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# Detection Algorithm of BPSK Signal of Parameter-Adjusted Bistable Stochastic Resonance Model Based on Scale Change

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**ABSTRACT** Given that signal is weakened to a certain extent in the process of noise suppression using mainstream method, and that new noise is introduced by signal processing system, causing the decrease of detection performance, to improve the performance of detection to BPSK signal under the condition of strong noise and no prior information, the detection algorithm of BPSK signal of bistable stochastic resonance (BSR) system, only low-amplitude and low-frequency periodic signal can be processed. Scale change is first made to BSR in this study, verifying that BSR can be applied to high-frequency BPSK signal under high sampling frequency condition, and nonlinear threshold detection system is designed following Neyman-Pearson criterion to deduce and quantitatively show error rate of detector. Besides, complete flow for signal detection was built by taking it as feedback quantity to adjust the system parameters adaptively. Scale change feasibility and applicability of algorithm proposed in this study were verified through simulation experiment, which lays the theoretical basis for the detection of weak BPSK signal under low signal-to-noise ratio (SNR).

**INDEX TERMS** Strong noise, detection to BPSK signal, bistable stochastic resonance, scale change, Neyman-Pearson criterion.

#### I. INTRODUCTION

Binary Phase Shift Keying (BPSK) has been broadly adopted in numerous existing communication protocols, e.g., IEEE 802.11a [1] and the second generation digital terrestrial television broadcasting system [2], to ensure the quality of service (QoS) against poor wireless channels. BPSK has a higher tolerance to nonlinear degradation and provides an ideal receiver sensitivity improvement with balanced detection [3]. It is easy to achieve with mature technology, and BPSK signal and its modulation types are widely applied to photology and communication [4], [5]. Accordingly, detection to BPSK signal has always been research hotspot of academic circle.

With the rapid development of the communication field, new requirements have been imposed on the regulation of wireless communication. As an important step in wireless communication supervision, signal recognition in non-cooperative mode is to estimate and judge information such as the modulation method of the received signal in the absence of the sender and channel information. Weak signal detection in non-cooperative mode is a key technology for monitoring and managing communication systems such as software radio, multiple transmission and multiple reception systems, and orthogonal frequency division multiplexing. It is widely used in civilian and military fields. In the civilian field, the BPSK signal is a very widely used signal. Detection without prior information is mainly used in radio management and spectrum monitoring. Supervise the frequency usage of radio users to prevent illegal frequency usage and protect legitimate radio users from interference [6], [8]. At present, there are instruments for weak signal recognition in non-cooperative mode, such as Keysight's N9041B, Rohde & Shwarz's FSP series, and so on. In electronic countermeasures in the military field, BPSK signals are more typical LPI radar signals and common communication signals [9], [10]. Detection of BPSK signals without prior information can effectively

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monitor and interfere with enemy wireless signals. Therefore, it is of positive significance to study the detection of BPSK signals without prior information.

Scholars primarily focused on the relevant research and gained numerous results. Literature [11] utilized low-noise amplification of phase-sensitive amplifiers (PSAs) to improve power of BPSK signal, which is conducive to signal detection. Literature [12], [13] uses PSA to regenerate the BPSK signal to obtain the phase size information of the signal, thereby improving the detection efficiency. However, this method is limited by the bandwidth of electronic components and can only be used for single-channel operation. Literature [14] modified above scheme but was still affected by bandwidth. Literature [15], [16] designed signal transmission system based on applying physical-layer network coding (PNC), of which literature [15] restrained inter-carrier interference generated by carrier frequency offset by utilizing BP-PNC algorithm, thus lowering transmission error rate of BPSK signal. Literature [17], [18] utilized Cramer-Rao low bound (CRLB) for signal-to-noise ratio (SNR) estimation from BPSK modulated signals. Literature [19] estimated channel based on minimum bit error ratio criterion, and above methods lay a theoretical foundation for improving system output SNR and improve estimation accuracy of BPSK signal. Literature [20] deduced bit error rate (BER) expression of BPSK signal under Ricean-Faded Cochannel interference theoretically, and BER of signal can be improved through improvement of Rice factor, but method is significantly affected by SNR. Literature [21] has proposed full rank transmit covariance matrices and maximum achievable SNR of BPSK signal which satisfy constant modulus constraint. But covariance matrix full rank will be ensured in this method. Literature [22] improved Constant Modulus Algorithm (CMA) by utilizing a simple modification of the Godard cost function, to realize blind separation to BPSK signal having the same rates and the same carrier frequencies, but the method requires knowledge of the BPSK sources frequency offsets at the receiver side. Literature [23] utilized real-valued property of BPSK-OFDM signals to devise an accurate real-valued minimum mean square error (MMSE) in time-varying channels. The method provided more accurate detection and lower computational complexity. The method requires a priori information of the BPSK-OFDM signals in the equalization matrix construction, which cannot be obtained easily in real time in reality. Literature [24] decreased BER of BPSK signal under multiple tone interference by increasing the subband order to enhance system performance, and performance of the method is significantly affected by system parameter. Literature [25] detected BPSK signal through adaptive windowed cross Wigner-Ville distribution method, and its result is close to CRLB in certain condition. But kernel function selection and parameter setting under different application background conditions, and conflict between time frequency resolutions restrict performance of time-frequency analysis (TFA) method. Utilizing Compressed Sensing (CS)

theory, literature [26] made sampling to BPSK signal with sampling frequency lower than Nyquist, which decreases demand to storage resource, improves processing speed and weakens effect of Iterative Soft Interference on BPSK signal. But in accordance with CS theory, signal will possess sparsity and signal is not concerned with sparsity space where it is located, which restricts application of the method in reality. Literature [27] transformed signal to circular stationary dimension through cyclic autocorrelation and extracted signal feature to realize detection to BPSK signal, but efficiency of the method decreases under non-Gaussian noise environment.

Above scheme has good performance under its background set, but three common problems restrict further improvement of performance of above method:

① When noise is suppressed, signal energy is also weakened, and even partial information is lost, which is not as serious as noise weakening in degree. Under low SNR, SNR of mixed signal processed may still be unsatisfactory, thus affecting subsequent signal detection.

<sup>(2)</sup> Signal processing system is also noise source itself, and in the process of restraining background noise, noise generated in the internal of system, higher harmonic appearing in processing and image signal etc. also affect signal detection.

<sup>(3)</sup> With increase of noise intensity, efficiency of above system is worsened obviously, and detection efficiency decreases substantially.

The reason for the above three problems is that scholars believe that noise is completely harmful to signal detection. In this view, the detection method for weak signals is mainly to improve the signal characteristics, or to reduce the noise characteristics, or both. Therefore, the proposed method is always limited by the above three problems.

In traditional linear theory, noise negatively affects signal. Yet within bistable stochastic resonance (BSR) system of nonlinear theory [28], [29], when system, signal and noise have a matching relation, stochastic resonance will be generated by system, and seen macroscopically, its effect is that noise energy moves towards weak signal, suggesting that signal energy is not lost, but strengthened, thus improving SNR of signal and being more conducive to signal detection.

