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Bayesian Compressive Sensing Based on Importance Models

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Abstract: To solve the problem that all row signals use the same reconstruction algorithm, a type of Bayesian compressive sensing based on importance models is proposed, which reconstructs more important signals firstly even if losing some unimportant signals. Compared to Bayesian compressive sensing whose performances is not well when sampling ratio is lower, the proposed algorithms can improve reconstruction quality effectively. The importance models include two processes, one is judging whether the signal is important and the other is how to reconstruct important signals better. In this paper, the improved reconstruction algorithm is based on sparse important signal and assigning measures by important weights. The two algorithms give priority to the more important column coefficient signals in the reconstruction process. The experimental results show that the proposed algorithms have better reconstruction effect than the traditional Bayesian compressive sensing, and especially, the performance of reconstruction algorithm based on assigning measures by important weights is improved obviously when the sampling rate is relatively low. *Copyright © 2013 IFSA.*

Keywords: Bayesian compressive sensing, Importance model, Wavelet.

1. Introduction

Compressive Sensing [1-5] has an extensive application perspective in the field of signal processing because it breaks through Nyquist theory which the sampling rate must be more than two times the highest frequency. Sparse representation of image is a research hotspot in the compressive sensing. Currently, there are many improvements of sparse representation of image, which are based on different transformations, such as wavelet [1, 6], curvelet [7], bandlet [8], contourlet [9], dual-tree complex wavelet etc [10]. These algorithms are all only concerned on improving the sparse representation of image and use the unified reconstruction algorithm to all coefficients after sparse transformation, without considering the coefficient features.

Wavelet coefficients have many features [12], such as spatial frequency and direction selection, energy concentration of frequency domain and energy attenuation, spatial clustering of high frequency, the similarity between subband coefficients, the relative between amplitude etc. The scale features of wavelet coefficients [13] and modeling sparse prior by using the similarity between subband coefficients [14, 15] are used to improve the reconstruction effect of Bayesian compressive sensing successfully. In this paper, we firstly try to study the feature of wavelet energy distribution and wavelet coefficient statistics. Then we take full advantage of wavelet coefficient features to propose two importance models. They are based on sparse important signals and measures assigned by important weights respectively. The proposed algorithms give priority to the more important column coefficient signals in the reconstruction process. The experiment results show that compared to the traditional Bayesian compressive sensing [11], the proposed algorithms have improvements in some extent. Especially, the performance of reconstruction algorithm based on measures assigned by important weights is improved obviously when the sampling rate is relatively low.

The paper is organized as follows. In section two, we introduce the energy distribution feature of wavelet coefficients and coefficient statistics in each wavelet level. In section three, we introduce the importance model. In section four and five, we propose the improved algorithms based on sparse important signals and measure assigned by important weight, respectively. They are two different perspectives to reconstruct mainly important row signals. Simulation results which are presented in section six testify the improved effect of Bayesian compressive sensing based on importance models. Conclusions and discussions of future work are provided in section seven.

2. The Feature of Wavelet Coefficients

2.1. Wavelet Energy Distribution

After wavelet transformation like Fig. 1, the energy distribution of image is changed. The energy formula can be given by:

$$E = \frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{N} A(i,j)^2 , \quad (1)$$

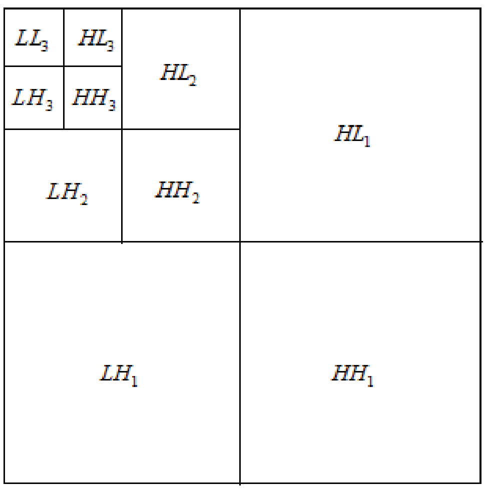


Fig. 1. Three levels of wavelet transformation.

