\widetilde{D}_i\widetilde{D}_i P + 2\alpha r_3 I + 2\alpha\widetilde{H}_i^T\widetilde{H}_i + \lambda[P\widetilde{G}_i + \widetilde{G}_i^T P + \alpha^2 I + r_4 I + \alpha\widetilde{G}_i^T\widetilde{G}_i + \alpha^2\widetilde{G}_i^T\widetilde{G}_i + r_4 I + \alpha r_4 I]$ *with a fixed constant $\alpha > 0$ and fuzzy approximation constants $\{r_i > 0\}_{i=1}^4$, then the H_∞ observer-based attack-tolerant guidance control performance in (15) is achieved with disturbance attenuation level ρ^2.*

Proof. See Appendix C. □

Remark 13. *In Theorem 3, for the fuzzy-model-based (FMB) control, the common quadratic Lyapunov functional is chosen, i.e., $V(\tilde{x}(t)) = \tilde{x}(t)P\tilde{x}(t)$ with positive definite symmetric matrix P. In this case, the derived design conditions in (35) are a set of Riccati-like inequalities and they can be further transformed to solvable LMIs by the proposed Two-Step design procedure in the sequel. If the non-quadratic nonlinear Lyapunov functional is chosen, the derived design conditions in (35) will become a set of nonlinear matrix inequalities which can not be easily solved via current convex optimization methods for practical application.*

Remark 14. *To reduce the conservative of matrix inequalities in (35), (36), the selection of operation points and local linearized systems becomes an important issue. Clearly, (35) is hard to be successfully solved if the fuzzy approximation error parameters $\{r_i\}_{i=1}^4$ in (34) are large. If a large number of local linearzied systems (i.e., Fuzzy IF-THEN model rules) are used to interpolate the original missile guidance system, the fuzzy approximation error parameters $\{r_i\}_{i=1}^4$ can be effectively decreased. However, at the same time, the number of matrix inequalities in (35) will increase and the corresponding feasibility will reduce. As a result, in the fuzzy-model-based control, there exists a trade-off between the number of IF-THEN model rules in (19) and the feasibility of derived matrix inequalities in (35).*

For the matrix inequalities in (35), due to the coupling of design variables (i.e., P, $\{L_i\}_{i=1}^l$ and $\{K_i\}_{i=1}^l$), the matrix inequalities are bilinear inequalities and it could not be solved via any current optimization method. To deal with this problem, the following two-step design procedure is developed. To begin with, by choosing the Lyapunov function as $V(\tilde{x}(t)) = \overline{x}^T(t)P_1\overline{x}(t) + e^T(t)P_2 e(t) = \tilde{x}^T(t)P\tilde{x}(t)$ with $P = diag\{P_1, P_2\}$ and applying Schur complement to (35), the matrix inequality in (35) can be written as:

$$\begin{bmatrix}\Pi_{ij}^1 & \Pi_{ij}^2\\ * & \Pi_{ij}^3\end{bmatrix} < 0,\ \forall i,j = 1,\cdots,l \quad (37)$$

where $\Pi_{ij}^1 = Q + P\widetilde{A}_{ij} + \widetilde{A}_{ij}^T P + \alpha^2 I + r_1 I + 2\alpha r_3 I + r_2 I + 2\alpha\widetilde{H}_i^T\widetilde{H}_i + \lambda[P\widetilde{G}_i + \widetilde{G}_i^T P + \alpha^2 I + r_4 I + \alpha\widetilde{G}_i^T\widetilde{G}_i + \alpha^2\widetilde{G}_i^T\widetilde{G}_i + r_4 I + \alpha r_4 I]$, $\Pi_{ij}^2 = [\bar{K}_j^T\ P\widetilde{I}_i\ P\widetilde{D}_i]$ and $\Pi_{ij}^3 = diag\{-R^{-1}, -I, -\rho^2 I\}$.

The detailed two-step design procedure is investigated as follows:

(STEP 1) By the definition of negative-definite matrix, all the diagonal terms in (37) must be negative if (37) hold. As a result, we firstly solve the (2,2) term in Π_{ij}^1 to obtain the design variables P_2 and $\{L_i\}_{i=1}^l$. By substituting the system matrices in (33), Π_{ij}^1 can be unfolded as follows:

$$\Pi_{ij}^1 = \begin{bmatrix}\Pi_{ij}^{1,1} & \Pi_{ij}^{1,2}\\ * & \Pi_{ij}^{1,3}\end{bmatrix} \quad (38)$$

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where $\Pi_{ij}^{1,1} = P_1(\overline{A}_i + \overline{B}_i K_j) + (\overline{A}_i + \overline{B}_i K_j)^T P_1 + Q_1' + 2\alpha \overline{H}_i^T \overline{H}_i + 2\lambda(\alpha+\alpha^2)\overline{G}_i^T \overline{G}_i + \lambda(P_1\overline{G}_i + \overline{G}_i^T P_1) + [\alpha^2 + r_1 + 2\alpha r_3 + r_2 + \lambda(\alpha^2 + 2r_4 + \alpha r_4)]I$, $\Pi_{ij}^{1,2} = -P_1\overline{B}_i K_j + \lambda \overline{G}_i^T P_2$ and $\Pi_{ij}^{1,3} = P_2(\overline{A}_i - L_i\overline{C}_j) + (\overline{A}_i - L_i\overline{C}_j)^T P_2 + Q_2 + [\alpha^2 + r_1 + 2\alpha r_3 + r_2 + \lambda(\alpha^2 + 2r_4 + \alpha r_4)]I$

Then, the constraints $\Pi_{ij}^{1,3}$ in Π_{ij}^1 is considered to be solved:

$$\begin{aligned}&[\alpha^2 + r_1 + 2\alpha r_3 + r_2 + \lambda(\alpha^2 + 2r_4 + \alpha r_4)]I \\ &+Q_2 + P_2(\overline{A}_i - L_i\overline{C}_j) + (\overline{A}_i - L_i\overline{C}_j)^T P_2 < 0\end{aligned} \tag{39}$$

Choose the slack variables $\{Y_i = P_2 L_i\}_{i=1}^l$, the matrix inequalities in (39) become the following LMIs:

$$\begin{aligned}&[\alpha^2 + r_1 + 2\alpha r_3 + r_2 + \lambda(\alpha^2 + 2r_4 + \alpha r_4 I)]I \\ &+Q_2 + P_2\overline{A}_i - Y_i\overline{C}_j + \overline{A}_i^T P_2 - \overline{C}_j^T Y_i^T < 0 \\ &\forall i,j = 1,\cdots,l\end{aligned} \tag{40}$$

Also, the eigenvalue constraint in (36) associated with P_2 can be written as following LMI:

$$P_2 < \alpha I \tag{41}$$

Then, we could easily solve (40) and (41) via MATLAB LMI TOOLBOX to obtain $\{Y_i = P_2 L_i\}_{i=1}^l$ and P_2. Moreover, the fuzzy observer gains can be obtained by $\{L_i = P_2^{-1} Y_i\}_{i=1}^l$.