The existing methods are divided into noise characteristics [30]-[35] and system parameters [36]-[41] adjustment. In the study of noise characteristics. Literature s [30], [31] designed different detectors based on the statistical characteristics of noise. Literature [32] combined traditional empirical mode decomposition (EMD) method with SR to achieve the decomposition of multi-frequency signals in colored noise. Literature [33] proposed a method combining adaptive bistable stochastic resonance and multi-scale noise tuning based on the noise characteristics to improve the detection ability for weak signals. Literature [34] studied the characteristics of SR system where the background noise is a mixture of color noise and white noise. Reference [35] studied the SNR characteristics of SR system output under the background of additive white noise and multiplicative white noise. In the study of parameter adaptability. Literature [36] used

particle swarm algorithm to match parameters. Literature [37] studied the resonance parameter characteristics in the Second-Order Underdamped System. Literature [38] uses the Hilbert transform and High-pass filter to combine the signal with the artificial fish swarm algorithm to improve the SNR of the output signal of the SR system. Literature [39] is based on the fact that asymmetry can improve the enhancement ability of asymmetric bistable SR in weak feature extraction. An underdamped well-width asymmetric bistable SR was constructed to improve signal detection capabilities. Literature [40] proposed a second-order matched stochastic resonance (SMSR) method. By combining the noise intensity optimization and signal frequency synchronization with duffing system, matching parameters are obtained, and weak signals are detected. Literature [41] proposed an underdamped step-varying second-order SR. In this method, you can adjust the second-order parameters to achieve resonance.

The above research results have made great achievements in researching the characteristics of noise and improving the matching of resonance, which has a guiding role in the research of this paper. Based on the above literatures, this paper studies the BPSK statistical characteristics under the condition of no prior information, unknown noise types and signal parameters, and performs signal detection based on Neiman-Pearson criterion.

Structure arrangement of this study is as follows: introduce basic concepts of BPSK and BSR in Section II; improve BSR through scale change and construct complete flow for signal detection in Section III; design nonlinear detector based on Neyman-Pearson criterion in Section IV; deduce output SNR of system and error rate in Section V; verify effectiveness of method through simulation and compare with mainstream method to highlight advantages of algorithm in Section VI; draw a conclusion and summarize the whole paper in Section VII.

The contribution of this study is to extend the traditional SR algorithm for small amplitude periodic signals to the detection of non-periodic high-frequency signals based on scale transformation, which broadens the application range of the algorithm, and significantly improves the ability of weak signal detection. Besides, the signal processing flow based on Neyman-Pearson criterion is built. In theory, the approximate probability density distribution function of the signal and noise processed by SR system is deduced, thereby achieving the constant detection of false alarm. In the meantime, the calculating process and ideal value of bit error rate are derived, and the signal detection process is optimized. The error rate constructs the negative feedback mechanism, design control law and the dynamic adaptive adjustment system parameters for the feedback quantity to achieve the optimal parameter matching and further improve the detection efficiency.

# **II. BASIC CONCEPTS**

### A. BPSK SIGNAL MODEL

BPSK signals have wide applications in the field of signal processing due to their large time-width bandwidth product,

high resolution and low probability of interception. [42]. Signal model of typical superimposed Gaussian white noise is as follows:

$$x(n) = A \exp \left[ j \left( 2\pi f \Delta tn + \theta(n) \right) \right] * rect(n) + \omega(n) \quad (1)$$

where, *rect*() denotes rectangular square wave, f is the carrier frequency of BPSK signal,  $\Delta t$  is sampling interval and  $\theta(n)$  is phase encoding, and  $\omega(n)$  is Gaussian white noise sequence subject to standard normal distribution. Modulation mode of BPSK signal is mainly embodied in phase function, where,

$$\theta(n) = \pi^* d(n) \tag{2}$$

d(n) denotes binary sequence and  $d(n) \in \{0,1\}$ , with equal probability valuing.

## B. BISTABLE STOCHASTIC RESONANCE SYSTEM

A completely different approach to increase the SNR can be adopted based on stochastic resonance (SR) theory. It can take advantage of the noise to enhance the weak signals by some nonlinear systems [43], [44]. Langevin equation having double-well potential property is typical bistable nonlinear system, and then system model can be expressed as:

$$\begin{cases} \frac{dx}{dt} = -\frac{\partial U(x)}{\partial x} + s(t) + N(t) \\ U(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4 \end{cases}$$
(3)

In above equation, U(x) is the potential function of BSR system. s(t) is an input signal, which will be the BPSK signal to be detected later. a, b are the structural parameters of the BSR system, all of which are real numbers greater than zero. N(t) stands for the background white Gaussian noise with zero mean and variance D, x is the output variable of the system. It is used to describe the output of the BSR system after the signal and noise have resonated under the influence of the system.

$$\begin{cases} E[N(t)] = 0\\ E[N(t_1)N(t_1)] = 2D\delta(t_1 - t_2) \end{cases}$$
(4)

where  $\delta(t)$  is impulse function.

When D = 0, it is assumed that there is no noise, only when the signal is input. The system has two steady states at  $\pm \sqrt{a/b}$ , the well bottom of the potential well. There is a barrier  $\Delta U$  between the two well bottoms. Only beyond the barrier can the system resonate between the two wells. The height of the barrier is:

$$\Delta U = \frac{a^2}{4b} \tag{5}$$

To go beyond this barrier, the input signal amplitude needs to exceed the threshold  $A_c$ . Let the pole of the potential function in equation (3) coincide with the inflection point, namely:

$$\begin{cases} \frac{\partial U(x)}{\partial x} = ax + bx^3 + A_0 = 0\\ \frac{\partial^2 U(x)}{\partial x^2} = a + 3bx^2 = 0 \end{cases}$$
(6)



FIGURE 1. Schematic diagram on bistable potential well.



FIGURE 2. Schematic diagram on tilt of potential well.

The threshold  $A_c$  that can be solved is:

$$A_c = \sqrt{\frac{4a^3}{27b}} \tag{7}$$

System output is considered as particles moving on potential function curve. After signal is input to system, potential well will make periodic tilt change in accordance with signal frequency under modulation drive of signal, which is as shown in Fig.1and Fig. 2.

When  $A < A_c$ , particle can only make periodic motion within 2 potential wells,  $\pm \sqrt{a/b}$ , and cannot jump to another potential well; particle can only jump between 2 potential wells when  $A > A_c$ . After noise is input to system, because of synergistic effect of signal and noise, tilt of potential well increases gradually, so that particle can jump from original potential well to another potential well even when  $A < A_c$ . When noise intensity increases to an optimum value, frequency of jumping between 2 potential wells by particle will be the same with frequency of input signal and output SNR will reach to the maximum value; when noise intensity increases continuously, law of particle motion will disappear, and output SNR will decrease obviously.

However, there are corresponding requirements for signals and noise. For the signal, there is a barrier as shown in equation (5) due to the BSR system. Stochastic resonance can only be excited when the energy of the signal exceeds the barrier. That is, the amplitude A of the signal is required to exceed the threshold  $A_0$  of equation (7). Otherwise, the system will not generate random resonance. For noise, although the stochastic resonance system directs the energy of the noise to the signal, the energy of the signal is boosted. But the noise



FIGURE 3. Algorithm principle block diagram.

is not as strong as possible. Once SNR is lower than the SNR wall, the detection of the signal cannot be achieved by any processing method. Therefore, the requirement for noise is that the SNR cannot be made lower than the SNR wall.

