On Secrecy Capacity of Fast Fading MIMOME Wiretap Channels With Statistical CSIT

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Abstract-In this paper, we consider secure transmissions in ergodic Rayleigh fast-faded multiple-input multiple-output multiple-antenna-eavesdropper (MIMOME) wiretap channels with only statistical channel state information at the transmitter (CSIT). When the legitimate receiver has more (or equal) antennas than the eavesdropper, we prove the first MIMOME secrecy capacity with partial CSIT by establishing a new secrecy capacity upper-bound. The key step is to form an MIMOME degraded channel by dividing the legitimate receiver's channel matrix into two submatrices, and setting one of the submatrices to be the same as the eavesdropper's channel matrix. Next, under the total power constraint over all transmit antennas, we analytically solve the channel-input covariance matrix optimization problem to fully characterize the MIMOME secrecy capacity. Typically, the MIMOME optimization problems are non-concave. However, thank to the proposed degraded channel, we can transform the stochastic MIMOME optimization problem to be a Schur-concave one and then find its solution. Besides total power constraint, we also investigate the secrecy capacity when the transmitter is subject to the practical per-antenna power constraint. The corresponding optimization problem is even more difficult since it is not Schuar-concave. Under the two power constraints considered, the corresponding MIMOME secrecy capacities can both scale with the signal-to-noise ratios (SNR) when the difference between numbers of antennas at legitimate receiver and eavesdropper are large enough. However, when the legitimate receiver and eavesdropper have a single antenna each, such SNR scalings do not exist for both cases.

I. INTRODUCTION

Key-based enciphering is a well-adopted technique to ensure the security in current data transmission system. However, for secure communications in wireless networks, the distributions and managements of secret keys may be challenging tasks [1]. The physical-layer security introduced in [2] [3] is appealing due to its keyless nature. The basic building block of physicallayer security is the so-called wiretap channel. In this channel, a source node wants to transmit confidential messages securely to a legitimate receiver and to keep the eavesdropper as ignorant of the message as possible. Wyner [3] characterized the secrecy capacity of the discrete memoryless wiretap channel, in which the secret key was not used. The secrecy capacity is the largest secrecy rate of communication between the source and the destination nodes with the eavesdropper knowing no information of the messages. In order to meet the demands of high data rate transmissions and improve the connectivities of the secure networks [4], the multiple antenna

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systems with security concerns are considered by several authors. In [5], the secrecy capacity of a Gaussian channel with two-input, two-output, single-antenna-eavesdropper was first characterized. This result was extended by [6], in which the secrecy capacities of general Gaussian multiple-input multiple-output multiple-antenna-eavesdropper (MIMOME) channels are proved. In wireless environments, with perfect knowledge of the legitimate receiver's channel state information at the transmitter (CSIT), the time-varying characteristic of fading eavesdropper channels can be further exploited to enhance the secrecy [7]–[10].

However, to attain the secrecy capacity results in [5]–[7], at least the perfect knowledge of the legitimate receiver's CSIT is required. For the fast fading channels, it may be hard to track the rapidly varying channel coefficients because of the limited feedback bandwidth and the delay caused by the channel estimation. Thus for fast-fading channels, it is more practical to consider the case with only partial CSIT of the legitimate channel. For the setting where only statistical CSIT of both legitimate and eavesdropper channels is available, the secrecy capacity is only rigorously characterized for multipleinput single-output single-antenna-eavesdropper (MISOSE) Rayleigh fast-faded channels [11]. A negative phenomenon, revealed in [11], showed that for the MISOSE channels with only statistical CSIT, the secrecy capacities would neither scale with the number of transmit antennas nor the signal-to-noise ratio (SNR). Thus using multiple transmitter antennas in the MISOSE system limitedly helps increase the secrecy capacity, compared with the system using single transmitter antenna.

In this paper, we want to overcome the aforementioned drawbacks of the MISOSE system. We then focus on the MIMOME fast Rayleigh-faded channels where the transmitter only have the statistical CSIT of the legitimate and eavesdropper channels. Two different power constraints are considered. One is the total power constraint over all transmit antennas, and the other is the more practical per-antenna power constraint. Under the total power constraint over all transmit antennas, the MIMOME secrecy capacity is characterized when antennas of the legitimate receiver are more than (or equal to) those of the eavesdropper. To the best of the author's knowledge, this is the first MIMOME secrecy capacity result with partial CSIT. Compared with our previous works [11], new proof techniques are developed for the MIMOME channels. First, we establish a new secrecy capacity upperbound by dividing the channel matrix of legitimate receiver into two submatrices, one of the submatrices has dimensions equal to those of the eavesdropper's channel matrix, to form an MIMOME degraded channel. Second, instead of using

completely monotone property as [11], which may only exists for the MISOSE problem, we solve the stochastic MIMOME optimization problem by transforming it to an equivalent Schur-concave problem. The key to this transformation is using the proposed equivalent MIMOME degraded channel. According to our secrecy capacity results, on the contrary to [11], we observe that the SNR scaling of secrecy capacity can be obtained in the MIMOME channels when the number of antenna of the legitimate receiver is larger than (but not equal to) that of the eavesdropper. Also when the number of transmit antennas is fixed, increasing the difference between numbers of antennas of legitimate receiver and eavesdropper helps to increase the secrecy capacities.

For the cases where the transmitters are subject to the practical per-antenna power constraints, we also fully characterize the secrecy capacities for the MISOSE wiretap channels with statistical CSIT. Because the optimization problem subject to the per-antenna power constraint is not Schur-concave, only numerical algorithm is developed to find the secrecy capacity for the MIMOME channel. However, we can still show that under this constraint, when there are sufficient transmit antennas with large enough transmitted power each, the MIMOME secrecy capacities can also scale with SNRs. Such a scaling cannot be obtained for the MISOSE wiretap channels under per-antenna power constraint, even when all transmit antennas are allowed to transmit with large power.

Under total power constraint over all transmit antennas, several works have studied the secrecy capacities in various channel settings. For channels with full CSIT, the secrecy capacities were found in [5], [6]; and for ergodic slow fading channels with partial CSIT, they were found in [7]. However, for fast fading channels with full legitimate CSIT and statistical eavesdropper CSIT, the secrecy capacities are unknown. And several works instead studied the achievable secrecy rate in these channels [8]-[10]. The settings in [5]-[10] are fundamentally different to ours. In this paper, fast fading channels with only statistical CSIT of both legitimate receiver and eavesdropper are considered. Our works are the first secrecy capacity results for MIMOME channels with statistical CSIT. More related works and the comparisons with our works can be found in our MISOSE previous work [11] and references within. Subject to the per-antenna power constraint, secrecy rate optimization with full CSIT was studied in [12]. Besides the difference between CSIT assumptions compared to ours, the channel input matrix in [12] was optimized numerically and the optimality was not guaranteed. However, in our work, optimal channel input covariance matrix for the MISOSE channel is analytically solved, and our numerical algorithm for the MIMOME channel can guarantee the optimality.

The rest of the paper is organized as follows. In Section III we introduce the considered system model. In Section III, under total power constraint over all tranmitter antennas, we prove the secrecy capacities for fast fading MIMOME wiretap channels with statistical CSIT. In Section IV, we investigate secrecy capacities under per-antenna power constraints. Simulation results are provided in Section V, and the conclusion is given in Section VI.

Notations: In this paper, lower and upper case bold al-

phabets denote vectors and matrices, respectively. The zero-mean complex Gaussian random vector with covariance matrix Σ is denoted as $CN(0,\Sigma)$. The mutual information between two random vectors \mathbf{x} and \mathbf{y} is $I(\mathbf{x};\mathbf{y})$, while the conditional differential entropy is $h(\mathbf{x}|\mathbf{y})$. The superscript $(.)^H$ denotes the transpose complex conjugate. $|\mathbf{A}|$ and |a| represent the determinant of the square matrix \mathbf{A} and the absolute value of the scalar variable a, respectively. The trace of \mathbf{A} is denoted by $\mathrm{tr}(\mathbf{A})$. The element of \mathbf{A} in the ith row and jth column is $\{\mathbf{A}\}_{i,j}$. A diagonal matrix whose diagonal entries are $a_1 \dots a_k$ is denoted by $diag(a_1 \dots a_k)$. The positive semidefinite ordering between Hermitian matrices \mathbf{A} and \mathbf{B} are denoted by $\mathbf{A} \succeq \mathbf{B}$ ($\mathbf{A} \succ \mathbf{B}$), where $\mathbf{A} - \mathbf{B}$ is a positive semi-definite (definite) matrix.

II. SYSTEM MODEL

In the considered MIMOME wiretap channel, we study the problem of reliably communicating a secret message w from the transmitter to the legitimate receiver, subject to a constraint on the information attainable by the eavesdropper (in upcoming (6)). The transmitter has n_t antennas, while the legitimate receiver and eavesdropper respectively have n_r and n_e antennas as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}_{\mathbf{y}},\tag{1}$$

$$\mathbf{z} = \mathbf{G}\mathbf{x} + \mathbf{n}_7,\tag{2}$$

where $\mathbf{x} \in \mathbb{C}^{n_t \times 1}$ represents the transmitted vector signal; the legitimate channel matrix is $\mathbf{H} \in \mathbb{C}^{n_r \times n_t}$ while the eavesdropper channel matrix is $\mathbf{G} \in \mathbb{C}^{n_e \times n_t}$; \mathbf{n}_y and \mathbf{n}_z are additive white Gaussian noise vectors at the legitimate receiver and eavesdropper, respectively, with each element independent and identically distributed (i.i.d.), circularly symmetric, and having zero mean and unit variance. The channels are assumed to be fast Rayleigh fading, that is, each element of \mathbf{H} and \mathbf{G} is i.i.d distributed as

$$CN(0, \sigma_{\mathbf{h}}^2)$$
 and $CN(0, \sigma_{\mathbf{g}}^2)$, (3)

respectively, while the channel coefficients change in each symbol time. The \mathbf{H} , \mathbf{G} , \mathbf{n}_y and \mathbf{n}_z are independent. We assume that the legitimate receiver knows the instantaneous channel state information of \mathbf{H} perfectly, while the eavesdropper knows those of \mathbf{H} and \mathbf{G} perfectly. As for the CSIT, only the distributions of \mathbf{H} and \mathbf{G} are known while the realizations of \mathbf{H} and \mathbf{G} are unknown at the transmitter.

In this work, we consider two kinds of constraints for the channel input \mathbf{x} . The first one is the total power constraint over all transmitter antennas as

$$\operatorname{Tr}(\Sigma_{\mathbf{x}}) \le P,$$
 (4)

where $\Sigma_{\mathbf{x}}$ is the covariance matrix of \mathbf{x} . Note that under (4), the transmitter can perform power allocation between transmitter antennas to increase the secrecy capacity. In addition to (4), we also consider a more practical per-antenna constraint for the transmitter antennas as

$$\{\Sigma_{\mathbf{x}}\}_{ii} \le P_i,\tag{5}$$

for $i = 1, ..., n_t$. Note that the per-antenna constraint (5) is more stringent than the total power constraint (4) when $\sum_{i=1}^{N_t} P_i \leq P$.

