

On the Reduction of Variance and Overestimation of Deep Q-Learning

Mohammed Sabry

University of Khartoum, Faculty of Engineering
Department of Electrical and Electronic Engineering
Khartoum, Sudan
mhmd.sabry.ab@gmail.com

Amr M. A. Khalifa

African Institute for Mathematical Sciences
Kigali, Rwanda
amr.khalifa.m.a@gmail.com

Abstract—The breakthrough of deep Q-Learning on different types of environments revolutionized the algorithmic design of Reinforcement Learning to introduce more stable and robust algorithms, to that end many extensions to deep Q-Learning algorithm have been proposed to reduce the variance of the target values and the overestimation phenomena. In this paper, we examine new methodology to solve these issues, we propose using Dropout techniques on deep Q-Learning algorithm as a way to reduce variance and overestimation. We further present experiments on some of the benchmark environments that demonstrate significant improvement of the stability of the performance and a reduction in variance and overestimation.

Index Terms—Dropout, Reinforcement Learning, DQN

I. INTRODUCTION

Reinforcement Learning (RL) is a learning paradigm that solves the problem of learning through interaction with environments, this is a totally different approach from the other learning paradigms that have been studied in the field of Machine Learning namely the supervised learning and the unsupervised learning. Reinforcement Learning is concerned with finding a sequence of actions an agent can follow that could lead to solve the task on the environment [1] [2] [3]. Most of Reinforcement Learning techniques estimate the consequences of actions in order to find an optimal policy in the form of sequence of actions that can be followed by the agent to solve the task. The process of choosing the optimal policy is based on selecting actions that maximize the future payoff of an action. Finding an optimal policy is the main concern of Reinforcement Learning for that reason many algorithms have been introduced over a course of time, e.g, Q-learning [4], SARSA [5], and policy gradient methods [6]. These methods use linear function approximation techniques to estimate action value, where convergence is guaranteed [7]. However real world problems usually involve high dimensional features, linear function approximation methods diminish the agent's flexibility to learn the appropriate representation, and so the need of an expressive and flexible non-linear function approximation emerges. The recent advances in deep neural networks helped to develop artificial agent named deep Q-network(DQN) [8] that can learn successful policies directly from high-dimensional features. Despite the remarkable flexibility and the huge representative capability of DQN, there are some issues that arise from the combination of

Q-learning and neural networks. Thrun and Schwartz (1993) were the first to investigate one of these issues which they have termed as the overestimation phenomena [9]. The maximization of the action space in Q-learning algorithm and the generalization errors in neural networks can lead to overestimation and variance in of state-action values. To mitigate these problems additional modifications and extensions to the basic algorithm are needed for further increase in training stability and reduction in overestimation. Van Hasselt et al. suggest the Double-DQN [10] which is an extension that uses double Q-learning estimator [11] as a solution to to the variance and the overestimation phenomena.

In this work we propose and empirically study a different solution to the variance and the overestimation phenomena that uses Dropout techniques.

We summarize the main contribution of this paper as follows:

- An extension to the DQN algorithm which stabilizes training, and improves the attained performance, using Dropout methods and demonstrating our solution effectiveness on classic control environment ran on computer simulations.

II. BACKGROUND

A. Dropout

Deep neural networks are the state of the art learning models used in artificial intelligence. The large number of parameters in neural networks make them very good at modelling and approximating any arbitrary function. However the larger number of parameters also make them particularly prone to over-fitting, requiring regularization methods to combat this problem. Dropout was first introduced in 2012 as a regularization technique to avoid over-fitting [12], and was applied in the winning submission for the Large Scale Visual Recognition Challenge that revolutionized deep learning research [13]. Over course of time a wide range of Dropout techniques inspired by the original method have been proposed. The term Dropout methods was used to refer to them in general [14]. They include variational Dropout [15], Max-pooling Dropout [16], fast Dropout [17], Cutout [18], Monte Carlo Dropout [19], Concrete Dropout [20] and many others.

