Appendix A. Derivation of formulas [1], [3], and [4]

Consider a vector $\vec{v}$ in $\mathbb{R}^3$ with coordinates (x, y, z). The angles (α, β, γ) between its projection onto the coordinate planes $e_2 - e_1$, $e_2 - e_3$, and $e_3 - e_1$, and the first coordinate axis in each of these planes are given by

$$\tan\alpha = \frac{x}{y}, \quad \tan\beta = \frac{z}{y}, \quad \tan\gamma = \frac{x}{z},$$

which shows that

$$\tan\alpha = \tan\beta\tan\gamma. \tag{A.1}$$

Let us now consider the coordinates (x', y', z') and angles $(\alpha', \beta', \gamma')$ of $\vec{v}$ with respect to a new coordinate system that is obtained by rotating the original coordinate system around the z-axis, that is, around the origin in the $e_2 - e_1$-plane through an angle δ (the *pelvic tilt*). Since such a rotation neither changes the z-coordinate nor the distance of (x, y) to the origin, we have

$$\alpha' = \alpha + \delta, \quad z' = z, \quad {x'}^2 + {y'}^2 = x^2 + y^2.$$

Below we derive an expression for the new angles β' and γ' in terms of the known quantities α', β, and γ. From $x = z\tan\gamma$, $y = z/\tan\beta$, we find that

$$\sin\alpha' = \frac{x'}{\sqrt{{x'}^2 + {y'}^2}} = \frac{x'}{\sqrt{x^2 + y^2}} = \frac{x'}{|z|\sqrt{\tan^2\gamma + 1/\tan^2\beta}}.$$

It is no restriction to assume that $z > 0$. Then from $\tan\gamma' = \frac{x'}{z'} = \frac{x'}{z}$, we have

$$\tan\gamma' = \sin\alpha'\sqrt{\tan^2\gamma + 1/\tan^2\beta}, \tag{A.2}$$

so that $\tan\beta' = \frac{\tan\alpha'}{\tan\gamma'}$ shows that

$$\tan\beta' = \frac{1}{\cos\alpha'\sqrt{\tan^2\gamma + 1/\tan^2\beta}}. \tag{A.3}$$

Reading α, β and γ as *sagittal tilt*, *coronal inclination* and *transverse version*, respectively, formulas (A.1), (A.2), and (A.3) correspond to [1], [3], and [4], respectively

Remarks. • Obviously similar formulas can be obtained for rotations around the x- or y-axis. By applying such formulas successively, composite rotations around multiple axes can be handled as well.

• There are more options to define angles in each of the coordinate planes. For example, in view of the conventions in acetabular cup orientation, in the e_2-e_1 plane we measured angles between vectors and the e_2-axis. When replacing the e_2- by the e_1-axis, above formulas should be adapted by reading $(\sin\alpha, \cos\alpha, \tan\alpha)$ as $(\cos\alpha, \sin\alpha, 1/\tan\alpha)$, and of course similar for angles involving α'.

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