

Interplay Between Delayed CSIT and Network Topology for Secure MISO BC

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Abstract

We study the problem of secure transmission over a Gaussian two-user multi-input single-output (MISO) broadcast channel under the assumption that links connecting the transmitter to the two receivers may have unequal strength *statistically*. In addition to this, the state of the channel to each receiver is conveyed in a strictly causal manner to the transmitter. We focus on a two state topological setting of strong v.s. weak links. Under these assumptions, we first consider the MISO wiretap channel and establish bounds on generalized secure degrees of freedom (GSDoF). Next, we extend this model to the two-user MISO broadcast channel and establish inner and outer bounds on GSDoF region with different topology states. The encoding scheme sheds light on the usage of both resources, i.e., topology of the model and strictly causal channel state information at the transmitter (CSIT); and, allows digitization and multi-casting of overheard side information, while transmitting confidential message over the stronger link. Furthermore, for a special class of channels, we show that the established bounds agree and so we characterize the sum GSDoF.

I. INTRODUCTION

In communication networks, due to the scarcity of available resources and increase in the demand of higher data rates imposed by the consumers, multiple nodes communicate with each other over a shared medium. This in turn leads to a fundamental problem of interference in networks. A key ingredient to eradicate the detrimental effect of interference efficiently is by means of CSIT. In existing literature, for instance [2], different schemes are proposed which require perfect knowledge of CSIT to align or cancel interference. In practice, wireless medium is exposed to various random effects; thus, conveying perfect CSIT is difficult. Recently, in [3] Maddah-Ali *et al.* study a MISO broadcast channel and show a rather

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surprising result that strictly causal (delayed) CSIT is still useful in the sense that it enlarges the degrees of freedom (DoF) compared to a similar model with no CSIT. The model studied in [3] is generalized to a variety of settings namely, two- and three-user multi-input multi-output (MIMO) broadcast channel in [4], [5], two-user interference channel in [6], [7], and X-channel in [8], [9] all from DoF perspective. In cellular networks due to mobility, communication links are subjected to different topological effects, e.g., inter-cell interference, wave propagation path loss, jamming. These physical factors influence links in an *asymmetric* manner, that lead to some links being stronger than others *statistically*. A fundamental issue with DoF analysis is that it ignores the diversity of links strength and implicitly assumes that all non-zero channels are equally strong in the sense that each link is capable of carrying 1 DoF, irrespective of the magnitude of channel coefficients. The GDoF metric solves this limitation by taking diversity of links strength into account [10], [11]. In [12], Chen *et al.* study a two-user MISO broadcast channel by considering the two state topological setting of strong v.s. weak links and assume that CSI conveyed by both receivers can vary over time. For this model the authors establish bounds on GDoF region.

As said before, due to the broadcast nature of wireless medium, communication can be over heard by unintended nodes in the network. Wyner in [13], introduced a basic wiretap channel to study secrecy by taking physical layer attributes of the channel into account. In Wyner's setup, the source wants to communicate a confidential message to the legitimate receiver and this message is meant to be concealed from the eavesdropper. For the degraded setting, in which the channel to legitimate receiver is stronger than to the eavesdropper secrecy capacity is established. In the last decade, the wiretap channel has attracted significant interest in the research community and is extended to study a variety of multi-user channels, e.g., the broadcast channel [14], [15], the multi-access channel [16]–[20], the relay channel [21]–[23], the interference channel [24], [25], and the multi-antenna channel [26]–[29]. For a review of other related contributions the reader may refer to [30] (and references therein). Due to the difficulty in characterizing the complete secrecy capacity region, a number of recent contributions has focused on characterizing the *approximate* capacity of these networks. The approximate capacity is measured by the notion of secure degrees of freedom (SDoF). Similar to the model with no security constraints, the SDoF metric captures the asymptotic behavior of secure data rates in high signal-to-noise ratio (SNR) regime. Thus, SDoF can be equivalently understood as the secure spatial multiplexing gain, number of secure signaling dimensions, or the secrecy capacity pre-log factor. From DoF perspective, the authors in [31] study a K -user interference channel and establish a lower bound on the sum SDoF, where perfect non-causal CSI is available at all nodes. Recently, Yang *et al.* in [32] study the two-user MIMO broadcast channel under a relaxation that instead of non-causal CSI, strictly causal CSI (delayed) is provided to the transmitter

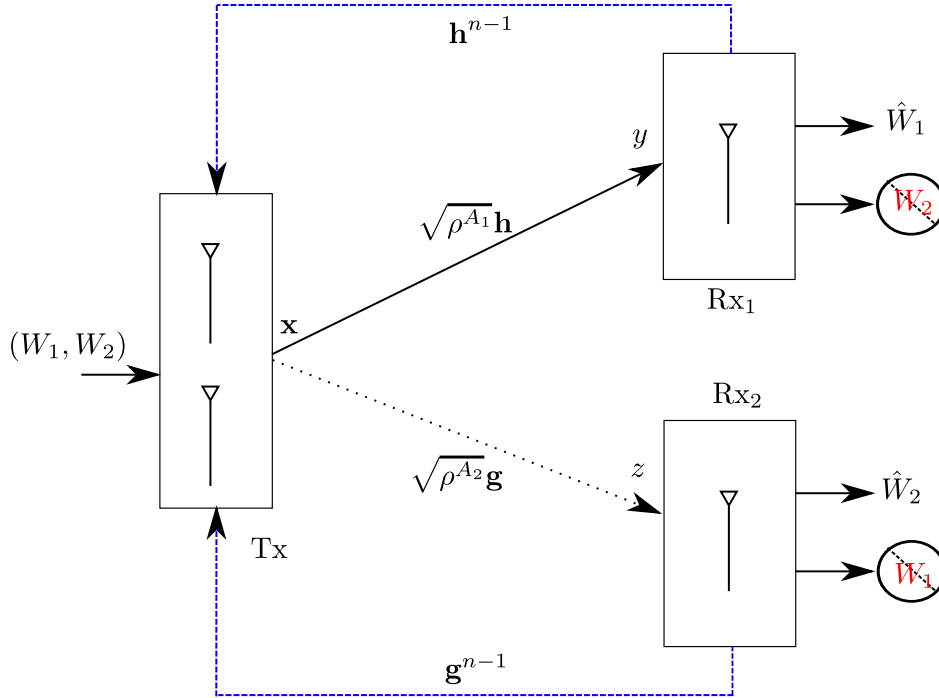


Fig. 1. $(2, 1, 1)$ -MISO broadcast channel, where the link power exponent to receiver 1 is $A_1 \in \{1, \alpha\}$ and to receiver 2 is $A_2 \in \{1, \alpha\}$ and $0 \leq \alpha \leq 1$.

from both receivers. For this model the authors characterize the SDoF region. The coding scheme in [32] follows by a careful extension of Maddah Ali-Tse scheme [3] with additional noise injection to account for secrecy constraints. Zaidi *et al.* in [33], [34] study the two-user MIMO X-channel with asymmetric feedback and delayed CSIT, and characterize the complete sum SDoF region. In [35], the authors studied the MISO broadcast channel and assume that CSI conveyed by two receivers can vary over time and establish bounds on SDoF region. Recall that, similar to DoF — SDoF metric ignores the diversity of links strength at the receivers which may be beneficial to strengthen the secrecy in certain situations. Thus, going beyond the SDoF metric to the GSDoF will be useful to gain further insights and is the focus of this work.

In this paper, we consider a Gaussian $(2, 1, 1)$ -MISO broadcast channel which consists of three nodes — a transmitter and two receivers as shown in Figure 1. The transmitter is equipped with two antennas and each receiver is equipped with a single antenna. The transmitter wants to reliably transmit message W_1 to the receiver 1 and message W_2 to the receiver 2. In investigating this model we make three assumptions, namely, 1) each receiver knows the perfect instantaneous CSI and also the CSI of the other

receiver with a unit delay, 2) each receiver is allowed to convey the past or delayed CSI to the transmitter, and 3) links connecting two receivers may have different strength, statistically. We restrict our attention to the two state topological setting of stronger v.s. weaker links; thus, the topology of this network is allowed to alternate between four possible states and is known at the transmitter. Furthermore, message W_1 intended for the receiver 1 is meant to be kept secret from the receiver 2 and message W_2 intended for the receiver 2 is meant to be kept secret from the receiver 1. Thus, each receiver plays two different roles, not only 1) it acts as a legitimate receiver for the message intended for itself, 2) it also acts as an eavesdropper for the message intended for the other receiver. We assume that the eavesdroppers are passive and are not capable to modify the communication.

The MISO broadcast channel that we study in this paper relates to a number of works studied previously. Compared to the MIMO broadcast channel with secrecy constraints studied in [32], in this work the links connecting two receivers may observe different strength. The $(2, 1, 1)$ -MISO broadcast channel that we study can be seen as a special case of the one in [12] but with imposed security constraints. From a practical viewpoint, the channel shown in Figure 1 may be useful to model the down-link phase of a cellular network in which the base station wants to securely communicate with two receivers and the messages are meant to be kept secret from each other — where both receivers are subjected to jamming from an external interferer.

The main contributions of this work are summarized as follows. We first consider the $(2, 1, 1)$ -MISO wiretap channel and establish bounds on the GSDoF. Next, we extend the MISO wiretap model to the broadcast setting with strictly causal (delayed) CSIT as shown in Figure 1. For this model, we first establish an outer bound on the GSDoF region. The techniques used to establish the outer bound are essentially similar to the one that we use to prove the upper bound for the wiretap model. Concentrating on the role of topological diversity, we consider two elementary settings. In the first scenario, we consider a setting in which the link to one receiver is comparatively stronger than the other. We refer to this as *fixed* topology. In the second scenario, we consider the *symmetric alternating* topology. This setting refers to the case in which the link to one receiver is stronger than to the other receiver, half of the duration of communication time. For these two models, we establish inner bounds on the GSDoF region. The encoding scheme is based on an appropriate extension of Maddah Ali scheme [3] with noise injection [32], and carefully utilizes the topology of the network. The key ingredients of the coding scheme are, as opposed to [3] where side information is conveyed in an analog manner, digitized side information is multicasted, and in supplement to this, fresh confidential information is sent to the receiver with the stronger link.

We also study a special class of $(2, 1, 1)$ -MISO broadcast channel with integer channel coefficients.

We first consider the MISO wiretap channel with fixed topology, where the link to legitimate receiver is stronger than to the eavesdropper and characterize the GSDoF. The coding scheme in this case follows by appropriately combining compute-and-forward scheme [36] and also uses some elements of the schemes that we have previously developed for the general case. Next, we consider the $(2, 1, 1)$ -MISO broadcast channel with *symmetric alternating* topology and characterize the sum GSDoF. Furthermore, we specialize our results for the case in which there are no security constraints. In particular, for the *fixed* topology setting in which the link to one receiver is comparatively stronger than to the other, we characterize the GDoF region. The coding scheme in this case follows by specializing the scheme that we have developed previously for a related model by removing the secrecy constraints. Finally, we illustrate our results with the help of some numerical examples.

We structure this paper as follows. Section II provides a formal description of the channel model along with some useful definitions. In Section III, we consider the MISO wiretap channel and state the upper and lower bounds on GSDoF. In Section IV, we study the MISO broadcast channel and state the outer and inner bounds on the GSDoF region. In Section V, we specialize our results in previous sections to integer channels. Section VI provides some numerical examples. Finally, in Section VII we conclude this paper by summarizing its contributions.

Notations: A few words about notations. Boldface upper case letter \mathbf{X} denote matrices, boldface lower case letter \mathbf{x} denote vectors, and calligraphic letter \mathcal{X} designate alphabets. At each time instant t , \mathbf{x}_t denote $[x_{t1}, \dots, x_{tn}]$, and $\mathbb{E}[\cdot]$ denote the expectation operator. We use \doteq to denote an exponential equality, such that given $f(\rho) \doteq \rho^\beta$ implies $\lim_{\rho \rightarrow \infty} \log f(\rho) / \log(\rho) = \beta$. We use $\mathcal{O}(f(\rho))$ to denote the asymptotic behaviour of the function $f(\rho)$. The term $o(n)$ is some function $g(n)$ such that $\lim_{n \rightarrow \infty} \frac{g(n)}{n} = 0$. The Gaussian distribution with mean μ and variance σ^2 is denoted by $\mathcal{CN}(\mu, \sigma^2)$. Finally, throughout the paper, logarithms are taken to base 2.

II. SYSTEM MODEL AND DEFINITIONS

We consider a two-user $(2, 1, 1)$ -Gaussian MISO broadcast channel, as shown in Figure 1. In this model, the transmitter is equipped with two transmit antennas and each of the receiver is equipped with a single antenna. The transmitter wants to reliably transmit message $W_1 \in \mathcal{W}_1 = \{1, \dots, 2^{nR_1(A_1, \rho)}\}$ to receiver 1, and message $W_2 \in \mathcal{W}_2 = \{1, \dots, 2^{nR_2(A_2, \rho)}\}$ to receiver 2, respectively; and, in doing so, it wishes to conceal the message W_1 , intended to receiver 1, from receiver 2 and the message W_2 , intended to receiver 2, from receiver 1, respectively. Thus, receiver 2 not only is a legitimate receiver for confidential message W_2 , it acts as an eavesdropper for the MISO channel to receiver 1. Similarly,

receiver 1 not only is a legitimate receiver for confidential message W_1 , it is an eavesdropper for the MISO channel to receiver 2. For this setting, we consider a fast fading environment and assume that each receiver is fully aware of its own perfect instantaneous CSI and also the CSI of the other receiver with a unit delay. In addition to this, each receiver is allowed to convey only the past or outdated CSI to the transmitter, i.e., at time instant t , transmitter has perfect knowledge of *only* the past $(t - 1)$ channel states from both receivers. It is easy to see that by setting $W_i = \phi$ for $i = 1$ or 2 , the model in Figure 1 reduces to the $(2, 1, 1)$ -MISO wiretap channel.