BSR can transform noise energy into signal energy, which reserves information of original signal and is also conducive to its detection. But classical BSR is only applicable to small-intensity and low-frequency periodic signal, and if one of above three conditions is not met, matching work of system will not be done, which restricts application of SR theory seriously. But in reality, signal intensity and frequency band property are relative values in background, and based on Fourier transform thought, non-periodic signal can also be considered as signal with infinite period. Accordingly, applicable scope of BSR may be expanded further. This study will expand applicability of BSR through scale change thought in next section.

Therefore, this paper builds a detection algorithm for BPSK signals based on BSR. The functional block diagram is shown in Figure 3.

First, according to the estimated frequency range of the signal to be detected, a preliminary scale transformation is performed on the BSR. Based on the output parameter characteristics of BSR, a detector was designed based on Neyman Pearson criterion. When the detection result meets the error requirements, a signal is output, otherwise the BSR system parameters are further adjusted according to the errors until the conditions are met. Next, this article discusses each of these parts in detail.

## III. SIGNAL PROCESSING FLOW BASED ONTOCHASTIC RESONANCE OF SCALE CHANGE

#### A. SCALE CHANGE MODEL AND DEMONSTRATION

The theoretical analysis and quantitative derivation of stochastic resonance systems are based on the theory of adiabatic approximation and linear response. The above theory assumes that the frequency of the signal to be detected is extremely low and the amplitude is extremely small. Therefore, the classical stochastic resonance theory can only deal with low-frequency signals with small amplitude, generally handle no more than a few Hz [28], [32]. But in actual scientific research and project application, center frequency of signal is greater than 1 and frequency band is wide. Thus, to improve applicability of algorithm proposed in this study, this section introduces normalization scale change of bistable system model, and based on combination with derivation demonstration, demonstrates that applicable scope of BSR can be expanded by adjusting parameters.

This study makes variable substitution to equation (3), i.e., scale change, assumed that:

$$\begin{cases} z = x \sqrt{\frac{b}{a}} \\ \tau = at \end{cases}$$
(8)

Substitute equation (8) into equation (3) to obtain following equation through sorting:

$$a\sqrt{\frac{a}{b}}\frac{dz}{d\tau} = a\sqrt{\frac{a}{b}}z - a\sqrt{\frac{a}{b}}z^3 + s\left(\frac{\tau}{a}\right) + N\left(\frac{\tau}{a}\right) \tag{9}$$

Assumed that:

$$\zeta(\tau) = \frac{1}{\sqrt{2Da}} N\left(\frac{\tau}{a}\right) \tag{10}$$

Then above equation meets:

$$\begin{cases} E\left[\zeta\left(t\right)\right] = 0\\ E\left[\zeta\left(t_{1}\right)\zeta\left(t_{1}\right)\right] = \delta\left(t_{1} - t_{2}\right) \end{cases}$$
(11)

Substitute equation (11) into equation (9) to obtain:

$$a\sqrt{\frac{a}{b}}\frac{dz}{d\tau} = a\sqrt{\frac{a}{b}}z - a\sqrt{\frac{a}{b}}z^3 + s\left(\frac{\tau}{a}\right) + \sqrt{2Da}\zeta(\tau) \quad (12)$$

Following equation can be obtained by sorting to above equation:

$$\frac{dz}{d\tau} = z - z^3 + \sqrt{\frac{b}{a^3}}s\left(\frac{\tau}{a}\right) + \sqrt{\frac{b}{a^3}}\sqrt{2Da}\zeta(\tau) \quad (13)$$

Above equation is normalization form of equation (3), both equations are equivalent, and signal frequency is 1/a of frequency of original signal. Accordingly, for high-frequency signal, signal to be detected can be normalized as equivalent low-frequency signal by choosing increased system parameter *a* and improving sampling frequency of signal, thus making analysis and solution by utilizing SR theory. Signal and noise will be multiplied by the same scale factor, and SNR of original system is not changed.

#### **B. ALGORITHM FLOW**

Last section discusses feasibility to expand stochastic resonance by utilizing scale change method, and this section constructs signal processing flow based on bistable system. Its flow chart is as Fig.4:



FIGURE 4. Signal processing flow based on bistable stochastic resonance.

The algorithm steps can be summarized as following 6 steps:

Step 1: The BSR system is constructed according to the Langevin equation. Initialize the structural parameters of the BSR system  $a_0 = b_0 = 1$ , signal amplitude  $A_0 = 0.5$ , frequency  $f_0 = 0.5$ , noise standard deviation  $\sigma_0 = 1$ .

Step 2: The magnitude  $f_1$  of the frequency  $f_{\text{BPSK}}$  of the BPSK signal to be detected is estimated. Set the sampling frequency  $f_s$  to 100 times  $f_1$ . Even if frequency  $f_1$  of signal to be detected is lower than sampling frequency  $f_s$ , it is ensured that signal can be transformed to equivalent low frequency range after scale change.

Step 3: Utilize  $a_1 \doteq a_0 (f_s/f_0)$  and assume  $b_1 \doteq a_1$ , to obtain bistable system parameters  $a_1$  and  $b_1$  after scale change, and to construct stochastic resonance system.

Step 4: Make sampling to mixed signal to be detected and calculate root-mean-square (RMS) value  $\sigma_1$  of mixed signal. Because low SNR is set under this study, root-mean-square value of mixed signal is set to be equal to root-mean-square value of noise almost. Scale factor *K* is obtained by utilizing  $K = a_1(\sigma_0/\sigma_1)$ .

Step 5: Multiply mixed signal to be detected by scale factor K and input the result into stochastic resonance system. Input output result into signal detection system to obtain error rate of system, and make trimming to  $a_1$  based on it, return to Step 3, and make recalculation to lower error rate of result.



FIGURE 5. Spectrogram before and after mixed signal passing SR.



FIGURE 6. Schematic diagram of nonlinear threshold system.

Step 6: Adjust parameter to minimize error rate, and signal processing result will be the most ideal one. The detection waveform and the corresponding bit error rate of the system are detected to realize the detection of the BPSK signal. Signal detection part and error rate calculation part will be discussed below in detail.

# IV. DETECTION FLOW OF NONLINEAR THRESHOLD SYSTEM BASED ON NEYMAN-PEARSON CRITERION

In order to further extract the weak signal from the noise. This paper designs a signal detector based on the Neyman-Pearson criterion. This is the signal detection part of Figure 4.

Mixed signal of BPSK signal and noise are affected by nonlinear interaction when it passes BSR system, which changes Probability Density Function (PDF) of noise. This section sets relatively high SNR to make simulation experiment, input signal is 0.03Hz and SNR is -10dB, and result obtained is as shown in Fig. 5.