(STEP 2) By pre-multiplying and post-multiplying the matrix $diag\{\bar{W}_1, I, I, I\}$ to (37) with $\bar{W}_1 = diag\{W_1, W_1\}$ and $W_1 = P_1^{-1}$ and using Schur Complement, (37) is equivalent to the following matrix inequalities

$$\begin{bmatrix} \bar{\Pi}_{ij}^1 & \bar{\Pi}_{ij}^2 \\ * & \bar{\Pi}_{ij}^3 \end{bmatrix} < 0,\ \forall i,j = 1,\cdots,l \tag{42}$$

with

$$\begin{aligned}\bar{\Pi}_{ij}^1 &= \bar{W}_1 P\widetilde{A}_{ij}\bar{W}_1 + \bar{W}_1\widetilde{A}_{ij}^T P\bar{W}_1 + \lambda[\bar{W}_1 P\widetilde{G}_i\bar{W}_1 \\ &\quad + \bar{W}_1\widetilde{G}_i^T P\bar{W}_1] \\ \bar{\Pi}_{ij}^2 &= [\bar{W}_1\bar{K}_j^T\ \bar{W}_1 P\widetilde{I}_i\ \bar{W}_1 P\widetilde{D}_i \\ &\qquad \bar{W}_1 Q^{\frac{1}{2}}\ \bar{W}_1\widetilde{H}_i^T\ \bar{W}_1\widetilde{G}_i^T\ \bar{W}_1] \\ \bar{\Pi}_{ij}^3 &= diag\{-R^{-1}, -I, -\rho^2 I, -I, -\tfrac{1}{2\alpha}I\cdots \\ &\qquad \cdots, -\tfrac{1}{\alpha+\alpha^2}I, -\bar{r}^{-1}I\} \\ \bar{r} &= \alpha^2 + r_1 + 2\alpha r_3 + r_2 + \lambda(r_4 + \alpha r_4 + \alpha^2 + r_4)\end{aligned}$$

Then, to decouple the bilnear term $\lambda[W_1\overline{G}_i^T P_2 W_1]$ and $\lambda W_1 P_2\overline{G}_i W_1$ in $\lambda[\bar{W}_1 P\widetilde{G}_i\bar{W}_1 + \bar{W}_1\widetilde{G}_i^T P\bar{W}_1]$, the following inequality can be obtained by Lemma 2:

$$\begin{aligned}&x^T[\lambda W_1\overline{G}_i^T P_2 W_1]y + y^T[\lambda W_1 P_2\overline{G}_i W_1]x \\ &\le x^T[\lambda W_1\overline{G}_i^T\overline{G}_i W_1]x + y^T[\lambda W_1 P_2 P_2 W_1]y\end{aligned} \tag{43}$$

where x and y are arbitrary vectors with appropriate dimensions. Then, by (43), the matrix inequalities $\lambda[\bar{W}_1 P\widetilde{G}_i\bar{W}_1 + \bar{W}_1\widetilde{G}_i^T P\bar{W}_1]$ can be relaxed as:

$$\begin{aligned}&\lambda[\bar{W}_1 P\widetilde{G}_i\bar{W}_1 + \bar{W}_1\widetilde{G}_i^T P\bar{W}_1] \\ &= \lambda\begin{bmatrix} W_1\overline{G}_i^T + \overline{G}_i W_1 & W_1\overline{G}_i^T P_2 W_1 \\ * & 0 \end{bmatrix} \\ &\le \lambda\begin{bmatrix} W_1\overline{G}_i^T + \overline{G}_i W_1 \\ +W_1\overline{G}_i^T\overline{G}_i W_1 & 0 \\ * & W_1 P_2 P_2 W_1 \end{bmatrix}\end{aligned} \tag{44}$$

By applying Schur complement to (42) with (44) and setting new variable $\{N_j = K_j W_1\}_{j=1}^l$, the matrix inequalities (42) become the following matrix inequalities

$$\begin{bmatrix} \tilde{\Pi}_{ij}^1 & \tilde{\Pi}_{ij}^2 & \tilde{\Pi}_{ij}^3 & \tilde{\Pi}_{ij}^4 \\ * & \tilde{\Pi}_{ij}^5 & \tilde{\Pi}_{ij}^6 & \tilde{\Pi}_{ij}^7 \\ * & * & \tilde{\Pi}_{ij}^8 & 0 \\ * & * & * & \tilde{\Pi}_{ij}^9 \end{bmatrix} < 0,\ \forall i,j = 1,\cdots,l \tag{45}$$

with

$$\begin{aligned}\tilde{\Pi}_{ij}^1 &= \lambda(\overline{G}_i W_1 + W_1\overline{G}_i^T) + \overline{A}_i W_1 + \overline{B}_i N_j \\ &\quad + W_1\overline{A}_i^T + N_j^T\overline{B}_i^T \\ \tilde{\Pi}_{ij}^2 &= -\overline{B}_i N_j \\ \tilde{\Pi}_{ij}^3 &= \begin{bmatrix} N_j^T & 0 & \overline{D}_i & W_1(Q_1')^{\frac{1}{2}} & 0 \end{bmatrix} \\ \tilde{\Pi}_{ij}^4 &= \begin{bmatrix} W_1\overline{H}_i^T & W_1\overline{H}_i^T & W_1\overline{G}_i^T & W_1\overline{G}_i^T & \cdots \\ & \cdots & W_1 & 0 & W_1\overline{G}_i^T & 0 \end{bmatrix} \\ \tilde{\Pi}_{ij}^5 &= W_1 P_2(\overline{A}_i - L_i\overline{C}_j)W_1 + W_1(\overline{A}_i - L_i\overline{C}_j)^T P_2 W_1 \\ \tilde{\Pi}_{ij}^6 &= \begin{bmatrix} N_j^T & W_1 Y_i & W_1 P_2(\overline{D}_i - L_i\overline{E}_j) & 0 & W_1 Q_2{}^{\frac{1}{2}} \end{bmatrix} \\ \tilde{\Pi}_{ij}^7 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & W_1 & 0 & W_1 P_2 \end{bmatrix} \\ \tilde{\Pi}_{ij}^8 &= diag\{-R^{-1}, -I, -\rho^2 I, -I, -I\} \\ \tilde{\Pi}_{ij}^9 &= diag\{-\tfrac{1}{2\alpha}I, -\tfrac{1}{2\alpha}I, -\tfrac{1}{\alpha+\alpha^2}I, -\tfrac{1}{\alpha+\alpha^2}I \\ &\qquad , -\bar{r}^{-1}I, -\bar{r}^{-1}I, -\lambda^{-1}I, -\lambda^{-1}I\}\end{aligned}$$

Furthermore, by choosing a set of positive one dimension slack variables $\{\varphi_{ij} > 0\}_{i,j=1}^l$ and a predefined constant $\alpha_1 > 0$, the following constraints are made to decouple the bilinear term in $\tilde{\Pi}_{ij}^5$:

$$\begin{aligned}&P_2(\overline{A}_i - L_i\overline{C}_j) + (\overline{A}_i - L_i\overline{C}_j)^T P_2 < -\varphi_{ij} \\ &\alpha_1 I < W_1 \\ &\forall i,j = 1,\cdots,l\end{aligned} \tag{46}$$

From the second inequality in (46), we immediately have following result:

$$-W_1 W_1 < -\alpha_1^2 I \tag{47}$$

By the above constraints in (46), (47), $\tilde{\Pi}_{ij}^5$ can be released as follows:

$$\begin{aligned}&W_1(P_2(\overline{A}_i - L_i\overline{C}_j) + (\overline{A}_i - L_i\overline{C}_j)^T P_2)W_1 \\ &< -\varphi_{ij} W_1 W_1 \\ &< -\varphi_{ij}\alpha_1^2 I \\ &\forall i,j = 1,\cdots,l\end{aligned} \tag{48}$$

Then, with the LMIs in (48), the matrix inequalities in (45) can be reformulated as following LMIs:

$$\begin{bmatrix} \tilde{\Pi}_{ij}^1 & \tilde{\Pi}_{ij}^2 & \tilde{\Pi}_{ij}^3 & \tilde{\Pi}_{ij}^4 \\ * & \tilde{\Pi}_{ij}^{5*} & \tilde{\Pi}_{ij}^6 & \tilde{\Pi}_{ij}^7 \\ * & * & \tilde{\Pi}_{ij}^8 & 0 \\ * & * & * & \tilde{\Pi}_{ij}^9 \end{bmatrix} < 0,\ \forall i,j = 1,\cdots,l \tag{49}$$

with $\tilde{\Pi}_{ij}^{5*} = -\varphi_{ij}\alpha_1^2 I$.