Due to the inherent randomness of the wireless channel and topological changes that may arise, for instance — due to the mobility of the users or interference (jamming) from unintended nodes, some elements of the network can experience more interference compared to the others. These factors in turn originate two fundamental classes of links, where few links are comparatively stronger than others statistically. Let $A_1 \in \{1, \alpha\}$ denote the link power exponent from the transmitter-to-receiver 1 and $A_2 \in \{1, \alpha\}$ denote the link power exponent from transmitter-to-receiver 2, respectively, for $0 \leq \alpha \leq 1$; where we denote the stronger link by $A_i := 1$ and weaker link by $A_i := \alpha$, $i = 1, 2$. As alluded before, the notion of stronger v.s. weaker links implies a statistical comparison, so for instance, $A_1 > A_2$ refers to the case in which link connecting the transmitter to receiver 1 is stronger than to the receiver 2 statistically. For convenience, without loss of generality in the rest of the paper, we will refer to this as receiver 1 being *stronger* than receiver 2. Then, based on the topology of the network, the model that we study belongs to any of the four possible states, $(A_1, A_2) \in \{1, \alpha\}^2$. We denote $\lambda_{A_1 A_2}$ be the fraction of time topology state (A_1, A_2) occurs, such that

$$\sum_{(A_1, A_2) \in \{1, \alpha\}^2} \lambda_{A_1 A_2} = 1. \quad (1)$$

The channel input-output relationship at time instant t is then given by

$$\begin{aligned} y_t &= \sqrt{\rho^{A_{1,t}}} \mathbf{h}_t \mathbf{x}_t + n_{1t} \\ z_t &= \sqrt{\rho^{A_{2,t}}} \mathbf{g}_t \mathbf{x}_t + n_{2t}, \quad t = 1, \dots, n \end{aligned} \quad (2)$$

where $\mathbf{x} \in \mathbb{C}^{2 \times 1}$ is the channel input vector, $\mathbf{h} \in \mathcal{H} \subseteq \mathbb{C}^{1 \times 2}$ is the channel vector connecting receiver 1 to the transmitter and $\mathbf{g} \in \mathcal{G} \subseteq \mathbb{C}^{1 \times 2}$ is the channel vector connecting receiver 2 to the transmitter. The parameter ρ is subject to input power constraint and the channel output noise n_i is assumed to be independent and identically distributed (i.i.d.) white Gaussian noise, with $n_i \sim \mathcal{CN}(0, 1)$ for $i = 1, 2$. For convenience, we normalize the channel input vector, $\|\mathbf{x}_t\|^2 \leq 1$, then the average received signal-to-noise

ratio (SNR) for each link at time instant t is given by

$$\begin{aligned}\mathbb{E}_{\mathbf{h}_t, \mathbf{x}_t} \left[\|\sqrt{\rho^{A_{1,t}}} \mathbf{h}_t \mathbf{x}_t\|^2 \right] &= \rho^{A_{1,t}} \\ \mathbb{E}_{\mathbf{g}_t, \mathbf{x}_t} \left[\|\sqrt{\rho^{A_{2,t}}} \mathbf{g}_t \mathbf{x}_t\|^2 \right] &= \rho^{A_{2,t}}.\end{aligned}$$

For ease of exposition, we denote $\mathbf{S}_t = \begin{bmatrix} \mathbf{h}_t \\ \mathbf{g}_t \end{bmatrix}$ as the channel state matrix and $\mathbf{S}^{t-1} = \{\mathbf{S}_1, \dots, \mathbf{S}_{t-1}\}$ captures the collection of channel state matrices over the past $(t-1)$ symbols, respectively, where $\mathbf{S}^0 = \emptyset$. We assume that, at each time instant t , the channel state matrix \mathbf{S}_t is full rank almost surely. Furthermore, at each time instant t , the past states of the channel matrix \mathbf{S}^{t-1} are known to all nodes. However, the instantaneous states \mathbf{h}_t and \mathbf{g}_t are known only to receiver 1, and receiver 2, respectively.

Definition 1: A code for the Gaussian two-user $(2, 1, 1)$ -MISO broadcast channel with delayed CSIT and alternating topology consists of sequence of stochastic encoders at the transmitter,

$$\{\varphi_t : \mathcal{W}_1 \times \mathcal{W}_2 \times \mathcal{S}^{t-1} \longrightarrow \mathcal{X}_1 \times \mathcal{X}_2\}_{t=1}^n \quad (3)$$

where the messages W_1 and W_2 are drawn uniformly over the sets \mathcal{W}_1 and \mathcal{W}_2 respectively; and two decoding functions at receivers

$$\begin{aligned}\psi_1 &: \mathcal{Y}^n \times \mathcal{S}^{n-1} \times \mathcal{H}_n \longrightarrow \hat{\mathcal{W}}_1 \\ \psi_2 &: \mathcal{Z}^n \times \mathcal{S}^{n-1} \times \mathcal{G}_n \longrightarrow \hat{\mathcal{W}}_2.\end{aligned} \quad (4)$$

Definition 2: A rate pair $(R_1(A_1, \rho), R_2(A_2, \rho))$ is said to be achievable if there exists a sequence of codes such that

$$\limsup_{n \rightarrow \infty} \Pr\{\hat{W}_i \neq W_i\} = 0, \quad \forall i \in \{1, 2\}. \quad (5)$$

Definition 3: A GSDoF pair $(d_1(A_1), d_2(A_2))$ is said to be achievable if there exists a sequence of codes satisfying following

1) Reliability condition:

$$\limsup_{n \rightarrow \infty} \Pr\{\hat{W}_i \neq W_i\} = 0, \quad \forall i \in \{1, 2\}, \quad (6)$$

2) Perfect secrecy condition:¹

$$\limsup_{n \rightarrow \infty} \frac{I(W_2; y^n, \mathbf{S}^n)}{n} = 0, \quad (7)$$

$$\limsup_{n \rightarrow \infty} \frac{I(W_1; z^n, \mathbf{S}^n)}{n} = 0, \quad (8)$$

¹For convenience, with a slight abuse in notations, we replace $\mathbf{S}^n := (\mathbf{S}^{n-1}, \mathbf{h}_n)$, $\mathbf{S}^n := (\mathbf{S}^{n-1}, \mathbf{g}_n)$ in (7) and (8), respectively.

3) and communication rate condition:

$$\lim_{\rho \rightarrow \infty} \liminf_{n \rightarrow \infty} \frac{\log |\mathcal{W}_i(n, \rho, A_i)|}{n \log \rho} \geq d_i(A_i), \quad \forall i \in \{1, 2\}. \quad (9)$$

III. GSDoF OF MISO WIRETAP CHANNEL WITH DELAYED CSIT

In this section, we investigate the GSDoF of the MISO wiretap channel with delayed CSIT. Before proceeding to state the results, we first digress to provide a useful lemma which we will repetitively use in this work.

Lemma 1: For the Gaussian MISO channel in (2), following inequalities hold

$$h(y^n, z^n | \mathbf{S}^n) \dot{\leq} 2h(z^n | \mathbf{S}^n) + n\lambda_{1\alpha}(1 - \alpha) \log(\rho), \quad (10a)$$

$$h(y^n, z^n | \mathbf{S}^n) \dot{\leq} 2h(y^n | \mathbf{S}^n) + n\lambda_{\alpha 1}(1 - \alpha) \log(\rho), \quad (10b)$$

$$h(y^n | \mathbf{S}^n) \dot{\leq} 2h(z^n | \mathbf{S}^n) + n\lambda_{1\alpha}(1 - \alpha) \log(\rho), \quad (10c)$$

$$h(z^n | \mathbf{S}^n) \dot{\leq} 2h(y^n | \mathbf{S}^n) + n\lambda_{\alpha 1}(1 - \alpha) \log(\rho). \quad (10d)$$

Proof: The proof of Lemma 1 appears in Appendix I. The inequalities in Lemma 1 also hold with additional conditioning over message W . \square

A. Upper Bound

We now establish an upper bound on the GSDoF of the MISO wiretap channel with delayed CSIT and alternating topology.

Theorem 1: For the (2, 1, 1)–MISO wiretap channel with delayed CSIT and alternating topology $(\lambda_{A_1 A_2})$, an upper bound on GSDoF is given by

$$d(\lambda_{A_1 A_2}) \leq \frac{(3 - \alpha)\lambda_{1\alpha} + 2(\lambda_{11} + \alpha\lambda_{\alpha\alpha}) + (1 + \alpha)\lambda_{\alpha 1}}{3}. \quad (11)$$

Proof: The proof of Theorem 1 appears in Appendix II. \square

B. Coding schemes with fixed topology

Next, we provide some encoding schemes for fixed topology states. For simplicity of analysis and in accordance with DoF framework, in this work we neglect the additive Gaussian noise and only mention the asymptotic behavior of the inputs by ignoring the exact power allocations.

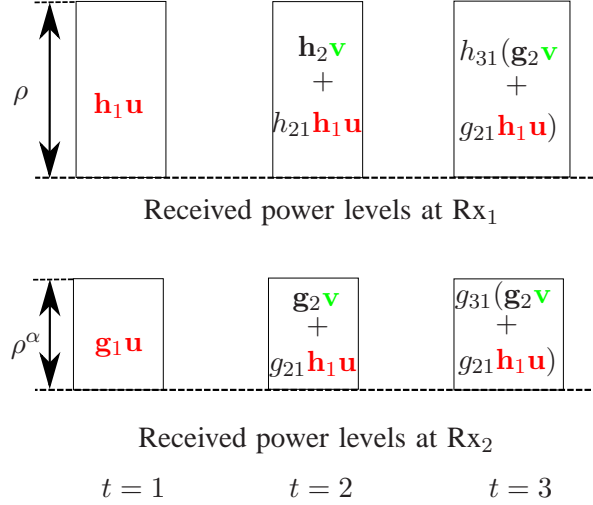


Fig. 2. Received power levels at receiver 1 and receiver 2.

1) *Fixed Topology* ($\lambda_{1\alpha} = 1$): We now focus our attention on the case in which receiver 1 (legitimate receiver) is stronger than receiver 2 (eavesdropper), comparatively and state a lower bound on GSDoF. From practical viewpoint, this case may be useful to model a setting in which the legitimate receiver is geographically located at a more favorable position compared to the eavesdropper, and observes less interference from an external interferer (jammer) as opposed to the eavesdropper.

Proposition 1: The GSDoF of (2, 1, 1)–MISO wiretap channel with delayed CSIT and fixed topology ($\lambda_{1\alpha} = 1$) is given by

$$\frac{2}{3} \leq d \leq 1 - \frac{\alpha}{3}. \quad (12)$$

Proof: The upper bound follows immediately from the bound established in Theorem 1 by setting $\lambda_{1\alpha} := 1$ in (11). We now provide the description of the encoding scheme that we use to prove the lower bound in Proposition 1. The coding scheme in this case is an adaptation of the scheme developed by [32], and, so we outline it briefly. In this scheme, the transmitter sends two symbols (v_1, v_2) to receiver 1 and wishes to conceal them from receiver 2. The coding scheme consists of three time slots. In the first time slot the transmitter injects uncoded Gaussian noise ($\mathbf{u} := [u_1, u_2]^T$) from both antennas. Note that, due to the topology of the network, the channel output at receiver 2 (eavesdropper) is available at a lower power level compared to the receiver 1 as shown in Figure 2. The channel output at the receiver 1 can be interpreted as a secret key that helps to secure the confidential messages in the next timeslot. At the end of timeslot 1, by means of past CSI the transmitter can *learn* the channel output at receiver 1 and

sends it with confidential symbols ($\mathbf{v} := [v_1, v_2]^T$) intended for receiver 1. At the end of second timeslot, receiver 1 gets the confidential symbols embedded in with secret key ($\mathbf{h}_1 \mathbf{u}$). Since it knows the CSI and $\mathbf{h}_1 \mathbf{u}$, it subtracts out the contribution of $\mathbf{h}_1 \mathbf{u}$ from the channel output at the end of second timeslot to get one equation with two variables and requires one extra equation to decode the confidential symbols, which is being available as side information at receiver 2. By means of past CSI, the transmitter can construct the channel output at receiver 2 in timeslot 2 and in the third time slot sends it to the receiver 1, that helps to decode the two symbols securely. \square

Remark 1: It can be easily seen from (12) that the lower and upper bounds do not coincide in general. However, for a special class of channels where the channel coefficients belong to $\mathbf{h}, \mathbf{g} \in \mathbb{Z}^{1 \times 2}$, we show that by combining some elements of this scheme with compute-and-forward scheme [36] the bounds agree and GSDoF is characterized. The special case will be discussed in Section V.

2) *Fixed Topology* ($\lambda_{\alpha 1} = 1$): This setting refers to the case in which link connecting transmitter-to-legitimate receiver is statistically weaker than to the eavesdropper, for which $2\alpha/3$ SDoF is achievable. The coding scheme follows along similar lines as in Proposition 1 and is omitted.

IV. GSDoF OF MISO BROADCAST CHANNEL WITH DELAYED CSIT

Next, we extend the MISO wiretap channel model to the two-user broadcast setting and establish bounds on GSDoF region.

A. Outer Bound

The following theorem provides an outer bound on the GSDoF region of the MISO broadcast channel with delayed CSIT.

Theorem 2: For the $(2, 1, 1)$ -MISO broadcast channel with delayed CSIT and alternating topology $(\lambda_{A_1 A_2})$, an outer bound on GSDoF region $\mathcal{C}_{\text{GSDoF}}(\lambda_{A_1 A_2})$ is given by the set of all non-negative pairs (d_1, d_2) satisfying

$$3d_1 + d_2 \leq (3 - \alpha)\lambda_{1\alpha} + 2(\lambda_{11} + \alpha\lambda_{\alpha\alpha}) + (1 + \alpha)\lambda_{\alpha 1} \quad (13a)$$

$$d_1 + 3d_2 \leq (3 - \alpha)\lambda_{\alpha 1} + 2(\lambda_{11} + \alpha\lambda_{\alpha\alpha}) + (1 + \alpha)\lambda_{1\alpha}. \quad (13b)$$

Proof: The proof of Theorem 2 appears in Appendix III. \square

Timeslot	1	2	3	4
\mathbf{x}	\mathbf{u}	$[v_1 \ v_2]^T + L_1(\mathbf{u})$	$[w_1 \ w_2]^T + M_1(\mathbf{u})$	$L_3(w_1, w_2, M_1(\mathbf{u}))$ $+ M_2(v_1, v_2, L_1(\mathbf{u}))$
Rx ₁	$y_1 = \sqrt{\rho}L_1(\mathbf{u})$	$y_2 = \sqrt{\rho}L_2(v_1, v_2, L_1(\mathbf{u}))$	$y_3 = \sqrt{\rho}L_3(w_1, w_2, M_1(\mathbf{u}))$	$y_4 = \sqrt{\rho}L_3(w_1, w_2, M_1(\mathbf{u}))$ $+ \sqrt{\rho}M_2(v_1, v_2, L_1(\mathbf{u}))$
Rx ₂	$z_1 = \sqrt{\rho^\alpha}M_1(\mathbf{u})$	$z_2 = \sqrt{\rho^\alpha}M_2(v_1, v_2, L_1(\mathbf{u}))$	$z_3 = \sqrt{\rho^\alpha}M_3(w_1, w_2, M_1(\mathbf{u}))$	$z_4 = \sqrt{\rho^\alpha}L_3(w_1, w_2, M_1(\mathbf{u}))$ $+ \sqrt{\rho^\alpha}M_2(v_1, v_2, L_1(\mathbf{u}))$

TABLE I

YANG *et al.* SCHEME FOR FIXED TOPOLOGY ($\lambda_{1\alpha} = 1$).