The perfect secrecy and the corresponding secrecy capacity are defined as follows. Consider a $(2^{NR}, N)$ -code with an encoder that maps the message $w \in \mathcal{W}_N = \{1, 2, \dots, 2^{NR}\}$ into a length-N codeword, and a decoder at the legitimate receiver that maps the received sequence y^N (the collections of y over the code length N) from the legitimate channel (1) to an estimated message $\hat{w} \in \mathcal{W}_N$. We then have the following definitions, where \mathbf{z}^N , \mathbf{H}^N , and \mathbf{G}^N are the collections of \mathbf{z} , \mathbf{H} , and \mathbf{G} over the code length N, respectively.

Definition 1 (Secrecy Capacity [1] [3] [7]): Perfect secrecy is achievable with rate R if, for any $\varepsilon > 0$, there exists a sequence of $(2^{NR}, N)$ -codes and an integer N_0 such that for any $N > N_0$

$$R_e = h(w|\mathbf{z}^N, \mathbf{H}^N, \mathbf{G}^N)/N \ge R - \varepsilon,$$
 and $\Pr(\hat{w} \ne w) \le \varepsilon,$

where R_e in (6) is the equivocation rate and w is the secret message. The **secrecy capacity** is the supremum of all achievable secrecy rates.

Note that the perfect secrecy requirement in (6) is measured by $I(w; \mathbf{z}^N, \mathbf{H}^N, \mathbf{G}^N)/N = R - R_e$, which is based on all the observations $(\mathbf{z}^N, \mathbf{H}^N, \mathbf{G}^N)$ the eavesdropper has.

From Csiszár and Körner's seminal work [2], we know that the secrecy capacity of MIMOME channel (1) and (2) is

$$\max_{U} I(U; \mathbf{y}, \mathbf{H}) - I(U; \mathbf{z}, \mathbf{H}, \mathbf{G}),$$

$$= \max_{U} I(U; \mathbf{y}|\mathbf{H}) - I(U; \mathbf{z}|\mathbf{H}, \mathbf{G}), \tag{7}$$

where U is an auxiliary random variable satisfying the Markov relationship $U \to \mathbf{x} \to (\mathbf{y}, \mathbf{H}), (\mathbf{z}, \mathbf{H}, \mathbf{G})$, and (7) results from the fact that the transmitter does not have the knowledge of the realizations of \mathbf{H} and \mathbf{G} . However, the optimal choice of U which maximizes the secrecy capacity of considered fast fading MIMOME channel is unknown. In this paper, we want to fully characterize the optimal U. Note that the secrecy capacity considered here is achieved by encoding over multiple channel states and the perfect secrecy constraint must be satisfied for all $N > N_0$. This implies that no secrecy outage [13] [9] is allowed. In delay-limited applications, such perfect secrecy condition may not be achievable and, thus, a tradeoff exists between secrecy rate and secrecy outage probability. These issues have been discussed in [13] [9] and are beyond the scope of this paper.

III. SECRECY CAPACITY UNDER THE TOTAL POWER CONSTRAINT

In this section, we explicitly find the optimal U in (7) for wiretap channel (1)(2), and fully characterize the MIMOME secrecy capacity with statistical CSIT in the upcoming Theorem 1. When there is full CSIT [6], one can find the optimal auxiliary random variable U by constructing an equivalent degraded MIMOME channel to upper-bound the secrecy capacity. However, with only statistical CSIT, if one naively applies the degraded channel construction method in [6], the

resulting secrecy capacity upper bound will depend on the realizations of (\mathbf{H}, \mathbf{G}) and become very loose. Thus in general it is very hard to find the optimal U maximizing (7)

Due to the difficulty mentioned in the previous paragraph, with statistical CSIT, finding the optimal U maximizing (7) is very hard in general. However, in the following Lemma, for the special cases where the MIMOME channels are Rayleigh faded as (3), we show that one may still construct a degraded channel for upper-bounding and find the optimal U maximizing (7). The key for building this equivalent degraded MIMOME channel for (1)(2) is replacing the legitimate channel \mathbf{H} with equivalent \mathbf{H}' in upcoming (9) as follows. When $n_r \geq n_e$, one can divide the legitimate channel matrix \mathbf{H} in (1) as two submatrices

$$\mathbf{H} = [\mathbf{H}_{(n_r - n_e)}^T \ \mathbf{H}_{n_e}^T]^T, \tag{8}$$

where $\mathbf{H}_{(n_r-n_e)} \in \mathbb{C}^{(n_r-n_e)\times n_t}$ and $\mathbf{H}_{n_e} \in \mathbb{C}^{n_e\times n_t}$, with each element of $\mathbf{H}_{(n_r-n_e)}$ and \mathbf{H}_{n_e} distributed as i.i.d. Gaussian $CN(0,\sigma_h^2)$. From the properties of complex Gaussian distributions, the distribution of \mathbf{H} is the same as that of

$$\mathbf{H}' = \left[\mathbf{H}_{(n_r - n_e)}^T \left(\frac{\sigma_{\mathbf{h}}}{\sigma_{\mathbf{g}}}\right) \mathbf{G}^T\right]^T, \tag{9}$$

because each element of eavesdropper channel matrix G is distributed as $CN(0,\sigma_g^2)$ according to (3). With (9), one can build an equivalent degraded MIMOME channel for (1)(2) (see upcoming (10) (12)), where the received signal at eavesdropper can be treated as a degraded version of that at the legitimate receiver. Then we can obtain a tight secrecy capacity upper bound and have the following property of the optimal U for (7).

Lemma 1: For the MIMOME fast Rayleigh fading wiretap channel (1)(2) with the statistical CSIT of **H** and **G**, using Gaussian **x** without prefixing $U \equiv \mathbf{x}$ is the optimal transmission strategy for (7) under (4) when $n_r \geq n_e$ and $\sigma_h \geq \sigma_g$, where n_r and n_e respectively are the number of antennas at the legitimate receiver and eavesdropper, while σ_h and σ_g respectively are the variances of the legitimate and eavesdropper channels as (3).

Proof: We first form the degraded MIMOME channel with respect to (1)(2). Here we will only consider the cases where $\sigma_g^2 > 0$, since the case with $\sigma_g^2 = 0$ is a trivial one which corresponds to channel without eavesdropper. Note that **H** in (8) has the same distribution as **H**' in (9). Then we can form an equivalent received signal

$$\mathbf{y}' = \mathbf{H}'\mathbf{x} + \mathbf{n}_{\mathbf{y}},\tag{10}$$

which has the same marginal distribution as the legitimate received signal \mathbf{y} in (1). Now, rewrite the above \mathbf{y}' using (9)

$$\mathbf{y}' = [\mathbf{H}_{(n_r - n_e)}^T \left(\frac{\sigma_{\mathbf{h}}}{\sigma_{\mathbf{g}}}\right) \mathbf{G}^T]^T \mathbf{x} + \mathbf{n}_y$$

$$= \left[\left(\mathbf{y}'_{(n_r - n_e)}\right)^T \left(\mathbf{y}'_{n_e}\right)^T \right]^T, \tag{11}$$

where $\mathbf{y}'_{(n_r-n_e)} \in \mathbb{C}^{(n_r-n_e)\times 1}$ and $\mathbf{y}'_{n_e} \in \mathbb{C}^{n_e\times 1}$. Now we consider

$$\frac{\sigma_{\mathbf{g}}}{\sigma_{\mathbf{h}}}\mathbf{y}' = \left[\frac{\sigma_{\mathbf{g}}}{\sigma_{\mathbf{h}}}\left(\mathbf{y}'_{(n_r - n_e)}\right)^T \left(\mathbf{G}\mathbf{x} + (\sigma_{\mathbf{g}}/\sigma_{\mathbf{h}})\mathbf{n}_{\mathbf{y},n_e}\right)^T\right]^T,$$

where the noise vector $\mathbf{n}_{y,n_e} \in \mathbb{C}^{n_e \times 1}$ comes from dividing the noise vector at the legitimate receiver as $\mathbf{n}_y = [\mathbf{n}_{y,(n_r-n_e)}^T \ \mathbf{n}_{y,n_e}^T]^T$. From (2), it is clear that when $\sigma_{\mathbf{h}} \geq \sigma_{\mathbf{g}}$, we have the markov relationship that given \mathbf{H}'

$$\mathbf{x} \rightarrow \frac{\sigma_{\mathbf{g}}}{\sigma_{\mathbf{h}}} \mathbf{y}' \rightarrow \mathbf{z}.$$
 (12)

The degraded MIMOME wiretap channel $(x, (\sigma_g/\sigma_h)y', z)$ is then formed.

Now based on the proposed degraded channel $(x,(\sigma_g/\sigma_h)y',z)$, we know that the secrecy capacity is upper-bounded by

$$C_s^t \le \max_{\mathbf{x}} I(\mathbf{x}; \frac{\sigma_{\mathbf{g}}}{\sigma_{\mathbf{h}}} \mathbf{y}' | \mathbf{H}') - I(\mathbf{x}; \mathbf{z} | \mathbf{G}), \tag{13}$$

$$= \max_{\mathbf{x}} I(\mathbf{x}; \mathbf{y}|\mathbf{H}) - I(\mathbf{x}; \mathbf{z}|\mathbf{G}), \tag{14}$$

where the inequality follows [11], with the right-hand-side (RHS) being the secrecy capacity of our degraded channel but forcing the eavesdropper knowing (\mathbf{z}, \mathbf{G}) instead of $(\mathbf{z}, \mathbf{H}', \mathbf{G})$; and the equality comes from that $(\mathbf{y}', \mathbf{H}')$ and (\mathbf{y}, \mathbf{H}) have the same distributions. From [2], we also know that the RHS of (14) is achievable. Thus the RHS of (14) is the secrecy capacity C_s^t . Furthermore, from [6], we know that Gaussian \mathbf{x} is optimal for the secrecy capacity in (14) under total power constraint (4). Then our claim is valid.

Note that on the contrary to the upper-bound with full CSIT in [6], our secrecy capacity upper bound (13) is independent of the realizations of \mathbf{H} and \mathbf{G} . This is why our bounds are tight for MIMOME channels with only statistical CSIT. Compared to the proof for the MISOSE secrecy capacity [11], the key for deriving the tight MIMOME upper-bound is separating the legitimate channel matrix \mathbf{H} by two submatrices $\mathbf{H}_{(n_r-n_e)}$ and \mathbf{H}_{n_e} as (8), and only introducing correlations between \mathbf{H}_{n_e} and \mathbf{G} as (9). One can treat the unchanged submatrix $\mathbf{H}_{(n_r-n_e)}$ as a "safe" channel matrix without being eavesdropped, which provides SNR scaling for the secrecy capacity. In MISOSE channel, such a $\mathbf{H}_{(n_r-n_e)}$ does not exist because $n_r = n_e = 1$, and there is no SNR scaling. This intuition is verified rigorously from the upcoming Theorem 1 and Corollary 1.