Fig. 2 shows energy distribution of wavelet coefficients in the three level subbands. X-coordinate is corresponding subband and Y-coordinate is energy ratio. The left figure shows all subbands. In order to better observe the energy distribution in the three levels, the subbands except $LL3$ subband are showed in the right of Fig. 2. We can see that wavelet energy decrease level by level and concentrate on high level subbands. $LL3$ subband is highest. If subband coefficients of the higher level can be reconstructed better and LL subband coefficients can be reconstructed better, the quality of reconstructed image will be improved certainly.

2.2. Wavelet Coefficient Statistics

Analyzing the wavelet coefficients in different subbands, it is easy concluded that the coefficient values in HL subband and LH subband are larger than those in HH subband in the same level. As the level is higher, the wavelet coefficients are higher. Large coefficients are mainly in the LL subband, few in the HL subband and LH suband and very rare in the HH subband. Fig. 3 shows the absolute subtraction value between HL coefficients and HH coefficients of each level for a 256 × 256 Lena image. Above the dotted line, the coefficient absolute values of HL subband are larger than HH subband, otherwise the opposite. We can conclude that most HL subband coefficients are larger than HH subband coefficients in the same level.

3. Importance Reconstruction Model

Most of energy is gathered in the high level subband and many large coefficients are in high level subband. If we can improve the reconstruction quality of column coefficient signals which contain much energy or large coefficients, even losing quality column coefficient signals containing few energy or small coefficients, the overall reconstruction quality will be improved. Importance reconstruction model can be expressed like Fig. 4.

The main core of importance reconstruction model is how to judge the importance of row coefficient signals and how to focus on the reconstruction of these important signals. The following section, we consider from two directions and propose two reconstruction algorithms based on sparse important signals and more measures assigned to important signals, respectively.

4. Improved Algorithm Based on Sparse Important Signals

The reconstruction of Bayesian compressive sensing [12] can be expressed a problem that suppose the prior probability distribution for θ is a kind of sparse distribution like Laplace distribution and then maximize posterior probability to solve l_1 norm problem.

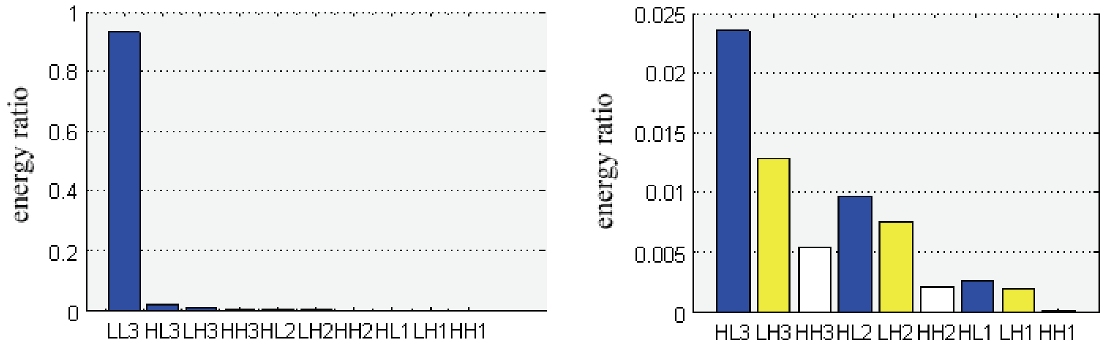


Fig. 2. Energy distribution of wavelet coefficients.