Also, by using schur complement to (36), the eigenvalue constraint in (36) associated with $W_1 = P_1^{-1}$ can be written as following LMI:

$$P_1 \le \alpha I \Leftrightarrow \begin{bmatrix} \alpha I & I \\ I & W_1 \end{bmatrix} \ge 0 \tag{50}$$

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By using the MATLAB LMI TOOLBOX, the LMIs in (46), (49), (50) could be easily solved to obtain the design variables to obtain $\{\varphi_{ij}, N_j\}_{i,j=1}^{l}$ and W_1. Moreover, the fuzzy controller gains can be obtained by $\{K_j = N_j W_1^{-1}\}_{i=1}^{l}$.

Based on the above discussion, the robust H_∞ fuzzy observer-based attack-tolerant control for stochastic missile guidance system can be solved via the proposed two-step design procedure. Moreover, to achieve the optimal disturbance attenuation level, the following optimization problem is formulated:

$$\min_{W_1>0, P_2>0, \{Y_i, N_j, \varphi_{ij}>0\}_{i,j=1}^{l}} \rho^2 \quad s.t. \ (40),(46),(49) \tag{51}$$

The above LMIs-constrained optimization problem is also called eigenvalue problem (EVP) and can be solved very efficiently by convex optimization algorithm [48]. More specifically, this problem can be solved by decreasing ρ^2 until $W_1 > 0$ and $P_2 > 0$ cannot be found in (40) and (49).

The optimal H_∞ observer-based attack-tolerant missile guidance control design procedure is summarized as follows:

STEP I: Select fuzzy plant rules and membership functions for nonlinear missile guidance system (5), and model the actuator attack signal in (8) and sensor attack signal in (9) with appropriate extrapolation parameters $\{a_i, b_j\}_{i,j=0}^{k}$.

STEP II: Construct the fuzzy approximation fuzzy approximation constants $\{r_i \geq 0\}_{i=1}^{4}$ and select fixed constants $\alpha, \alpha_1 > 0$.

STEP III: Select the weighting matrices Q_1, Q_2,and R in the H_∞ observer-based attack-tolerant guidance control performance in (15).

STEP IV: Select the attenuation level ρ^2 and solve LMIs in (40), (41) to obtain P_2 and $\{Y_i\}_{i=1}^{l}$.

STEP V: Substitute P_2 and $\{Y_i\}_{i=1}^{l}$ into (46), (49), and solve (46), (49), (50) to obtain W_1, $\{\varphi_{ij}\}_{i,j=1}^{l}$ and $\{N_j\}_{j=1}^{l}$

STEP VI: Decrease ρ^2 and repeat Steps IV–VI until W_1 and P_2 can not be found.

STEP VII: Construct the fuzzy observer gains $\{L_i = P_2^{-1} Y_i\}_{i=1}^{l}$ in (28) and fuzzy guidance control law $\{K_j = N_j W_1^{-1}\}_{i=1}^{l}$ in (32).

V. SIMULATION EXAMPLE

A. SIMULATION PARAMETER SETTING

The following example is given to verify the missile guidance performance of the proposed robust H_∞ fuzzy observer-based attack-tolerant missile guidance control law of nonlinear stochastic missile guidance system in (5) under actuator and sensor attack signals and external disturbance. To confirm the guidance performance and the robustness of the fuzzy H_∞ observer-based attack-tolerant missile guidance control law, we set the external disturbances $w(t)$ due to step maneuvering as follows [10], [44]:

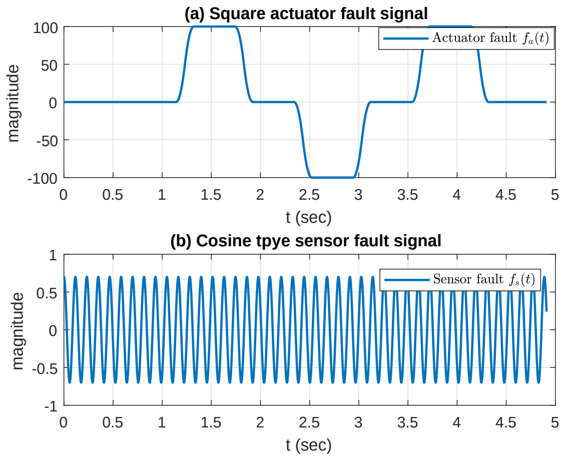

FIGURE 2: (a) the square actuator attack signal. (b) the cosine-type sensor attack signal.

$$\begin{aligned} w_r(t) &= \chi_T r(t) \\ w_\theta(t) &= \chi_T \frac{-\dot{\phi}(t)}{\sqrt{\dot{\phi}^2(t) + \dot{\theta}(t)\cos^2\phi(t)}} \theta(t) \\ w_\phi(t) &= \chi_T \frac{\dot{\theta}(t)\cos\phi(t)}{\sqrt{\dot{\phi}^2(t) + \dot{\theta}(t)\cos^2\phi(t)}} \phi(t) \end{aligned} \tag{52}$$

where χ_T denotes the target's navigation random gain within 0 to 20. The initial condition of the missile guidance system in (5) is given as:

$$x_0 = [4900, \pi/3, \pi/3, -1000, 500, 500]^T, \tag{53}$$

with the linear output matrix $C(x(t)) = I_6$ and the measurement noise $n(t) = 0.5 \times \cos(0.05t) \times [1, 1, 1, 1, 1, 1]^T$.

In the nonlinear stochastic missile guidance system in (5), the matrices of stochastic effect are defined as $H(x(t)) = 0.3 \times [0\,0\,0\,0\,V_\theta\,V_\phi]^T$, $G(x(t)) = 0.1 \times [0\,0\,0\,0\,V_\theta\,V_\phi]^T$, and Poisson jump intensity $\lambda = 0.7$. Besides, the actuator attack matrix is set as $D_a = [0\;0\;0\;0\;1\;1]^T$ and the sensor attack matrix is set as $D_s = [0\;0\;0\;2\;2\;2]^T$. Suppose the missile suffers an equivalent actuator attack signal from target by side-step maneuvering due to two-side jets of target and the sensor of seeker of missile also suffers from the sinusoid signal attack from target as shown in Fig.2.

Based on the design procedure, we design a robust H_∞ observer-based attack-tolerant guidance control law via the following steps.

Step I) In this study, we use 48 rules based on the following fuzzy premise variables $z_1(t) = r(t)$, $z_2(t) = \phi(t)$, $z_3(t) = V_\theta(t)$, and $z_4(t) = V_\phi(t)$ with the corresponding

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fuzzy operation points:

$$
\begin{aligned}
& r_i = 598.2,\ for\ i = 1-24 \\
& r_i = 2558.5,\ for\ i = 25-48 \\
& \phi_i = -0.6441,\ \text{for } i = 1-12, i = 25-36 \\
& \phi_i = 1.2771,\ \text{for } i = 13-24, i = 37-48 \\
& V_{\theta,i} = -49.2, \\
& \text{for } i = 1-4, i = 13-16, i = 25-38, i = 37-40 \\
& V_{\theta,i} = 77, \\
& \text{for } i = 5-8, i = 17-20, i = 29-32, i = 41-44 \\
& V_{\theta,i} = 555.1, \\
& \text{for } i = 9-12, i = 20-24, i = 33-36, i = 45-48 \\
& V_{\phi,i} = -121, \text{for } i = 1+4d,\ V_{\phi,i} = 0, \text{for } i = 2+4d \\
& V_{\phi,i} = 135.3, \text{for } i = 3+4d,\ V_{\phi,i} = 310.5, \text{for } i = 4+4d \\
& where\ d = 0-11
\end{aligned}
$$

and the ith IF-THEN rule of T-S fuzzy system for the stochastic nonlinear missile guidance system in (5) is given as follows:

$$
\begin{aligned}
& \text{If } z_1(t) \text{ is } F_{i1}, \text{ and ..., and } z_g(t) \text{ is } F_{ig} \\
& \text{Then} \\
& dx(t) = (A_i x(t) + B_i u(t) + D_i w(t) + D_a \gamma_a(t))dt \\
& \qquad + H_i x(t) dW(t) + G_i x(t) dN(t) \\
& y(t) = C_i x(t) + n(t) + D_s \gamma_s(t),\ i = 1, ..., 48
\end{aligned} \tag{54}
$$