Now we fully characterize the MIMOME secrecy capacity based on Lemma 1. Typically, the MIMOME secrecy capacity optimization problems like the upcoming (16) are nonconcave. This is due to that the MIMOME secrecy capacities, such as (16), are a difference of two concave functions. However, with aids of the degraded MIMOME channels formed by (9), the stochastic MIMOME optimization problem (16) can be transformed to be a Schur-concave problem. This transformation helps a lot to find the optimal solution (17). Note that the completely monotone property for MISOSE optimization problem [11] may not exist for the MIMOME one, and thus the method in [11] is hard to be extended to the MIMOME cases.

Theorem 1: Under the total power constraint

$$\operatorname{Tr}(\Sigma_{\mathbf{x}}) \leq P,$$
 (15)

the MIMOME secrecy capacity C_s^t when $n_r \ge n_e$ and $\sigma_h \ge \sigma_g$ is

$$\max_{\Sigma_{\mathbf{x}}} \left(\mathbb{E}_{\mathbf{H}} \left[\log \left| \mathbf{I} + \mathbf{H} \Sigma_{\mathbf{x}} \mathbf{H}^{\dagger} \right| \right] - \mathbb{E}_{\mathbf{G}} \left[\log \left| \mathbf{I} + \mathbf{G} \Sigma_{\mathbf{x}} \mathbf{G}^{\dagger} \right| \right] \right), \quad (16)$$

and the optimal channel input covariance matrix subject to (15) is

$$\Sigma_{\mathbf{x}}^* = \frac{P}{n_t} \mathbf{I}.\tag{17}$$

Proof: First we show that under our setting, subject to (15), the stochastic MIMOME optimization problem (16) can be transformed to an equivalent concave problem. The key is cleverly using the same marginal channel formed by (9). Note that the objective function in (16) can be rewritten as $I(\mathbf{x}; \mathbf{y}|\mathbf{H}) - I(\mathbf{x}; \mathbf{z}|\mathbf{G})$ with $\mathbf{x} \sim CN(0, \Sigma_{\mathbf{x}})$ as

$$I(\mathbf{x}; \mathbf{y}|\mathbf{H}) - I(\mathbf{x}; \mathbf{z}|\mathbf{G}) \stackrel{(a)}{=} I(\mathbf{x}; \frac{\sigma_{\mathbf{g}}}{\sigma_{\mathbf{h}}} \mathbf{y}'|\mathbf{H}') - I(\mathbf{x}; \mathbf{z}|\mathbf{G}),$$

$$\stackrel{(b)}{=} I(\mathbf{x}; \frac{\sigma_{\mathbf{g}}}{\sigma_{\mathbf{h}}} \mathbf{y}'|\mathbf{z}, \mathbf{H}'), \tag{18}$$

and then from [6], we know that the RHS of (18 b) is concave in Σ_x . The (18 a) comes from (14), while (18 b) comes from (9) and the fact that

$$I(\mathbf{x}; \frac{\sigma_{\mathbf{g}}}{\sigma_{\mathbf{h}}} \mathbf{y}' | \mathbf{H}') - I(\mathbf{x}; \mathbf{z} | \mathbf{G})$$

$$\stackrel{(a)}{=} I(\mathbf{x}; \frac{\sigma_{\mathbf{g}}}{\sigma_{\mathbf{h}}} \mathbf{y}', \mathbf{z} | \mathbf{H}_{(n_r - n_e)}, \mathbf{G}) - I(\mathbf{x}; \mathbf{z} | \mathbf{H}_{(n_r - n_e)}, \mathbf{G}),$$

$$\stackrel{(b)}{=} I(\mathbf{x}; \frac{\sigma_{\mathbf{g}}}{\sigma_{\mathbf{h}}} \mathbf{y}' | \mathbf{z}, \mathbf{H}_{(n_r - n_e)}, \mathbf{G}).$$
(19)

In the above, (19 a) comes from that

$$\begin{split} I(\mathbf{x}; \frac{\sigma_{\mathbf{g}}}{\sigma_{\mathbf{h}}} \mathbf{y}' | \mathbf{H}') &= I(\mathbf{x}; \frac{\sigma_{\mathbf{g}}}{\sigma_{\mathbf{h}}} \mathbf{y}', \mathbf{z} | \mathbf{H}_{(n_r - n_e)}, \mathbf{G}), \\ \text{and} \quad I(\mathbf{x}; \mathbf{z} | \mathbf{G}) &= I(\mathbf{x}; \mathbf{z} | \mathbf{H}_{(n_r - n_e)}, \mathbf{G}), \end{split}$$

with the former resulting from (9) and the Markov relationship (12) and the latter resulting from the independence of $\mathbf{H}_{(n_r-n_e)}$ with $(\mathbf{x}, \mathbf{z}, \mathbf{G})$; while (19 b) comes from the chain rule of the mutual information [14].

After showing that (16) can be transformed to an equivalent concave problem, now we show that (16) can be further transformed to a symmetric optimization problem. Then we can explore the Schuar-concavity of the secrecy capacity (16) to show that the optimal $\Sigma_{\mathbf{x}} = \alpha \mathbf{I}$ where $0 \le \alpha \le P/n_t$. For any non-diagonal $\Sigma_{\mathbf{x}}$, we can apply the eigenvalue decomposition on it as $\Sigma_{\mathbf{x}} = \mathbf{U}\mathbf{D}\mathbf{U}^{\dagger}$, where \mathbf{U} is unitary and \mathbf{D} is diagonal. Then for the objective function in (16), setting $\Sigma_{\mathbf{x}} = \mathbf{U}\mathbf{D}\mathbf{U}^{\dagger}$ and $\Sigma_{\mathbf{x}} = \mathbf{D}$ will result in the same value since \mathbf{H} and \mathbf{G} are Rayleigh distributed as (3). Also the total power constraint (15) can be transformed as

$$\operatorname{Tr}\left(\mathbf{U}\mathbf{D}\mathbf{U}^{\dagger}\right) = \operatorname{Tr}\left(\mathbf{U}^{\dagger}\mathbf{U}\mathbf{D}\right) = \operatorname{Tr}\left(\mathbf{D}\right) \le P.$$
 (20)

In the following, we then focus on the following optimization problem instead

$$\max_{\mathbf{D}} \left(\mathbb{E}_{\mathbf{H}} \left[\log \left| \mathbf{I} + \mathbf{H} \mathbf{D} \mathbf{H}^{\dagger} \right| \right] - \mathbb{E}_{\mathbf{G}} \left[\log \left| \mathbf{I} + \mathbf{G} \mathbf{D} \mathbf{G}^{\dagger} \right| \right] \right), \quad (21)$$

where the diagonal matrix \mathbf{D} satisfying (20). Because \mathbf{H} and \mathbf{G} are Rayleigh faded as (3), (21) is symmetric for \mathbf{D} . That is, all permutations of the diagonal terms of \mathbf{D} will result in the same value for the objective function in (21). Together with the fact that (21) is concave as shown previously, we know that the objective function (21) is Schur-concave. Subject to the constraint $\mathrm{Tr}(\mathbf{D}) \leq P$, from [15], it can be easily shown that the optimal \mathbf{D} (and thus $\Sigma_{\mathbf{x}}$) is $\alpha \mathbf{I}$ where $0 \leq \alpha \leq P/n_t$. This result comes from the fact that the diagonal entries $\{\alpha, \ldots, \alpha\}$ are majorized by any other diagonal entries $\{d_1, \ldots, d_{n_t}\}$ of \mathbf{D} if $\frac{1}{n_t} \sum_{i=1}^{n_t} d_i = \alpha$, and properties of the Schur-concave function [15].

Finally, we will show that using all available power, that is, $\Sigma_{\mathbf{x}} = (P/n_t)\mathbf{I}$ is optimal for (16). To do this, we will first show an important property of the objective function in (16) with respect to the matrix partial ordering as in the following Lemma.

Lemma 2: For the objective function of the optimization problem in (16),

$$R_{s}(\Sigma_{\mathbf{x}}) \triangleq \mathbb{E}_{\mathbf{H}} \left[\log \left| \mathbf{I} + \mathbf{H} \Sigma_{x} \mathbf{H}^{\dagger} \right| \right] - \mathbb{E}_{\mathbf{G}} \left[\log \left| \mathbf{I} + \mathbf{G} \Sigma_{x} \mathbf{G}^{\dagger} \right| \right], \quad (22)$$

we have the following properties when $n_r \ge n_e$ and $\sigma_h \ge \sigma_g$

(I) If two channel input covariance matrices satisfy $\Sigma_x^2 \succeq \Sigma_x^1 \succeq \mathbf{0}$, then

$$R_s(\Sigma_r^2) > R_s(\Sigma_r^1). \tag{23}$$

(II) If
$$\Sigma_r^1 \succ \mathbf{0}$$
, then $R_s(\Sigma_r^1) > 0$.

The proof of Lemma 2 is given in Appendix A, which results from the equivalent degraded MIMOME channels described in the proof of Lemma 1. As aforementioned, we know the optimal $\Sigma_{\mathbf{x}} = \alpha \mathbf{I}$ for (16) subject to (15). Now we can show that $\alpha = P/n_t$ is optimal for (16) using Lemma 2 as follows. Let us substitute $\Sigma_{\mathbf{x}} = \alpha \mathbf{I}$ into the objective function in (16), it becomes

$$\mathbb{E}_{\mathbf{H}}\left[\log\left|\mathbf{I} + \alpha \mathbf{H} \mathbf{H}^{\dagger}\right|\right] - \mathbb{E}_{\mathbf{G}}\left[\log\left|\mathbf{I} + \alpha \mathbf{G} \mathbf{G}^{\dagger}\right|\right] = C(\alpha), \quad (24)$$

where $0 \le \alpha \le P/n_t$. If $\alpha > 0$, we know that

$$(P/n_t)\mathbf{I} \succeq \alpha \mathbf{I} \succeq \mathbf{0}$$
.

Then from Property (I) of Lemma 2, we know that $C(P/n_t) \ge C(\alpha)$ for any $\alpha > 0$. As for the case $\alpha = 0$, we know that C(0) = 0 from (24). However, if $\alpha > 0$, we have $C(\alpha) > 0$ from Property (II) of Lemma 2. Then we know that

$$C(P/n_t) \geq C(\alpha) > C(0)$$
,

for any $\alpha > 0$. Thus $\alpha = P/n_t$ is optimal for (24). It concludes our proof.