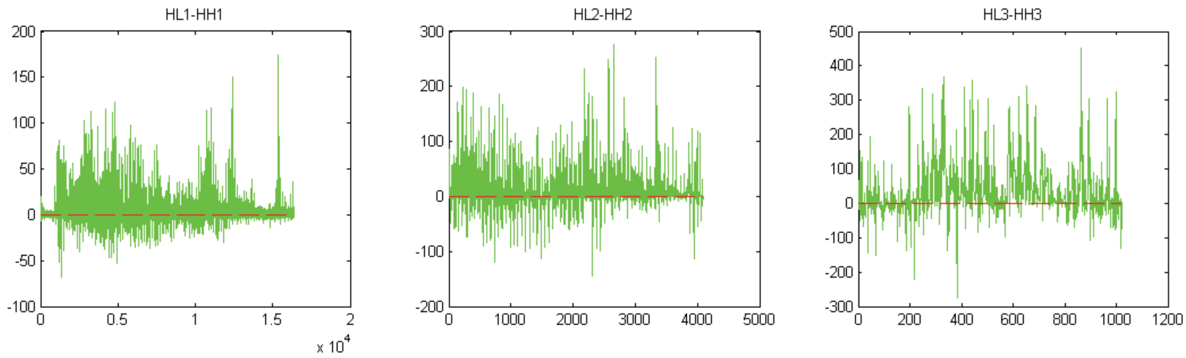


Fig. 3. HL coefficients vs. HH coefficients in every level.

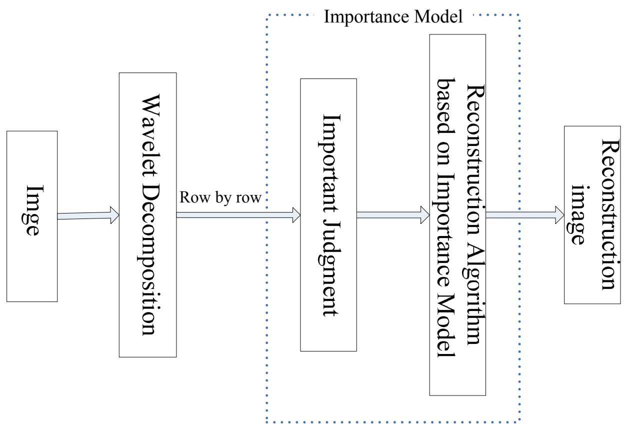


Fig. 4. Importance reconstruction model.

Bayesian compressive sensing mostly uses sparse prior. Therefore, if θ is sparser, the modeling effect of prior distribution is better.

Suppose image size is $M \times N$ pixel. Reconstruct wavelet coefficients row by row, which can decrease quantity of objection matrix [16]. In fact, this reconstruction method is a kind of block compressive sensing [17]. Wavelet coefficients θ can be represented $(\theta_1, \theta_2, ..., \theta_M)$ and the wavelet coefficients in the third level are $(\theta_1, \theta_2, ..., \theta_{M/4})$. According to the above section, if we reasonably use wavelet energy features and wavelet coefficient statistics to make $(\theta_1, \theta_2, ..., \theta_{M/4})$ sparser, the whole reconstruction effect will be improved. Exchange HL subband and HH subband in the same level. Fig. 5 shows the kurtosis difference between before and after wavelet coefficients exchange of 256×256 Lena image. From the Fig. 5 we can see that kurtosis change of the former thirty two line exchanged wavelet coefficients $(w_1, w_2, ..., w_{M/4})$ is not obvious. Because after wavelet translation, the LL subband coefficients in the three level are enough large. Even if after translation, the kurtosis is not affected. But we can see that kurtosis change of the last thirty two line exchanged wavelet coefficients $(w_1, w_2, ..., w_{M/4})$ is obvious which illustrate that the exchange $(w_1, w_2, ..., w_{M/4})$ is sparser than $(\theta_1, \theta_2, ..., \theta_{M/4})$.

We judge the row coefficient signals included high level subband as important signals and make them sparser. The main procedures of algorithm can be described as:

Step 1: Apply sparse transformation to the original image x and get wavelet coefficients θ. In his paper we use DWT transformation. Wavelet basis is $sym8$.

Step 2: Exchange HL subband coefficients and HH coefficients in each level of wavelet coefficients θ and get modified wavelet coefficients w.

Step 3: Reconstruct using BCS row by row and get reconstructed modified wavelet coefficients $\hat{w}$.

Step 4: Exchange HL subband coefficients and HH coefficients in each level of wavelet coefficients $\hat{w}$ and obtain wavelet coefficients $\hat{\theta}$.

Step 5: Apply DWT inverse transformation to wavelet coefficients $\hat{\theta}$ and obtain reconstructed image $\hat{x}$.