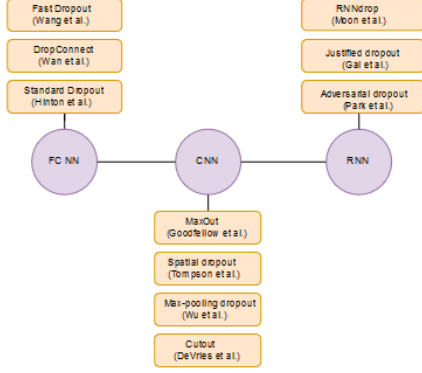


Fig. 1: Some proposed Dropout methods for different neural networks architectures.

1) *Standard Dropout*: It's the original Dropout method. It was introduced in 2012. Standard Dropout provides a simple technique for avoiding over-fitting in fully connected neural networks [12]. During each training phase, each neuron is excluded from the network with a probability p . Once trained, in the testing phase the full network is used, but each of the neurons' output is multiplied by the probability p that the neuron was excluded with. This approach gives approximately the same result as averaging of the outcome of a great number of different networks which is very expensive approach to evaluate, this compensates that in the testing phase Dropout achieves a green model averaging. The probability can vary for each layer, the original paper recommend $p = 0.2$ for the input layer and $p = 0.5$ for hidden layers. Neurons in the output layer are not dropped. This method proved effective for regularizing neural networks, enabling them to be trained for longer periods without over-fitting and resulting in improved performance, and since then many Dropout techniques have been improved for different types neural networks architectures (Figure 1).

B. Reinforcement Learning

The general Reinforcement learning framework [21](Figure 2) introduces an agent which is faced with a sequential decision making problem through its interaction with the environment, where this interaction takes place at discrete time steps ($t = 0, 1, \dots$). At time t the agent observes state $s_t \in S$, selects an action $a_t \in A$, which results in a reward $r_t \in R$, and a transition to a next state $s_{t+1} \in S$. We consider a discounted cumulative reward as objective function $G_t = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$ where $\gamma \in [0, 1]$ is the discount factor. The goal of the agent is to find an optimal policy $\pi : S \rightarrow A$ that maximize the expected discounted cumulative reward. Reinforcement methods encode policies through the use of value functions, which use the cumulative reward value of state or state-action pair to find a policy π that maximize the

expected discounted cumulative reward from a given state s . Specifically we are interested in state-action value functions:

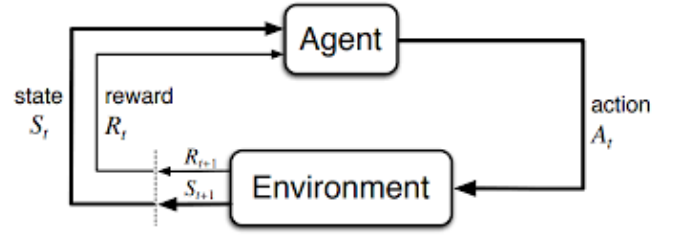


Fig. 2: Markov Decision Process (Famous RL framework).

$$Q^\pi(s, a) = E^\pi \left[\sum_{t=0}^{\infty} G_t | s_0 = s, a_0 = a \right] \quad (1)$$

The optimal Value function denoted as:

$$Q^*(s, a) = \max_{\pi} Q^\pi(s, a) \quad (2)$$

The above definition can be extended to other setups where we can have continuous time steps, continuous action space and continuous state space.

1) *Q-Learning*: The Q-learning is one of the most popular RL algorithms [4]. It's based on an incremental dynamic programming technique because of the step by step look-up table representation in which it determines the optimal policy [22]. Q-learning algorithm uses this table to estimate the optimal action value function Q^* , the table contains all states and actions on the environment and value function to estimate the quality(Q-function) of state-action pairs, then performs updates using the following update rule:

$$Q(s, a) \leftarrow Q(s_t, a_t) + \alpha (r + \gamma \max_{a_{t+1}} Q(s_{t+1}, a_{t+1}) - Q(s, a)) \quad (3)$$

where s_{t+1} is the resulting state after applying action a in the state s , r is the immediate reward observed for action a at state s , γ is the discount factor, and α is learning rate. The limitation of the look-up table representation emerges when the number of states and action is large, also maintaining a look-up table with all the possible state-action pairs values in memory is impractical. A common solution to this issue is to use other representations, i.e., the representations produced by the Neural Networks and the Deep Neural Networks.