Remark: Note that in the final step of Theorem 1's proof, we show that the optimal channel input covariance matrix happens when the total power constraint in (15) is met with equality. This fact is not always true for the wiretap channel and counter examples are given [16] [8]. When the transmit power increases, both the SNRs at the legitimate receiver and the eavesdropper increase. Then the secrecy capacity may not always be maximized with using all available power. Indeed, this property was also examined in [10]. However, in [10], the perfect CSIT of the legitimate receiver is assumed to be available, which is fundamentally different to our setting.

Now we have the following result which characterizes the secrecy capacity in Theorem 1 with respect to the number of antennas (n_t, n_r, n_e) at the transmitter, legitimate receiver and eavesdropper respectively. The proof is given in Appendix B.

Corollary 1: Under total power constraint (15), we have the following asymptotic results for the MIMOME secrecy capacity C_s^t as

$$\lim_{P_g \to \infty} \frac{C_s^t - n_r \log(\sigma_h^2 / \sigma_g^2)}{\log P_g} = \begin{cases} n_r - n_e, & n_t \ge n_r > n_e \\ n_t - n_e, & n_r \ge n_t > n_e \\ 0, & n_r \ge n_e \ge n_t \end{cases}$$

when $\sigma_h \ge \sigma_g > 0$, where $P_g = \sigma_g^2 P$ is the equivalent received SNR at the eavesdropper.

In [11], it was shown that when $n_r = n_e = 1$, the secrecy capacity does not scale with P. Our Corollary 1 shows that to make secrecy capacity scale with P, we must let $n_r - n_e$ larger than (but not equal to) zero. Also adding enough number of transmit antennas to make $n_t > n_e$ is very important for increasing the secrecy capacity. Indeed, from our numerical results in Section V, with fixed $n_t > n_e$, increasing the difference $n_r - n_e$ will help to increase the secrecy capacities for all SNR regimes (besides the high SNR regime proved in Corollary 1).

From Property (II) of Lemma 2, the secrecy capacity in Theorem 1 is always positive when P > 0, $n_r \ge n_e$ and $\sigma_h > \sigma_g$. It will also be interesting to see when the secrecy capacity is zero. We have the following result, where the proof is similar to that of Lemma 1 and neglected.

Corollary 2: For the MIMOME fast Rayleigh fading wiretap channel (1)(2) with the statistical CSIT of **H** and **G**, the secrecy capacity is zero when $n_r \le n_e$ and $\sigma_h \le \sigma_g$.

IV. SECRECY CAPACITY UNDER THE PER-ANTENNA POWER CONSTRAINT

After investigating the MIMOME secrecy capacity C_s^t under the total power constraint over all transmitter antennas, now we consider the MIMOME channel under the more practical per-antenna power constraint. First, we show that the MIMOME secrecy capacity under the per-antenna power constraint can be expressed as the following convex optimization problem, with proof given in Appendix C.

Proposition 1: Under the per-antenna power constraint

$$\{\Sigma_{\mathbf{x}}\}_{ii} < P_i, \tag{25}$$

 $i = 1,...,n_t$, when $n_r \ge n_e$ and $\sigma_h \ge \sigma_g > 0$, the MIMOME secrecy capacity C_s^p is

$$\max_{\Sigma_{\mathbf{x}}} \left(\mathbb{E}_{\mathbf{H}} \left[\log \left| \mathbf{I} + \mathbf{H} \Sigma_{\mathbf{x}} \mathbf{H}^{\dagger} \right| \right] - \mathbb{E}_{\mathbf{G}} \left[\log \left| \mathbf{I} + \mathbf{G} \Sigma_{\mathbf{x}} \mathbf{G}^{\dagger} \right| \right] \right), \quad (26)$$

which can be transformed to a concave optimization problem subject to (25) as

$$\max_{\Sigma_{\mathbf{x}}} \left(\mathbb{E}_{\mathbf{G}, \mathbf{H}_{(n_{r}-n_{e})}} \left[\log \left| \mathbf{I} + \Sigma_{\mathbf{x}} \left(\mathbf{H}_{(n_{r}-n_{e})}^{\dagger} \mathbf{H}_{(n_{r}-n_{e})} + \frac{\sigma_{h}^{2}}{\sigma_{g}^{2}} \mathbf{G}^{\dagger} \mathbf{G} \right) \right| \right] - \mathbb{E}_{\mathbf{G}} \left[\log \left| \mathbf{I} + \mathbf{G} \Sigma_{\mathbf{x}} \mathbf{G}^{\dagger} \right| \right] \right), \tag{27}$$

where **G** is the eavesdropper channel matrix and $\mathbf{H}_{(n_r-n_e)}$ is the sub-matrix of the legitimate channel as defined in (9).

Unlike the optimization in Theorem 1, subject to (25), the optimal $\Sigma_{\mathbf{x}}$ of problem (26) is very hard to find analytically for general n_r . The main difficulty is that the per-antenna power constraint (25) is not symmetric. Note that (25) is related to eigenvectors of $\Sigma_{\mathbf{x}}$, while the total power constraint (15) (Tr $\{\Sigma_{\mathbf{x}}\}$) is independent of these eigenvectors. Thus although the total power constraint can be transformed to (20) via the eigenvalue decomposition of $\Sigma_{\mathbf{x}}$, the same technique cannot be applied to the per-antenna constraint (15). Thus we only claim that the problem (26) subject to (25) can be transformed to a concave one as (27), but not the Schuar-concave one as in the proof of Theorem 1. Lack of symmetry in constraint (25) makes finding analytical solution of it difficult. Nevertheless, we still have the following result for the structure of the optimal $\Sigma_{\mathbf{x}}^*$ in Proposition 1 as

Proposition 2: Under the per-antenna power constraint $\{\Sigma_{\mathbf{x}}\}_{ii} \leq P_i, i = 1, \dots, n_t$, when $n_r \geq n_e$ and $\sigma_h \geq \sigma_g$, there exists an optimal channel input covariance matrix Σ_x^* for the MIMOME secrecy capacity optimization problem (26) which satisfies

$$\{\Sigma_{\mathbf{x}}^*\}_{ii} = P_i, \quad i = 1, \dots, n_t,$$
 (28)

that is, each antenna should fully use its available power.

Proof: We will prove it by showing that if there exists an optimal $\Sigma_{\mathbf{x}}$ which does not meet (28), one can find another $\Sigma_{\mathbf{x}}^*$ meeting (28) and having the same optimal value. Suppose that there exists an optimal $\Sigma_{\mathbf{x}}$ maximizing (26) which has some diagonal terms using powers smaller than their maximum available powers. We collect the indexes corresponding to these diagonal terms as a non-empty set $\mathbb{I}_s \neq \emptyset$, i.e., $\{\Sigma_{\mathbf{x}}\}_{ii} < P_i$, if $i \in I_s \subseteq \{1, \ldots, n_t\}$. Then we can form another $\Sigma_{\mathbf{x}}^*$ as

$$\{\Sigma_{\mathbf{x}}^*\}_{ij} = \left\{ \begin{array}{ll} P_i & \text{if } i=j \text{ and } i \in \mathbb{I}_s, \\ \{\Sigma_{\mathbf{x}}\}_{ij} & \text{otherwise.} \end{array} \right.$$

Then we know that $\Sigma_{\mathbf{x}}^* - \Sigma_{\mathbf{x}} \succ 0$, because $\Sigma_{\mathbf{x}}^* - \Sigma_{\mathbf{x}}$ is a diagonal matrix with $|\mathbb{I}_s|$ positive diagonal terms, while the other diagonal terms are zero. Note that the size of set \mathbb{I}_s meets

 $|\mathbb{I}_s| > 0$ and at least one diagonal terms of $\Sigma_{\mathbf{x}}^* - \Sigma_{\mathbf{x}}$ is positive. Then from Property (I) of Lemma 2 we know that for the objective function $R_s(.)$ in (26), $R_s(\Sigma_{\mathbf{x}}^*) \geq R_s(\Sigma_{\mathbf{x}})$. Since $\Sigma_{\mathbf{x}}$ is assumed to be optimal, $R_s(\Sigma_{\mathbf{x}}) \geq R_s(\Sigma_{\mathbf{x}}^*)$, and then we know that $R_s(\Sigma_{\mathbf{x}}^*) = R_s(\Sigma_{\mathbf{x}})$.

Although for general n_r , we only can obtain the above property for the optimization problem in Proposition 1, due to the aforementioned difficulty below Proposition 1. For the special MISOSE case $n_r = n_e = 1$, we can have the following secrecy capacity result as in the upcoming Theorem. The key proof steps come as follows. First, we use (27) in Proposition 1 to transform the objective function in (26) into a simpler one as in the upcoming (30). And then we can apply Proposition 2 and properties of random Gaussian vectors to obtain analytical solutions.

Theorem 2: Subject to the per-antenna power constraints $\{\Sigma_{\mathbf{x}}\}_{ii} \leq P_i$, $i = 1, ..., n_t$, the secrecy capacity optimization problem (27) has optimal channel input covariance matrix

$$\Sigma_{\mathbf{x}}^* = \operatorname{diag}(P_1, P_2, \dots, P_{n_t}) \tag{29}$$

for the MISOSE wiretap channel, where the legitimate receiver and eavesdropper has single antenna each $n_r = n_e = 1$.

Proof: For this special case $n_r = n_e = 1$, the sub-matrix of the legitimate channel $\mathbf{H}_{(n_r - n_e)}$ in (27) does not exist. Then the optimization problem in (27) can be rewritten as

$$\begin{aligned} & \max_{\Sigma_{\mathbf{x}}} \left(\mathbb{E}_{\mathbf{g}} \left[\log \frac{\sigma_{\mathbf{g}}^{2} / \sigma_{\mathbf{h}}^{2} + \mathbf{g}^{\dagger} \Sigma_{\mathbf{x}} \mathbf{g}}{\sigma_{\mathbf{g}}^{2} / \sigma_{\mathbf{h}}^{2}} \right] - \mathbb{E}_{\mathbf{g}} \left[\log \left(1 + \mathbf{g}^{\dagger} \Sigma_{\mathbf{x}} \mathbf{g} \right) \right] \right), \\ &= \max_{\Sigma_{\mathbf{x}}} \left(\mathbb{E}_{\mathbf{g}} \left[\log \left(1 + \frac{\sigma_{\mathbf{g}}^{2} / \sigma_{\mathbf{h}}^{2} - 1}{1 + \mathbf{g}^{\dagger} \Sigma_{\mathbf{x}} \mathbf{g}} \right) \right] \right) + \log \left(\sigma_{\mathbf{h}}^{2} / \sigma_{\mathbf{g}}^{2} \right), \end{aligned} (30)$$

where for clearness, we replace the notation G in (27) with $\mathbf{g}^{\dagger} \sim CN(0, \sigma_{\mathbf{g}}^2\mathbf{I})$ to emphasize that the eavesdropper channel is now a vector when $n_r = 1$. Note that the objective function in (30) is only related to the channel vector \mathbf{g} for the eavesdropper, but not the channel vector for the legitimate receiver. The above transformations make finding analytical solutions possible.