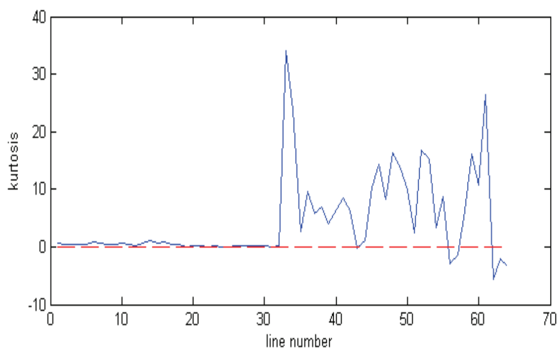


Fig. 5. The kurtosis difference between before and after exchange of the first 64 line wavelet coefficients.

5. Improved Algorithm Based on Measures Assigned by Important Weights

The previous section we improve the algorithm performance via sparse important signals. In this section, we will reconstruct better the row signals included much energy and large coefficients via assigning measure quantity.

According to the second section about wavelet features, high level subbands contain most of energy. We try to allocate more measures to high level subbands, and decrease corresponding measures to low level subbands. Therefore the total number of measures is not change. We lose the reconstruction quality of unimportant row signals in exchange for a better reconstruction of important signals to improve the whole reconstruction effect. But some energy is scattered in high subbands after wavelet decomposition of multilevel, we sort energy of all row signals to judge each row importance and then allocate more measures to row signals whose importance value is higher. The main procedures of algorithm can be described as:

Step 1: Apply sparse transform to the original image x and get wavelet coefficients θ. Here, we use DWT transformation. Wavelet basis is $sym8$.

Step 2: Obtain energy value of each row coefficient signal, sort it and get a sorting index I.

Step 3: Assign measures according to the index I.

Step 4: Reconstruct using BCS row by row and get reconstructed wavelet coefficients $\hat{\theta}$.

Step 5: Apply DWT inverse transform to wavelet coefficients $\hat{\theta}$ and obtain reconstructed image $\hat{x}$.

If the measure number of each row is M before reassigning, set a threshold $T \in (1, ..., N)$ to express the number of important signals. If the signal index satisfies to $I_i \le T$, we judge it as a important signal, then the measure set M_b. If the index $I_i > T$, it is an unimportant signal, then the measure set M_s. M, M_b, and M_s satisfy:

$$M = \frac{TM_b + (N-T)M_s}{N}, \tag{2}$$

where $M_b > M > M_s$. Using this assignment method, the whole measure number is not change, just reallocating once and give more measures to important signals to reconstruct accurately. This measure allocation method is a simple approach, but it is no full use of the energy order. Therefore, we give another measure allocation method based on energy weights.

If the index of row signal $I_i > T$, measure number still sets M_s.

If the index of row signal $I_i > T$, set a weight factor:

$$W_i = \frac{N - I_i}{\sum_{i=1}^{T} (N - I_i)} \tag{3}$$

for these row signals.

According to the weight factor W_i, get the corresponding measure:

$$M_i = Round\left(W_i T M_b\right), \tag{4}$$

where $Round(\cdot)$ is rounded value. The mean of summation M_i equals to M_b, which ensures the total measure unchanged.

Weight factor W_i can be viewed as importance factor determining how important of a row signal. It meets purpose that more important signals which include more energy reconstruct more sufficiently. Therefore, it is a good measure assignment method.

6. Experiment Results

In this section we mainly compare BCS algorithm and improved algorithms based on sparse important signals or measure assigned by important weights. To introduce easily, the two improved methods can be expressed as BCS-Sparsity and BCS-Measure. We reconstruct ten 256×256 MRI images, using DWT transformation. Wavelet basis is sym16 and observation matrix is Gaussian random matrix. Set the thresholds:

$$T = \frac{N}{2}, \tag{5}$$

$$M_b = \frac{3}{4}M, \tag{6}$$

$$M_s = \frac{1}{4}M, \tag{7}$$

Fig. 6 is the reconstruction results when the sampling rate is 0.5. Rerr is expressed as the reconstruction error, $\left\|\theta - \hat{\theta}\right\|_2 / \left\|\theta\right\|_2$. Rerr is closer to zero, the reconstruction effect is better. Fig. 6 shows that Rerrs of the two algorithms both decrease and the BCS-Measure algorithm improves effectively compared to the original BCS and is superior to BCS-Sparsity.