Through comparison to 2 figures, it can be seen obviously that after nonlinear processing by stochastic resonance system, noise power is gathered towards frequency point direction of signal, suggesting that noise is not subject to normal distribution again after passing stochastic resonance system. Thus, this study designs nonlinear threshold system (NTS) by utilizing Neyman-Pearson criterion, and its function model is as shown in Fig. 6.

Its expression is as follows:

$$y = \begin{cases} T & x \ge \theta \\ P & x < \theta \end{cases}$$
(14)

T and P are two constants, which will be given later. Because whether BPSK signal exists or not needs to be detected in this section, following equation can be obtained after discretization to it:

$$\begin{cases} s_1(k) = 1 \\ s_2(k) = -1, \end{cases} \quad 1 \le k \le N$$
 (15)

N is the number of sampling point in above equation.

Each code element of BPSK signal will last for some time, and sampling frequency is greater than signal frequency, suggesting that when BPSK signal exists, system will continuously collect relatively numerous mixed signal of  $s_1(k)$  and noise or mixed signal of  $s_2(k)$  and noise to be used as signal detection basis.

When signal exists, probability that system collects  $s_1(k)$  and  $s_2(k)$  is 0.5. Assumed that system collects  $s_1(k)$  with noise being n(k) first, existence or inexistence of signal can be expressed through binary hypothesis testing, i.e.,

$$\begin{cases} H_0: x(k) = n(k), & noise only \\ H_1: x(k) = s_1(k) + n(k), & signal and noise \end{cases}$$
(16)

Because PDF of Gaussian white noise is changed after it passes BSR system, and because it is difficult to obtain processing mechanism of nonlinear system through theoretical analysis and nonlinear system has sensitivity to initial conditions, this study will not deduce PDF of Gaussian white noise after it passes bistable system. To design detector, and to ensure universality of algorithm, assumed that noise is subject to generalized Gaussian noise, its PDF is as follows:

$$f(x; p, \beta) = \frac{p}{2\beta\sigma_n\Gamma\left(\frac{1}{p}\right)}e^{-\left(\left|\frac{x-\mu_n}{\beta\sigma_n}\right|^2\right)}, \quad p > 0, \beta > 0 \quad (17)$$

In above equation,  $\mu_n$  and  $\sigma_n$  respectively are mean value and variance of noise, and expression of  $\Gamma(x)$  and  $\beta$  is as follows:

$$\Gamma(x) = \int_{0}^{+\infty} u^{x-1} e^{-u} du$$
 (18)

$$\beta = \sqrt{\Gamma\left(\frac{1}{p}\right)\Gamma\left(\frac{3}{p}\right)} \tag{19}$$

The value of p is used to describe the noise type, and the different p values correspond to different noise PDF. Using the output of the BSR, the noise characteristics are analyzed, and the parameters are modeled to obtain a noise distribution model close to the result, so as to determine the value of p. When p is 1, the noise is Laplacian type noise. When p is 2, the noise is Gaussian noise. When p is  $\infty$ , the noise is uniformly distributed.

Assumed T = 1, P = -1, and  $\theta = s_1/2 + n$  in this study, threshold system can be regarded as a symbolic function and expressed as:

$$y = sgn(s_1 + n - \theta) = \begin{cases} 1 & s_1 + n - \theta \ge 0\\ -1 & s_1 + n - \theta < 0 \end{cases}$$
(20)

Calculate mean value of output signal *y* first, which can be expressed as:

$$E[y] = P(y = 1) - P(y = -1)$$
  
=  $P(s_1 + n \ge \theta) - P(s_1 + n < \theta)$   
=  $1 - 2F_n(\theta - s_1)$  (21)

where:

$$F_n(\theta - s_1) = \int_{-\infty}^{\theta - s_1} f_n(u) du$$
 (22)

 $f_n(u)$  is PDF of generalized Gaussian noise in equation (17). Calculate variance of output signal *y*:

$$E\left[y^{2}\right] = (1)^{2} P(y=1) + (-1)^{2} P(y=-1) = 1 \quad (23)$$

$$Var(y) = E[y^{2}] - E^{2}[y]$$
  
=  $4F_{n}(\theta - s_{1}) - 4F_{n}^{2}(\theta - s_{1})$  (24)

Test statistics of nonlinear threshold system obtained is:

$$l = \frac{1}{N} \sum_{k=1}^{N} y(k)$$
 (25)

Waveform received is different under existence or inexistence of signal, and accordingly, under  $H_0$  and  $H_1$  assumption condition, test statistics has different mean value and standard deviation, respectively set as  $\mu_0$ ,  $\sigma_0$ ,  $\mu_1$ ,  $\sigma_1$ . In accordance with property of test statistics, values respectively are:

$$\mu_0 = 1 - 2F_n\left(\theta\right) \tag{26}$$

$$\mu_1 = 1 - 2F_n \left(\theta - s_1\right) \tag{27}$$

$$\sigma_0 = \frac{1}{N} \left[ 4F_n(\theta) - 4F_n^2(\theta) \right]$$
(28)

$$\sigma_1 = \frac{1}{N} \left[ 4F_n \left( \theta - s_1 \right) - 4F_n^2 \left( \theta - s_1 \right) \right]$$
(29)

Because the first code element of BPSK signal will last for some time, and sampling frequency of system is high enough, suggesting that the number of point on sampling to system is large enough, in accordance with central-limit theorem, under above 2 assumptions, test statistics is subject to Gaussian distribution and its PDF is:

$$p(l|H_0) = \frac{1}{\sqrt{2\pi\sigma_0}} e^{-\frac{(l-\mu_0)^2}{2\sigma_0^2}}$$
(30)

$$p(l|H_1) = \frac{1}{\sqrt{2\pi\sigma_1}} e^{-\frac{(l-\mu_1)^2}{2\sigma_1^2}}$$
(31)

To improve efficiency of detection system to signal processing, this study designs detector by adopting Neyman-Pearson criterion, thus ensuring entry of useful information into system as much as possible and avoiding entry of overmuch false data into detector to affect work efficiency of system; when false alarm probability  $P_{\rm F}$  is constant, detection probability  $P_{\rm D}$  will be maximized, and they can be expressed as:

$$P_F = \int_{\alpha}^{\infty} p(l|H_0) dl \tag{32}$$

$$P_D = \int_{\alpha}^{\infty} p(l|H_1) dl \tag{33}$$

Because Neyman-Pearson criterion is to make lower limit integration to equation (32) after false alarm probability  $P_F$ is set, and make its value be  $P_F$ , threshold value  $\alpha$  can be obtained. It is assumed that  $P_F$  has been set in this study, and following equation can be obtained by substituting equation (30) into equation (33):

$$P_F = \int_{\alpha}^{\infty} \frac{1}{\sqrt{2\pi\sigma_0}} e^{-\frac{(l-\mu_0)^2}{2\sigma_0^2}} dl$$
(34)

Make variable substitution to above equation, assumed that:

$$v = \frac{l - \mu_0}{\sigma_0} \tag{35}$$

Then equation (21) can be expressed as:

$$P_F = \int_{\frac{\alpha - \mu_0}{\sigma_0}}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{v^2}{2}} dv$$
(36)

Above equation does not have theory resolution, but in practical application, auxiliary calculation can be made by combining with distribution table of standard normal function, i.e., function value corresponding to following equation:

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du$$
(37)

Then false alarm probability can be expressed as:

$$P_F = 1 - \Phi\left(\frac{\alpha - \mu_0}{\sigma_0}\right) \tag{38}$$

After false alarm probability is set, standard normal function value can be determined, and  $\mu_0$  and  $\sigma_0$  can be obtained through calculation, and threshold value  $\alpha$  can be obtained through table lookup and simple calculation, so that detection probability obtained is:

$$P_D = 1 - \Phi\left(\frac{\alpha - \mu_1}{\sigma_1}\right) \tag{39}$$

Fig. 7 is detection flow of mixed signal after processing by stochastic resonance system.