To model the actuator attack signal and sensor attack signal, a fourth order smoothed model in (8) is employed for actuator attack signal and a fourth order smoothed model in (9) is employed for sensor attack signal as follows:

$$
A_{\gamma_a} = \begin{bmatrix} \frac{\bar{a}_0}{\tau} & \frac{a_1}{\tau} & \frac{a_2}{\tau} & \frac{a_3}{\tau} \\ \frac{1}{\tau} & -\frac{1}{\tau} & 0 & 0 \\ 0 & \frac{1}{\tau} & -\frac{1}{\tau} & 0 \\ 0 & 0 & \frac{1}{\tau} & -\frac{1}{\tau} \end{bmatrix},
$$

$$
A_{\gamma_s} = \begin{bmatrix} \frac{\bar{b}_0}{\tau} & \frac{b_1}{\tau} & \frac{b_2}{\tau} & \frac{b_3}{\tau} \\ \frac{1}{\tau} & -\frac{1}{\tau} & 0 & 0 \\ 0 & \frac{1}{\tau} & -\frac{1}{\tau} & 0 \\ 0 & 0 & \frac{1}{\tau} & -\frac{1}{\tau} \end{bmatrix}.
$$

where $\bar{a}_0 = -1 + a_0$, $\bar{b}_0 = -1 + b_0$ with the specified extrapolation parameters $\tau = 0.001$, $a_0 = 0.5$, $a_1 = 0.35$, $a_2 = 0.1$, $a_3 = 0.05$, $b_0 = 0.6$, $b_1 = 0.25$, $b_2 = 0.12$, and $b_3 = 0.03$.

STEP II) By considering the approximation error, the upper bounds of fuzzy approximation errors in (34) are also known to be $r_1 = 2.5 \times 10^{-3}$, $r_2 = 0$, $r_3 = 0$, $r_4 = 0$. Besides, the constant α and α_1 are chosen as 10 and 1, respectively.

STEP III) The weighting matrices in the H_∞ observer-based attack-tolerant guidance control performance in (15) are respectively selected as follows:

$$
\begin{aligned}
& Q_1 = 0.0001 \times I_2, Q_2 = 0.0001 \times I_{14}, \\
& R = 0.001 \times I_2,
\end{aligned}
$$

STEP IV-VI) The optimal disturbance attenuation level $\rho^2 = 21.8$ is found after 122 iterations using the LMI optimization TOOLBOX in MATLAB. In this case, we obtain the common solution for $W_1, P_2, \{Y_i, N_j\}_{i,j=1}^{48}$.

STEP VII) Construct the fuzzy observer gains $\{L_i = P_2^{-1} Y_i\}_{i=1}^{48}$ in (28) and fuzzy guidance control law $\{K_j = N_j W_1^{-1}\}_{i=1}^{48}$ in (32).

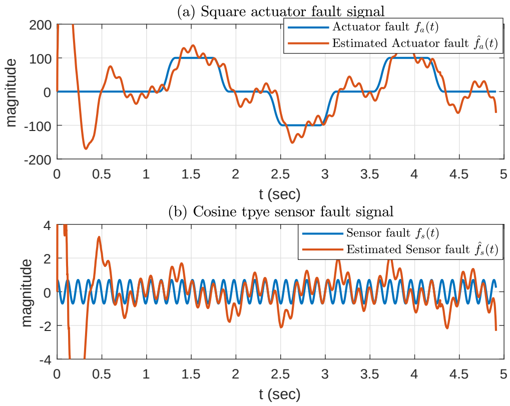

FIGURE 3: (a) the square actuator attack signal and its estimation. (b) the cosine-type sensor attack signal and its estimation.

B. SIMULATION RESULT

In the simulation example, the target tries to avoid the attack of missile by performing sudden side-step maneuvering by its two-side jets and transmit the jamming signal to interfere the sensor of seeker in missile, which could lead to equivalent actuator attack and sensor attack on missile. In Fig. 3, the equivalent actuator attack signal due to the sudden side-step maneuvering by two-side jets and sensor attack signal are shown as the square signal and cosine signal (blue line), respectively. In the beginning, the large estimation error on the missile guidance system state is found due to a large initial condition. Then, the robust H_∞ fuzzy observer can estimate the attack signals precisely. However, there still have some small fluctuations at steady state due to the effect of attack signals, Poisson and Wiener random fluctuation even they are significantly attenuated by the proposed H_∞ fuzzy observer-based attack-tolerant guidance control scheme. From the structure of Luenberger-type observer, the estimation of state variable interacts with the estimation of two attack signals. Thus, from the estimation result, the estimation of actuator attack signal is slightly fluctuated due to the effect of cosine sensor attack signal. Even there has a small estimation error of square actuator attack signal, the estimated square actuator attack signal can be used to efficiently eliminate the real square actuator attack signal.

The states of the stochastic missile guidance system and the corresponding estimated states by the proposed fuzzy observer in (29) are plotted in Figs. 4–8. From the results in Figs. 4–7, we can know that the fuzzy observer-based attack-tolerant controller of tactical missile could track the target successfully in a very short time. From Figs. 4–5, we can see

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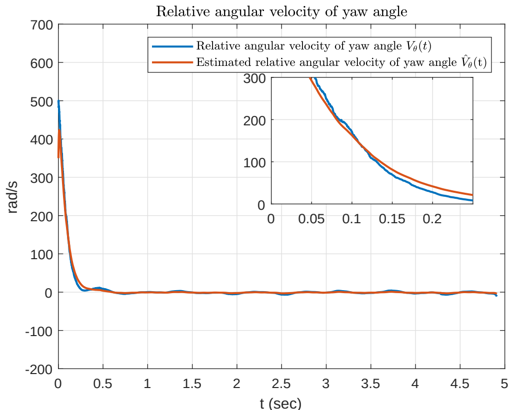

FIGURE 4: The relative yaw angular velocity and its estimation of stochastic missile guidance system by the proposed method under the effect of the square actuator attack signal and cosine-type sensor attack signal. The zoom-in figure shows the effect of stochastic fluctuation. By the proposed robust observer-based FTC control strategy, these effects can be efficiently attenuated.

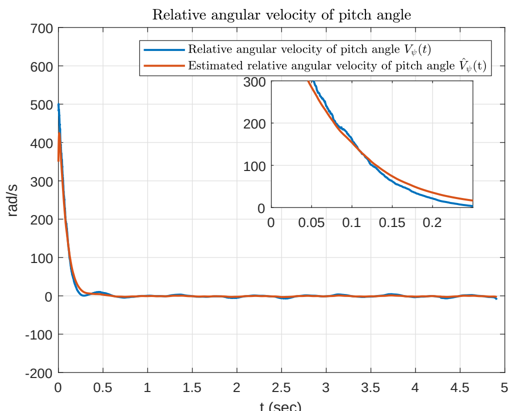

FIGURE 5: The relative pitch angular velocity and its estimation of stochastic missile guidance system by the proposed method under the effect of the square actuator attack signal and cosine-type sensor attack signal. The zoom-in figure shows the effect of stochastic fluctuation. By the proposed robust observer-based FTC control strategy, these effects can be efficiently attenuated.