B. Coding schemes with fixed topology

We now consider the $(2, 1, 1)$ -MISO broadcast channel with fixed topology ($\lambda_{1\alpha} = 1$ or $\lambda_{\alpha 1} = 1$), where one receiver is comparatively stronger than other. For sake of completeness, before providing the inner bound, we first revisit the scheme developed by Yang *et al.* [32, section V-B] for the model without topology consideration $(A_{1t}, A_{2t}) = (1, 1)$ to the fixed topology setting, i.e., $(A_{1t}, A_{2t}) = (1, \alpha)$, $\forall t$. A trivial inner bound on the GSDoF region $\mathcal{C}_{\text{GSDoF}}(\lambda_{A_1 A_2})$ of the two-user $(2, 1, 1)$ -MISO broadcast channel with fixed topology ($\lambda_{1\alpha} = 1$) is given by the set of all non-negative pairs (d_1, d_2) satisfying

$$3\alpha d_1 + d_2 \leq 2\alpha \quad (14a)$$

$$\alpha d_1 + 3d_2 \leq 2\alpha. \quad (14b)$$

The achievable GSDoF region is given by the corner points $(0, 2\alpha/3)$, $(2/3, 0)$, and the point $(1/2, \alpha/2)$ obtained by the intersection of line equations in (14). The corner points $(2/3, 0)$ and $(0, 2\alpha/3)$ are readily achievable by using the coding scheme as in Sections III-B1 and III-B2, since the model reduces to the equivalent $(2, 1, 1)$ -MISO wiretap channel — where the transmitter wants to send confidential message to one receiver and wishes to conceal it from the unintended receiver. We now give a sketch of the transmission scheme that is used to achieve the point $(1/2, \alpha/2)$. In this scheme, transmitter wants to transmit two symbols ($\mathbf{v} := [v_1, v_2]^T$) to receiver 1 and wishes to conceal them from receiver 2; and two symbols ($\mathbf{w} := [w_1, w_2]^T$) to receiver 2 and wishes to conceal them from receiver 1. The communication takes place in four phases, each comprising of only one timeslot. The encoding scheme is concisely shown in Table I. In the first timeslot the transmitter injects artificial noise ($\mathbf{u} := [u_1, u_2]^T$), where each

receiver gets a linear combination of artificial noise denoted by $L_1(\mathbf{u})$ and $M_1(\mathbf{u})$ but with different power levels, respectively. Due to the availability of strictly causal or delayed CSI, at the end of time slot 1 the transmitter can learn the channel output at both receivers and in the second timeslot transmits fresh information ($\mathbf{v} := [v_1, v_2]^T$) to receiver 1 along with the channel output at receiver 1 in timeslot 1, y_1 . At the end of timeslot 2, receiver 1 requires one equation to decode the intended symbols, being available as side information at receiver 2 (z_2). In the third timeslot, transmission scheme is similar to the one in timeslot 2 with the roles of receiver 1 and receiver 2 being reversed. In this phase, the transmitter sends fresh information ($\mathbf{w} := [w_1, w_2]^T$) to receiver 2 along with the past channel output at receiver 2 in timeslot 1 (z_1). By means of past CSIT, at the end of third time slot the transmitter can learn the side information at receiver 1 in timeslot 3 (y_3) and at receiver 2 in timeslot 2 (z_2), and sends them in timeslot 4. At the end of fourth time slot, since receiver 1 knows the CSI, it first subtracts out the contribution of side information y_3 seen at receiver 1 from channel output y_4 to get z_2 . Afterwards, it removes the contribution of y_1 from (y_2, z_2) and decodes \mathbf{v} through channel inversion. Receiver 2 can also perform similar operations to decode \mathbf{w} .

The equivocation analysis of this scheme, by proceeding as in Appendix IV-5, reveals that 2 symbols are securely transmitted over a total of four time slots yielding GSDoF of $2/4$ at receiver 1. Due to the symmetry of the problem, it can be readily shown that 2 symbols are securely send to receiver 2; where, due to the topology of the communication model each symbol is capable of carrying only α bits, which yields GSDoF of $2\alpha/4$ at receiver 2. In Figure 3, we plot the outer (13) and inner bounds (14) for the two-user MISO broadcast channel with fixed topology. It can be easily seen from Figure 3 that the bounds in (13) and (14) do not agree in general. In what follows, we provide an alternative coding scheme which gives an improved inner bound on GSDoF region.

The following proposition gives an inner bound on the GSDoF region of the MISO broadcast channel with fixed topology ($\lambda_{1\alpha} = 1$).

Proposition 2: An inner bound on the GSDoF region $\mathcal{C}_{\text{GSDoF}}(\lambda_{A_1 A_2})$ of the two-user $(2, 1, 1)$ -MISO broadcast channel with delayed CSIT and fixed topology ($\lambda_{1\alpha} = 1$) is given by the set of all non-negative pairs (d_1, d_2) satisfying

$$3(1 + \alpha)d_1 + 2d_2 \leq 2(1 + \alpha) \quad (15a)$$

$$\alpha(3 - \alpha)d_1 + 6d_2 \leq 4\alpha. \quad (15b)$$

Proof: The inner bound follows by generalizing the coding scheme developed in Proposition 1 for the wiretap channel to the broadcast setting. The region in (15) is characterized by the corner points

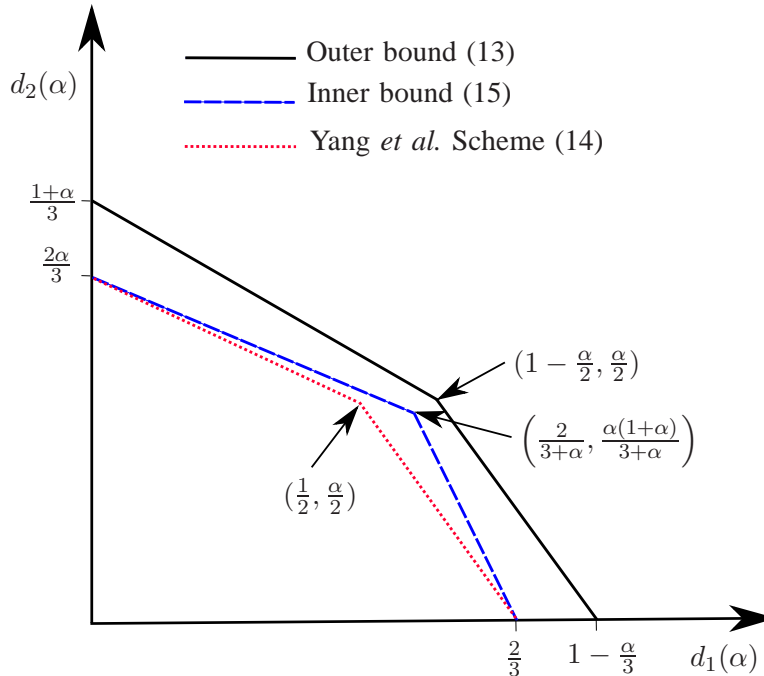


Fig. 3. Bounds on the GSDoF region of $(2, 1, 1)$ -MISO broadcast channel, where the link to receiver 2 is weaker than to receiver 1 ($\lambda_{1\alpha} = 1$).

$(2/3, 0)$, $(0, 2\alpha/3)$ and the point $(2/(3 + \alpha), \alpha(1 + \alpha)/(3 + \alpha))$ obtained by the intersection of line equations in (15). The achievability of the two corner points $(2/3, 0)$ and $(0, 2\alpha/3)$ follow by the coding scheme developed in Proposition 1 and in section III-B2, respectively, where the transmitter is interested to send message to only one receiver and the other receiver acts as an eavesdropper. The achievability of the point $(2/(3 + \alpha), \alpha(1 + \alpha)/(3 + \alpha))$ is provided in Appendix IV. \square

Remark 2: As seen in Figure 3, the outer (13) and inner (15) bounds do not meet in general; however, it is worth noting that the encoding scheme we have established in Proposition 2 provides a larger sum GSDoF compared to Yang *et al.* scheme (14), i.e.,

$$\underbrace{\frac{1 + \alpha}{2}}_{\text{sum GSDoF (14)}} \leq \underbrace{\frac{2 + \alpha(1 + \alpha)}{3 + \alpha}}_{\text{sum SDoF(15)}} \quad (16)$$

C. Coding scheme with symmetric alternating topology

We now turn our attention to the MISO broadcast channel with alternating topology and focus on the symmetric case where the fractions of time spent in state $\lambda_{1\alpha}$ and $\lambda_{\alpha 1}$ are equal. This communication channel may model a setting in which due to the mobility of the users each receiver experiences

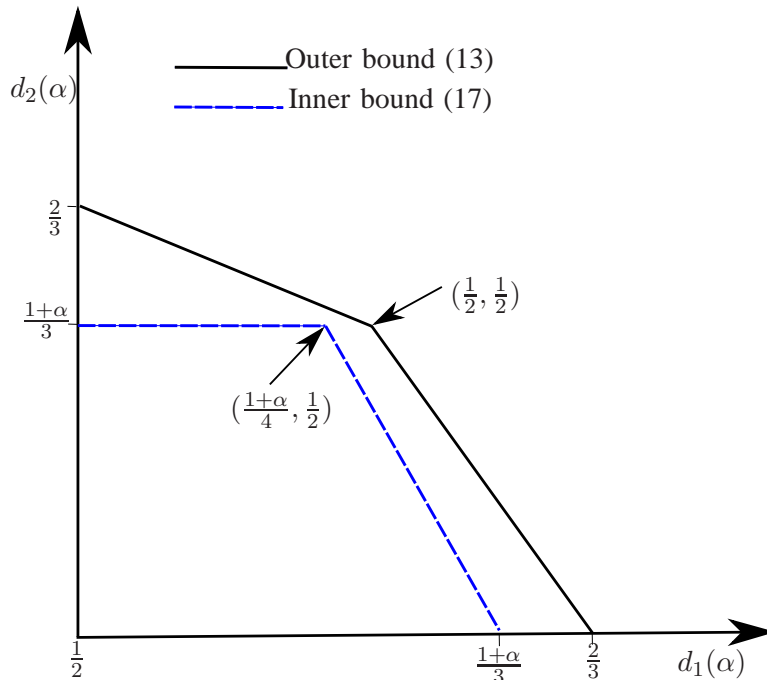


Fig. 4. Bounds on the GSDoF region of $(2, 1, 1)$ -MISO broadcast channel with symmetric alternating topology $(\lambda_{1\alpha}, \lambda_{\alpha 1}) = (\frac{1}{2}, \frac{1}{2})$.

strong interference from an external jammer half of the duration of communication time. The following proposition provides an inner bound on GSDoF region of the MISO broadcast channel with symmetric alternating topology $(\lambda_{1\alpha}, \lambda_{\alpha 1}) = (\frac{1}{2}, \frac{1}{2})$.

Proposition 3: An inner bound on the GSDoF region $\mathcal{C}_{\text{GSDoF}}(\lambda_{A_1 A_2})$ of the two user $(2, 1, 1)$ -MISO broadcast channel with delayed CSIT and symmetric alternating topology $(\lambda_{1\alpha} = \lambda_{\alpha 1} = \frac{1}{2})$ is given by the set of all non-negative pairs (d_1, d_2) satisfying

$$6d_1 + (1 + \alpha)d_2 \leq 2(1 + \alpha) \quad (17a)$$

$$2(2\alpha - 1)d_1 + 3(1 + \alpha)d_2 \leq (1 + \alpha)^2. \quad (17b)$$

Proof: The inner bound follows by specializing the coding scheme that we establish for the fixed topology setting to the alternating topology setting. As seen from Figure 4, it is sufficient to prove the SDoF pairs $(\frac{1+\alpha}{3}, 0)$, $(0, \frac{1+\alpha}{3})$ and $(\frac{1+\alpha}{3}, \frac{1}{2})$, since the entire region (17) can then be achieved by time sharing. The SDoF pairs $(\frac{1+\alpha}{3}, 0)$ and $(0, \frac{1+\alpha}{3})$ are readily achievable by using the combination of coding schemes developed in Proposition 1 and in Section III-B2, equal fractions of time. The achievability of

the point $(\frac{1+\alpha}{4}, \frac{1}{2})$ is relegated to Appendix V. \square

Remark 3 (Generalized sum SDoF Gains with Topological Diversity): Proposition 3 provides an inner bound on the GSDoF region of MISO broadcast channel with symmetric alternating topology, where each receiver observes a strong link half of the duration of communication time. We note that sum GSDoF in Proposition 3 can be larger than the one obtained by a similar model but with *non-diverse* topology consideration. For non-diverse topology setting, i.e., $\lambda_{11}, \lambda_{\alpha\alpha}$, the optimal sum GSDoF is given by 1 and α , respectively [32]. The non-diverse topology model of $(\lambda_{11}, \lambda_{\alpha\alpha}) = (\frac{1}{2}, \frac{1}{2})$ is equivalent to the set-up that we consider in Proposition 3 in the sense that the duration of communication time for stronger and weaker links for both receivers are same. The sum GSDoF with non-diverse topology is given by

$$\begin{aligned} \text{GSDoF} &= \frac{1}{2} \times \underbrace{1}_{\text{sum GSDoF}(A_1, A_2) = (1, 1)} + \frac{1}{2} \times \underbrace{\alpha}_{\text{sum GSDoF}(A_1, A_2) = (\alpha, \alpha)} \\ &= \frac{1 + \alpha}{2} \leq \underbrace{\frac{3 + \alpha}{4}}_{\text{sum GSDoF}(\lambda_{1\alpha}, \lambda_{\alpha 1}) = (\frac{1}{2}, \frac{1}{2})} \end{aligned} \quad (18)$$

which is clearly smaller than the sum GSDoF of Proposition 3. This result shows the benefits of topological diversity.

V. GSDoF (GDoF) CHARACTERIZATION IN FEW SPECIAL CASES

In this section, we study the two user Gaussian MISO broadcast channel (2) where we restrict our attention to integer channels, i.e., $\mathbf{h} \in \mathcal{H} \subseteq \mathbb{Z}^{1 \times 2}$ and $\mathbf{g} \in \mathcal{G} \subseteq \mathbb{Z}^{1 \times 2}$.² In what follows, we construct some elemental encoding schemes which characterize the (sum) GSDoF for various topology states.

A. Wiretap channel with Fixed Topology ($\lambda_{1\alpha} = 1$)

We first consider the MISO wiretap channel with fixed topology state ($\lambda_{1\alpha} = 1$) and establish the optimal GSDoF.

Proposition 4: The GSDoF of (2, 1, 1)-MISO wiretap channel with delayed CSIT and fixed topology state ($\lambda_{1\alpha} = 1$) is given by

$$d = 1 - \frac{\alpha}{3}. \quad (19)$$

Proof: The encoding scheme uses some elements from the compute-and-forward scheme in [36]

²The results established for the integer channels can be readily extended to hold for complex channels using standard techniques [36], [37].