Capability to detect BPSK signal can be enhanced further by utilizing Neyman-Pearson criterion under no prior information.

Because above derivation is based on the condition that signal received is +1, detection probability  $P_D$  can be obtained through above calculation. But in reality, the first code element of BPSK signal can be positive or negative.

Thus, this study adopts 2 sets of same detection system, the first set adopts normal processing, and signal is reversed and input to the second set of detection system, i.e., s(t)turning to -s(t), and under simultaneous working of bistable system, detection probability  $P_{Dv}$  of the second set of detection system is obtained; compare  $P_D$  with  $P_{Dv}$ , they differ significantly in size, and select the greater one, which is signal detection probability, and whether signal is positive or negative can also be determined. The first set of system only detects positive signal while the second set of system detects negative signal and add sequence of signal detected by them to obtain complete sequence of BPSK signal.



FIGURE 7. Detection flow of BPSK positive signal based on Neyman-Pearson criterion.

#### V. ANALYSIS ON ALGORITHM PERFORMANCE

#### A. ANALYSIS OF DETECTION CAPABILITIES

To analyze algorithm performance quantitatively, signal detection enhancement capability is described quantitatively by algorithm proposed in this study.

Through expression of BPSK signal, it can be seen that between each code element, signal is sine wave, and unilateral output power spectral density function of bistable system can be expressed as:

$$S(\omega) = S_{s}(\omega) + S_{n}(\omega)$$

$$= \frac{\frac{2a^{4}A^{4}}{\pi b^{2}D^{2}}e^{-\frac{a^{2}}{2bD}}}{\frac{2a^{2}}{\pi^{2}}e^{-\frac{a^{2}}{2bD}} + \omega_{c}}\delta(\omega - \omega_{c})$$

$$+ \left(1 - \frac{\frac{a^{3}A^{2}}{b\pi^{2}D^{2}}e^{-\frac{a^{2}}{2bD}}}{\frac{2a^{2}}{\pi^{2}}e^{-\frac{a^{2}}{2bD}} + \omega_{c}}\right) \left(\frac{\frac{4\sqrt{2}a^{2}}{\pi b}e^{-\frac{a^{2}}{4bD}}}{\frac{2a^{2}}{\pi^{2}}e^{-\frac{a^{2}}{2bD}} + \omega_{c}}\right) (40)$$

In above equation,  $\omega_c = 2\pi f_c$  denotes angular frequency of BPSK signal, and  $f_c$  is modulation carrier frequency of signal. In the meantime, under adiabatic approximation, to ensure that bistable system can generate stochastic resonance, modulation frequency of BPSK signal and Clay Moss rate  $R_k$  will meet:

$$f_c = \frac{1}{2}R_k = \frac{a}{2\sqrt{2}\pi}e^{-\frac{a^2}{4bD}}$$
(41)

Then output signal power  $P_s$  can be expressed as:

$$P_{s} = \frac{1}{2\pi} \int_{0}^{+\infty} S_{s}(\omega) d\omega = \frac{\frac{2a^{4}A^{4}}{\pi b^{2}D^{2}}e^{-\frac{a^{2}}{2bD}}}{\frac{2a^{2}}{\pi^{2}}e^{-\frac{a^{2}}{2bD}} + \omega_{c}}$$
(42)

Noise power  $P_n$  can be expressed as:

$$P_{n} = \frac{1}{2\pi} \int_{0}^{+\infty} S_{n}(\omega) d\omega$$
  
=  $\frac{2a}{b\pi} \left( 1 - \frac{\frac{a^{3}A^{2}}{b\pi^{2}D^{2}}e^{-\frac{a^{2}}{2bD}}}{\frac{2a^{2}}{\pi^{2}}e^{-\frac{a^{2}}{2bD}} + \omega_{c}} \right) \arctan\left(\frac{\omega}{\frac{\sqrt{2}a}{\pi}e^{-\frac{a^{2}}{4bD}}}\right) \Big|_{0}^{+\infty}$   
=  $\frac{a}{b} - \frac{\frac{a^{4}A^{2}}{b^{2}\pi^{2}D^{2}}e^{-\frac{a^{2}}{2bD}}}{\frac{2a^{2}}{\pi^{2}}e^{-\frac{a^{2}}{2bD}} + \omega_{c}}$  (43)

Through sorting, *SNR*<sub>out</sub> of output signal can be expressed as:

$$SNR_{\rm out} = \frac{P_s}{P_n} = \frac{2a^2A^2}{abD^2\left(4 + \pi^2\right) - 2a^2A^2}$$
(44)

SNR<sub>in</sub> of input signal is:

$$SNR_{\rm in} = \frac{P_s}{P_n} = \frac{A^2}{4D} \tag{45}$$

SNR gain of bistable system before and after signal input is:

$$G = \frac{SNR_{\text{out}}}{SNR_{\text{in}}} = \frac{8a^2D}{ab(4+\pi^2)D^2 - 2a^2A^2}$$
(46)

To ensure that G is greater than 1, above equation will meet:

$$0 < ab\left(4 + \pi^2\right)D^2 - 2a^2A^2 < 8a^2D \tag{47}$$

After solution to above one-variable quadratic inequality set related to *D*, following result can be obtained:

$$\sqrt{\frac{2aA^2}{(4+\pi^2)b}} < D < \frac{4a\sqrt{16a^2 + 2abA^2(4+\pi^2)}}{(4+\pi^2)b}$$
(48)

When original noise intensity meets above conditions, output SNR of system will be enhanced.

To research relationship between noise and system parameter, assumed that:

$$G(a) = \frac{8a^2D}{ab(4+\pi^2)D^2 - 2a^2A^2} = \frac{8D}{\frac{1}{a}b(4+\pi^2)D^2 - 2A^2}$$
(49)

From above equation, it can be seen that system gain is positively correlated with parameter a. To ensure that gain is greater than 1, parameter a will meet:

$$\begin{cases} \frac{1}{a}b(4+\pi^2)D^2 - 2A^2 > 0\\ \frac{1}{a}b(4+\pi^2)D^2 - 2A^2 < 8D \end{cases}$$
(50)

After solution, following result can be obtained:

$$\frac{\left(4+\pi^2\right)bD^2}{2\left(4D+A^2\right)} < a < \frac{\left(4+\pi^2\right)bD^2}{2A^2} \tag{51}$$

Because gain is monotone increasing function concerned with a, when value of a is close to upper limit, system gain will be maximized.