Fig. 7 shows the reconstruction effect for a MR image in the different sampling ratio using BCS, BCS-Sparsity and BCS-Measure, respectively.

Still set the thresholds as equations (5), (6), and (7). Use DWT transformation, wavelet basis is sym16 and observation matrix is Gaussian random matrix. We use PSNR to judge the three algorithms. The PSNR can be defined as:

$$PSNR = 10\log_{10}\left(\frac{255^2}{MSE}\right) \tag{8}$$

where

$$MSE = \frac{1}{A \times B}\sum_{i=0}^{A-1}\sum_{j=0}^{B-1}\left(x(i,j) - \hat{x}(i,j)\right)^2, \tag{9}$$

and (i,j) is the pixel point of a $A \times B$ image. Larger PSNR indicate more closely to the original MR image.

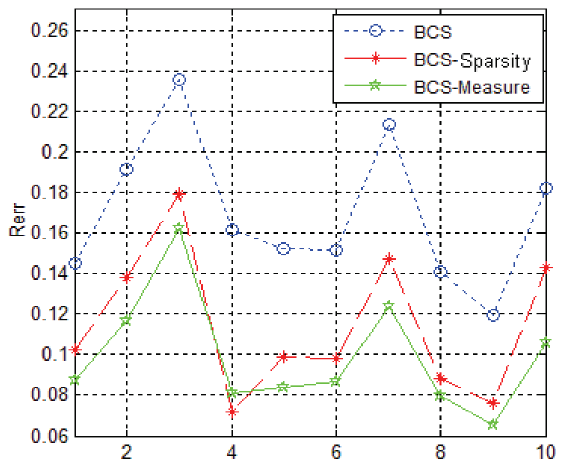


Fig. 6. Compare reconstruction effect using three different algorithms in the same sampling ratio.

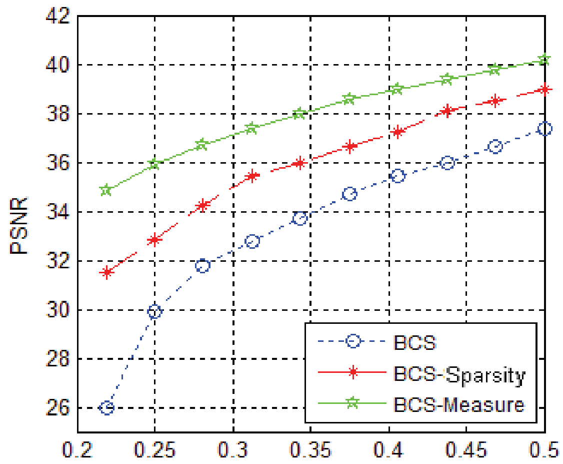


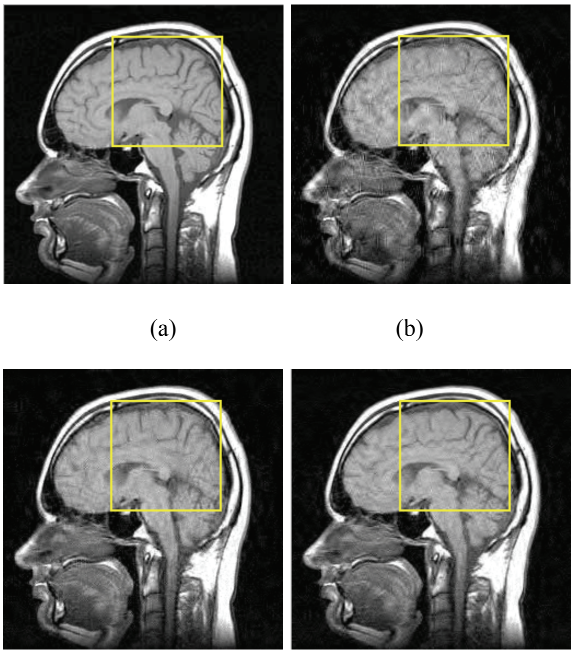
Fig. 7. Reconstruction effect using three algorithms in the different sampling ratio.