2) *Deep Q-learning (DQN)*: Deep Q-learning combines Q-learning algorithm with neural network approximation to approximate the action-value function $Q(s, a, \theta)$ [8]. Then the update rule of Q values become:

$$L_i(\theta_i) = (r + \gamma \max_{a'} Q(s', a'; \theta_i^-) - Q(s, a; \theta_i))^2 \quad (4)$$

θ_i and θ_i^- are the parameters of network and target network at iteration i respectively. The target network parameters θ_i^- are only updated with the Q-network parameters θ_i every C steps and are held fixed between individual updates. DQN uses a memory bank approach to store (s, a, r, s') as experiences from previous iterations sampled uniformly and used in next iterations this technique termed as Experience Reply(ER).

3) *Overestimation error*: The Q-learning overestimation phenomena was first investigated by Thrun and Schwartz (1993) [9]. It presents a positive bias that can cause asymptotically sub-optimal policies that can distort the overall cumulative rewards. Most of the analytical and empirical results conclude that the overestimation arises from the max operator in Q-learning value function, the noise of approximation methods that it's used and in some cases the environment affects the expected overestimation and yields a bigger uneven bias in states where the Q-values are similar for the different actions, or in states which are the start of along trajectory.

4) *DQN Variance*: The sources of DQN variance are Approximation Gradient Error(AGE) [23] and Target Approximation Error(TAE) [24]. In Approximation Gradient Error, the error in gradient direction estimation of the cost function leads to inaccurate and extremely different predictions on the learning trajectory through different episodes because of the unseen state transitions and the finite size of experience reply buffer. This type of variance leads to converging to sub-optimal policies and brutally hurts DQN performance. The second source of variance Target Approximation Error which is the error coming from the inexact minimization of DQN parameters. Many of the proposed extensions focus on minimizing the variance that comes from AGE by finding methods to optimize the learning trajectory or from TAE by using methods like averaging to exact DQN parameters. Dropout methods have the ability to assemble these two solutions which minimize different source of variance. Dropout methods can achieve a consistence learning trajectory and exact DQN parameters with averaging, which comes inherently with Dropout methods. In the experiments we detected variance using standard deviation from average score collected from many independent learning trails.

III. EXPERIMENTS

The experiments were designed to address the following questions:

- How does Dropout affect the Variance in DQN.
- How does Dropout affect the Overestimation phenomena in DQN.
- How does Dropout affect the learned policies quality.

To that end, we ran Dropout-DQN and DQN on one of the classic control environments to express the effect of Dropout on Variance and the learned policies quality. For the Overestimation phenomena, we ran Dropout-DQN and DQN on a Gridworld environment to express the effect of Dropout because in such environment the optimal value function can be computed exactly.

A. Classic Control Environment

To evaluate the Dropout-DQN, we adopt the vanilla RL methodology where agent's performance is measured at the end of training epochs. Thus we ran ten consecutive learning

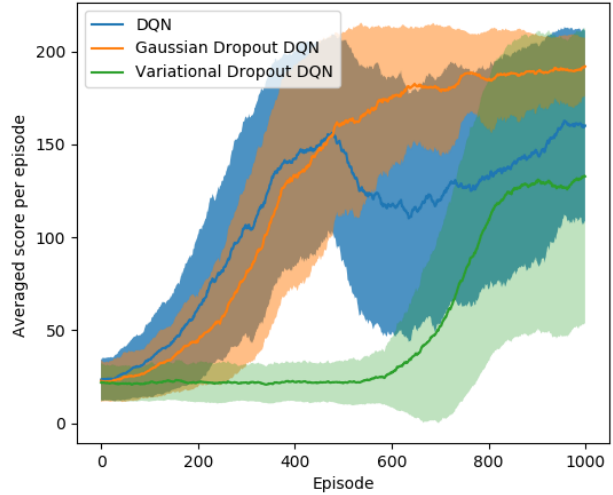


Fig. 3: Dropout DQN with different Dropout methods in CARTPOLE environment. The bold lines are averaged scores over 10 independent learning trials. The shaded area represents one standard deviation.



Fig. 4: Gridworld problem. The agent starts at the upper-right of the grid. In the bottom-left corner, a reward of +1 is obtained.

TABLE I: Variance comparison use Wilcoxon Sign-Ranked Test in CARTPOLE environment. DQN and its variants achieved scores(Rewards) are averaged over 10 independent learning trails.