From above equation, it can also be seen that increase of a is conducive to improvement of signal SNR, and scaling proposed in this study also requires a to be large as much as possible, suggesting that applicability of stochastic resonance can be expanded by increasing a, and system gain can also be enhanced simultaneously, which is also one advantage of scaling method proposed in this study.

## B. ANALYSIS OF BER AND OPTIMAL MATCHING OF SYSTEM PARAMETERS

SNR of signal is enhanced after it passes bistable system. To evaluate performance of detector, this study takes error rate as evaluation index and feedback quantity from signal processing system, and its expression is as follows:

$$P_e = P_0 P(0|1) + P_1 P(1|0)$$
(52)

In above equation,  $P_1$  and  $P_0$  respectively represent probability for existence or inexistence of code element.  $P_0$ +  $P_1 = 1$ . Under Neyman-Pearson criterion, and because this study transforms BPSK signal into 2 groups of binary detection, P(0|1) is expressed as false dismissal probability while P(1|0) is expressed as false alarm probability, i.e.,

$$\begin{cases} P(0|1) = 1 - P_D \\ P(1|0) = P_F \end{cases}$$
(53)

Accordingly, error rate of detector can be expressed as follows by combining with derivation result of last section:

$$P_e = P_0 (1 - P_D) + P_1 P_F$$
  
=  $P_0 \Phi\left(\frac{\alpha - \mu_1}{\sigma_1}\right) + P_1 \left[1 - \Phi\left(\frac{\alpha - \mu_0}{\sigma_0}\right)\right]$  (54)

In accordance with above equation, the greater the false alarm probability  $P_F$  is, the lower the detection threshold  $\alpha$ will be; the greater the detection probability  $P_D$  is, the lower the false dismissal probability will be, vice versa. Thus, above equation shows that error rate will present decrease first and then increase with decrease of false alarm probability set manually, suggesting that an ideal false alarm probability  $P_{Fopt}$  exists, to minimize error rate of system.

Seen from  $\Phi$  function, error rate  $P_e$  is concerned with noise intensity before detector passing, and after processing by bistable stochastic resonance system, SNR of output signal is improved, suggesting that value of  $\Phi$  function is lowered and error rate of detection signal is lowered thus. Above process is error rate calculation part in signal processing flow, and system parameter will be adjusted in accordance with error rate size to realize optimal detection.

In Step 3,  $a_1$  is approximated as  $f_s/f_0$  times of  $a_0$ . Though scale scaling can be achieved, it is necessary to achieve optimal parameter matching. Thus, the ratio of  $a_1$  to  $a_0$  should be adjusted.

Eq.(54) suggests that the system's error rate is related to the probability of transmitting signals,  $P_0$  and  $P_1$ , and to the

set probability of false alarm  $P_F$ . Also, based on Neyman-Pearson criterion, when the false alarm probability of a Receiver is set, its detection probability  $P_D$  can also be determined according to the Receiver Operating Characteristic curve (ROC).

To achieve the optimal matching of  $a_1$ , parameters of the receiver can be adjusted before signal detection. The parameter adjustment process can be described as the following steps.

Step 1: According to the application requirements, estimate the frequency band of the signal to be detected. Take any frequency  $f_e$  in this frequency band. BSRs with parameters  $a_0$  and  $b_0$  were constructed to generate stochastic resonance at low frequency  $f_0$ . Adjust the system parameters  $a_1 = b_1 =$  $f_e a_0/f_0$ . And initialize  $a_1 = b_1 = 0.5$ . Also set the system false alarm probability  $P_F$ .

Step 2: Take a detection signal of 100 sampling points and input it into the BSR. This group of signals was detected 9 times. Because the BSR system adds noise during each test, the results of these 9 times are not exactly the same. Comparing the number of 0 and 1 of the 9 test results at each position, the more is the test result at that position. Thereby, a preferable detection result of 100 sampling points is obtained.

Step 3: Count the number of 0 and 1 in the 100 preferred results, and replace the probability with the frequency to obtain  $P_0$  and  $P_1$  in formula (52). Use formulas (26)-(29) to calculate  $\mu_0$ ,  $\mu_1$ ,  $\sigma_0$ , and  $\sigma_1$ , and then use formulas (38) and (39) to obtain the detection probability  $P_D$ . Thus, the bit error rate  $P_e$  of formula (54) is obtained.

Step 4: Count the number of errors  $n_e$  in the 900 detection results to obtain the actual bit error rate  $P_{er} = n_e/900$ .

Step 5: Bring the above parameters into the control law, that is:

$$\begin{cases} err(q) = P_{er}(q) - P_{eopt} \\ u(q+1) = u(q) + k \times err(q) \\ a_1(q+1) = u(q+1) \times a_0 \end{cases}$$
(55)

The error is calculated as err(q). The ratio of  $a_1$  to  $a_0$  is adjusted in accordance with such error, k is a proportional constant, which is an order of magnitude with  $f_e$ , q is the number of iterations.

Step 6: Repeat Step 2-Step 5 until err(q) is less than 0.01.

Eq.(55) is the classical control of ratio, which is mature and easy to achieve. When the algorithm is finished, the number of record iterations is  $Q, q \in [1, Q]$ . In such a way,  $a_1$  can be adjusted, and the error rate of the detection can be ensured to decrease under such system parameter. The BSR system can then be used to detect similar subsequent signals.

The above process is also one of the reasons for the good effect of the algorithm in this paper. In the absence of prior information, the parameters of many detection algorithms remain unchanged, making it difficult to cope with signals of different parameters. During the detection process, the algorithm in this paper continuously obtains the signal information, and further adjusts its parameters according to the signal parameters to achieve better detection. This improves the algorithm's performance.

# C. ANALYSIS OF ALGORITHM COMPLEXITY

The algorithm flow suggests that following the algorithm of this study, the mixed noise signal is first processed through the bistable stochastic resonance system, and then the output signal is detected based on the Neyman-Pearson criterion. Subsequently, the parameters of bistable system are adjusted according to the detected error rate until the lowest error rate is achieved. Accordingly, it is necessary to analyze the superimposed amount of a random resonance and detection algorithm, and then carry out *Qth* iteration to get the times of optimal solutions, i.e., the complexity of this algorithm.

The part of BSR with changed scale should be first analyzed. The first step is scale change, i.e., Eq.(7) and Eq.(8). The frequency and scale change of the signal are changed as 1/a of the original values and then divided by *a. The complexity of this algorithm is O(n)*. Subsequently, the results of scale change is input into BER system, i.e., Eq.(3). The algorithm is complex primarily because of  $bx^3$ , with the complexity of  $O(n^3)$ . Thus, the complexity of the algorithm om BSR is  $O(n + n^3)$ , i.e.,  $O(n^3)$ .