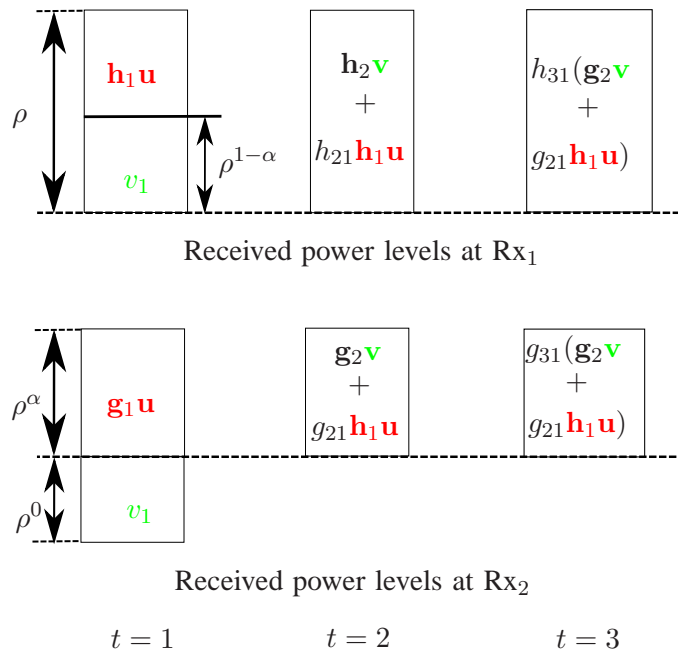


Fig. 5. Received power levels at receiver 1 and receiver 2.

and the one in Proposition 1. In this scheme the transmitter wants to send three symbols (v_1, v_2, v_3) to receiver 1 (legitimate receiver) and wishes to conceal them from receiver 2. The coding scheme consists of three phases, each comprising of only one timeslot. In the first phase, the leverage provided by the topology of the network is utilized as follows. The transmitter injects artificial noise from both antennas, where the output at the receiver 2 is obtained at a lower power level ($\mathcal{O}(\rho^\alpha)$) compared to the legitimate receiver ($\mathcal{O}(\rho)$). Thus, by reducing the transmission power of the artificial noise to the order of receiver 2 ($\mathcal{O}(\rho^\alpha)$), the transmitter can use the remaining power ($\mathcal{O}(\rho^{1-\alpha})$) to send a confidential symbol to receiver 1. The receiver 2 will receive the confidential symbol embedded in with artificial noise but at noise floor level; and hence, can not decode it. In this phase, as opposed to the scheme in Proposition 1, the transmitter uses structured codes to send artificial noise, where both receivers are able to compute a unique function of artificial noise. This unique function can be interpreted as a secret key which will be used in the next time slot to secure information. Let $u_i \in \Lambda$, where u_i denotes the noise codeword chosen from the lattice codebook Λ , for $i = 1, 2$.³ Note that each transmit antenna encodes an independent stream of noise — no coding across sub channels. After encoding, the transmitter sends structured noise (u_1, u_2)

³For details about lattice codes, the interested reader may refer to [36], [38].

along with a confidential symbol v_1 chosen from a Gaussian codebook as

$$\mathbf{x}_1 = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} v_1 \rho^{-\alpha/2} \\ \phi \end{bmatrix} \quad (20)$$

The channel input-output relationship is given by

$$y_1 = \underbrace{\sqrt{\rho} \mathbf{h}_1 \mathbf{u}}_{\mathcal{O}(\rho)} + \underbrace{\sqrt{\rho^{(1-\alpha)}} h_{11} v_1}_{\mathcal{O}(\rho^{1-\alpha})}, \quad (21a)$$

$$z_1 = \underbrace{\sqrt{\rho^\alpha} \mathbf{g}_1 \mathbf{u}}_{\mathcal{O}(\rho^\alpha)} + \underbrace{\sqrt{\rho^0} g_{11} v_1}_{\mathcal{O}(\rho^0)}. \quad (21b)$$

At the end of phase 1, each receiver conveys the past CSI to the transmitter. Figure 5 illustrates the received power levels at receiver 1 and 2, respectively. At the end of phase 1, receiver 2 gets a linear combination of noise codewords along with the confidential symbol v_1 at noise floor level; and, thus, can not decode it. Receiver 1 gets the confidential symbol embedded in with a linear combination of noise codewords $\mathbf{h}_1 \mathbf{u}$. It first re-constructs $\mathbf{h}_1 \mathbf{u}$ from the channel output y_1 by treating v_1 as noise, within bounded noise distortion. Since $\sum_{i=1}^2 a_{ji} u_i \in \Lambda$ where $a_{ji} \in \mathbb{Z}$, $(i, j) = \{1, 2\}^2$, the two receivers can decode the corresponding equations $(\mathbf{h}_1 \mathbf{u}, \mathbf{g}_1 \mathbf{u})$ as long the computation rate [36]

$$R_{\text{comp}} \leq \min \left\{ R(\mathbf{h}_1, \mathbf{a}_1 := \mathbf{h}_1), R(\mathbf{g}_1, \mathbf{a}_1 := \mathbf{g}_1) \right\} \quad (22)$$

where

$$R(\mathbf{h}_1, \mathbf{a}_1 := \mathbf{h}_1) = \log^+ \left(\left(\|\mathbf{a}_1\|^2 + \rho^{(1-\alpha)} \|h_{11}\|^2 - \frac{\rho \|\mathbf{h}_1 \mathbf{a}_1\|}{1 + \rho \|\mathbf{h}_1\|^2} \right)^{-1} \right)$$

$$R(\mathbf{g}_1, \mathbf{a}_1 := \mathbf{g}_1) = \log^+ \left(\left(\|\mathbf{a}_1\|^2 - \frac{\rho^\alpha \|\mathbf{g}_1 \mathbf{a}_1\|}{1 + \rho^\alpha \|\mathbf{g}_1\|^2} \right)^{-1} \right)$$

is satisfied.⁴ It can be readily shown from (22) that, R_{comp} yields a GDoF of α . After decoding $\mathbf{h}_1 \mathbf{u}$, the transmitter subtracts out the contribution of $\mathbf{h}_1 \mathbf{u}$ from y_1 and decodes v_1 through channel inversion. The information transmitted to receiver 1 via symbol v_1 is given by

$$\begin{aligned} R_{v_1} &= I(v_1; y_1 | \mathbf{h}_1 \mathbf{u}) \\ &= h(\sqrt{\rho} \mathbf{h}_1 \mathbf{u} + \sqrt{\rho^{(1-\alpha)}} h_{11} v_1 | \mathbf{h}_1 \mathbf{u}) \\ &= (1 - \alpha) \log(\rho). \end{aligned} \quad (23)$$

⁴In this scheme, it is sufficient if receiver 1 can only decode the linear combination of artificial noise. The condition that both receivers decode unique key is only required for the broadcast case.

In the second phase, the transmitter transmits fresh information ($\mathbf{v} := [v_2, v_3]^T$) to receiver 1 along with a linear combination of channel output ($\mathbf{h}_1 \mathbf{u}$) at receiver 1 during the first phase. Since the transmitter already knows \mathbf{u} and due to the availability of past CSI of receiver 1 (\mathbf{h}_1) in phase 1, it can easily construct $\mathbf{h}_1 \mathbf{u}$ and sends

$$\mathbf{x}_2 = \begin{bmatrix} v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} \mathbf{h}_1 \mathbf{u} \\ \phi \end{bmatrix}. \quad (24)$$

The channel input-output relationship is given by

$$y_2 = \sqrt{\rho} \mathbf{h}_2 \mathbf{v} + \sqrt{\rho} h_{21} \mathbf{h}_1 \mathbf{u}, \quad (25a)$$

$$z_2 = \sqrt{\rho^\alpha} \mathbf{g}_2 \mathbf{v} + \sqrt{\rho^\alpha} g_{21} \mathbf{h}_1 \mathbf{u}. \quad (25b)$$

At the end of phase 2, each receiver conveys the past CSI to the transmitter. Since the receiver 1 knows the CSI (\mathbf{h}_2) and also the channel output y_1 from phase 1, it subtracts out the contribution of $\mathbf{h}_1 \mathbf{u}$ from the channel output y_2 , to obtain one equation with two unknowns ($\mathbf{v} := [v_2, v_3]^T$). Thus, receiver 1 requires one extra equation to successfully decode the intended variables, being available as interference (side information) at receiver 2.

In the third phase, due to the availability of delayed CSIT, the transmitter can construct the side information z_2 at receiver 2 and sends

$$\mathbf{x}_3 = \begin{bmatrix} \mathbf{g}_2 \mathbf{v} + g_{21} \mathbf{h}_1 \mathbf{u} \\ \phi \end{bmatrix}. \quad (26)$$

The channel input-output relationship is given by

$$y_3 = \sqrt{\rho} h_{31} \mathbf{g}_2 \mathbf{v} + \sqrt{\rho} h_{31} g_{21} \mathbf{h}_1 \mathbf{u}, \quad (27a)$$

$$z_3 = \sqrt{\rho^\alpha} g_{31} \mathbf{g}_2 \mathbf{v} + \sqrt{\rho^\alpha} g_{31} g_{21} \mathbf{h}_1 \mathbf{u}. \quad (27b)$$

At the end of phase 3, by using y_1 receiver 1 subtracts out the contribution of $\mathbf{h}_1 \mathbf{u}$ from (y_2, y_3) and decodes \mathbf{v} through channel inversion. Thus, at the end of three timeslot, 3 symbols are transmitted to receiver 1 which contains $((1 - \alpha) + 2) \log(\rho)$ bits.

Leakage Analysis. We can write the channel output at the eavesdropper in compact form as

$$\mathbf{z} := \begin{bmatrix} \sqrt{\rho^0}g_{11} & \sqrt{\rho^\alpha}\mathbf{g}_1 & 0 \\ 0 & \sqrt{\rho^\alpha}g_{21}\mathbf{h}_1 & \sqrt{\rho^\alpha} \\ 0 & \underbrace{\sqrt{\rho^\alpha}g_{31}g_{21}\mathbf{h}_1}_{\mathbf{G} \in \mathbb{Z}^{3 \times 3}} & \sqrt{\rho^\alpha}g_{31} \end{bmatrix} \begin{bmatrix} v_1 \\ \mathbf{u} \\ \mathbf{g}_2\mathbf{v} \end{bmatrix}. \quad (28)$$

The information rate leaked to receiver 2 is bounded by

$$\begin{aligned} I(v_1, \mathbf{v}; \mathbf{z}|\mathbf{S}^n) &= I(v_1; \mathbf{z}|\mathbf{S}^n) + I(\mathbf{v}; \mathbf{z}|v_1, \mathbf{S}^n) \\ &\stackrel{(a)}{=} I(v_1; z_1|\mathbf{S}^n) + I(\mathbf{v}; \mathbf{z}|v_1, \mathbf{S}^n) \\ &\leq o(\log(\rho)) + I(\mathbf{g}_2\mathbf{v}, \mathbf{u}; \mathbf{z}|v_1, \mathbf{S}^n) - I(\mathbf{u}; \mathbf{z}|\mathbf{g}_2\mathbf{v}, v_1, \mathbf{S}^n) \\ &= o(\log(\rho)) + \text{rank}(\mathbf{G}) \cdot \log(\rho^\alpha) - 2\alpha \log(\rho) \\ &= o(\log(\rho)) + 2\alpha \log(\rho) - 2\alpha \log(\rho) \\ &= o(\log(\rho)). \end{aligned} \quad (29)$$

where (a) follows due to the Independence of v_1 and (z_2, z_3) . From the above analysis, it can be easily seen that 3 symbols are transmitted to receiver 1 over a total of 3 time slots, yielding $\frac{2+(1-\alpha)}{3}$ GSDoF at receiver 1. \square

Remark 4: From (29), the information leakage to the eavesdropper is

$$\limsup_{n \rightarrow \infty} \frac{I(W_1; z^n|\mathbf{S}^n)}{n} = o(\log(\rho)). \quad (30)$$

Next, we strengthen the scheme in Proposition 4 by combining it with random coding argument used in Wyner's wiretap coding [13] such that (8) is satisfied. We consider an equivalent \tilde{n} -block transmission model, where the total duration of the coding scheme in Proposition 4 denotes the block length. Let $\tilde{\mathbf{v}} := (v_1, v_2, v_3)$ denotes the input in each block, $\tilde{\mathbf{y}} := (y_1, y_2, y_3)$ denotes the channel output at receiver 1 and $\tilde{\mathbf{z}} := (z_1, z_2, z_3)$ denotes the channel output at receiver 2, where the inputs $\tilde{\mathbf{v}}$ are chosen independently from state sequence $\tilde{\mathbf{S}}$. This resulting model reduces to the Wyner's wiretap setup [13] where equivocation rate is given by

$$R_e = I(\tilde{\mathbf{v}}; \tilde{\mathbf{y}}|\tilde{\mathbf{S}}) - I(\tilde{\mathbf{v}}; \tilde{\mathbf{z}}|\tilde{\mathbf{S}})$$

which satisfies the perfect secrecy criteria of

$$\limsup_{n \rightarrow \infty} \frac{I(W_1; \tilde{\mathbf{z}}^n|\tilde{\mathbf{S}}^n)}{n} = 0.$$

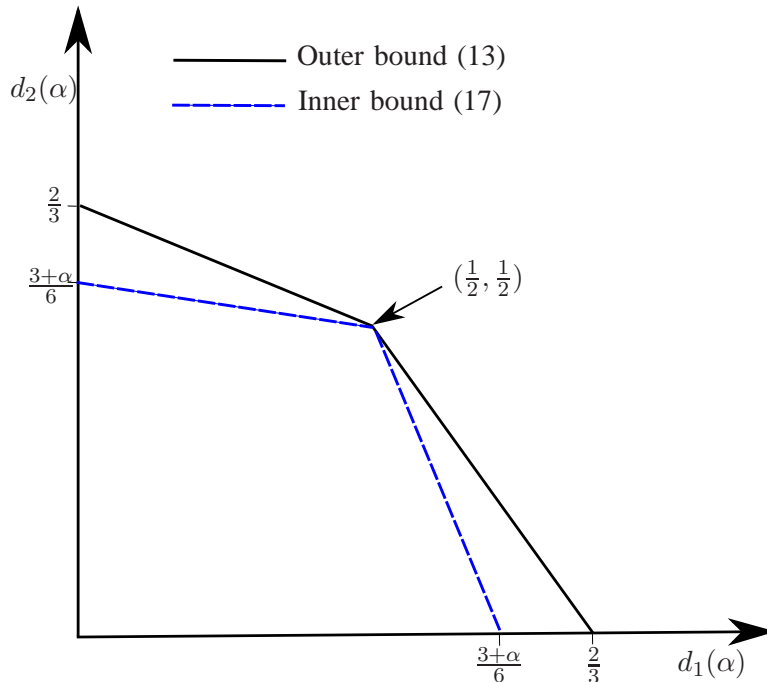


Fig. 6. Bounds on the GSDoF region of $(2, 1, 1)$ -MISO broadcast channel with symmetric alternating topology $(\lambda_{1\alpha}, \lambda_{\alpha 1}) = (\frac{1}{2}, \frac{1}{2})$.

It is worth noting that, by using similar arguments, we can strengthen the security of all schemes in this work which fulfill the perfect secrecy criteria (7) and (8).

B. Coding scheme with symmetric alternating topology

The following proposition provides the sum GSDoF of the MISO broadcast channel with symmetric alternating topology $(\lambda_{1\alpha}, \lambda_{\alpha 1}) = (\frac{1}{2}, \frac{1}{2})$.