Next, from Proposition 2, without loss of optimality, one can replace the inequality constraint (25) by the equality constraint on the diagonal entries of $\Sigma_{\mathbf{x}}$ as

$$\{\Sigma_{\mathbf{x}}\}_{ii} = P_i,\tag{31}$$

for $i = 1,...,n_t$. Now we show that for any $\beta \le 0$, $\sigma > 0$, $\beta + 2\sigma > 0$, subject to equality constraint (31), the optimization problem

$$\max_{\Sigma_{\mathbf{x}}} \left(\mathbb{E}_{\mathbf{g}} \left[\log \left(1 + \frac{\beta}{\sigma + \mathbf{g}^{\dagger} \Sigma_{\mathbf{x}} \mathbf{g}} \right) \right] \right)$$
 (32)

has optimal solution (29). This fact is proved in Appendix D, where the key step is cleverly applying properties of random Gaussian vectors. Note that the objective function in (30) is a special case of that in (32) with $\beta = \sigma_g^2/\sigma_h^2 - 1$ and $\sigma = 1$. And then subject to (25), the optimal Σ_x maximizing (27) (and the the original (26)) with $n_r = n_e = 1$ is (29). Our claim in Theorem 2 is valid.

Here we investigate the SNR scaling of the MISOSE channel under the per-antenna power constrains. From Theorem 2, we have

Corollary 3: Under the per-antenna power constraint $\{\Sigma_{\mathbf{x}}\}_{ii} \leq P_i$, $i=1,\ldots,n_t$, when $n_r=n_e=1$ and $\sigma_h \geq \sigma_g > 0$, we have the following asymptotic results for the MISOSE secrecy capacity C_s^p as

$$\lim_{P_{max}^g \to \infty} \frac{C_s^p - \log(\sigma_h^2/\sigma_g^2)}{\log P_{max}^g} = 0,$$
 (33)

where $P_{max}^g = \sigma_g^2 P_{max}$ with

$$P_{max} \triangleq \max_{i=1,\dots,n_t} P_i \tag{34}$$

being the maximum among per-antenna power constraints.

Proof: As (22), let us denote the objective function of optimization problem (26) as $R_s(\Sigma_{\mathbf{x}})$. From the Property (I) of Lemma 2, we know that the MISOSE secrecy capacity under the per-antenna power constraint $C_s^p = R_s(diag\{P_{1}, \ldots, P_{n_t}\}) \le R_s(diag\{P_{max}, \ldots, P_{max}\})$. This fact is due to that

$$diag\{P_{max},\ldots,P_{max}\} \succeq diag\{P_1,\ldots,P_{n_t}\} \succeq \mathbf{0},$$

because $diag\{P_{max}, \dots, P_{max}\} - diag\{P_1, \dots, P_{n_t}\}$ is a diagonal matrix with non-negative entries from (34). By Theorem 1 we know that $R_s(diag\{P_{max}, \dots, P_{max}\})$ is the MISOSE secrecy capacity under total power constraint $Tr(\Sigma_x) \leq n_t P_{max}$. Then by Corollary 1, for fixed n_t , (33) is valid since $n_r = n_e = 1$.

From Corollary 3, we know that the multiple transmitter antennas for the MISOSE channel limitedly help to increase the secrecy capacity under the per-antenna power constraint. Indeed, note that (33) is hold even when only one transmitter antenna, which has the maximum power constraint, is selected to transmit secret messages while the other ones are silent. In this case, the per-antenna power constraint corresponding to (25) are $P_i = P_{max}$ and $P_j = 0, j \neq i, \forall j$, where the *i*th transmitter antenna is selected. When SNR is large enough, the secrecy rate for this simple transmitter antenna selection scheme [17] is within constant gap compared with the secrecy capacity where all transmitter antennas are used.

To overcome the negative results for MISOSE channels revealed in Corollary 3, as in Section III, we now show that making the difference between numbers of antennas at the legitimate receiver and eavesdropper $n_r - n_e > 0$ will help to increase the secrecy capacity under the per-antenna power constraint. Although we cannot find the optimal $\Sigma_{\mathbf{x}}$ for (26) subject to (25), we use the following sub-optimal $\Sigma_{\mathbf{x}}$ to compute the secrecy capacity lower bound as $\Sigma_{\mathbf{x}} = \mathrm{diag}\left(P_1, P_2, \ldots, P_{n_t}\right)$. This selection of $\Sigma_{\mathbf{x}}$ is optimal for the MISOSE case from Theorem 2, and for the MIMOME case, this selection is also reasonable due to Proposition 2. Now we have the following result for the secrecy capacity in Proposition 1 as

Corollary 4: Under the per-antenna power constraint $\{\Sigma_{\mathbf{x}}\}_{ii} \leq P_i$, $i=1,\ldots,n_t$, when $\sigma_h \geq \sigma_g > 0$, we have the following asymptotic result for the MIMOME secrecy capacity

 C_s^p as

$$\lim_{P_{min}^{g} \to \infty} \frac{C_{s}^{p} - n_{r} \log(\sigma_{h}^{2}/\sigma_{g}^{2})}{\log P_{min}^{g}} \ge \begin{cases} n_{r} - n_{e}, & n_{t} \ge n_{r} > n_{e} \\ n_{t} - n_{e}, & n_{r} \ge n_{t} > n_{e}, \end{cases}$$
(35)

where $P_{min}^g = \sigma_g^2 P_{min}$ with

$$P_{min} \triangleq \min_{i=1,\dots,n_t} P_i \tag{36}$$

is the minimum among per-antenna power constraints.

Proof: From the definition of P_{min} in (36), we know that

$$diag\{P_1,\ldots,P_{n_t}\} \succeq diag\{P_{min},\ldots,P_{min}\} \succ \mathbf{0}.$$

Then we know that $C_s^p \geq R_s(diag\{P_1,\ldots,P_{n_t}\}) \geq R_s(diag\{P_{min},\ldots,P_{min}\})$, where the second inequality comes from Property (I) of Lemma 2. By Theorem 1 we know that $R_s(diag\{P_{min},\ldots,P_{min}\})$ is the MIMOME secrecy capacity under total power constraint $\text{Tr}(\Sigma_x) \leq n_t P_{min}$. Then by Corollary 1, for fixed n_t , (35) is valid.

From (35), as discussions under Corollary 1, adding enough number of legitimate-receiver antennas n_r (also enough number of transmit antennas n_t) is very important for increasing the secrecy capacity C_s^p . Moreover, (35) also reveals some antenna selection rules for MIMOME wiretap channel under the perantenna power constraint. For example, when $n_t > n_r > n_e$, the transmitter can select the largest n_r antennas to transmit the secret message. Each selected antenna transmits with all its allowable power. When the allowable power of all selected antenna is high enough, this simple transmit antenna selection scheme can achieve good secrecy rate performance according to (35).

Finally, we develop algorithms to solve the MIMOME secrecy capacity optimization problems (26) subject to the perantenna power constraint (25). To do this, as in Proposition 1, by using the same marginal channel matrix \mathbf{H}' in (9), we first transfer the stochastic optimization problem (26) into a concave one (27). Next, from Proposition 2, one can set the inequalities in constraints (25) with equalities as (28) to further simplify the problem. Then we can use algorithms similar to those in [18] to find the optimal values for (26) by matrix calculus [19] [20]. The gap to the optimal value is within n_t/t , where n_t is the number of transmit antenna and t>0is a parameter to prevent the algorithm approaching nonsemidefine Σ_x . Note that our problem can not be simplified by the multiple-access-channels-broadcast-channels duality as [18]. Though wiretap channels are similar to the broadcast channels, there may be no corresponding dualities for the wiretap channels.

Here comes the reformulation of the secrecy capacity optimization problem (26) subject to (25) for our algorithm. First, we use the following objective function as

$$f_t(\Sigma_{\mathbf{x}}) = \tilde{R}_s(\Sigma_{\mathbf{x}}) + \frac{1}{t} \log |\Sigma_{\mathbf{x}}|, \tag{39}$$

where $\tilde{R}_s(.)$ is the concave objective function in (27) and $\frac{1}{t} \log |\Sigma_x|$ is the logarithmic barrier. As for constraint (28) (perantenna power constraint (25) with equality), we also rewrite it as the following equality constraint

$$\mathbf{A} \cdot vec(\mathbf{\Sigma}_{\mathbf{x}}) = [P_1 \dots P_{n_t}]^T, \tag{40}$$

$$\nabla_{\mathbf{x}} f_{t} = \mathbb{E}_{\mathbf{H}'} \left[\left((\mathbf{H}')^{\dagger} \otimes (\mathbf{H}')^{T} \right) vec \left(\left((\mathbf{I} + \mathbf{H}' \Sigma_{\mathbf{x}} (\mathbf{H}')^{\dagger})^{-1} \right)^{T} \right) - \left(\mathbf{G}^{\dagger} \otimes \mathbf{G}^{T} \right) vec \left(\left((\mathbf{I} + \mathbf{G} \Sigma_{\mathbf{x}} \mathbf{G}^{\dagger})^{-1} \right)^{T} \right) \right] + \frac{1}{t} vec \left(\left(\Sigma_{\mathbf{x}}^{-1} \right)^{T} \right)$$
(37)

$$\nabla_{xx}^{2} f_{t} = \mathbb{E}_{\mathbf{H}'} \left[-((\mathbf{H}')^{\dagger} \otimes (\mathbf{H}')^{T}) \left((\mathbf{I} + \mathbf{H}' \Sigma_{\mathbf{x}} (\mathbf{H}')^{\dagger}) \otimes ((\mathbf{I} + \mathbf{H}' \Sigma_{\mathbf{x}} (\mathbf{H}')^{\dagger})^{T} \right)^{-1} \cdot \mathbf{K}_{p} \left(((\mathbf{H}')^{\dagger})^{T} \otimes \mathbf{H}' \right) \right.$$

$$\left. + (\mathbf{G}^{\dagger} \otimes \mathbf{G}^{T}) \left((\mathbf{I} + \mathbf{G} \Sigma_{\mathbf{x}} \mathbf{G}^{\dagger}) \otimes (\mathbf{I} + \mathbf{G} \Sigma_{\mathbf{x}} \mathbf{G}^{\dagger})^{T} \right)^{-1} \cdot \mathbf{K}_{p} \left((\mathbf{G}^{\dagger})^{T} \otimes \mathbf{G} \right) \right] - \frac{1}{t} (\Sigma_{\mathbf{x}} \otimes \Sigma_{\mathbf{x}}^{T})^{-1} \mathbf{K}_{p}$$

$$(38)$$

where the $n_t \times (n_t)^2$ matrix **A** has entries being 1 for those corresponds to $\{\Sigma_{\mathbf{x}}\}_{ii}, i = 1 \dots n_t$, and entries being 0 else, and the vectorization operator for a matrix vec(.) is defined as [19] [20]. For example, when $n_t = 2$

$$\mathbf{A} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right].$$

Now we focus on the detailed steps for the optimization of $f_t(\Sigma_{\mathbf{x}})$ in (39) subject to equality constraint (40). In the following, the ν is the $n_t \times 1$ Lagrange multiplier associated to equality constraint (40).