Variance Comparison	Before Dropout(DQN) Avg. (Std.)	After Dropout Avg. (Std.)	Wilcoxon Sign-Ranked Test Statistic (p-value)
DQN VS Gaussian Dropout DQN	108.020 (54.932)	117.871 (46.846)	175584.0 (3.005e-16)
DQN VS Variational Dropout DQN	108.020 (54.932)	51.490 (28.075)	30695.0 (1.256e-127)

trails and averaged them. We have evaluated Dropout-DQN algorithm on CARTPOLE problem from the Classic Control Environment. The game of CARTPOLE was selected because of its popularity as a benchmark for RL algorithms and because of the fact that it is easy for the DQN to reach a steady state policy. For the experiments, fully connected neural network architecture was used. It was composed of two hidden layers of 128 neurons and two Dropout layers between the input layer and the first hidden layer and between the two hidden layers. To minimize the DQN loss, ADAM optimizer was used [25].

We detected the variance between DQN and Dropout-DQN visually and numerically as figure 3 and table 1 show.

The results in figure 3 show that using DQN with different Dropout methods result in better-performing policies and less variability as the reduced standard deviation between the variants indicate to. In table 1, Wilcoxon Sign-Ranked test was used to analyze the effect of Variance before applying Dropout (DQN) and after applying Dropout (Dropout methods DQN). There was a statistically significant decrease in Variance (14.72% between Gaussian Dropout and DQN, 48.89% between Variational Dropout and DQN). Furthermore one of the Dropout methods outperformed DQN score.

Overall, the results suggest that in practice Dropout can be used to reduce the variance and the overestimation of DQN, which leads to stabilized learning curves and significantly improved performance.

B. Gridworld

The Gridworld problem (Figure 4) is a common RL benchmark. Gridworld has a smaller state space that allows the ER buffer to store all possible state action pairs. Additionally, it allows the optimal action value function Q to be exactly computed.

1) *ENVIRONMENT SETUP*: In this experiment we design a customized environment on the problem of Gridworld (Figure 4) the state space contains pairs of points from a 2D discrete grid ($S = (x, y)_{x, y \in 0, 1, 2, 3, 4}$). There are four actions corresponding to steps in each compass direction, a reward of $r = +1$ in state G (Goal) and $r = -1$ otherwise. Fully connected neural network architecture was used. It was composed of two hidden layers of 128 neurons and two Dropout layers between the input layer and the first hidden layer and between the two hidden layers. ADAM optimizer for the minimization [25].

2) *OVERESTIMATION*: Figure 5 demonstrates that using Dropout methods in DQN reduce the overestimation from the optimal policy. Despite that Gridworld environment is not suffering from intangible overestimation that can distort the

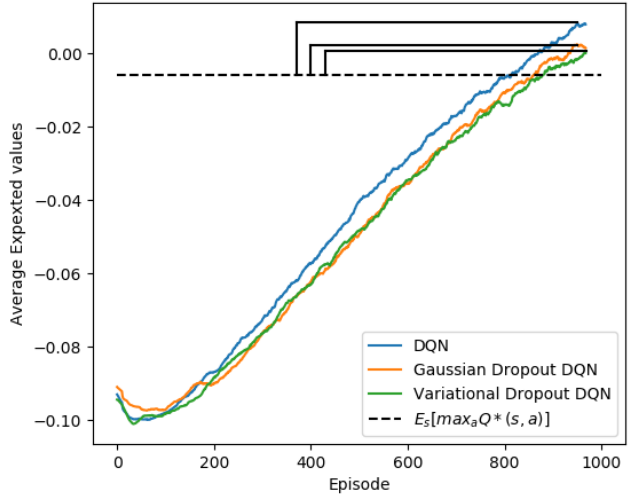


Fig. 5: Average Expected value in Gridworld. Dropout methods on DQN lead to less overestimation (positive-bias). The lines are averages over 50 independent learning trails. The dotted line represents the Optimal Policy.

overall cumulative rewards but reducing overestimation leads to more accurate predictions.