Proposition 5: An inner bound on the GSDoF region $\mathcal{C}_{\text{GSDoF}}(\lambda_{A_1 A_2})$ of the two user $(2, 1, 1)$ -MISO broadcast channel with delayed CSIT and symmetric alternating topology $(\lambda_{1\alpha} = \lambda_{\alpha 1} = \frac{1}{2})$ is given by the set of all non-negative pairs (d_1, d_2) satisfying

$$3d_1 + \alpha d_2 \leq \frac{3 + \alpha}{2} \quad (31a)$$

$$\alpha d_1 + 3d_2 \leq \frac{3 + \alpha}{2}. \quad (31b)$$

Proof: As shown in Figure 6, it is sufficient to prove the GSDoF pairs $(\frac{3+\alpha}{6}, 0)$, $(0, \frac{3+\alpha}{6})$ and $(\frac{1}{2}, \frac{1}{2})$, since the entire region (31) can then be achieved by time sharing. The GSDoF pairs $(\frac{3+\alpha}{6}, 0)$ and

$(0, \frac{3+\alpha}{6})$ are achievable by using the combination of coding schemes developed in Proposition 4 and in Section III-B2, equal fractions of time. The achievability of the point $(\frac{1}{2}, \frac{1}{2})$ is provided in Appendix V. \square

C. GDoF region with fixed topology

We now consider the MISO broadcast channel studied in Section IV-B with no secrecy constraints and characterize the GDoF region. The following theorem provides the GDoF region of the MISO broadcast channel with fixed topology ($\lambda_{1\alpha} = 1$).

Theorem 3: The GDoF region $\mathcal{C}_{\text{GDoF}}(\lambda_{A_1 A_2})$ of the two user $(2, 1, 1)$ -MISO broadcast channel with delayed CSIT and fixed topology ($\lambda_{1\alpha} = 1$) is given by the set of all non-negative pairs (d_1, d_2) satisfying

$$d_1 \leq 1 \tag{32a}$$

$$d_2 \leq \alpha \tag{32b}$$

$$2d_1 + d_2 \leq 2 \tag{32c}$$

$$d_1 + 2d_2 \leq 1 + \alpha. \tag{32d}$$

Proof: The converse immediately follows from the outer bound established in [12, eq. 30-31]. As seen in Figure 7, it suffices to prove that following GDoF pairs $(0, \alpha)$, $(1, 0)$, $(1 - \alpha, \alpha)$ and $(1 - \frac{\alpha}{3}, \frac{2\alpha}{3})$ are achievable. The GDoF pairs $(0, \alpha)$, and $(1, 0)$ are readily achievable even without utilizing the delayed CSI, by transmitting information to only one receiver, since the equivalent model reduces to a point-to-point channel with two transmit antennas and a single receive antenna.

The achievability of the GDoF pair $(1 - \alpha, \alpha)$ follows by sending one symbol (v) to receiver 1 and one symbol (w) to receiver 2 as follows

$$\mathbf{x}_1 = \begin{bmatrix} w + v\rho^{-\alpha/2} \\ \phi \end{bmatrix} \tag{33}$$

where $\mathbb{E}[|w|^2] \doteq 1$ and $\mathbb{E}[|v|^2] \doteq 1$. The channel input-output relationship is given by

$$y_1 = \underbrace{\sqrt{\rho}h_{11}w}_{\mathcal{O}(\rho)} + \underbrace{\sqrt{\rho^{(1-\alpha)}}h_{11}v}_{\mathcal{O}(\rho^{1-\alpha})}, \tag{34a}$$

$$z_1 = \underbrace{\sqrt{\rho^\alpha}g_{11}w}_{\mathcal{O}(\rho^\alpha)} + \underbrace{\sqrt{\rho^0}g_{11}v}_{\mathcal{O}(\rho^0)}. \tag{34b}$$

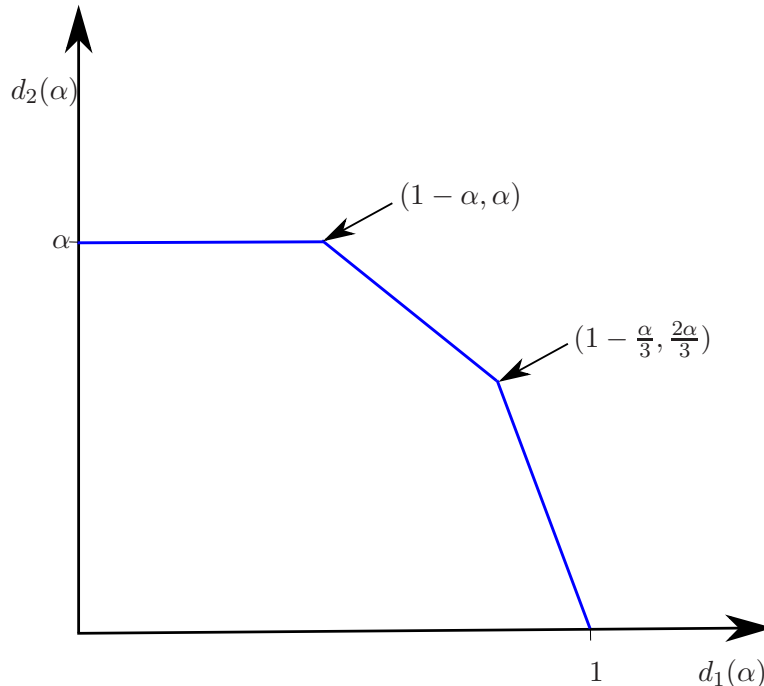


Fig. 7. GDoF region (32) of $(2, 1, 1)$ -MISO broadcast channel, where the link to receiver 2 is weaker than receiver 1 ($\lambda_{1\alpha} := 1$).

At the end of transmission, receiver 1 first re-constructs $h_{11}w$ from the channel output y_1 by treating v as noise, within bounded noise distortion. Afterwards, it decodes v by subtracting out the contribution of w from y_1 . The information transmitted to receiver 1 via symbol v is given by

$$\begin{aligned} R_{v_1} &= I(v; y_1 | h_{11}w) \\ &= (1 - \alpha) \log(\rho) \end{aligned} \quad (35)$$

which yields a GDoF of $1 - \alpha$ at receiver 1. Using similar reasoning and algebra, it can be readily shown that α GDoF is achievable at receiver 2.

The achievability of the point $(1 - \frac{\alpha}{3}, \frac{2\alpha}{3})$ follows by specializing the encoding scheme that we establish in Proposition 2 by removing the secrecy constraints. The proof of the achievability is relegated to Appendix VII. \square

VI. NUMERICAL EXAMPLES

In this section, we provide some numerical examples to illustrate the results presented in the previous sections. Figure 8 shows an inner bound on the GDoF region for the MISO broadcast channel with delayed CSIT and fixed network topology given by [12, Propostion 2]. In order to illustrate the loss

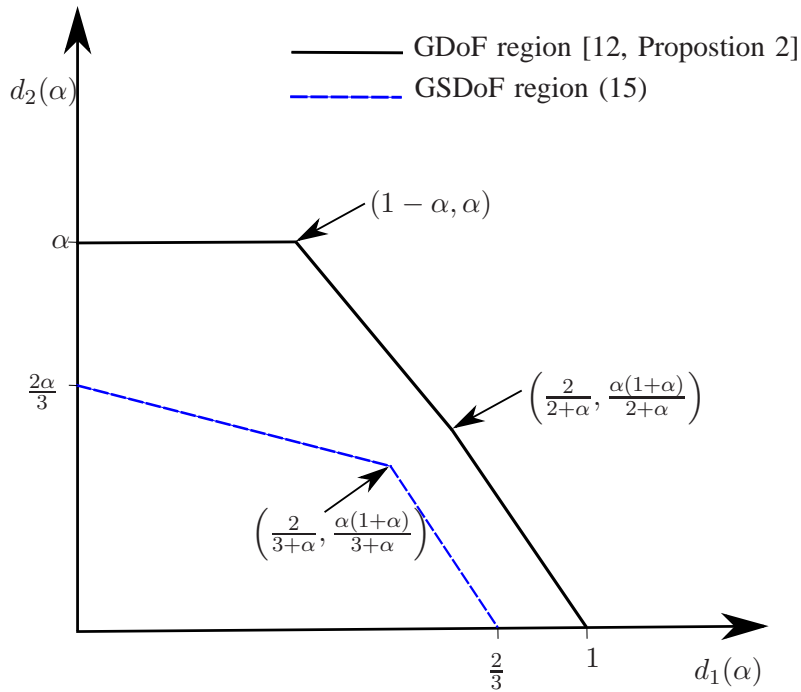


Fig. 8. GDoF and GSDoF regions of MISO broadcast channel with fixed topology ($\lambda_{1\alpha}$).

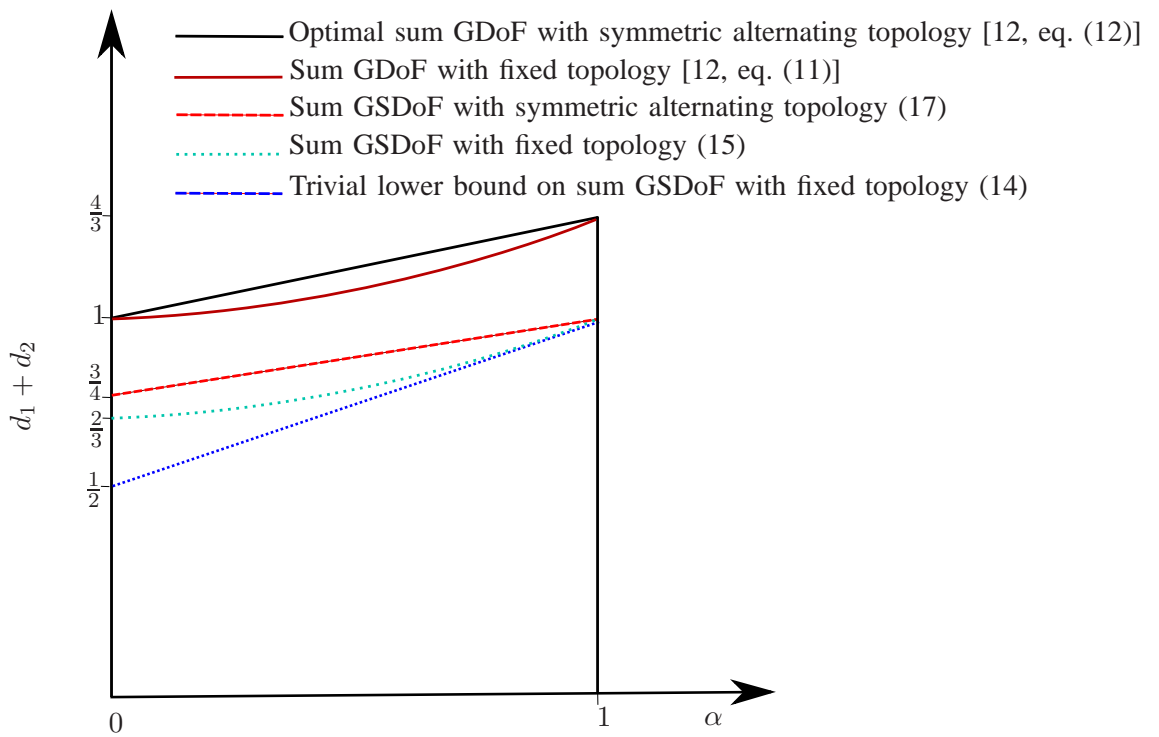


Fig. 9. Sum GDoF and GSDoF of MISO broadcast channel with different topology consideration.

incurred in GDoF region, we also plot the inner bound on GSDoF region of a similar model given by Proposition 2. As we have mentioned before, albeit the optimality of the inner bound of Proposition 2 is still to be shown, the visible gap between the two regions illustrates the loss in terms of GDoF due to secrecy constraints.

This loss is also reflected in Figure 9, where we plot the generalized sum (or total) DoF of MISO broadcast channel and delayed CSIT with fixed and symmetric alternating network topology given by [12, eq. (11)] and [12, eq. (12)] as a function of network topology parameter α . Figure 9 also shows the sum GSDoF of MISO broadcast channel with fixed and symmetric alternating topology given by Proposition 2 and 3, respectively. As a reference, we also plot a lower bound on sum GSDoF obtained by straightforward adaptation of Yang *et al.* scheme given by (14). It can be easily seen from the figure that, as the parameter α approaches 1, i.e., all links have equal strength, the inner bounds in Proposition 2 and 3 recover the sum SDoF of a similar model in the absence of network topology [32]. We note that this equivalence also holds for models without secrecy constraints. In particular, as shown in the Figure 9 as α approaches 1, one can recover the optimal sum DoF of Maddah-Ali-Tse (MAT) scheme [3].

VII. CONCLUSION

In this paper, we study the GSDoF of a MISO broadcast channel. We assume that perfect CSI is available at the receivers and each receiver only conveys the past CSI to the transmitter. In addition to this, links connecting both receivers may have unequal strength, statistically. We first consider the MISO wiretap channel and establish bounds on GSDoF. For the case in which the legitimate receiver is comparatively stronger than the eavesdropper, under certain conditions, the lower and upper bounds agree, and so, we characterize the GSDoF. Next, we extend this model to the broadcast setting and establish bounds on GSDoF region. The coding scheme is based on an appropriate extension of noise injection scheme [32], where the transmitter utilizes the knowledge of network topology and past CSI in a non-trivial manner. Furthermore, we specialize our result to the model with no secrecy constraints and characterize the DoF region for the topology state in which one receiver is stronger than the other. The results establish in this work highlight the interplay between network topology and CSI, and sheds light on how to efficiently utilize both resources in securing information.

APPENDIX I

PROOF OF LEMMA 1

Before proceeding to the proof of Lemma 1, for completeness, we first introduce a property [6, Lemma 4] which is used to establish the results in this work.

Recall that the channel output at receiver 2 is given by

$$z_t = \sqrt{\rho^{A_{2,t}}} \mathbf{g}_t \mathbf{x}_t + n_{2t}. \quad (36)$$

Now, let's consider an artificial channel \tilde{z}_t at receiver 2, such that the channel input-output relationship at i -th time instant is

$$\tilde{z}_t = \sqrt{\rho^{A_{2,t}}} \tilde{\mathbf{g}}_t \mathbf{x}_t + \tilde{n}_{2t} \quad (37)$$

where $\tilde{\mathbf{g}}_t$ and \tilde{n}_{2t} are independent from each other and identically distributed as \mathbf{g}_t and n_{2t} , respectively. Let $\lambda_{\mathbf{g}_t}$ denotes the probability distribution from which, \mathbf{g}_t and $\tilde{\mathbf{g}}_t$ are independent and identically drawn. Let $\mathbf{S}^n := \{\mathbf{g}_t, \tilde{\mathbf{g}}_t\}_{t=1}^n$.