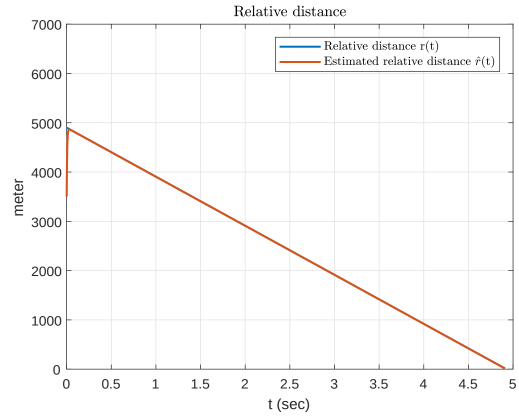

FIGURE 6: The relative distance and its estimation of stochastic missile guidance system by the proposed method under the effect of the square actuator attack signal and cosine-type sensor attack signal from the target.

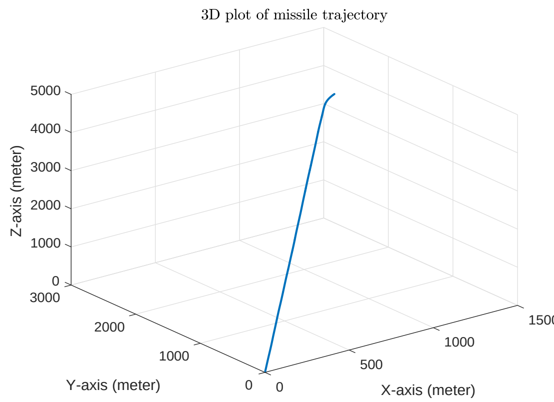

FIGURE 7: The 3D relative distance between the missile and the target. Due to the initial responses of angular velocities, the missile slight turns around to find the direction of the target. After the target's direction is locked (i.e., the relative angular velocities are controlled to zero), the missile approaches the target with the head on condition. Finally, the missile successfully hits the target at 4.9s.

the influence of random fluctuations, due to Wiener processes and Poisson jump processes, on two angular velocities can be eliminated by the proposed robust H_∞ observer-based guidance control method. As shown in Fig. 3, the target begins to perform sudden side-step maneuvering from about $1.5s$ and to interfere the sensor on the seeker of missile from the beginning. Because the proposed robust H_∞ observer-based attack-tolerant missile guidance controller can estimate the state variables of missile and actuator attack signal quickly for attack-tolerant guidance control of missile system, the effect of actuator attack can be cancelled out by the proposed H_∞ observer-based attack-tolerant guidance controller. For the sensor attack signal, Figs. 4–5 reveal that the relative angular velocity of yaw angle and relative angular velocity of pitch angle fluctuate around the real states slightly in the guidance control process.

In Fig. 6, by the proposed method, the missile can hit the target successfully at about $4.9s$ on the head-on condition. Once the target is hit, attack signals have vanished after $4.9s$. From the 3-D graph in Fig. 7, the missile slightly spins itself to locate the position of target at the initial and thereafter it can approach the target with a fixed direction in the rest of guidance process. In Fig. 8, both guidance control strategies $u_\theta(t)$, and $u_\phi(t)$ on two velocities $V_\theta(t)$ and $V_\phi(t)$ approach zero and fluctuate around zero quickly to eliminate the effect of acturator attack signal. Generally, because of external disturbance, attack signals, and continuous and discontinuous random fluctuations, it is much difficult to achieve the head-

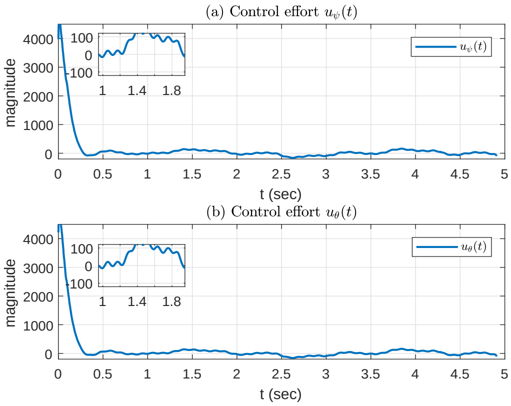

FIGURE 8: The control signal. Once the actuator attack signals appear in the system, the control inputs will employ estimated actuator attack signal to eliminate the effect of real actuator attack signal.

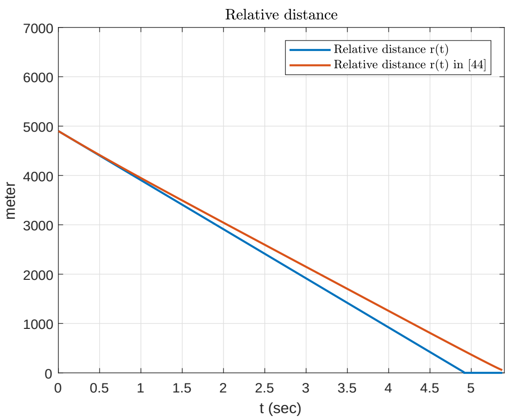

FIGURE 9: The relative distance controlled by the proposed method and conventional robust fuzzy missile guidance control strategy in [44].

on condition for the missile. By the proposed H_∞ observer-based attack-tolerant guidance controller, the effect of external disturbance, attack signals, Wiener process and Poisson jump processes on the missile guidance system is greatly reduced and the missile can hit the target successfully.

C. COMPARISON BETWEEN OUR METHOD AND CONVENTIONAL ROBUST FUZZY MISSILE GUIDANCE CONTROL STRATEGY

For the comparison between the conventional guidance control design and our design, the conventional robust fuzzy missile guidance control strategy in [44] is carried out. Since the attack signals are not estimated in the conventional robust fuzzy missile guidance control strategy, these attack signals are considered as a kind of external disturbance and their effects are passively attenuated by the conventional robust H_∞ guidnace controller.

The simulation results by the method in [44] are shown in

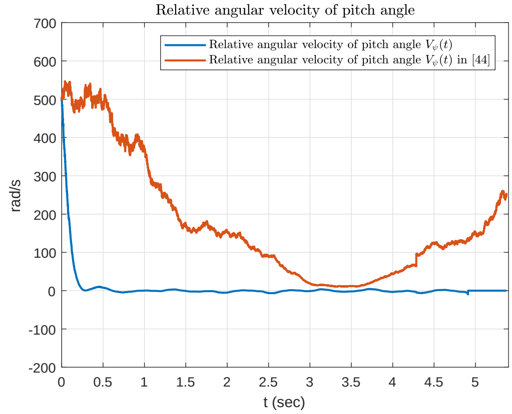

FIGURE 10: The relative angular velocity of pitch angle controlled by the proposed method and conventional robust fuzzy missile guidance control strategy in [44].

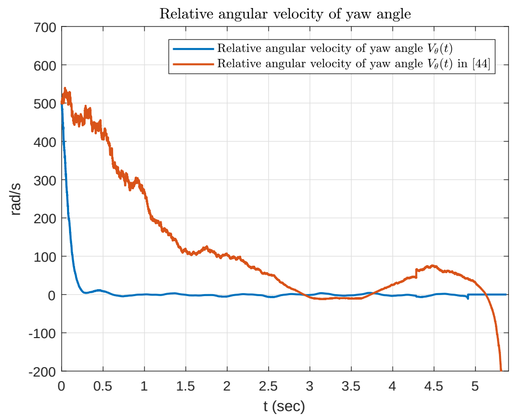

FIGURE 11: The relative angular velocity of yaw angle controlled by the proposed method and conventional robust fuzzy missile guidance control strategy in [44].