It shows that no matter the sampling ratio changes, the PSNRs using BCS-Sparsity and BCS-Measure are higher than BCS method. When the sampling ratio is 0.2188, BCS-Sparsity is improved above 5 dB and BCS-Measure is improved about 9 dB. Although the improvement using BCS-Sparsity method is smaller than BCS-Measure, the distance between the two algorithms grows smaller with the sampling ratio increasing. It illustrates that BCS-Measure method is more superior when the sampling ratio is low and BCS-Sparsity method also has some improvement effect.

Set the thresholds as equations (5), (6), (7) and

$$M = \frac{N}{2}, \tag{10}$$

wavelet basis is sym16. Fig. 8 compares the reconstruction effect for 256×256 brain MR image using three different algorithms. Fig. 8 (a) is the original image, Fig. 8 (b), Fig. 8 (c) and Fig. 8 (d) are reconstruction images using BCS, BCS-Sparsity or BCS-Measure respectively. It shows that the reconstruction images using BCS-Sparsity and BCS-Measure are clearer than BCS algorithm.

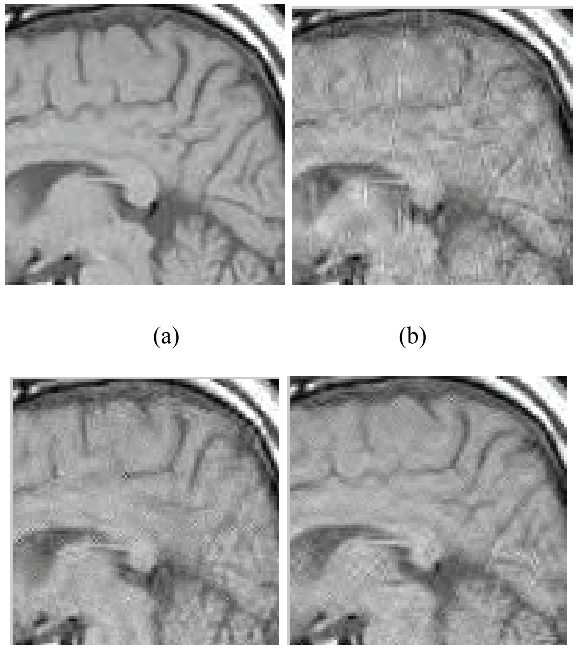


(a) (b) (c) (d)

Fig. 8. 256×256 MRI reconstructed images: (a) original image, (b) BCS algorithm, (c) BCS-Sparsity algorithm, (d) BCS-Measure algorithm.

To observe the image detail clearly, Fig. 9 shows partial enlarged detail around the eye. Fig. 9 (a) is the partial enlarged detail of original image, Fig. 9 (b), Fig. 9 (c) and Fig. 9 (d) are partial enlarged detail of reconstruction images using BCS, BCS-Sparsity or BCS-Measure respectively. It shows that the details is less clear used BCS algorithm, but the reconstructed images using the improved algorithms are almost similarly clear to original images. It is visual illustration of the improvement effect via Bayesian compressive sensing based on importance model.

To illustrate that the main idea of improved algorithm based on importance models which are emphasis on the reconstruction of important row signals and focus on reconstruction of LL subband and high level subband, we show reconstruction wavelet coefficients $\hat{\theta}$ of 128×128 brain image in Fig. 10. Set the thresholds as equations (5), (6) and (7). Use DWT transformation, wavelet basis is sym8 and observation matrix is Gaussian random matrix. Fig. 10 (a) shows the whole reconstruction wavelet coefficients using three algorithms. In Fig. 10 (b), we compare the reconstruction wavelet coefficients using three algorithms in high level. Fig. 10 (c) shows the reconstruction wavelet coefficients in HH_1 subband using three algorithms. Fig. 11 shows reconstruction error using BCS, BCS-Sparsity and BCS-Measure algorithm in different subbands. The reconstruction error using BCS-Measure algorithm is all lower than BCS in the high level subbands. The reconstruction error using BCS-Sparsity algorithm is lower than BCS in the high level subbands except HH_2 subband, but the difference is not large. From the Fig. 10 and Fig. 11, we can see that the two improved methods reconstruct better in high level subbands and worse in low level subbands than BCS.