Property 1: The channel output symmetry states that

$$h(z_t|z^{t-1}, \mathbf{S}^n) = h(\tilde{z}_t|z^{t-1}, \mathbf{S}^n). \quad (38)$$

Proof: We begin the proof as follows.

$$\begin{aligned} h(z_t|z^{t-1}, \mathbf{S}^n) &= h(z_t|z^{t-1}, \mathbf{g}_t, \tilde{\mathbf{g}}_t, \mathbf{S}^n \setminus \mathbf{S}_t) \\ &= \mathbb{E}_{\lambda_{\mathbf{g}_t}} [h(\sqrt{\rho^{A_{2,t}}} \mathbf{g}_t \mathbf{x}_t + n_{2t}|z^{t-1}, \mathbf{g}_t = \mathbf{g}, \tilde{\mathbf{g}}_t, \mathbf{S}^n \setminus \mathbf{S}_t)] \\ &\stackrel{(a)}{=} \mathbb{E}_{\lambda_{\tilde{\mathbf{g}}_t}} [h(\sqrt{\rho^{A_{2,t}}} \mathbf{g}_t \mathbf{x}_t + \tilde{n}_{2t}|z^{t-1}, \mathbf{S}^n \setminus \mathbf{S}_t)] \\ &\stackrel{(b)}{=} \mathbb{E}_{\lambda_{\tilde{\mathbf{g}}_t}} [h(\sqrt{\rho^{A_{2,t}}} \mathbf{g}_t \mathbf{x}_t + \tilde{n}_{2t}|z^{t-1}, \tilde{\mathbf{g}}_t = \mathbf{g}, \mathbf{g}_t, \mathbf{S}^n \setminus \mathbf{S}_t)] \\ &\stackrel{(c)}{=} \mathbb{E}_{\lambda_{\tilde{\mathbf{g}}_t}} [h(\tilde{z}_t|z^{t-1}, \mathbf{g}_t, \tilde{\mathbf{g}}_t = \mathbf{g}, \mathbf{S}^n \setminus \mathbf{S}_t)] \\ &= h(\tilde{z}_t|z^{t-1}, \mathbf{S}^n) \end{aligned} \quad (39)$$

where (a) follows because n_{2t} and \tilde{n}_{2t} are independent from $(\mathbf{x}_t, \mathbf{g}_t, \tilde{\mathbf{g}}_t)$ and have same statistics, (b) follows since \mathbf{g}_t and $\tilde{\mathbf{g}}_t$ belongs to $\lambda_{\mathbf{g}_t}$ and have the same alphabet set; and (c) follows due to the independence of \mathbf{x}_t and $(\mathbf{g}_t, \tilde{\mathbf{g}}_t)$. \square

We now provide the proof of (10a) and (10c); due to the symmetry the rest of the inequalities follow straightforwardly. For convenience, we first denote the channel output as

$$z^n := (z_{11}^n, z_{1\alpha}^n, z_{\alpha 1}^n, z_{\alpha\alpha}^n)$$

where $z_{A_1 A_2}^n$ denotes the part of channel output, when $(A_1, A_2) \in \{1, \alpha\}^2$ channel state occurs.

We begin the proof by defining an auxiliary random variable u_t , such that, given $(z_t, \tilde{z}_t, u_t, \mathbf{S}_t)$, y_t is recovered fully. From (36) and (37), the channel outputs at receiver 2 and artificial receiver are given by

$$\begin{bmatrix} z_t \\ \tilde{z}_t \end{bmatrix} = \sqrt{\rho^{A_{2,t}}} \begin{bmatrix} \mathbf{g}_t \\ \tilde{\mathbf{g}}_t \end{bmatrix} \mathbf{x}_t + \begin{bmatrix} n_{2t} \\ \tilde{n}_{2t} \end{bmatrix}. \quad (40)$$

Then, scaling (40) with $\sqrt{\rho^{(A_{1,t}-A_{2,t})}} \begin{bmatrix} \mathbf{g}_t \\ \mathbf{h}_t \end{bmatrix} \begin{bmatrix} \mathbf{g}_t \\ \tilde{\mathbf{g}}_t \end{bmatrix}^{-1}$, we get

$$\begin{aligned} &= \underbrace{\sqrt{\rho^{A_{1,t}}} \begin{bmatrix} \mathbf{g}_t \\ \mathbf{h}_t \end{bmatrix} \mathbf{x}_t + \begin{bmatrix} 0 \\ n_{1t} \end{bmatrix}}_{= \begin{bmatrix} \phi_1 \\ y_t \end{bmatrix}} + \underbrace{\sqrt{\rho^{(A_{1,t}-A_{2,t})}} \begin{bmatrix} \mathbf{g}_t \\ \mathbf{h}_t \end{bmatrix} \begin{bmatrix} \mathbf{g}_t \\ \tilde{\mathbf{g}}_t \end{bmatrix}^{-1} \begin{bmatrix} n_{2t} \\ \tilde{n}_{2t} \end{bmatrix} + \begin{bmatrix} 0 \\ -n_{1t} \end{bmatrix}}_{:= \begin{bmatrix} \phi_2 \\ u_t \end{bmatrix}} \end{aligned} \quad (41)$$

where $\mathbb{E}[\|u_t\|^2] \doteq \rho^{(A_{1,t}-A_{2,t})^+}$. Then, it can be easily seen from (41) that, given \mathbf{S}^n , by subtracting the contribution of u_t from (z_t, \tilde{z}_t) , suffices to construct y_t .

Now, we proceed as follows.

$$h(z^n | \mathbf{S}^n) = \sum_{t=1}^n h(z_t | z^{t-1}, \mathbf{S}^n) \quad (42)$$

$$h(z^n | \mathbf{S}^n) = \sum_{t=1}^n h(\tilde{z}_t | z^{t-1}, \mathbf{S}^n) \quad (43)$$

where (43) follows due to the property of channel output symmetry (38). Then, by combining (42) and (43), we get

$$\begin{aligned} 2h(z^n | \mathbf{S}^n) &= \sum_{t=1}^n h(z_t | z^{t-1}, \mathbf{S}^n) + h(\tilde{z}_t | z^{t-1}, \mathbf{S}^n) \\ &\stackrel{(d)}{\geq} \sum_{t=1}^n h(z_t, \tilde{z}_t | z^{t-1}, \mathbf{S}^n) \\ &= \sum_{t=1}^n h(z_t, \tilde{z}_t, y_t, u_t | z^{t-1}, \mathbf{S}^n) - h(y_t, u_t | z^n, \tilde{z}_t, \mathbf{S}^n) \\ &= \sum_{t=1}^n h(z_t, y_t | z^{t-1}, \mathbf{S}^n) + h(\tilde{z}_t, u_t | z^n, y_t, \mathbf{S}^n) - h(u_t | z^n, \tilde{z}_t, \mathbf{S}^n) - h(y_t | z^n, \tilde{z}_t, u_t, \mathbf{S}^n) \\ &= \sum_{t=1}^n h(z_t, y_t | z^{t-1}, \mathbf{S}^n) + h(\tilde{z}_t | z^n, y_t, \mathbf{S}^n) + \underbrace{h(u_t | z^n, \tilde{z}_t, y_t, \mathbf{S}^n)}_{=0} - h(u_t | z^n, \tilde{z}_t, \mathbf{S}^n) \\ &\quad - \underbrace{h(y_t | z^n, \tilde{z}_t, u_t, \mathbf{S}^n)}_{=0} \end{aligned}$$

$$\begin{aligned}
&\stackrel{(e)}{=} \sum_{t=1}^n h(z_t, y_t | z^{t-1}, \mathbf{S}^n) + \underbrace{h(\tilde{z}_t | z^n, y_t, \mathbf{S}^n)}_{=no(\log(\rho))} - h(u_t | z^n, \tilde{z}_t, \mathbf{S}^n) \\
&\stackrel{(f)}{\geq} \sum_{t=1}^n h(z_t, y_t | z^{t-1}, \mathbf{S}^n) - h(u_t | \mathbf{S}^n) + no(\log(\rho)) \\
&\stackrel{(g)}{\geq} \sum_{t=1}^n h(z_t, y_t | z^{t-1}, y^{t-1}, \mathbf{S}^n) - h(u_t | \mathbf{S}^n) + no(\log(\rho)) \\
&\stackrel{(h)}{=} h(z^n, y^n | \mathbf{S}^n) - h(u^n | \mathbf{S}^n) + no(\log(\rho)) \\
&= h(z^n, y^n | \mathbf{S}^n) - h(u_{11}^n, u_{1\alpha}^n, u_{\alpha 1}^n, u_{\alpha\alpha}^n | \mathbf{S}^n) + no(\log(\rho)) \\
&\stackrel{(i)}{\geq} h(z^n, y^n | \mathbf{S}^n) - \sum_{A_1, A_2 \in (1, \alpha)^2} n\lambda_{A_1 A_2} \log(\rho^{(A_1 - A_2)^+}) + no(\log(\rho)) \\
&= h(z^n, y^n | \mathbf{S}^n) - n\lambda_{1\alpha}(1 - \alpha) \log(\rho) + no(\log(\rho)) \tag{44}
\end{aligned}$$

where (d), (g) and (i) follow from the fact that conditioning reduces entropy, (e) follows because y_t and u_t can be fully recovered from $(z_t, \tilde{z}_t, u_t, \mathbf{S}^n)$ and $(z_t, \tilde{z}_t, y_t, \mathbf{S}^n)$, respectively, (f) follows because \tilde{z}_t is constructed within bounded noise distortion from (z_t, y_t, \mathbf{S}^n) ; and, (h) follows due to the independence of u_t and u^{t-1} .

We can also bound (44) as follows

$$\begin{aligned}
2h(z^n | \mathbf{S}^n) + n\lambda_{1\alpha}(1 - \alpha) \log(\rho) &\geq h(z^n, y^n | \mathbf{S}^n) + no(\log(\rho)) \\
&\geq h(y^n | \mathbf{S}^n) + no(\log(\rho)). \tag{45}
\end{aligned}$$

This concludes the proof.

APPENDIX II

PROOF OF THEOREM 1

We now provide the proof of Theorem 1. We begin the proof as follows.

$$\begin{aligned}
nR_e &= H(W | z^n, \mathbf{S}^n) \\
&= H(W | \mathbf{S}^n) - I(W; z^n | \mathbf{S}^n) \\
&= I(W; y^n | \mathbf{S}^n) + H(W | y^n, \mathbf{S}^n) - I(W; z^n | \mathbf{S}^n) \\
&\stackrel{(a)}{\leq} I(W; y^n | \mathbf{S}^n) - I(W; z^n | \mathbf{S}^n) + n\epsilon_n \\
&\leq I(W; y^n, z^n | \mathbf{S}^n) - I(W; z^n | \mathbf{S}^n) + n\epsilon_n \tag{46}
\end{aligned}$$

$$\begin{aligned}
&= h(y^n, z^n | \mathbf{S}^n) - h(y^n | W, z^n, \mathbf{S}^n) - h(z^n | \mathbf{S}^n) + n\epsilon_n \\
&\leq h(y^n, z^n | \mathbf{S}^n) - h(z^n | \mathbf{S}^n) - \underbrace{h(y^n | W, \mathbf{x}^n, z^n, \mathbf{S}^n)}_{\geq n\alpha \log(\rho)} + n\epsilon_n \\
&\stackrel{(b)}{\leq} h(z^n | \mathbf{S}^n) + n\lambda_{1\alpha}(1 - \alpha) \log(\rho) + n\epsilon_n
\end{aligned} \tag{47}$$

where $\epsilon_n \rightarrow 0$ as $n \rightarrow \infty$; (a) follows from Fano's inequality, (b) follows because (y^n) can be obtained within noise distortion form $(\mathbf{x}^n, \mathbf{S}^n)$, and using (10a).

We can also bound R_e as follows. From (46), we get

$$\begin{aligned}
nR_e &\leq I(W; y^n | \mathbf{S}^n) - I(W; z^n | \mathbf{S}^n) + n\epsilon_n \\
&\stackrel{(c)}{\leq} h(y^n | \mathbf{S}^n) - \frac{1}{2}h(z^n | W, \mathbf{S}^n) + \frac{n\lambda_{\alpha 1}(1 - \alpha)}{2} \log(\rho) - h(z^n | \mathbf{S}^n) + h(z^n | W, \mathbf{S}^n) + n\epsilon_n \\
&\stackrel{(d)}{\leq} h(y^n | \mathbf{S}^n) - \frac{1}{2}h(z^n | \mathbf{S}^n) + \frac{n\lambda_{\alpha 1}(1 - \alpha)}{2} \log(\rho) + n\epsilon_n
\end{aligned} \tag{48}$$

where (c) follows from (10d) and (d) follows from the fact that conditioning reduces entropy.

Then combining these two upper bounds in (47) and (48), we get

$$nR_e \leq \min \left\{ h(z^n | \mathbf{S}^n) + n\lambda_{1\alpha}(1 - \alpha) \log(\rho), h(y^n | \mathbf{S}^n) - \frac{1}{2}h(z^n | \mathbf{S}^n) + \frac{n\lambda_{\alpha 1}(1 - \alpha)}{2} \log(\rho) \right\} + n\epsilon_n \tag{49}$$

$$\begin{aligned}
&\stackrel{(e)}{\leq} \max_{h(y^n)} \frac{2}{3} h(y^n | \mathbf{S}^n) + \frac{n(1 - \alpha)(\lambda_{1\alpha} + \lambda_{\alpha 1})}{3} \log(\rho) + n\epsilon_n \\
&\stackrel{(f)}{\leq} \max_{h(y^n)} \frac{2}{3} \left(h(y_{11}^n | \mathbf{S}^n) + h(y_{1\alpha}^n | \mathbf{S}^n) + h(y_{\alpha 1}^n | \mathbf{S}^n) + h(y_{\alpha\alpha}^n | \mathbf{S}^n) \right) + \frac{n(1 - \alpha)(\lambda_{1\alpha} + \lambda_{\alpha 1})}{3} \log(\rho) + n\epsilon_n \\
&\leq \frac{(3 - \alpha)\lambda_{1\alpha} + 2(\lambda_{11} + \alpha\lambda_{\alpha\alpha}) + (1 + \alpha)\lambda_{\alpha 1}}{3} n \log(\rho) + n\epsilon_n
\end{aligned} \tag{50}$$

where (e) follows by maximizing (49) with respect to $h(z^n | \mathbf{S}^n)$, and (f) follows from the fact that conditioning reduces entropy. Then, dividing both sides by $n \log(\rho)$ and taking $\lim \rho \rightarrow \infty$ and $\lim n \rightarrow \infty$, in (50), we get (11).

This concludes the proof.

APPENDIX III

PROOF OF THEOREM 2

The converse follows by generalizing the proof established in Theorem 1 in the context of MISO wiretap channel with delayed CSIT and alternating topology, to the two-user MISO broadcast channel;

and, so we outline it briefly. We begin the proof as follows.

$$\begin{aligned}
nR_1 &= H(W_1|W_2, z^n, \mathbf{S}^n) \\
&\stackrel{(a)}{\leq} I(W_1; y^n|W_2, \mathbf{S}^n) - I(W_1; z^n|W_2, \mathbf{S}^n) + n\epsilon_n \\
&\stackrel{(b)}{\leq} h(z^n|W_2, \mathbf{S}^n) + n\lambda_{1\alpha}(1 - \alpha)\log(\rho) + n\epsilon_n
\end{aligned} \tag{51}$$

where $\epsilon_n \rightarrow 0$ as $n \rightarrow \infty$; (a) follows from Fano's inequality, (b) follows by following similar steps leading to (47) and by replacing W with W_1 .