- 1) Initialize $\Sigma_{\mathbf{x}}^{(0)}$ and $\mathbf{v}^{(0)}$
- 2) By invoking the matrix calculus [19] [20], compute the residue as

$$\mathbf{r} = \begin{bmatrix} \nabla_{x} f_{t} + \mathbf{A}^{T} \mathbf{v} \\ \mathbf{A} \cdot vec(\mathbf{\Sigma}_{\mathbf{x}}) - [P_{1} \dots P_{n_{t}}]^{T} \end{bmatrix}$$

where ∇_x means the gradient of function respect to vector $vec(\Sigma_{\mathbf{x}})$, and $\nabla_x f_t$ is given in (37) with \mathbf{H}' from (9) and \otimes being the Kronecker product. The direction $[\Delta_x, \Delta_v]^T$ to update $[vec(\Sigma_{\mathbf{x}}), v]^T$ is computed by the KKT matrix [18] as

$$\left[egin{array}{c} \Delta_x \ \Delta_{
m v} \end{array}
ight] = - \left[egin{array}{cc}
abla_{xx}^2 f_t & {f A}^T \ {f A} & 0 \end{array}
ight]^{-1} {f r},$$

where $\nabla_{xx}^2 f_t$ is explicitly given in (38), with \mathbf{K}_p being the permutation matrix such that $vec(\mathbf{A}^T) = \mathbf{K}_p vec(\mathbf{A})$ as defined in [20, (51)].

- 3) For the n+1 step, we compute $\left(vec\left(\Sigma_{\mathbf{x}}^{(n+1)}\right), \mathbf{v}^{(n+1)}\right) = \left(vec\left(\Sigma_{\mathbf{x}}^{(n)}\right) + s\Delta_{x}, \mathbf{v}^{(n)} + s\Delta\mathbf{v}\right)$, where s is the step size found by the backtracking line search.
- 4) If the norm of residue $\|\mathbf{r}\|_2 < \varepsilon$, increase t by a factor $\gamma = 1.5$ and go to step 5. If not, go back to step 2.
- 5) Stop when the gap $\frac{n_t}{t} < \varepsilon$, if not, go back to step 2.

Note that steps 2 to 4 are the infeasible start Newton steps, and the algorithm will converge since objective function $f_t(.)$ in (39) is concave (both $\tilde{R}_s(.)$ from (27) and the logarithmic barrier are concave) [18]. And following [18], one can show that the gap to the optimal value of (26) is $\frac{n_t}{t}$ as in step 5, because the dimension of $\Sigma_{\mathbf{x}}$ in the logarithmic barrier $\frac{1}{t} \log |\Sigma_{\mathbf{x}}|$ is $n_t \times n_t$.

Compared with the algorithm in [12] which also considers the per-antenna power constraint, our algorithm can guarantee that the gap to the optimal value is vanishing when $t \to \infty$ while that in [12] can not guarantee such an optimality. Moreover, [12] requires full CSIT of both legitimate channel

and eavesdropper while our statistical CSIT requirement is more practical.

V. SIMULATIONS

In this section, we provide numerical and simulation results for our theoretical claims in previous sections. In all figures presented, noise at the legitimate receiver and eavesdropper has unit variance each. The SNR is then defined as the total transmitted power over all antennas in the dB scale. The ratio of the legitimate channel variance over the eavesdropper channel variance in (3) is $\sigma_h^2/\sigma_g^2 = 4$. All channel matrices are Rayleigh faded.

First, in Fig. 1-4, we show the numerical results for secrecy capacities C_s^t under the total power constraints (15) in Theorem 1. In Fig. 1, we compare the MIMOME secrecy capacities C_s^t under different combinations of number of antennas. Three different combinations (n_t, n_r, n_e) with fixed $n_t = 4$ and $n_t \ge n_r$ are considered, where n_t, n_r and n_e respectively are the number of antennas at the transmitter, the legitimate receiver, and the eavesdropper. Consistent with [11], when $n_r = n_e = 1$, the MISOSE secrecy capacities do not scale with SNR and converges at high SNR. Same phenomenon also happens even when $n_r = n_e = 2$. Thus it may be a waste of resource by increasing the SNR for the MIMOME channel with $n_r = n_e$. To overcome this drawback, one can increase n_r to make it larger than n_e . These observations meet our results in Corollary 1. Also with fixed number of transmit antennas $n_t > n_e$, increasing the difference $n_r - n_e$ is very helpful to increase the secrecy capacities in all SNR regimes. Similarly, as in Fig. 1, in Fig. 2 we compare the MIMOME secrecy capacities C_s^t in Theorem 1 but with fixed $n_r = 5$ and $n_r \ge n_t$. The MISOSE secrecy capacities with $(n_t, n_r, n_e) = (5, 1, 1)$ are also depicted in Fig 2, and again they do not scale with the SNR. The SNR scaling in MIMOME channels can be obtained when $n_r > n_e$ but not $n_r = n_e$. And with fixed $n_r > n_e$ (but not $n_r = n_e$), increasing the difference $n_t - n_e$ is very helpful to increase the secrecy capacities even at medium SNR regimes.

Next, we consider the MIMOME secrecy capacities C_s^t in Theorem 1 with single transmit antenna in Fig. 3. As predicted in Corollary 1, when $n_t = 1$ there will be no SNR scaling no matter $n_r > n_e$ or $n_r = n_e$. However, one can observe that channels with larger $n_r - n_e$ (but not larger n_r) will have higher secrecy capacities C_s^t . Next, in Fig. 4, we show that all available transmit power P should be used for MIMOME channels with total power constraints (15). We plot the $R_s(\alpha I)$ s in (22), the secrecy rates with channel input matrices $\Sigma_x = \alpha I$. The maximum allowable transmit power P = 20 dB and

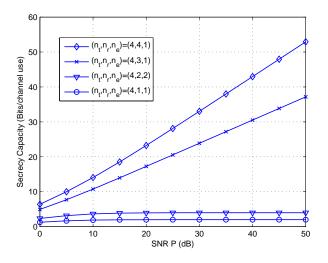


Fig. 1. Under total power constraints over all transmit antennas (15), the secrecy capacities versus SNRs with different number of antennas $(n_t \ge n_r)$.

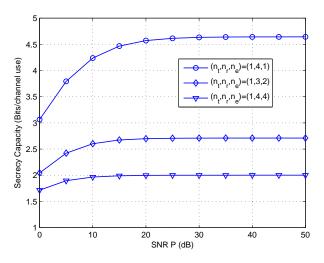


Fig. 3. Under total power constraints over all transmit antennas (15), the secrecy capacities versus SNRs with single transmit antenna ($n_t = 1$).

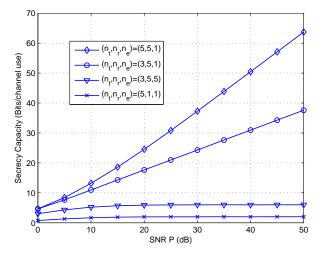


Fig. 2. Under total power constraints over all transmit antennas (15), the secrecy capacities versus SNRs with different number of antennas $(n_r \ge n_t)$.

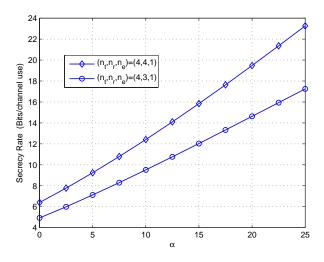


Fig. 4. Under total power constraints over all transmit antennas (15), the MIMOME secrecy rates $R_s(\alpha \mathbf{I})$ in (22) (with channel input matrix $\Sigma_{\mathbf{x}} = \alpha \mathbf{I}$) versus α where $0 \le \alpha \le P/n_t$.

 $0 \le \alpha \le P/n_t$. From Fig.4, the secrecy rate $R_s(\alpha \mathbf{I})$ increases monotonically with α . Thus when α equals to its maximum P/n_t , the secrecy rate $R_s(P/n_t\mathbf{I})$ is the secrecy capacity, as in the proof of Theorem 1.

In Fig. 5 and 6, we studies the wiretap channels under the per-antenna power constraints (25). In Fig.5, the secrecy rates of wiretap channels with $(n_t, n_r, n_e) = (2, 2, 1)$ (the objection function of (26) with channel input covariance matrices $\Sigma_{\mathbf{x}} = diag(P_1, P_2)$) are compared with the secrecy capacities C_s^p of wiretap channels with $(n_t, n_r, n_e) = (2, 1, 1)$ (characterized in Theorem 2). As predicted in Corollary 3, the MISOSE channels under constraints (25) cannot have SNR scaling. However, as stated in Corollary 4, the SNR scaling can be obtained for the wiretap channel with $(n_t, n_r, n_e) = (2, 2, 1)$ even we use the suboptimal $\Sigma_{\mathbf{x}}$ to compute the secrecy rate in Fig.5. In Fig.6, we use the algorithm in Section IV to numerically solve the secrecy capacity optimization problem (26) subject to constraint (25), where $(n_t, n_r, n_e) = (2, 2, 1)$ and

 $\varepsilon = 10^{-4}$. The SNR is $P_1 + P_2$ in dB scale. The proposed algorithms converge in few iterations. Note that the final converged values are very close the secrecy rates computed using channel input covariance matrices $\Sigma_{\mathbf{x}} = (P_1, P_2)$. This fact implies that the secrecy rates for $(n_t, n_r, n_e) = (2, 2, 1)$ plotted in Fig. 5 are very close to the secrecy capacities under per-antenna power constraints (25).