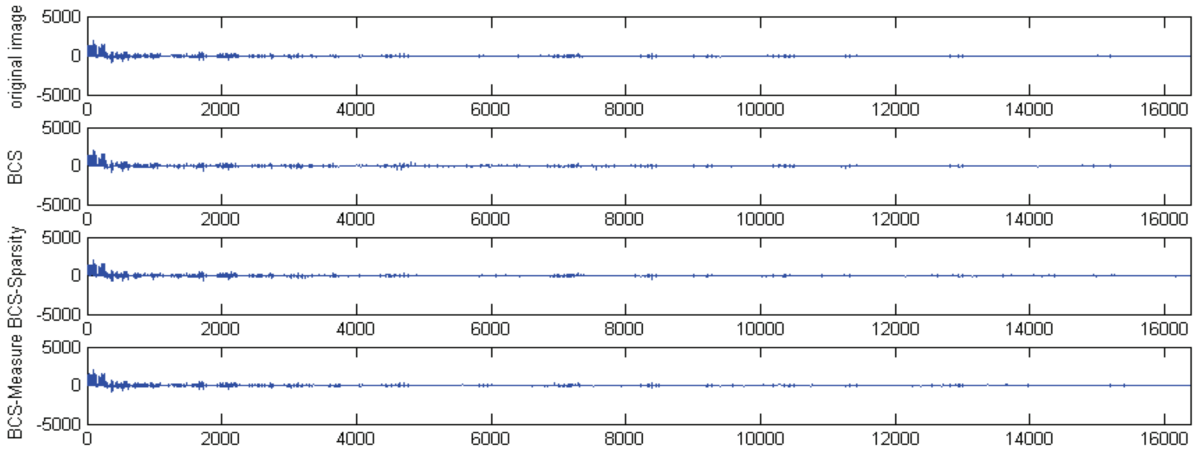


(a) (b) (c) (d)

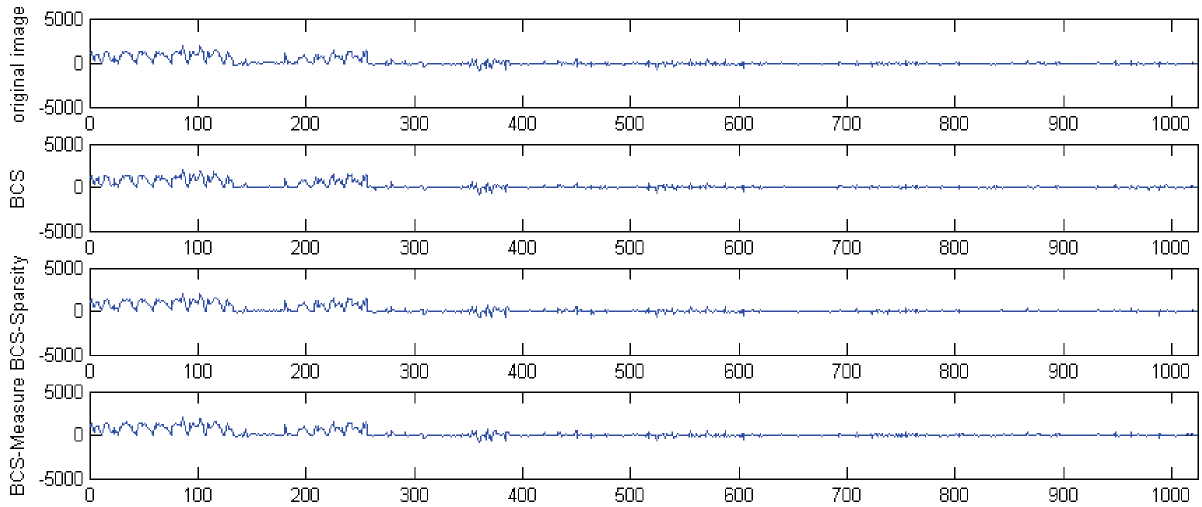
Fig. 9. 256×256 MRI partial enlarged detail around eye of reconstructed images.