We can also bound R_1 as follows.

$$\begin{aligned}
nR_1 &\leq I(W_1; y^n|\mathbf{S}^n) - I(W_1; z^n|\mathbf{S}^n) + n\epsilon_n \\
&\stackrel{(c)}{\leq} h(y^n|\mathbf{S}^n) - \frac{1}{2}h(z^n|\mathbf{S}^n) + \frac{n\lambda_{\alpha 1}(1 - \alpha)}{2}\log(\rho) + n\epsilon_n.
\end{aligned} \tag{52}$$

where (c) follows by similar steps leading to (48) and by replacing W with W_1 .

Then combining these two upper bounds in (51) and (52), we get

$$nR_1 \leq \min \left\{ h(z^n|W_2, \mathbf{S}^n) + n\lambda_{1\alpha}(1 - \alpha)\log(\rho), h(y^n|\mathbf{S}^n) - \frac{1}{2}h(z^n|\mathbf{S}^n) + \frac{n\lambda_{\alpha 1}(1 - \alpha)}{2}\log(\rho) \right\} + n\epsilon_n. \tag{53}$$

We can bound the rate R_2 as follows.

$$\begin{aligned}
nR_2 &= H(W_2|\mathbf{S}^n) \\
&= I(W_2; z^n|\mathbf{S}^n) + H(W_2|\mathbf{S}^n) \\
&\stackrel{(d)}{\leq} I(W_2; z^n|\mathbf{S}^n) + n\epsilon_n \\
&= h(z^n|\mathbf{S}^n) - h(z^n|W_2, \mathbf{S}^n) + n\epsilon_n
\end{aligned} \tag{54}$$

where (d) follows from Fano's inequality.

Next, by scaling (53) with 3 and combining it with (54), we obtain

$$\begin{aligned}
&n(3R_1 + R_2) \\
&\leq \min \left\{ 2h(z^n|W_2, \mathbf{S}^n) + h(z^n|\mathbf{S}^n) + 3n\lambda_{1\alpha}(1 - \alpha)\log(\rho), 3h(y^n|\mathbf{S}^n) - \frac{1}{2}h(z^n|\mathbf{S}^n) - h(z^n|W_2, \mathbf{S}^n) \right. \\
&\quad \left. + \frac{3n\lambda_{\alpha 1}(1 - \alpha)}{2}\log(\rho) \right\} + n\epsilon'_n \\
&= \max_{\beta} \min \left\{ \beta + 3n\lambda_{1\alpha}(1 - \alpha)\log(\rho), 3h(y^n|\mathbf{S}^n) - \frac{\beta}{2} + \frac{3n\lambda_{\alpha 1}(1 - \alpha)}{2}\log(\rho) \right\} + n\epsilon'_n \\
&\stackrel{(e)}{\leq} \max_{h(y^n)} 2h(y^n|\mathbf{S}^n) + n(1 - \alpha)(\lambda_{1\alpha} + \lambda_{\alpha 1})\log(\rho) + n\epsilon'_n
\end{aligned} \tag{55}$$

$$\leq \left[(3 - \alpha)\lambda_{1\alpha} + 2(\lambda_{11} + \alpha\lambda_{\alpha\alpha}) + (1 + \alpha)\lambda_{\alpha 1} \right] n \log(\rho) + n\epsilon'_n \quad (56)$$

where we define $\beta := 2h(z^n|W_2, \mathbf{S}^n) + h(z^n|\mathbf{S}^n)$; and (e) follows by maximizing (55) with respect to β .

Due to the symmetry of the problem, and following similar steps leading to (56), we get

$$n(R_1 + 3R_2) \leq \max_{h(z^n)} 2h(z^n|\mathbf{S}^n) + n(1 - \alpha)(\lambda_{1\alpha} + \lambda_{\alpha 1}) \log(\rho) + n\epsilon'_n \quad (57)$$

$$\stackrel{(f)}{\leq} \left[(3 - \alpha)\lambda_{\alpha 1} + 2(\lambda_{11} + \alpha\lambda_{\alpha\alpha}) + (1 + \alpha)\lambda_{1\alpha} \right] n \log(\rho) + n\epsilon'_n \quad (58)$$

where (f) follows by maximizing (57) with respect to $h(z^n)$ and due to the fact that conditioning reduces entropy. Then, dividing both sides by $n \log(\rho)$ and taking $\lim \rho \rightarrow \infty$ and $\lim n \rightarrow \infty$, in (56) and (58), we get (13a) and (13b), respectively.

This concludes the proof.

APPENDIX IV

CODING SCHEME ACHIEVING $\left(\frac{2}{3+\alpha}, \frac{\alpha(1+\alpha)}{3+\alpha} \right)$ GSDOF PAIR IN PROPOSITION 2

We now provide the proof of the coding scheme which gives the GSDoF pair $(2/(3+\alpha), \alpha(1+\alpha)/(3+\alpha))$. The transmission scheme consists of four phases.

1) *Phase 1*: In this phase communication takes place in T_1 channel uses, where the transmitter injects artificial noise from both antennas. Let $\mathbf{u} := [\mathbf{u}_1, \dots, \mathbf{u}_{T_1}]^T$, where $\mathbf{u}_t = [u_{t,1}, u_{t,2}]^T \forall t \in T_1$ denotes the noise injected by the transmitter. The channel input-output relationship is given by

$$\mathbf{y}_1 = \underbrace{\sqrt{\rho} \tilde{\mathbf{h}}_1 \mathbf{u}}_{\mathcal{O}(\rho)} \quad (59a)$$

$$\mathbf{z}_1 = \underbrace{\sqrt{\rho^\alpha} \tilde{\mathbf{g}}_1 \mathbf{u}}_{\mathcal{O}(\rho^\alpha)} \quad (59b)$$

where $\tilde{\mathbf{h}}_1 = \text{diag}(\{\mathbf{h}_t\}) \in \mathbb{C}^{T_1 \times 2T_1}$, $\tilde{\mathbf{g}}_1 = \text{diag}(\{\mathbf{g}_t\}) \in \mathbb{C}^{T_1 \times 2T_1}$, $\mathbf{y}_1 \in \mathbb{C}^{T_1}$ and $\mathbf{z}_1 \in \mathbb{C}^{T_1}$, for $t = 1, \dots, T_1$. At the end of phase 1, both receivers feed back the past CSI to the transmitter. By the help of past CSI, the transmitter can learn the channel outputs at both receivers.

2) *Phase 2*: In this phase communication takes place in T_1 channel uses. The transmitter sends fresh symbols $\mathbf{v} := [\mathbf{v}_1, \dots, \mathbf{v}_{T_1}]^T$, where $\mathbf{v}_t = [v_{t,1}, v_{t,2}]^T \forall t \in T_1$ to receiver 1 along with the past channel output at receiver 1 in phase 1 as

$$\mathbf{x}_2 = \mathbf{v} + \Theta_1 \mathbf{y}_1 \quad (60)$$

where $\Theta_1 \in \mathbb{C}^{2T_1 \times T_1}$ is a matrix, that is assumed to be known at all nodes. The channel input-output relationship is given by

$$\mathbf{y}_2 = \underbrace{\sqrt{\rho} \tilde{\mathbf{h}}_2 (\mathbf{v} + \Theta_1 \mathbf{y}_1)}_{\mathcal{O}(\rho)}, \quad (61a)$$

$$\mathbf{z}_2 = \underbrace{\sqrt{\rho^\alpha} \tilde{\mathbf{g}}_2 (\mathbf{v} + \Theta_1 \mathbf{y}_1)}_{\mathcal{O}(\rho^\alpha)} \quad (61b)$$

where $\tilde{\mathbf{h}}_2 = \text{diag}(\{\mathbf{h}_t\}) \in \mathbb{C}^{T_1 \times 2T_1}$, $\tilde{\mathbf{g}}_2 = \text{diag}(\{\mathbf{g}_t\}) \in \mathbb{C}^{T_1 \times 2T_1}$, $\mathbf{y}_2 \in \mathbb{C}^{T_1}$ and $\mathbf{z}_2 \in \mathbb{C}^{T_1}$, for $t = 1, \dots, T_1$. At the end of phase 2, both receivers feed back the past CSI to the transmitter. Since the receiver 1 knows the CSI ($\tilde{\mathbf{h}}_2$) and the channel output at receiver 1 in phase 1 (\mathbf{y}_1), it subtracts out the contribution of \mathbf{y}_1 from \mathbf{y}_2 to obtain T_1 equations with $2T_1$ \mathbf{v} -variables and requires T_1 extra equations being available as side information at receiver 2 to decode the intended variables. Notice that the side information at receiver 2 is available at a reduced power level ($\mathcal{O}(\rho^\alpha)$) compared to the receiver 1.

3) *Phase 3*: This phase is similar to phase 2, with the roles of receiver 1 and receiver 2 being reversed. In this phase communication takes place in $T_2 := \alpha T_1$ channel uses. The transmitter sends confidential symbols $\mathbf{w} := [\mathbf{w}_1, \dots, \mathbf{w}_{T_2}]^T$, where $\mathbf{w}_t = [w_{t,1}, w_{t,2}]^T \forall t \in T_2$ to receiver 2 along with the past channel output at receiver 2 as

$$\mathbf{x}_3 = \mathbf{w} + \Theta_2 \mathbf{z}_1 \quad (62)$$

where $\Theta_2 \in \mathbb{C}^{2T_2 \times T_2}$ is a matrix, that is assumed to be known at all nodes. The channel input-output relationship is given by

$$\mathbf{y}_3 = \underbrace{\sqrt{\rho} \tilde{\mathbf{h}}_3 (\mathbf{w} + \Theta_2 \mathbf{z}_1)}_{\mathcal{O}(\rho)}, \quad (63a)$$

$$\mathbf{z}_3 = \underbrace{\sqrt{\rho^\alpha} \tilde{\mathbf{g}}_3 (\mathbf{w} + \Theta_2 \mathbf{z}_1)}_{\mathcal{O}(\rho^\alpha)} \quad (63b)$$

where $\tilde{\mathbf{h}}_3 = \text{diag}(\{\mathbf{h}_t\}) \in \mathbb{C}^{T_2 \times 2T_2}$, $\tilde{\mathbf{g}}_3 = \text{diag}(\{\mathbf{g}_t\}) \in \mathbb{C}^{T_2 \times 2T_2}$, $\mathbf{y}_3 \in \mathbb{C}^{T_2}$ and $\mathbf{z}_3 \in \mathbb{C}^{T_2}$, for $t = 1, \dots, T_2$. At the end of phase 3, both receivers feed back the past CSI to the transmitter. Since the receiver 2 knows the CSI ($\tilde{\mathbf{g}}_3$) and the channel output at receiver 1 in phase 1 (\mathbf{z}_1), it subtracts out the contribution of \mathbf{z}_1 from \mathbf{z}_3 to obtain T_2 equations with $2T_2$ \mathbf{w} -variables and requires T_2 equations being available as side information at receiver 1 — however at a higher power level ($\mathcal{O}(\rho)$) compared to the receiver 2.

4) *Phase 4*: At the end of phase 3, receiver 1 requires side information (\mathbf{z}_2) at receiver 2 in phase 2 and receiver 2 requires side information (\mathbf{y}_3) at receiver 1 in phase 3 to successfully decode the intended variables. The transmitter can easily *learn* them by means of past CSI; and, the goal of this phase is to communicate these side informations for interference alignment à-la MAT scheme [3]. The key ingredients of the transmission scheme are, 1) by opposition to classical MAT scheme where side information is conveyed in an analog manner, digitized side information is multicasted, and 2) transmission of fresh information to receiver 1. Recall that, at the end of phase 3, the side information at receiver 2 in phase 2 is available at a reduced power level $\mathcal{O}(\rho^\alpha)$ compared to the side information at receiver 1 $\mathcal{O}(\rho)$ in phase 3. After learning the side information, the transmitter perform following operations — it quantizes the channel output at receiver 2 (\mathbf{z}_2) into $\alpha T_1 \log(\rho) + o(\log \rho)$ bits within bounded noise distortion. A similar operation is performed at the receiver 1 channel output in phase 3 — the transmitter first generates ($\mathbf{y}'_3 := \Psi_1 \mathbf{y}_3$) where $\Psi_1 \in \mathbb{C}^{T_1 \times T_2}$ and is assumed to be known everywhere, and quantize it to $T_2 \log(\rho) + o(\log \rho)$ bits within bounded noise distortion. Since $T_2 := \alpha T_1$, the transmitter performs a bit wise XOR operation to generate $\alpha T_1 \log(\rho) + o(\log \rho)$ bits which are then mapped to a common message $\{c_t\}$ where $c_t \in \mathcal{C} = \{1, \dots, \rho^\alpha\} \forall t \in T_1$ and transmits it with fresh information ($v_{t,3}$) for receiver 1 as

$$\mathbf{x}_{t,4} = \begin{bmatrix} c_t + v_{t,3} \rho^{-\alpha/2} \\ \phi \end{bmatrix} \quad (64)$$

where $\mathbb{E}[|v_{t,3}|^2] \doteq 1$ and $\mathbb{E}[|c_t|^2] \doteq 1, \forall t \in T_1$. At the end of phase 4, the channel input-output relationship is given by

$$\mathbf{y}_4 = \underbrace{\sqrt{\rho} \mathbf{h}_{41} \mathbf{c}}_{\mathcal{O}(\rho)} + \underbrace{\sqrt{\rho^{(1-\alpha)}} \mathbf{h}_{41} \mathbf{v}_3}_{\mathcal{O}(\rho^{1-\alpha})} \quad (65a)$$

$$\mathbf{z}_4 = \underbrace{\sqrt{\rho^\alpha} \mathbf{g}_{41} \mathbf{c}}_{\mathcal{O}(\rho)} + \underbrace{\sqrt{\rho^0} \mathbf{g}_{41} \mathbf{v}_3}_{\mathcal{O}(\rho^0)} \quad (65b)$$

where $\mathbf{h}_{41} = \text{diag}(\{\mathbf{h}_t\}) \in \mathbb{C}^{T_1 \times T_1}$, $\mathbf{g}_{41} = \text{diag}(\{\mathbf{g}_t\}) \in \mathbb{C}^{T_1 \times T_1}$, $\mathbf{c} = [c_1, \dots, c_{T_1}]^T$, $\mathbf{v}_3 = [v_{1,3}, \dots, v_{T_1,3}]^T$, $\mathbf{y}_4 \in \mathbb{C}^{T_1}$ and $\mathbf{z}_4 \in \mathbb{C}^{T_1}$, for $t = 1, \dots, T_1$. At the end of phase 4, receiver 2 gets the confidential symbols \mathbf{v}_3 intended for receiver 1 at noise floor level and is unable to decode it. Receiver 1 first constructs $\mathbf{h}_{41} \mathbf{c}$ from the channel output \mathbf{y}_4 by treating \mathbf{v}_3 as noise, and, afterwards can easily decode \mathbf{v}_3 . Through straightforward algebra, it can be readily shown that $(1-\alpha)T_1 \log(\rho)$ bits are securely conveyed by symbol \mathbf{v}_3 to receiver 1. By using \mathbf{c} , both receivers can learn the side information required to decode the intended symbols as follows. After decoding \mathbf{c} receiver 1 performs following operations —

1) since the receiver 1 knows \mathbf{y}_3 and Ψ_1 , it first performs XOR operation between the quantized version of \mathbf{y}'_3 and \mathbf{c} to recover the side information (\mathbf{z}_2) within bounded noise distortion, and then 2) it subtracts out the contribution of \mathbf{y}_1 from ($\mathbf{y}_2, \mathbf{z}_2$) to decode \mathbf{v} through channel inversion. Due to the symmetry of the problem receiver 2 can also perform similar operations to first recover side information \mathbf{y}_3 and from ($\mathbf{z}_3, \mathbf{y}_3$) decodes \mathbf{w} through channel inversion.