Finally, in Fig. 7, we compare C_s^t and C_s^p , the MISOSE secrecy capacities under the total power constraints (15) and per-antenna power constraints (25) in Theorem 1 and 2, respectively. Since under total power constraints (15), the transmitters are free to allocate power between transmit antennas, the corresponding secrecy capacities are higher than those under per-antennas power constraints (25) $(C_s^t \ge C_s^p)$ when $P = P_1 + P_2$. We also observe that when the ratio P_1/P_2 are larger, the secrecy capacities P_s^p become smaller when $P_s^p = P_s^p = P_s^p = P_s^p$. This is due to that the multiple transmit

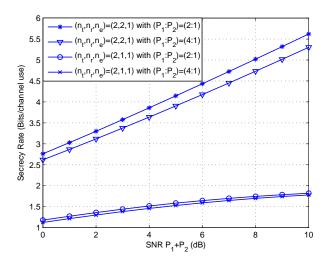


Fig. 5. Under per-antenna power constraints (25), secrecy rates of wiretap channels with $(n_t, n_r, n_e) = (2, 2, 1)$ versus secrecy capacities of wiretap channels with $(n_t, n_r, n_e) = (2, 1, 1)$.

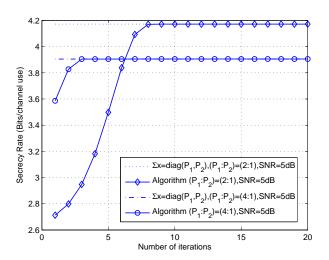


Fig. 6. Convergence of the proposed iterative algorithms for computing the secrecy capacities under constraints (25), where $(n_t, n_r, n_e) = (2, 2, 1)$.

antennas act more like a single antenna when P_1/P_2 is larger. Similar observations can be obtained from the secrecy rates under power constraints (25) and $(n_t, n_r, n_e) = (2, 2, 1)$ in Fig. 5.

VI. CONCLUSION

In this paper, under two different power constraints, the secrecy capacities in fast fading Rayleigh MIMOME wiretap channels with only the statistics of CSIT of both the legitimate and eavesdropper channels were considered. When antennas of the legitimate receiver were more than (or equal to) those of the eavesdropper, under the total power constraint, we fully characterized the MIMOME secrecy capacity. Under the perantenna power constraint, we also showed the secrecy capacity for the MISOSE channel. These results are the first secrecy capacity results for multiple antenna wiretap channels with partial CSIT.

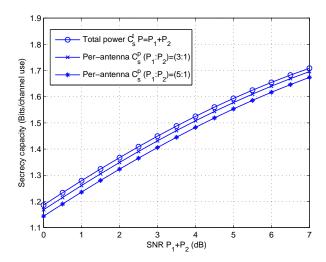


Fig. 7. MISOSE secrecy capacities under the total power constraints (15) and per-antennas power constraints (25), where $(n_t, n_r, n_e) = (2, 1, 1)$.

APPENDIX

A. Proof of Lemma 2

We first focus on Property (I). First, let us consider the case where $\Sigma_x^2 \succeq \Sigma_x^1 \succ \mathbf{0}$. We begin our proof by transforming the objective function in (16) into the upcoming (44). Since \mathbf{H}' in (9) has the same distribution as \mathbf{H} , we rewrite the objective function in (16) as

$$\mathbb{E}_{\mathbf{H}}\left[\log\left|\mathbf{I}+\mathbf{H}'\boldsymbol{\Sigma}_{x}(\mathbf{H}')^{\dagger}\right|\right] - \mathbb{E}_{\mathbf{G}}\left[\log\left|\mathbf{I}+\mathbf{G}\boldsymbol{\Sigma}_{x}\mathbf{G}^{\dagger}\right|\right].$$

And by the matrix equality |I + AB| = |I + BA|, the above equation equals to

$$\mathbb{E}_{\mathbf{H}'} \left[\log \left| \mathbf{I} + \Sigma_{\mathbf{x}} (\mathbf{H}')^{\dagger} \mathbf{H}' \right| \right] - \mathbb{E}_{\mathbf{G}} \left[\log \left| \mathbf{I} + \Sigma_{\mathbf{x}} \mathbf{G}^{\dagger} \mathbf{G} \right| \right]. \tag{41}$$

For any $\Sigma_{\mathbf{x}} \succ \mathbf{0}$, from (9),

$$\begin{split} & \mathbb{E}_{\mathbf{H}'}\left[\log\left|\mathbf{I} + \boldsymbol{\Sigma}_{x}(\mathbf{H}')^{\dagger}\mathbf{H}'\right|\right] \\ = & \mathbb{E}_{\mathbf{H}'}\left[\log\left|\mathbf{I} + \boldsymbol{\Sigma}_{x}\left(\mathbf{H}_{(n_{r}-n_{e})}^{\dagger}\mathbf{H}_{(n_{r}-n_{e})} + \frac{\sigma_{h}^{2}}{\sigma_{g}^{2}}\mathbf{G}^{\dagger}\mathbf{G}\right)\right|\right]. \end{split}$$

Substituting the above equalities into (41), it then equals to

$$\mathbb{E}_{\mathbf{H}'} \left[\log \left| \mathbf{I} + \left(\mathbf{I} + \Sigma_{\mathbf{x}} \mathbf{G}^{\dagger} \mathbf{G} \right)^{-1} \Sigma_{\mathbf{x}} \left((\tilde{\mathbf{H}}')^{\dagger} \tilde{\mathbf{H}}' \right) \right| \right], \quad (42)$$

where

$$(\tilde{\mathbf{H}}')^{\dagger}\tilde{\mathbf{H}}' = \mathbf{H}_{(n_r - n_e)}^{\dagger} \mathbf{H}_{(n_r - n_e)} + \left(\frac{\sigma_h^2}{\sigma_g^2} - 1\right) \mathbf{G}^{\dagger} \mathbf{G}. \tag{43}$$

Note that since since $\sigma_h^2 \ge \sigma_g^2$, the RHS of (43) is positive semi-definite. And thus we can always find $\tilde{\mathbf{H}}'$ to make (43) valid [21]. Then we can rewrite (42) as

$$\mathbb{E}_{\mathbf{H}'} \left[\log \left| \mathbf{I} + \tilde{\mathbf{H}}' \left(\Sigma_{\mathbf{x}}^{-1} + \mathbf{G}^{\dagger} \mathbf{G} \right)^{-1} (\tilde{\mathbf{H}}')^{\dagger} \right| \right]. \tag{44}$$

Now for any $\Sigma_x^2 \succeq \Sigma_x^1 \succ \mathbf{0}$, we know that $((\Sigma_x^2)^{-1} + \mathbf{G}^{\dagger}\mathbf{G})^{-1} \succeq ((\Sigma_x^1)^{-1} + \mathbf{G}^{\dagger}\mathbf{G})^{-1} \succ \mathbf{0}$, and thus

$$\tilde{\mathbf{H}}'\left((\Sigma_{x}^{2})^{-1} + \mathbf{G}^{\dagger}\mathbf{G}\right)^{-1} (\tilde{\mathbf{H}}')^{\dagger} \\
\succeq \tilde{\mathbf{H}}'\left((\Sigma_{x}^{1})^{-1} + \mathbf{G}^{\dagger}\mathbf{G}\right)^{-1} (\tilde{\mathbf{H}}')^{\dagger} \stackrel{(a)}{\succ} \mathbf{0}.$$
(45)

Note that the above relationship is valid for every realization of the random channels, and from [21], we know that

$$\begin{split} & \mathbb{E}_{\mathbf{H}'} \left[\log \left| \mathbf{I} + \tilde{\mathbf{H}}' \left((\boldsymbol{\Sigma}_{x}^{2})^{-1} + \mathbf{G}^{\dagger} \mathbf{G} \right)^{-1} (\tilde{\mathbf{H}}')^{\dagger} \right] \\ \geq & \mathbb{E}_{\mathbf{H}'} \left[\log \left| \mathbf{I} + \tilde{\mathbf{H}}' \left((\boldsymbol{\Sigma}_{x}^{1})^{-1} + \mathbf{G}^{\dagger} \mathbf{G} \right)^{-1} (\tilde{\mathbf{H}}')^{\dagger} \right] \end{split}.$$

Thus (23) is valid if $\Sigma_x^2 \succeq \Sigma_x^1 \succ \mathbf{0}$.

Now we only need to further prove that if $\Sigma_x^2 \succeq \Sigma_x^1 \succeq \mathbf{0}$, when Σ_x^1 is singular, $R_s(\Sigma_x^2) \geq R_s(\Sigma_x^1)$. Then our claim in Property (I) is valid. Here we prove the case where both Σ_x^1 and Σ_x^2 are singular, while the case where only Σ_x^1 is singular can be proved similarly. First, $\forall \alpha > 0$, as shown in the previous paragraph, $R_s(\Sigma_x^2 + \alpha \mathbf{I}) - R_s(\Sigma_x^1 + \alpha \mathbf{I}) \geq 0$ when $\Sigma_x^2 \succeq \Sigma_x^1 \succeq \mathbf{0}$ since $\Sigma_x^2 + \alpha \mathbf{I} \succeq \Sigma_x^1 + \alpha \mathbf{I} \succeq \mathbf{0}$. Next, following methods in [22, P.3957], we have $\lim_{\alpha \to 0} R_s(\Sigma_x^1 + \alpha \mathbf{I}) = R_s(\Sigma_x^1)$ with $\alpha > 0$. This fact comes from the continuity of the log det functions in (22) over positive definite matrices [22]. Then we know that $R_s(\Sigma_x^2) - R_s(\Sigma_x^1) = \lim_{\alpha \to 0} \left(R_s(\Sigma_x^2 + \alpha \mathbf{I}) - R_s(\Sigma_x^1 + \alpha \mathbf{I})\right) \geq 0$. This concludes our proof for Property (I). As for Property (II), it can be easy obtained from (45 a)

B. Proof of Colloray 1

We only prove the case where $n_t \ge n_r > n_e$, since the rest two cases can be proved similarly. From Theorem 1, we know that the MIMOME secrecy capacity C_s^t equals to

$$C_s^t = \mathbb{E}_{\mathbf{H}} \left[\log \left| \mathbf{I} + \frac{P}{n_t} \mathbf{H} \mathbf{H}^{\dagger} \right| \right] - \mathbb{E}_{\mathbf{G}} \left[\log \left| \mathbf{I} + \frac{P}{n_t} \mathbf{G} \mathbf{G}^{\dagger} \right| \right]$$

From [23], one can transform the previous equation as

$$C_s^t = n_r \mathbb{E}\left[\log\left(1 + \frac{\sigma_h^2 P}{n_t}\lambda\right)\right] - n_e \mathbb{E}\left[\log\left(1 + \frac{\sigma_g^2 P}{n_t}\lambda\right)\right],$$

where λ is an unordered eignvalue of a complex Wishart matrix with n_t degrees of freedom and covariance matrix **I**. With the definition $P_g = \sigma_g^2 P$,

$$C_s^t = n_r \mathbb{E}\left[\log\left(1 + \frac{\sigma_h^2}{\sigma_g^2} \frac{P_g}{n_t} \lambda\right)\right] - n_e \mathbb{E}\left[\log\left(1 + \frac{P_g}{n_t} \lambda\right)\right].$$

Note that $\sigma_h^2/\sigma_\varrho^2 \ge 1$, then

$$\lim_{P_g \to \infty} \frac{C_s^t - n_r \log(\sigma_h^2 / \sigma_g^2)}{\log P_g} = n_r - n_e,$$

for fixed n_t .