Next, the signal detection is to be analyzed. When determining the generalized noise distribution in PDF, p value is selected according to the BSR output result to achieve an approximate description of noise. The corresponding algorithm complexity is O(1). Then the parameters of  $\mu_0, \sigma_0, \mu_1, \sigma_1$  will be calculated. (26)-(29) suggest that the algorithm for  $\sigma$  is more complex than that for  $\mu$ . Thus, we only investigates the complexity of algorithm for  $\sigma$ .  $F_n$ is obtained by the integral calculation of Eq. (22). The integrand and the bound of the integral are known here, and the integrand is a typical distribution function. The value of  $F_n$  can be determined by referring to the table, and the computational complexity is O(n). Then,  $\sigma$  is calculated using Eq. (28) and (29). Thus, the complexity for calculating  $\sigma$  is  $O(n^3)$ . Next, the threshold  $\alpha$  is derived by the established  $P_F$ . Eq.(36) suggests that the integrand and bounds are known, and the integrand meets the typical Gaussian distribution. The threshold  $\alpha$  can be obtained by viewing table and through simple operation. The computational complexity is  $O(n^2)$ . Finally, the test statistics are analyzed, and according to Eq.(25), the test statistics are the mean value of the output signal, and then the statistics are compared with the threshold. The algorithm complexity of the two parts is O(n). The algorithm complexity of the signal detection part is therefore  $O(1+n+n^3+n^2+n)$ , i.e.,  $O(n^3)$ .

Finally, in combination with the algorithm process in Figure 4, most of the rest are simple four operations and assignment except for the BSR and signal detection. Only the algorithm for noise variance in the computing environment has a complexity of  $O(n^2)$ . Then, the overall algorithm complexity of the system is  $O[Q(n^3 + n^3 + n^2)]$ , i.e.,  $O(2 Qn^3)$ . Nonlinear detection with similar complexity of most signal



FIGURE 8. Figure on comparison to previous and late waveform after BPSK signal passing bistable system.

detection algorithms' can significantly improve the detection ability of weak signals, and the algorithm has better performance.

# **VI. SIMULATION VERIFICATION**

# A. SCALE CHANGE MODEL

Scale change feasibility has been deduced and discussed in above content. To verify its feasibility, this paper makes simulation experiment to it. This paper makes simulation verification to scale change method by utilizing BPSK signal.

Sampling frequency of system is 20MHz, system parameter  $a = b = 10^7$ , and carrier frequency of BPSK signal is 200kHz, base band frequency is 20 kHz, and initial code element can be selected stochastically, initial phase is random, and random equal probability of +1 and -1 code element can change stochastically, and noise intensity is equal to signal intensity. Simulation environment is I7-4960, with 2.60GHz dominant frequency and 16G memory, and simulation experiment is made to platform based on Matlab 2014a. Figure on comparison to previous and late waveform obtained after BPSK signal passes above bistable system is:

In Fig.8, signal from top to bottom successively is BPSK signal modulated, input signal of bistable system mixed with equal intensity noise and output signal of bistable system. Through observation to Fig.8, it can be visually seen that above parameters set can make system generate stochastic resonance, which means that scaling method in this paper is feasible through experimental verification.

Through comparison to original signal and output signal of system, output signal can be modulated and matched with original signal well, and especially at  $1.2*10^{-5s}$  and  $3.9*10^{-5s}$  when 2 signal amplitudes are switched, i.e., the positive and negative of code element of BPSK signal change, output signal of bistable system embodies original signal feature well. Through comparison to mixed signal and output signal, under the condition that SNR is 0dB, output signal of system can be interpreted visually, because system transforms noise energy into signal energy, thus enhancing signal intensity, which shows that system has potential to detect weak signal under low SNR.



**FIGURE 9.** Figure on comparison to frequency spectrum before and after BPSK signal passing bistable system.

Make fast Fourier transform (FFT) to above mixed signal and output signal, and analyze the frequency spectrum, which is as shown in Fig. 9:

Through comparison to above 2 figures, it can be seen that intensity of signal at its frequency point increases obviously, and is 11 times of original signal intensity roughly, and SNR of output signal is improved obviously, which is beneficial to signal detection. Seen from Fig.2, original noise is Gaussian white noise, distributed at the whole frequency band uniformly with certain intensity, and after processing by stochastic resonance system, its distribution is gathered towards signal frequency point obviously, which also verifies that noise distribution is changed and is not subject to normal distribution again after noise passes stochastic resonance system based on scale change. Therefore, good detector not under Gaussian white noise background shall be used to make further detection to output signal of bistable stochastic resonance.

#### **B. DETECTION ANALYSIS OF BPSK SIGNAL**

To ensure stochastic resonance, the number of sampling point of received signal set in the experiment is large enough, and in accordance with central-limit theorem, test statistics is subject to approximate Gaussian distribution. System parameter and simulation environment are almost equal to simulation condition in last section that SNR is -10dB and false alarm probability is 0.01, assumed that generalized Gaussian noise parameter p=1, suggesting that it is assumed that noise is subject to Laplace noise distribution after passing stochastic BSR system. Assumed that time span of BPSK signal is 0.1s, and 1000 amplitude transformations are made during the period, make the signal pass bistable system with parameter set in last section, make output signal of system pass nonlinear detector, and demodulate output signal of bistable system and output signal of detector. For the convenience of observation, waveform within 0.01s is chosen, which is as shown in Fig.10.

Through comparison to above 2 curves, it can be seen that bistable system has good capability to enhance signal energy, so signal feature is further highlighted, which is conducive



FIGURE 10. Figure on output waveform of bistable system and detector.



FIGURE 11. Figure on comparison between original signal and output signal.

to further detection of nonlinear detector. Make output signal pass nonlinear detector to obtain waveform of BPSK signal detected. To further evaluate correctness of output result of detector and error rate of calculating system, draw relational graph of original pure signal and output signal of detector, which is as shown in Fig.11.

Through comparison to above 2 groups of point, it can be seen that in the  $26^{\text{th}}$  signal modulation, system meets detection failure, and correct detection is obtained in other 99 times. In sequence of 1000 modulations to the whole band, 6 times of detection failure appear with relatively low error rate. Make 100 times of Monte Carlo simulation experiment to above flow, and average error rate of detection system obtained is 0.0076. Thus, under -10dB condition and Laplace noise background condition, algorithm proposed in this study has good detection efficiency.

### C. COMPARISON OF ALGORITHM PERFORMANCE

In order to further reflect the performance of this algorithm, under the condition of no prior information and low signal-to-noise ratio, the algorithm in this paper is compared with the existing algorithms to reflect the advantages of the method. At the same time, through the following three sets of comparative experiments, the algorithm in this paper demonstrates that the method has better results in a strong noise environment, for BPSK signal detection and without prior information.



**FIGURE 12.** Figure on comparison between original signal and output signal.

Firstly, the advantages of this algorithm in low SNR environment are discussed. To further measure performance of algorithm proposed in this study and embody advantage of algorithm proposed in this study, under low SNR, adjust parameter value of *p*, and compare detector in this study with Best Linear Detection (BLD), simulation environment and parameter value are kept unchanged, and simulation SNR is [-30,0]dB, and simulation figure obtained by taking 3 dB as step is as Fig. 12.

Through above figure, it can be seen that nonlinear detector in this study designed based on Neyman-Pearson criterion has good detection efficiency under low SNR and with increase of SNR, detection efficiency will increase obviously. In the meantime, in most cases, performance of detector in this study is superior to optimal linear detection algorithm. When p = 2, noise is Gaussian white noise, and optimal linear detector has the best detection performance under Gaussian white noise background, but when p is changed gradually, performance advantage of detector designed in this study appears gradually with more obvious error rate decrease.