Figs. 9–11. At the initial stage, due to the effect of continuous Wiener process, the relatively velocities of pitch angle and yaw angle are fluctuated and the conventional guidance controller slowly controls these two state variables. When the Poisson jump occurs in the system, the conventional controller spends more time to reduce this discontinuous effect. Once the actuator attack signal enters the missile guidance control system, the conventional controller can not directly eliminate the effect of these attack signals but passively attenuates them. From Figs. 9–11, if the stochastic Wiener and Poisson processes and attack signals are considered in the missile guidance control system, it can be inferred that the conventional robust H_∞ controller is hard to meet the head-on condition during the guidance control process.

If the head-on condition is not satisfied during the entire guidance control process, the relative velocity will decrease and it will increase the terminal time of guidance control. Compared with our method, the conventional robust H_∞

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fuzzy controller in [44] spends more 0.5s to enter the area of the explosion than our methods. It is worth to point out that if the missile is very close to the target and the head-on condition is not satisfied at that time, the relative angular velocities of yaw angle and pitch angle are much difficult to be controlled and these two angular velocities are more likely to diverge, i.e., the missile is more like to pass through the target.

VI. CONCLUSION

Based on the proposed dynamic smoothed attack signal model, the H_∞ observer-based guidance control technique and T-S fuzzy interpolation technique are combined to achieve the optimal H_∞ observer-based attack-tolerant guidance control performance for stochastic nonlinear dynamic missile systems with external disturbance as well as actuator and sensor attack signals. Through the proposed Two-Step design procedure, the H_∞ fuzzy observer-based attack-tolerant missile guidance control design problem is reduced to solving a set of LMIs. The optimal H_∞ fuzzy observer-based attack-tolerant control of missile guidance system is formulated as an EVP, which could be efficiently solved with the help of LMI TOOLBOX in MATLAB via convex optimization algorithm. Then, a design procedure is also proposed for the fuzzy observer-based attack-tolerant control to achieve the optimal robust H_∞ guidance control design of the stochastic nonlinear missile guidance control system under actuator and sensor attack signals as well as external disturbance and continuous and discontinuous random fluctuations. Simulations results indicate that the desired robust H_∞ observer-based guidance control performance for stochastic nonlinear missile guidance system with actuator and sensor attack signals and external disturbance can be achieved via the proposed method. Recently, due to the advance of network control system, the information of missile control system can be transmitted to ground control center (GCC) and the guidance control command can be calculated at GCC. In this case, the power consumption in the missile guidance control can be effectively reduced. However, since the information of missile guidance control system is transmitted via wireless channel, there will have some undesired effects during the information transmission, e.g., data dropout (packet drop) and network-induced delay. On the other hand, different than the conventional single missile system, the multi-missile system is a popular issue and it has been widely addressed by researchers in recent years [49]–[51]. In this situation, to achieve more difficult missions, a set of missile systems are controlled to maintain the desired formation during the maneuvering process. Hence, the future research direction will focus on the missile guidance design with network control mechanism and multi-missile formation guidance control design.

Appendix A: Proof of Theorem 1

By using Lemma 1, the numerator of the H_∞ observer-based attack-tolerant guidance control performance in (15) can be written as:

$$\begin{aligned}
&E\left\{\int_0^{t_f} \widetilde{x}^T(t)Q\widetilde{x}(t)+u^T(t)Ru(t)\,dt\right\}\\
&=E\{V(\widetilde{x}(0))-V(\widetilde{x}(t_f))\}+E\{\textstyle\int_0^{t_f}(\widetilde{x}^T(t)Q\widetilde{x}(t)\\
&+u^T(t)Ru(t))dt+\textstyle\int_0^{t_f}dV(\widetilde{x}(t))\}\\
&\le E\{V(\widetilde{x}(0))\}+E\{\textstyle\int_0^{t_f}(\widetilde{x}^T(t)Q\widetilde{x}(t)\\
&+u^T(t)Ru(t))dt+\textstyle\int_0^{t_f}dV(\widetilde{x}(t))\}\\
&=E\{V(\widetilde{x}(0))\}+E\{\textstyle\int_0^{t_f}\{\widetilde{x}^T(t)Q\widetilde{x}(t)+u^T(t)Ru(t)\\
&+V_{\widetilde{x}}^T[\widetilde{F}(\widetilde{x}(t))+\widetilde{B}u(t)+\widetilde{D}(\widetilde{x}(t))\bar{w}(t)]+\tfrac{1}{2}\widetilde{H}^T(\widetilde{x}(t))\\
&\times V_{\widetilde{x}\widetilde{x}}\widetilde{H}(\widetilde{x}(t))+\lambda[V(\widetilde{x}(t)+\widetilde{G}(\widetilde{x}(t)))-V(\widetilde{x}(t))]\}dt\}
\end{aligned} \tag{55}$$

Then, by using the quadratic inequality in (17) in Lemma 2, the following inequality holds:

$$\begin{aligned}
&\tfrac{1}{2}V_{\widetilde{x}}^T\widetilde{D}(\widetilde{x}(t))\bar{w}(t)+\tfrac{1}{2}\bar{w}^T(t)\widetilde{D}^T(\widetilde{x}(t))V_{\widetilde{x}}\\
&\le\rho^2\bar{w}^T(t)\bar{w}(t)+\tfrac{1}{4\rho^2}V_{\widetilde{x}}^T\widetilde{D}(\widetilde{x}(t))\widetilde{D}^T(\widetilde{x}(t))V_{\widetilde{x}}
\end{aligned} \tag{56}$$

where ρ is a positive number.

By the inequality in (56), (55) can be written as:

$$\begin{aligned}
&E\left\{\int_0^{t_f} \widetilde{x}^T(t)Q\widetilde{x}(t)+u^T(t)Ru(t)\,dt\right\}\\
&\le E\{V(\widetilde{x}(0))\}+E\{\textstyle\int_0^{t_f}\{\widetilde{x}^T(t)Q\widetilde{x}(t)\\
&+u^T(t)Ru(t)+V_{\widetilde{x}}^T[\widetilde{F}(\widetilde{x}(t))+\widetilde{B}u(t)]+\rho^2\bar{w}^T(t)\bar{w}(t)\\
&+\tfrac{1}{4\rho^2}V_{\widetilde{x}}^T\widetilde{D}(\widetilde{x}(t))\widetilde{D}^T(\widetilde{x}(t))V_{\widetilde{x}}+\tfrac{1}{2}\widetilde{H}^T(\widetilde{x}(t))V_{\widetilde{x}\widetilde{x}}\\
&\times\widetilde{H}(\widetilde{x}(t))+\lambda[V(\widetilde{x}(t)+\widetilde{G}(\widetilde{x}(t)))-V(\widetilde{x}(t))]\}dt\}
\end{aligned} \tag{57}$$

If the following HJI holds:

$$\begin{aligned}
&E\{\widetilde{x}^T(t)Q\widetilde{x}(t)+u^T(t)Ru(t)+V_{\widetilde{x}}^T[\widetilde{F}(\widetilde{x}(t))+\widetilde{B}u(t)]\\
&+\tfrac{1}{4\rho^2}V_{\widetilde{x}}^T\widetilde{D}(\widetilde{x}(t))\widetilde{D}^T(\widetilde{x}(t))V_{\widetilde{x}}+\tfrac{1}{2}\widetilde{H}^T(\widetilde{x}(t))\\
&\times V_{\widetilde{x}\widetilde{x}}\widetilde{H}(\widetilde{x}(t))+\lambda[V(\widetilde{x}(t)+\widetilde{G}(\widetilde{x}(t)))-V(\widetilde{x}(t))]\}\\
&\le 0
\end{aligned} \tag{58}$$

then (57) can be written as:

$$\begin{aligned}
&E\left\{\int_0^{t_f} \widetilde{x}^T(t)Q\widetilde{x}(t)+u^T(t)Ru(t)\,dt\right\}\\
&\le E\{V(\widetilde{x}(0))\}+E\{\textstyle\int_0^{t_f}\rho^2\bar{w}^T(t)\bar{w}(t)\,dt\}\\
&\forall\bar{w}(t)\in\mathcal{L}_2\{[0,t_f]\}
\end{aligned} \tag{59}$$

From (59) the H_∞ observer-based attack-tolerant guidance control performance in (15) is achieved with disturbance attenuation level ρ^2, i.e., $J_\infty \le \rho^2$ for all possible $\widetilde{w}(t)\in\mathcal{L}_2[0,t_f]$. The proof is done.