7. Conclusions

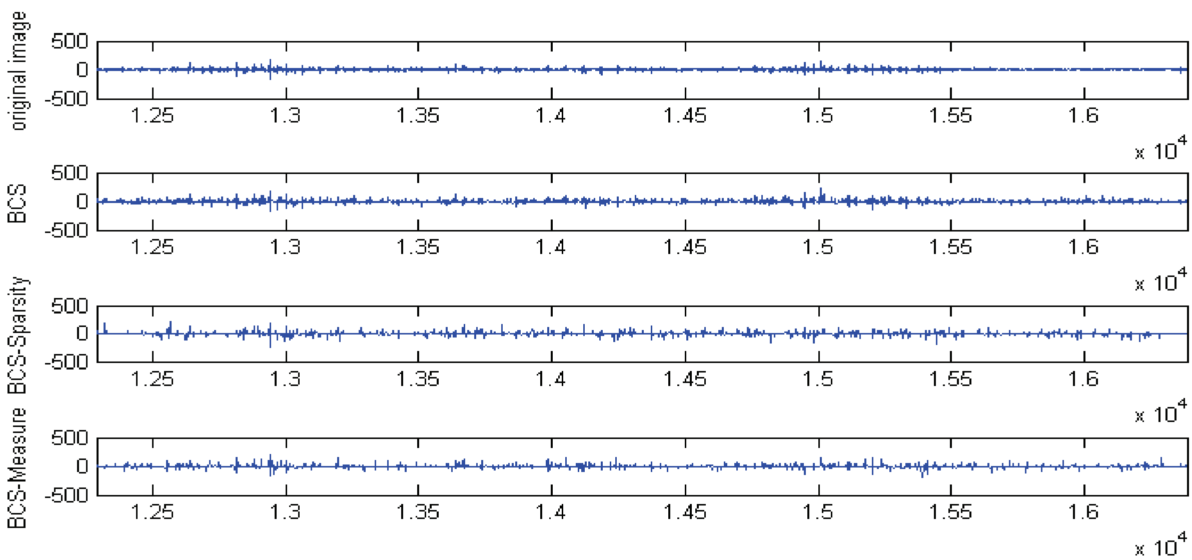
Wavelet coefficients have many features. In this paper, we study energy feature of wavelet coefficients to see that subband energy in high level is the highest and the influence to reconstructed quality is the largest. According to the wavelet coefficients statistics, we can see that large coefficients concentrate mostly in high subband of high energy. Taking full advantage of wavelet coefficient features, we propose two reconstruction algorithms based on importance models. The main idea utilizes the wavelet features to judge which row coefficient signals are important. And then according the judgment, we propose two reconstruction algorithms from two directions which are more sparsing or give more measures to important signals.



(a) The whole wavelet coefficients.



(b)The wavelet coefficients in high level.



(c) The wavelet coefficients in HH_1 subband.

Fig. 10. Compare the reconstructed wavelet coefficients using BCS, BCS-Sparsity or BCS-Measure.

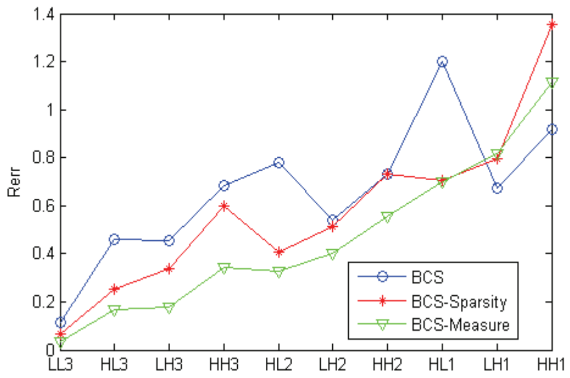


Fig. 11. Reconstruction error in different subband using BCS, BCS-Sparsity and BCS-Measure algorithm.

The experiment results analyze the performance of the two improved algorithms. When the sampling ratio is low, Bayesian compressive sensing based on assigning measures by important weight shows the relatively good performance compared to BCS or the other improved algorithm. But with the sampling ratio increase, Bayesian compressive sensing based on sparse important signals gradually shows its performance. When the sampling ratio reaches some value, the two methods have good performance for different images. But in general, the two algorithms are better than BCS algorithm, which indicate that introducing the importance models is effective. In the future research, we can introduce wavelet coefficient features deeply into compressive sensing to improve reconstruction quality.

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