However, with the further increase of the signal-to-noise ratio, the performance of the algorithm in this paper will be inferior to BLD at about 10dB. There are two main reasons. One is that the signal-to-noise ratio is increased, the resonance effect is weakened, and the ability to increase the signal energy is also reduced. Although the bit error rate is still decreasing, the performance is not as good as BLD. Another reason is that the signal detection algorithm in this paper is designed based on Neyman-Pearson criterion. The false alarm probability needs to be set, and the false alarm probability is negatively related to the bit error rate. And setting false alarm probability means that the system allows bit error rate. Due to the inherent errors of the system, the performance of this algorithm is limited under high SNR conditions. Therefore, the algorithm in this paper is more suitable for low SNR environments.

Secondly, it discusses the advantages of this algorithm to BPSK signal detection. The algorithm in this study is compared with [21]. The simulation parameters are the same as those shown in Figure 10 in reference [21]. The false alarm



FIGURE 13. Comparison between the performance of algorithms.

probability is set as 0.05. The Monte-Cario Experiment is performed 200 times. The simulation results are averaged, and the results are compared as shown in Fig. 13.

As suggested from the above figure, compared with [21], the algorithm in this study can provide higher SINR. This is because the algorithm in this study can transform the energy of noise into the energy of signal through nonlinear action under the condition of low signal-to-noise ratio. Thus, it has better detection ability than the method of [21].

Due to the non-linearity, the detection performance of the parameter-adjusted Algorithm (PAA) here is not the same as that of the method in [21]. According to the definitions of formulas (1), (7) in [21], and the derivation results of (20) and (23), combined with the simulation results of the literature, Figure 4 in [21], we can see that in the method of [21], The SINR of the output is linearly related to the SNR of the input. The detection capability is linearly correlated with the SNR. In the meantime, Figure 13 shows that with the rise of SNR, the detection efficiency of PAA algorithm is slower than that of the other two methods. Even with the improvement of SNR, the algorithm in this paper is almost overtaken by the other two methods. Since the efficiency of PAA algorithm is related to the intensity of noise, the noise energy decreases, and the detection efficiency decreases naturally. In other words, under the condition of high signal-to-noise ratio (SNR), the detection efficiency of PAA is similar to or even worse than that of the mainstream method, while under the condition of low SNR, the detection efficiency of PAA is obviously better than that of the mainstream method. Therefore, this method is more suitable for weak signal detection.

The method in this paper has a good effect on BPSK signal detection, because this paper designs a detector based on Neyman-Pearson criterion. The BPSK signal is a 0-1 coded signal, which corresponds to the presence or absence of the signal, which is a typical binary test problem. In the absence of prior information, a detector designed based on Neyman- Pearson criterion has an excellent effect on such binary detection. However, if it becomes a problem of multivariate hypothesis testing, the performance of the algorithm in this paper will decline, and the detector needs to be



FIGURE 14. Comparison of the performance of detection algorithms.

redesigned, or the multivariate test is converted into multiple binary test problems before it can be used. Therefore, the algorithm in this paper has a strong advantage for the detection of BPSK signals.

Finally, the performance of the proposed method without prior information is discussed. To further compare the performance of the method in this study, the algorithm here is compared with the detection algorithm in [45]. For the comparison of the detection algorithms optimized also based on SR resonance, the setting of simulation parameters is consistent with the simulation conditions in Fig. 6 in [45], and the false alarm probability is 0.1. Monte Carlo experiment is also performed 200 times, and the results are averaged. The results of comparison are shown in Fig. 14.

It is suggested from the above comparison that though Parameter-adjusted Algorithm (PAA) is slightly better than IED (Improved Energy Detection) and superior to the classical ED (Energy Detection), there is no significant improvement in terms of the results. This is because the IED method proposed in [45] needs to know the parameters of the signal to be tested, and then set a, b in SR system. The method in this study is not required to know all parameters of the signal to be detected, and the signal detection can be realized only by estimating the frequency band of the signal, which can reduce the dependency of prior information and relax the applicable conditions. Thus, the method has higher applicability.

In the meantime, another reason why the detection result of this method is slightly better than that of IED method when the prior information is less than that of IED method is that, IED method strives to reduce the influence of SNR Wall and achieves good results. Yet this method and corresponding parameters are difficult to achieve both the reduction of the influence of SNR Wall and the dual goal of optimal detection, so the improvement of detection performance is actually limited. The algorithm proposed in this paper takes the bit error rate as the feedback quantity, adaptively adjusts the system parameters according to the parameter characteristics of BPSK signal, and dynamically matches the parameters, thereby achieving better signal detection. Therefore, though the prior information is less than that of the IED method, the detection result can be better than that of the IED method.

TABLE 1. Improvements compared to the existing algorithms.

	Improvements
Detection algorithm	A detection method for BPSK signals was constructed without prior information and low signal-to-noise ratio.
BSR	Propose and prove the scale transformation method A method for adjusting SR system parameters based on the characteristics of the signal to be detected is given.
Signal detector	Derive the characteristics of SR system output parameters Detector designed based on Neyman-Pearson criterion
Parameter adjustment	Derived theoretical error Design feedback regulation law based on error

For the secure communication in the form of cooperation, the efficiency of this method is obviously not as good as that of the cooperative receiver. Therefore, this method is more suitable for signal detection without prior information.

Through above simulation, signal processing and detection flow built in this study can realize good detection and demodulation to BPSK signal under low SNR, to enhance capability to detect weak signal under strong noise background obviously.

In summary, the proposed algorithm can achieve better detection of BPSK signals. The main improvement points of this paper are shown in Table 1.

#### VII. CONCLUSIONS

(1) A set of bistable stochastic resonance model based on scale change is built by utilizing BSR system, efficiency of detection to BPSK signal is improved and nonlinear signal detector is built based on Neyman-Pearson criterion in this study. A set of complete signal detection flow is built by combining them, which realizes detection to weak BPSK signal under strong noise and no prior information.

(2) Aimed at the restriction that classical bistable stochastic resonance system can only process low-amplitude periodic signal at low-frequency stage, this study improves stochastic resonance model and expands its applicable scope by referring to scale change thought and verifies effectiveness of method proposed through theoretical derivation and experimental verification.

(3) Because existence or inexistence of signal will be determined first to detect BPSK signal under strong noise background, priori knowledge on BPSK signal is limited, and PDF of noise is changed after noise passes stochastic resonance system, nonlinear detector is designed based on Neyman-Pearson criterion, and complete flow on detector construction and parameter determination is given based on generalized Gaussian noise, and detector design under different noise types is realized.

(4) Quantitatively expressing algorithm efficiency of system proposed in this study through theoretical analysis and equation derivation has promotion significance to theoretical research of stochastic resonance. Construct feed-back system based on expression of algorithm efficiency to gain optimum value of detection efficiency. Algorithm proposed in this study is not only applicable to BPSK signal, but also applicable to other communication, photology and even fault signal etc., which provides good reference for subsequent signal detection theory and practical project application.

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