Appendix B: Proof of Theorem 2

By the rank test in [47], the ith augmented fuzzy system in (22) is observable if the following rank condition holds:

$$\begin{aligned}
&rank\begin{bmatrix} sI_{n+(k+1)(n_a+n_s)}-\overline{A}_i \\ \overline{C}_i\end{bmatrix}\\
&=rank\begin{bmatrix} sI_{(k+1)n_a}-A_{\gamma a} & 0 & 0\\ 0 & sI_{(k+1)n_s}-A_{\gamma_s} & 0\\ -D_aC_{\gamma_a} & 0 & sI_n-A_i\\ 0 & D_sC_{\gamma s} & C_i\end{bmatrix}\\
&=n+n_a(k+1)+n_s(k+1),\forall s\in S.
\end{aligned} \tag{60}$$

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where S denotes the set of complex $s-$ domain. In the following, the proof is separated into two cases with (i) $s \in S\backslash(eig\{A_i\} \cup eig(A_{\gamma_a}) \cup eig(A_{\gamma s}))$ and (ii) $s \in eig\{A_i\} \cup eig(A_{\gamma_a}) \cup eig(A_{\gamma s})$.

In the case (i), we immediately have following condition:

$$\begin{array}{l} rank[sI_{(k+1)n_a} - A_{\gamma_a}] = n_a(k+1) \\ rank[sI_{(k+1)n_s} - A_{\gamma_s}] = n_s(k+1) \\ rank[sI_{n\times n} - A_i] = n \\ \forall s \in S\backslash(eig\{A_i\} \cup eig(A_{\gamma_a}) \cup eig(A_{\gamma s})) \end{array}. \tag{61}$$

As a result, by (61), (60) is satisfied for $s \in S\backslash(eig\{A_i\} \cup eig(A_{\gamma_a}) \cup eig(A_{\gamma s}))$.

In the case (ii), by the assumption in (24) that the eigenvalues of $(A_i, A_{\gamma_a}, A_{\gamma_s})$ are mutually independent and (25), we can decouple the rank condition in (60) as the sum of three rank conditions

$$\begin{array}{l} rank\begin{bmatrix} sI_{(k+1)n_a} - A_{\gamma a} & 0 & 0 \\ 0 & sI_{(k+1)n_s} - A_{\gamma_s} & 0 \\ -D_aC_{\gamma_a} & 0 & sI_n - A_i \\ 0 & D_sC_{\gamma s} & C_i \end{bmatrix} \\ = rank\begin{bmatrix} sI_n - A_i \\ C_i \end{bmatrix} + rank\begin{bmatrix} sI_{(k+1)n_s} - A_{\gamma_s} \\ D_sC_{\gamma s} \end{bmatrix} \\ +rank\begin{bmatrix} sI_{(k+1)n_a} - A_{\gamma a} \\ -D_aC_{\gamma_a} \end{bmatrix} \\ \forall s \in eig\{A_i\} \cup eig(A_{\gamma_a}) \cup eig(A_{\gamma s}) \end{array}. \tag{62}$$

By applying the rank conditions in (23), (26) and (27), the rank condition in (62) can be written as:

$$\begin{array}{l} rank\begin{bmatrix} sI_{n\times n} - A_i \\ C_i \end{bmatrix} + rank\begin{bmatrix} sI_{(k+1)n_s} - A_{\gamma_s} \\ D_sC_{\gamma s} \end{bmatrix} \\ +rank\begin{bmatrix} sI_{(k+1)n_a} - A_{\gamma a} \\ -D_aC_{\gamma_a} \end{bmatrix} \\ = n + n_a(k+1) + n_s(k+1) \\ \forall s \in eig\{A_i\} \cup eig(A_{\gamma_a}) \cup eig(A_{\gamma s}) \end{array}. \tag{63}$$

Thus, the observability for the ith augmented fuzzy system in (22) is guaranteed. The proof is done.

Appendix C: Proof of Theorem 3

By selecting the Lyapunov function as $V(\widetilde{x}(t)) = \widetilde{x}^T(t)P\widetilde{x}(t)$ with positive matrix $P > 0$, the numerator of the H_∞ observer-based attack-tolerant guidance control performance in (15) can be written as

$$\begin{array}{l} E\left\{\int_0^{t_f} \widetilde{x}^T(t)Q\widetilde{x}(t) + u^T(t)Ru(t)\,dt\right\} \\ = E\left\{\widetilde{x}^T(0)P\widetilde{x}(0) - \widetilde{x}^T(t_f)P\widetilde{x}(t_f)\right\} \\ +E\{\int_0^{t_f}(\widetilde{x}^T(t)Q\widetilde{x}(t) + u^T(t)Ru(t))dt \\ +\int_0^{t_f} d\widetilde{x}^T(t)P\widetilde{x}(t)\} \\ \le E\left\{\widetilde{x}^T(0)P\widetilde{x}(0)\right\} + E\{\int_0^{t_f}\sum_{i,j=1}^{l} h_i(z(t))h_j(z(t)) \\ \times\{[\widetilde{x}^T(t)Q\widetilde{x}(t) + u^T(t)Ru(t) + \widetilde{x}^T(t)P(\widetilde{A}_{ij}\widetilde{x}(t) + \widetilde{D}_i \\ \times\bar{w}(t) + \Delta\widetilde{f}(\widetilde{x}(t)) + \widetilde{I}_i\Delta\widetilde{C}(\widetilde{x}(t)) + (\widetilde{A}_{ij}\widetilde{x}(t) + \widetilde{D}_i\bar{w}(t) \\ +\Delta\widetilde{f}(\widetilde{x}(t)) + \widetilde{I}_i\Delta\widetilde{C}(\widetilde{x}(t)))^TP\widetilde{x}(t) + (\widetilde{H}_i\widetilde{x}(t) \\ +\Delta\widetilde{H}(\widetilde{x}(t)))^TP(\widetilde{H}_i\widetilde{x}(t) + \Delta\widetilde{H}(\widetilde{x}(t))) \\ +\lambda[(\widetilde{x}(t) + \widetilde{G}_i\widetilde{x}(t) + \Delta\widetilde{G}(\widetilde{x}(t)))^TP(\widetilde{x}(t) + \widetilde{G}_i\widetilde{x}(t) \\ +\Delta\widetilde{G}(\widetilde{x}(t))) - \widetilde{x}^T(t)P\widetilde{x}(t)]\}dt\} \end{array} \tag{64}$$