C. The Learned Policies

Figure 6 shows the loss metrics of the three algorithms in CARTPOLE environment, this implies that using Dropout-DQN methods introduce more accurate gradient estimation of policies through iterations of different learning trails than DQN. The rate of convergence of one of Dropout-DQN methods has done more iterations than DQN under the same assumption for both algorithms. It has been theoretically proven [26] that a large number of iterations creates a good quality policy that its corresponding Q values function converge to the optimal Q function, and now it's empirically demonstrated. Both Dropout-DQN algorithms have lower loss than DQN, this means that more accurate predictions of the value of the current policy which might not be the optimal policy but at least have a small deviation of loss between different policies and with all mentioned factors above lead to less variance in cumulative rewards and less overestimation of certain policies.

IV. CONCLUSION AND FUTURE DIRECTIONS

In this work we have proposed and experimentally analyzed the advantages of Dropout technique on DQN algorithm, as

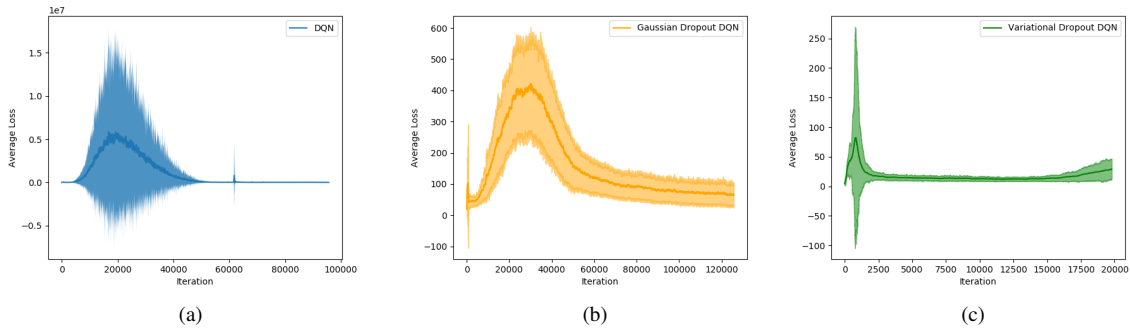


Fig. 6: The Average Loss metrics on Dropout DQN and DQN in CARTPOLE environment, iterations represent minimum times of soft updated loss of 10 independent learning trails.

an extension to DQN to stabilize the training, improving the performance, and reducing the variance. We have shown that the proposed scheme is superior in reducing the variance and the overestimation. However we have experimented on simple problems in simple environments using small networks architectures and only two of Dropout methods. Future work is going to including experimenting more Dropout methods on more challenging problems in complex Environments, e.g. Arcade Learning Environment(ALE).

Dropout-DQN is a simple extension that can be easily integrated with other DQN variants such as Prioritized experience replay [27], Double Q learning [10], Dueling Q learning [28], Optimality tightening [29] and Unifying count-based exploration and intrinsic motivation [30]. Indeed, it would be of interest to study the added value of Dropout when combined with these variants.