C. Proof of Proposition 1

Following [22], we can show that the result in Lemma 1 is still valid under (25), with proof based on replacing (25) with the covariance matrix constraint $\mathbb{E}(\mathbf{x}\mathbf{x}^{\dagger}) \prec \mathbf{S}$. To be more specific, $C_s^p(P_1, \dots, P_{n_t}) = \max_{\mathbf{S} \in \mathbb{S}} C_s(\mathbf{S})$, where $C_s^p(P_1, \dots, P_{n_t})$ is the secrecy capacity under per-antenna power constraint (25), $C_s(\mathbf{S})$ is the secrecy capacity under the covariance matrix constraint $\mathbb{E}(\mathbf{x}\mathbf{x}^{\dagger}) \leq \mathbf{S}$, and the set $\mathbb{S} = {\mathbf{S}|{\mathbf{S}}_{ii} \leq P_i, i = 1}$ $1, \ldots, n_t, \mathbf{S} \succeq \mathbf{0}$. Next, we can show that Gaussian signal **x** without prefixing $U \equiv \mathbf{x}$ is still secrecy capacity achieving with respect to $C_s(\mathbf{S})$. The proof is the same as that of Lemma 1, expect for proving that under covariance matrix constraint instead of (15), Gaussian x maximizes (14). To prove this fact, from (18 b), we know that the maximization in (14) equals to $\max_{\mathbf{x}} h(\frac{\sigma_{\mathbf{g}}}{\sigma_{\mathbf{h}}} \mathbf{y}' | \mathbf{z}, \mathbf{H}')$. Given a realization of $\mathbf{H}' = \underline{\mathbf{H}}'$, we can show that Gaussian x will make (y',z) jointly Gaussian and maximize $h(\frac{\sigma_g}{\sigma_h} \mathbf{y}' | \mathbf{z}, \mathbf{H}' = \underline{\mathbf{H}}')$ for all \mathbf{x} satisfying $\mathbb{E}(\mathbf{x}\mathbf{x}^\dagger) = \mathbf{S}'$ and $S' \leq S$, by modifying the proof of [6, Lemma 1] with [14, Theorem 8.6.5]. Moreover, since \mathbf{x} is independent of \mathbf{H}' , we know that Gaussian \mathbf{x} will maximize $h(\frac{\sigma_{\mathbf{g}}}{\sigma_{\mathbf{h}}}\mathbf{y}'|\mathbf{z},\mathbf{H}')$ and thus maximize (14) subject to the $\mathbb{E}(\mathbf{x}\mathbf{x}^{\dagger}) \leq \mathbf{S}$ (thus also for the per-antenna power constraint (25)).

Substitute the optimal Gaussian \mathbf{x} with covariance matrix $\Sigma_{\mathbf{x}}$ into $I(\mathbf{x};\mathbf{y}|\mathbf{H}) - I(\mathbf{x};\mathbf{y}|\mathbf{G})$, we have the secrecy capacity formulae as (26) subject to (25). Next, by replacing channel matrix \mathbf{H} in (26) with the same distribution one \mathbf{H}' in (9), we have (27) according to the proof in Appendix A (steps for reaching (42)). Finally, following the first paragraph of Theorem 1's proof, we know that the secrecy capacity (27) under (25) is concave in $\Sigma_{\mathbf{x}}$.

D. Solution of the optimization problem (32) subject to the equality constraint (31)

In this appendix, we show that subject to constraint (31), the optimization problem (32) has optimal solution (29). We only consider the case with $\beta < 0$ because $\beta = 0$ is a trivial case. The following proof is given by mathematical induction, which shows that the off-diagonal terms of Σ_x subject to (31) should be zeros.

(i) Let us first consider the case with $n_t = 2$. By defining $f(x) \triangleq \log(1+x)$ to simplify the notations, the objective function of (32) can be written as

$$\mathbb{E}_{\mathbf{g}} \left[f \left(\frac{\beta}{\sigma + |g_1|^2 \{ \Sigma_{\mathbf{x}} \}_{11} + |g_2|^2 \{ \Sigma_{\mathbf{x}} \}_{22} + g_1^* \{ \Sigma_{\mathbf{x}} \}_{12} g_2 + g_2^* \{ \Sigma_{\mathbf{x}} \}_{21} g_1 } \right) \right]. \tag{46}$$

Notice that, since g_1 and g_2 are independent zero-mean Gaussian, the expectation of (46) over $\mathbf{g} = [g_1 \ g_2]^T$ would not change by replacing g_1 with $-g_1$. Therefore, with $\{\Sigma_{\mathbf{x}}\}_{11} = P_1$

and $\{\Sigma_{\mathbf{x}}\}_{22} = P_2$, the expectation in (46) can be written as

$$\begin{split} &\mathbb{E}_{\mathbf{g}}\left[f\left(\frac{\beta}{\sigma + |g_{1}|^{2}P_{1} + |g_{2}|^{2}P_{2} + g_{1}^{*}\{\Sigma_{\mathbf{x}}\}_{12}g_{2} + g_{2}^{*}\{\Sigma_{\mathbf{x}}\}_{21}g_{1}}\right)\right] \\ &= \frac{1}{2}\left\{\mathbb{E}_{\mathbf{g}}\left[f\left(\frac{\beta}{\sigma + |g_{1}|^{2}P_{1} + |g_{2}|^{2}P_{2} + 2\operatorname{Re}\{g_{1}^{*}\{\Sigma_{\mathbf{x}}\}_{12}g_{2}\}}\right)\right] \\ &+ \mathbb{E}_{\mathbf{g}}\left[f\left(\frac{\beta}{\sigma + |g_{1}|^{2}P_{1} + |g_{2}|^{2}P_{2} - 2\operatorname{Re}\{g_{1}^{*}\{\Sigma_{\mathbf{x}}\}_{12}g_{2}\}}\right)\right]\right\} \\ &= \frac{1}{2}\mathbb{E}_{\mathbf{g}}\left[f\left(\frac{\beta\left[\beta + 2\sigma + 2\left(|g_{1}|^{2}P_{1} + |g_{2}|^{2}P_{2}\right)\right]}{\left(\sigma + |g_{1}|^{2}P_{1} + |g_{2}|^{2}P_{2}\right)^{2} - \left(2\operatorname{Re}\{g_{1}^{*}\{\Sigma_{\mathbf{x}}\}_{12}g_{2}\}\right)^{2}}\right)\right]. \end{split}$$

In this case, the expectation is maximized by choosing

$$\{\Sigma_{\mathbf{x}}\}_{12} = \{\Sigma_{\mathbf{x}}\}_{21}^* = 0$$

since $\beta < 0$ and $\beta + 2\sigma > 0$. Hence, subject to (31), (32) is maximized by choosing $\Sigma_{\mathbf{x}} = \operatorname{diag}(P_1, P_2)$, for the case with

(ii) Suppose that the statement holds for $n_t = k$, where $k \ge 2$. We need to show that it also holds for $n_t = k + 1$. Specifically, for $n_t = k + 1$, with eavesdropper channel being $\mathbf{g} =$ $[g_1,\ldots,g_{k+1}]^T$, the objective function $\mathbb{E}_{\mathbf{g}}\left|\log\left(1+\frac{\beta}{\sigma+\mathbf{g}^{\dagger}\Sigma_{\mathbf{x}}\mathbf{g}}\right)\right|$ in (32) can be written as

$$\mathbb{E}_{\mathbf{g}} \left[\log \left(1 + \frac{\beta}{\sigma + \sum_{i=1}^{k+1} \sum_{j=1}^{k+1} g_i^* \{ \Sigma_{\mathbf{x}} \}_{ij} g_j } \right) \right]$$

$$= \mathbb{E}_{\mathbf{g}} \left[\log \left(1 + \frac{\beta}{\sigma_{k+1} + 2 \operatorname{Re} \{ \sum_{i=1}^{k} g_i^* \{ \Sigma_{\mathbf{x}} \}_{i,k+1} g_{k+1} \} } \right) \right], \quad (47)$$

where

$$\sigma_{k+1} \triangleq \sigma + \tilde{\mathbf{g}}^{\dagger} \tilde{\Sigma}_{\mathbf{x}} \tilde{\mathbf{g}} + |g_{k+1}|^2 P_{k+1},$$

with $\tilde{\mathbf{g}} = [g_1, \dots, g_k]^T$ and $\tilde{\Sigma}_{\mathbf{x}}$ being a $k \times k$ matrix with $\{\tilde{\Sigma}_{\mathbf{x}}\}_{i,j} = \{\Sigma_{\mathbf{x}}\}_{i,j}, \text{ for } i,j=1,\ldots,k.$ Then, by the fact that $-g_{k+1}$ has the same distribution as g_{k+1} , the expectation in (47) becomes

$$\frac{1}{2}\mathbb{E}_{\mathbf{g}}\left[\log\left(1+\frac{\beta\left(\beta+2\sigma+2(\tilde{\mathbf{g}}^{\dagger}\tilde{\boldsymbol{\Sigma}}_{\mathbf{x}}\tilde{\mathbf{g}}+|g_{k+1}|^{2}P_{k+1})\right)}{(\sigma_{k+1})^{2}-(2\operatorname{Re}\{\boldsymbol{\Sigma}_{i=1}^{k}g_{i}^{*}\{\boldsymbol{\Sigma}_{\mathbf{x}}\}_{i,k+1}g_{k+1}\})^{2}}\right)\right].$$

Here, the expectation is maximized by choosing

$$\{\Sigma_{\mathbf{x}}\}_{i,k+1} = \{\Sigma_{\mathbf{x}}\}_{k+1}^* = 0$$

for i = 1, ..., k, due to the assumptions $\beta < 0$ and $\beta + 2\sigma > 0$. In this case, (47) becomes

$$\mathbb{E}_{\mathbf{g}} \left[\log \left(1 + \frac{\beta}{\sigma + |g_{k+1}|^2 P_{k+1} + \tilde{\mathbf{g}}^{\dagger} \tilde{\Sigma}_{\mathbf{v}} \tilde{\mathbf{g}}} \right) \right]. \tag{48}$$

Then because $\sigma + |g_{k+1}|^2 P_{k+1} \ge 0$, $\beta + 2(\sigma + |g_{k+1}|^2 P_{k+1}) > 0$, and $\tilde{\Sigma}_x$ is positive semi-definite, it follows from the inductive hypothesis that (48) is maximized by choosing $\hat{\Sigma}_{\mathbf{x}}$ = $\operatorname{diag}(P_1,\ldots,P_k)$. This concludes that subject to the equality constraint (31), the optimal solution maximizing the objective function of (32) is (29).

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