By Assumption 2 and Lemma 2, the following inequalities are constructed to deal with the time-varying fuzzy approximation error:

$$\begin{array}{l} \widetilde{x}^T(t)P\Delta\widetilde{f}(\widetilde{x}(t)) + \Delta\widetilde{f}^T(\widetilde{x}(t))P\widetilde{x}(t) \\ \quad \le \widetilde{x}^T(t)(r_1I + PP)\widetilde{x}(t) \\ \widetilde{x}^T(t)P\widetilde{I}_i\Delta\widetilde{C}(\widetilde{x}(t)) + \Delta\widetilde{C}^T(\widetilde{x}(t))\widetilde{I}_i^TP\widetilde{x}(t) \\ \quad \le \widetilde{x}^T(t)(r_2I + P\widetilde{I}_i\widetilde{I}_i^TP)\widetilde{x}(t) \\ \Delta\widetilde{H}^T(\widetilde{x}(t))P\widetilde{H}_i\widetilde{x}(t) + \widetilde{x}^T(t)\widetilde{H}_i^TP\Delta\widetilde{H}(\widetilde{x}(t)) \\ \quad \le \widetilde{x}^T(t)(\widetilde{H}_i^TP\widetilde{H}_i)\widetilde{x}(t) + \Delta\widetilde{H}_i^TP\Delta\widetilde{H}(\widetilde{x}(t)) \\ \widetilde{x}^T(t)\widetilde{G}_i^TP\Delta\widetilde{G}(\widetilde{x}(t)) + \Delta\widetilde{G}^T(\widetilde{x}(t))P\widetilde{G}_i\widetilde{x}(t) \\ \quad \le \widetilde{x}^T(t)(\widetilde{G}_i^TPP\widetilde{G}_i + r_4I)\widetilde{x}(t) \\ \widetilde{x}^T(t)P\Delta\widetilde{G}(\widetilde{x}(t)) + \Delta\widetilde{G}^T(\widetilde{x}(t))P\widetilde{x}(t) \\ \quad \le \widetilde{x}^T(t)(PP + r_4I)\widetilde{x}(t) \end{array} \tag{65}$$

On the other hand, to decouple the bilinear terms $\Delta\widetilde{H}^T(\widetilde{x}(t))P\Delta\widetilde{H}(\widetilde{x}(t))$ and $\Delta\widetilde{G}^T(\widetilde{x}(t))P\Delta\widetilde{G}(\widetilde{x}(t))$ in (64), the following inequality constraint is set with a predefined scalar α

$$P \le \alpha I \tag{66}$$

Then, by using the constraint in (66), we immediately have following results:

$$\begin{array}{l} \Delta\widetilde{H}^T(\widetilde{x}(t))P\Delta\widetilde{H}(\widetilde{x}(t)) \le \alpha r_3\widetilde{x}^T(t)\widetilde{x}(t) \\ \Delta\widetilde{G}^T(\widetilde{x}(t))P\Delta\widetilde{G}(\widetilde{x}(t)) \le \alpha r_4\widetilde{x}^T(t)\widetilde{x}(t) \end{array} \tag{67}$$

By using the inequalities in (65) and (67), the terms associated with stochastic process in (64) can be relaxed as follows:

$$\begin{array}{l} (\widetilde{H}_i\widetilde{x}(t) + \Delta\widetilde{H}(\widetilde{x}(t)))^TP(\widetilde{H}_i\widetilde{x}(t) + \Delta\widetilde{H}(\widetilde{x}(t))) \\ \le \widetilde{x}^T(t)(2\alpha r_3I + 2\alpha\widetilde{H}_i^T\widetilde{H}_i)\widetilde{x}(t) \end{array} \tag{68}$$

$$\begin{array}{l} \lambda[(\widetilde{x}(t) + \widetilde{G}_i\widetilde{x}(t) + \Delta\widetilde{G}(\widetilde{x}(t)))^TP(\widetilde{x}(t) + \widetilde{G}_i\widetilde{x}(t) \\ +\Delta\widetilde{G}(\widetilde{x}(t))) - \widetilde{x}^T(t)P\widetilde{x}(t)] \\ \le \widetilde{x}^T(t)\lambda[P\widetilde{G}_i + \widetilde{G}_i^TP + \alpha^2I + r_4I + \alpha\widetilde{G}_i^T\widetilde{G}_i \\ +\alpha^2\widetilde{G}_i^T\widetilde{G}_i + r_4I + \alpha r_4I]\widetilde{x}(t) \end{array} \tag{69}$$

Furthermore, the disturbance terms $\widetilde{x}^T(t)P\widetilde{D}_i\bar{w}(t)$ and $\bar{w}^T(t)\widetilde{D}_iP\widetilde{x}(t)$ in (64) can be estimated as follows:

$$\begin{aligned}&\widetilde{x}^T(t)P\widetilde{D}_i\bar{w}(t)+\bar{w}^T(t)\widetilde{D}_i^TP\widetilde{x}(t)\\&\le\rho^2\bar{w}^T(t)\bar{w}(t)+\frac{1}{\rho^2}\widetilde{x}^T(t)P\widetilde{D}_i\widetilde{D}_i^TP\widetilde{x}(t)\end{aligned}\tag{70}$$

for some positive number $\rho > 0$.

By the inequalities in (65), (67), (68), (69) and (70), (64) can be rewritten as follows:

$$\begin{aligned}&E\left\{\int_0^{t_f}\widetilde{x}^T(t)Q\widetilde{x}(t)+u^T(t)Ru(t)\,dt\right\}\\&\le E\left\{\widetilde{x}^T(0)P\widetilde{x}(0)\right\}+E\{\int_0^{t_f}\sum_{i,j=1}^{l}h_i(z(t))h_j(z(t))\\&\times\{\widetilde{x}^T(t)[Q+\bar{K}_j^TR\bar{K}_j+P\widetilde{A}_{ij}+\widetilde{A}_{ij}^TP+r_1I+\alpha^2I\\&+r_2I+P\widetilde{I}_i\widetilde{I}_i^TP+\frac{1}{\rho^2}P\widetilde{D}_i\widetilde{D}_i^TP+2\alpha r_3I+2\alpha\widetilde{H}_i^T\widetilde{H}_i\\&+\lambda[P\widetilde{G}_i+\widetilde{G}_i^TP+\alpha^2I+r_4I+\alpha\widetilde{G}_i^T\widetilde{G}_i+\alpha^2\widetilde{G}_i^T\widetilde{G}_i\\&+r_4I+\alpha r_4I]\}\widetilde{x}(t)+\rho^2\bar{w}^T(t)\bar{w}(t)dt\}\end{aligned}\tag{71}$$

where $\bar{K}_j = [K_j, -K_j]$. Then, if the following Riccati like inequalities hold:

$$\Pi_{ij} < 0,\ \forall i,j = 1,\cdots,l \tag{72}$$

where $\Pi_{ij} = Q+\bar{K}_j^TR\bar{K}_j+P\widetilde{A}_{ij}+\widetilde{A}_{ij}^TP+r_1I+\alpha^2I+r_2I+P\widetilde{I}_i\widetilde{I}_i^TP+\frac{1}{\rho^2}P\widetilde{D}_i\widetilde{D}_iP+2\alpha r_3I+2\alpha\widetilde{H}_i^T\widetilde{H}_i+\lambda[P\widetilde{G}_i+\widetilde{G}_i^TP+\alpha^2I+r_4I+\alpha\widetilde{G}_i^T\widetilde{G}_i+\alpha^2\widetilde{G}_i^T\widetilde{G}_i+r_4I+\alpha r_4I]$, (71) can be furthered represented as:

$$\begin{aligned}&E\left\{\int_0^{t_f}\widetilde{x}^T(t)Q\widetilde{x}(t)+u^T(t)Ru(t)\,dt\right\}\\&\le E\left\{\widetilde{x}^T(0)P\widetilde{x}(0)\right\}+E\{\int_0^{t_f}\rho^2\bar{w}^T(t)\bar{w}(t)dt\}\end{aligned}\tag{73}$$

i.e., the H_∞ observer-based attack-tolerant guidance control performance in (15) is achieved with disturbance attenuation level ρ^2. The proof is done.

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