REFERENCES

- [1] Learning and sequential decision making AG Barto, R. S. Sutton, CJCH Watkins - Learning and computational neuroscience, 1989
- [2] Learning to act using real-time dynamic programming AG Barto, SJ Bradtke, SP Singh - Artificial intelligence, 1995
- [3] C. J. C. H. Watkins, Learning from Delayed Rewards. PhD thesis, Kings College, Cambridge, England, 1989.
- [4] Watkins, Christopher JCH and Dayan, Peter. Q-learning. Machine Learning, 8(3-4):279292, 1992.
- [5] Rummery, Gavin A and Niranjan, Mahesan. On-line Q learning using connectionist systems. University of Cambridge, Department of Engineering, 1994.
- [6] Sutton, Richard S, McAllester, David A, Singh, Satinder P, and Mansour, Yishay. Policy gradient methods for reinforcement learning with function approximation. In NIPS, volume 99, pp. 10571063, 1999.
- [7] Tsitsiklis, John N and Van Roy, Benjamin. An analysis of temporal-difference learning with function approximation. IEEE transactions on automatic control, 42(5): 674690, 1997.
- [8] Mnih, Volodymyr, Kavukcuoglu, Koray, Silver, David, Rusu, Andrei A, Veness, Joel, Bellemare, Marc G, Graves, Alex, Riedmiller, Martin, Fidjeland, Andreas K, Ostrovski, Georg, et al. Human-level control through deep reinforcement learning. Nature, 518(7540):529 533, 2015.
- [9] Thrun, Sebastian and Schwartz, Anton. Issues in using function approximation for reinforcement learning. In Proceedings of the 1993 Connectionist Models Summer School Hillsdale, NJ. Lawrence Erlbaum, 1993.
- [10] Van Hasselt, Hado, Guez, Arthur, and Silver, David. Deep reinforcement learning with double Q-learning. arXiv preprint arXiv: 1509.06461, 2015.
- [11] Van Hasselt, Hado. Double Q-learning. In Lafferty, J. D., Williams, C. K. I., Shawe-Taylor, J., Zemel, R. S., and Culotta, A. (eds.), Advances in Neural Information Processing Systems 23, pp. 26132621. 2010.
- [12] G. E. Hinton, N. Srivastava, A. Krizhevsky, I. Sutskever, and R. R. Salakhutdinov, Improving neural networks by preventing co-adaptation of feature detectors, arXiv preprint arXiv:1207.0580, 2012.
- [13] A. Krizhevsky, I. Sutskever, and G. E. Hinton, Imagenet classification with deep convolutional neural networks, in Advances in Neural Information Processing Systems 25, 2012, pp. 1097 1105.
- [14] Alex Labach, Hojjat Salehinejad and Shahrokh Valaee, Survey of Dropout Methods for Deep Neural Networks, arXiv preprint arXiv:1904.13310, 2019.
- [15] D. P. Kingma, T. Salimans, and M. Welling, Variational Dropout and the local reparameterization trick, arXiv preprint arXiv:1506.02557, 2015.
- [16] H. Wu and X. Gu, Towards Dropout training for convolutional neural networks, Neural Networks, vol. 71, no. C, pp. 110, 2015.
- [17] S. Wang and C. Manning, Fast Dropout training, in Proceedings of the 30th International Conference on Machine Learning. PLMR, 2013.
- [18] T. DeVries and G. W. Taylor, Improved regularization of convolutional neural networks with cutout, arXiv preprint arXiv:1708.04552, 2017.
- [19] Y. Gal and Z. Ghahramani, Dropout as a bayesian approximation: Representing model uncertainty in deep learning, in Proceedings of the 33rd International Conference on Machine Learning. PLMR, 2016.
- [20] Y. Gal, J. Hron, and A. Kendall, Concrete Dropout, in Advances in Neural Information Processing Systems 30, 2017, pp. 35813590.
- [21] Sutton, Richard S and Barto, Andrew G. Reinforcement Learning: An Introduction. MIT Press Cambridge, 1998.
- [22] Bellman, Richard. A Markovian decision process. Indiana Univ. Math. J., 6:679684, 1957.
- [23] Wei-Ye Zhao1, Xi-Ya Guan3, Yang Liu2, Xiaoming Zhao2, Jian Peng2: Stochastic Variance Reduction for Deep Q-learning. arXiv preprint arXiv:1905.08152, 2019
- [24] Anshel, Baram, and Shimkin 2016] Anshel, O.; Baram, N.; and Shimkin, N. 2016. Averaged-dqn: Variance reduction and stabilization for deep reinforcement learning. arXiv preprint arXiv:1611.01929.
- [25] Kingma, Diederik P. and Ba, Jimmy. Adam: A method for stochastic optimization. arXiv preprint arXiv: 1412.6980, 2014.
- [26] Yang, Yuchen Xie, Zhaoran Wan. A Theoretical Analysis of Deep Q-Learning. arXiv:1901.00137, 2019.
- [27] Schaul, Tom, Quan, John, Antonoglou, Ioannis, and Silver, David. Prioritized experience replay. arXiv preprint arXiv:1511.05952, 2015.
- [28] Wang, Ziyu, de Freitas, Nando, and Lanctot, Marc. Dueling network architectures for deep reinforcement learning. arXiv preprint arXiv: 1511.06581, 2015.
- [29] He, Frank S., Yang Liu, Alexander G. Schwing, and Peng, Jian. Learning to play in a day: Faster deep reinforcement learning by optimality tightening. arXiv preprint arXiv:1611.01606, 2016.
- [30] Bellemare, Marc G, Srinivasan, Sriram, Ostrovski, Georg, Schaul, Tom, Saxton, David, and Munos, Remi. Unifying count-based exploration and intrinsic motivation. arXiv preprint arXiv:1606.01868, 2016.