5) *Equivocation Analysis:* We can write the channel input-output relationship as

$$\mathbf{y} := \underbrace{\begin{bmatrix} \sqrt{\rho}\tilde{\mathbf{h}}_2 & \sqrt{\rho}\tilde{\mathbf{h}}_2\Theta_1 & \mathbf{0} & \mathbf{0} \\ \sqrt{\rho^\alpha}\mathbf{h}_{41}\tilde{\mathbf{g}}_2 & \sqrt{\rho^\alpha}\mathbf{h}_{41}\tilde{\mathbf{g}}_2\Theta_1 & \sqrt{\rho}\mathbf{h}_{41}\Psi_1 & \sqrt{\rho^{1-\alpha}}\tilde{\mathbf{h}}_{41} \\ \mathbf{0} & \sqrt{\rho}\mathbf{I}_{T_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \sqrt{\rho}\mathbf{I}_{T_2} & \mathbf{0} \end{bmatrix}}_{\mathbf{H} \in \mathbb{C}^{3T_1+T_2 \times 4T_1+T_2}} \begin{bmatrix} \mathbf{v} \\ \tilde{\mathbf{h}}_1 \mathbf{u} \\ \tilde{\mathbf{h}}_3 \mathbf{w} + \tilde{\mathbf{h}}_3 \Theta_2 \tilde{\mathbf{g}}_1 \mathbf{u} \\ \mathbf{v}_3 \end{bmatrix}, \quad (66)$$

$$\mathbf{z} := \underbrace{\begin{bmatrix} \mathbf{0} & \sqrt{\rho^\alpha}\mathbf{I}_{T_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \sqrt{\rho^\alpha}\mathbf{I}_{T_1} & \mathbf{0} \\ \sqrt{\rho^\alpha}\mathbf{g}_3 & \sqrt{\rho^\alpha}\mathbf{g}_3\Theta_2 & \mathbf{0} & \mathbf{0} \\ \sqrt{\rho}\mathbf{g}_{41}\Psi_1\tilde{\mathbf{h}}_3 & \sqrt{\rho}\mathbf{g}_{41}\Psi_1\tilde{\mathbf{h}}_3\Theta_2 & \sqrt{\rho^\alpha}\mathbf{g}_{41} & \sqrt{\rho^0}\mathbf{g}_{41} \end{bmatrix}}_{\mathbf{G} \in \mathbb{C}^{3T_1+T_2 \times 3T_1+2T_2}} \begin{bmatrix} \mathbf{w} \\ \tilde{\mathbf{g}}_1 \mathbf{u} \\ \tilde{\mathbf{g}}_2 \mathbf{v} + \tilde{\mathbf{g}}_2 \Theta_1 \mathbf{h}_1 \mathbf{u} \\ \mathbf{v}_3 \end{bmatrix}. \quad (67)$$

The information rate to receiver 1 is bounded by

$$\begin{aligned} I(\mathbf{v}, \mathbf{v}_3; \mathbf{y} | \mathbf{S}^n) &= I(\mathbf{v}, \mathbf{v}_3; \mathbf{y}_1, \mathbf{y}_3 | \mathbf{S}^n) + I(\mathbf{v}; \mathbf{y}_2, \mathbf{y}_4 | \mathbf{v}_3, \mathbf{y}_1, \mathbf{y}_3, \mathbf{S}^n) + I(\mathbf{v}_3; \mathbf{y}_2, \mathbf{y}_4 | \mathbf{y}_1, \mathbf{y}_3, \mathbf{S}^n) \\ &\stackrel{(a)}{=} I(\mathbf{v}; \mathbf{y}_2, \mathbf{y}_4 | \mathbf{v}_3, \mathbf{y}_1, \mathbf{y}_3, \mathbf{S}^n) + (1 - \alpha)T_1 \log(\rho) \\ &= (1 + \alpha)T_1 \log(\rho) + (1 - \alpha)T_1 \log(\rho) \end{aligned} \quad (68)$$

where (a) follows due to the independence of $(\mathbf{v}, \mathbf{v}_3)$ and (y_1, y_3) .

We can bound the information leakage to receiver 2 as

$$\begin{aligned} I(\mathbf{v}, \mathbf{v}_3; \mathbf{z} | \mathbf{w}, \mathbf{S}^n) &= I(\mathbf{v}; \mathbf{z} | \mathbf{v}_3, \mathbf{w}, \mathbf{S}^n) + \underbrace{I(\mathbf{v}_3; \mathbf{z} | \mathbf{w}, \mathbf{S}^n)}_{o(\log(\rho))} \\ &\leq I(\tilde{\mathbf{g}}_2 \mathbf{v}, \mathbf{u}; \mathbf{z} | \mathbf{v}_3, \mathbf{w}, \mathbf{S}^n) - I(\mathbf{u}; \mathbf{z} | \tilde{\mathbf{g}}_2 \mathbf{v}, \mathbf{v}_3, \mathbf{w}, \mathbf{S}^n) + o(\log(\rho)) \\ &\leq I(\tilde{\mathbf{g}}_2 \mathbf{v} + \tilde{\mathbf{g}}_2 \Theta_1 \mathbf{h}_1 \mathbf{u}, \mathbf{u}; \mathbf{z} | \mathbf{v}_3, \mathbf{w}, \mathbf{S}^n) - I(\mathbf{u}; \mathbf{z} | \tilde{\mathbf{g}}_2 \mathbf{v}, \mathbf{v}_3, \mathbf{w}, \mathbf{S}^n) + o(\log(\rho)) \\ &\stackrel{(b)}{=} 2\alpha T_1 \log(\rho) - 2\alpha T_1 \log(\rho) + o(\log(\rho)) \end{aligned}$$

$$= o(\log(\rho)) \quad (69)$$

where (b) follows from [32, Lemma 2].

From the analysis above, it can be readily seen that $(1 + \alpha)T_1 \log(\rho)$ bits are securely sent by \mathbf{v} ; and $(1 - \alpha)T_1 \log(\rho)$ bits are securely sent by \mathbf{v}_3 , to receiver 1 over a total of $3T_1 + T_2 = (3 + \alpha)T_1$ time slots, yielding $d_1 = \frac{2}{3+\alpha}$ GSDoF at receiver 1. Similar analysis shows that $(1 + \alpha)T_2 \log(\rho)$ bits are securely transmitted via \mathbf{w} to receiver 2 over a total of $(3 + \alpha)T_1$ time slots, yielding $d_2 = \frac{\alpha(1+\alpha)}{3+\alpha}$ GSDoF at receiver 2.

This concludes the proof.

APPENDIX V

CODING SCHEME ACHIEVING $(\frac{1+\alpha}{4}, \frac{1}{2})$ SDOF PAIR IN PROPOSITION 3

In this coding scheme communication takes place in four phases, each consisting of only one time slot. The transmitter alternates between different states and uses $\lambda_{1\alpha}$ state at $t = 1, 2$, and $\lambda_{\alpha 1}$ state at $t = 3, 4$, respectively. In this scheme, transmitter wants to send two symbols (v_1, v_2) to receiver 1 that are meant to be kept secret from receiver 2 and three symbols (w_1, w_2, w_3) to receiver 2 that are meant to be kept secret from receiver 1.

In the first and second phase, topology state $\lambda_{1\alpha}$ occurs. The transmission scheme in this case is similar to that in phase 1 and 2 of the coding scheme of Appendix IV. The channel inputs-outputs relationship at receiver 1 (y_1, y_2) and receiver 2 (z_1, z_2) are given by (59a), (61a) and (59b), (61b), respectively, where $T_1 := 1$ and $\Theta_1 := [1, 0]^T$. Figure 11 illustrates the power levels at both receivers. In the third phase, the transmitter switches to $\lambda_{\alpha 1}$ topology state. Recall that, in this state receiver 2 is comparatively stronger than receiver 1. The transmission scheme in this phase follows by reversing the roles of receiver 1 and 2, respectively. In this phase, transmitter sends fresh information $(\mathbf{w} := [w_1, w_2]^T)$ to receiver 2 along with a linear combination of channel output $(\mathbf{g}_1 \mathbf{u})$ at receiver 2 during the first phase, i.e.,

$$\mathbf{x}_3 = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} \mathbf{g}_1 \mathbf{u} \\ \phi \end{bmatrix}. \quad (70)$$

The channel input-output relationship is given by

$$y_3 = \sqrt{\rho^\alpha}(\mathbf{h}_3 \mathbf{w} + h_{31} \mathbf{g}_1 \mathbf{u}) \quad (71a)$$

$$z_3 = \sqrt{\rho}(\mathbf{g}_3 \mathbf{w} + g_{31} \mathbf{g}_1 \mathbf{u}). \quad (71b)$$

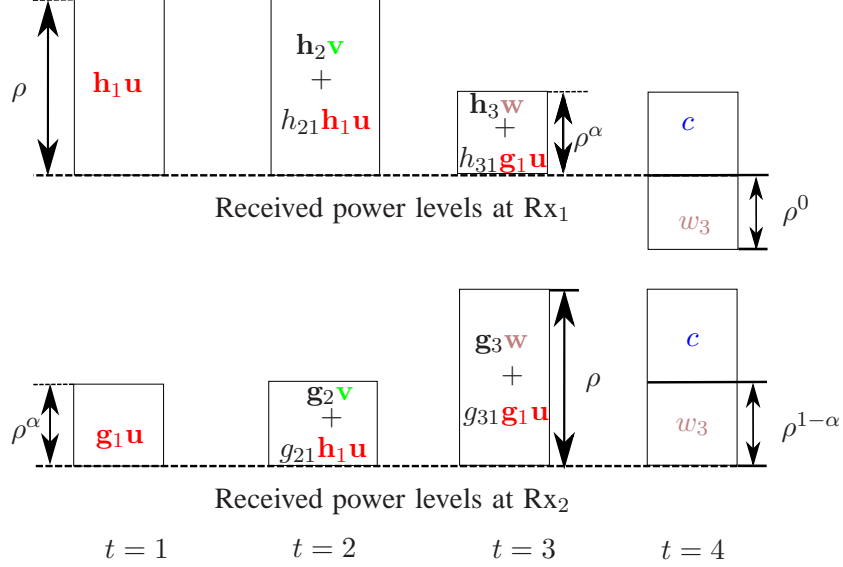


Fig. 10. Received power levels at receiver 1 and receiver 2 with alternating topology $(\lambda_{1\alpha}, \lambda_{\alpha 1}) := (\frac{1}{2}, \frac{1}{2})$.

At the end of phase 3, receiver 2 gets the confidential symbols ($\mathbf{w} := [w_1, w_2]^T$) embedded in with artificial noise. Since receiver 2 knows the CSI (\mathbf{g}_3), and also the channel output z_1 from phase 1, it subtracts out the contribution of z_1 from the channel output z_3 , to obtain one equation with two unknowns ($\mathbf{w} := [w_1, w_2]^T$) and requires one extra equation to successfully decode the intended variables being available as side information at receiver 1. This equation is conveyed by the transmitter to receiver 2 in phase 4.

In phase 4, the channel remains in $\lambda_{\alpha 1}$ topology state. At the end of phase 3, receiver 1 requires side information z_2 at receiver 2 in phase 2 and receiver 2 requires side information y_3 at receiver 1 in phase 3 to successfully decode the intended variables. Notice that the side information available at the unintended receivers are available at reduced power levels ($\mathcal{O}(\rho^\alpha)$). In this phase, after learning (y_3, z_2) , transmitter first generates a new symbol $s := y_3 + z_2$ where $\mathbb{E}[|s|^2] = \mathcal{O}(\rho^\alpha)$. Afterwards, it quantizes $s = \hat{s} + \Delta$, where Δ is the quantization error, \hat{s} is the quantized value and contains

$$\begin{aligned}
 R_{\hat{s}} &= I(s; \hat{s}) \\
 &\stackrel{(a)}{\approx} \alpha \log(\rho) \text{ bits}
 \end{aligned} \tag{72}$$

where (a) follows because the quantization error $\mathbb{E}[\Delta^2] = \mathbb{E}[|\hat{s} - s|^2]$ is bounded and does not scale asymptotically with $\log(\rho)$. The transmitter then maps the quantization index \hat{s} to a common symbol c using a Gaussian codebook, where $c \in \mathcal{C} = \{1, \dots, \rho^\alpha\}$, and transmits it with fresh information (w_3) for

receiver 2 as

$$\mathbf{x}_4 = \begin{bmatrix} c + w_3 \rho^{-\alpha/2} \\ \phi \end{bmatrix} \quad (73)$$

where $\mathbb{E}[|w_3|^2] \doteq 1$ and $\mathbb{E}[|c|^2] \doteq 1$. At the end of phase 4, the channel input-output relationship is given by

$$y_4 = \underbrace{\sqrt{\rho^\alpha} h_{41} c}_{\mathcal{O}(\rho^\alpha)} + \underbrace{\sqrt{\rho^0} h_{41} w_3}_{\mathcal{O}(\rho^0)} \quad (74a)$$

$$z_4 = \underbrace{\sqrt{\rho} g_{41} c}_{\mathcal{O}(\rho)} + \underbrace{\sqrt{\rho^{(1-\alpha)}} g_{41} w_3}_{\mathcal{O}(\rho^{1-\alpha})}. \quad (74b)$$

At the end of phase 4, receiver 1 gets the confidential symbol w_3 intended for receiver 2 at noise floor level and is unable to decode it. Receiver 2 can easily reconstruct c and subtracts out its contribution from z_4 to decode the confidential symbol w_3 through channel inversion. Subsequently, by using c and following steps as mentioned before in subsection IV-4, receiver 1 can learn the side information z_2 and receiver 2 can learn the side information y_3 . Thus, with the help of (y_1, y_2, z_2) receiver 1 can successfully decode the symbols (v_1, v_2) that are intended to it. By using the side information y_3 and the channel outputs (z_1, z_3) , receiver 2 decodes the intended variables (w_1, w_2) .

Following steps similar to in Appendix IV-5, it can be readily seen that $(1 + \alpha) \log(\rho)$ bits are securely transmitted to receiver 1 via \mathbf{v} over a total of 4 time slots, yielding $d_1 = \frac{1+\alpha}{4}$ SDoF at receiver 1. Due to the symmetry of the problem, it can be readily shown that $(1 + \alpha + 1 - \alpha) \log(\rho)$ bits are transmitted securely to receiver 2 over a total of 4 time slots, yielding $d_2 = \frac{1}{2}$ SDoF at receiver 2.

This concludes the proof.

APPENDIX VI

CODING SCHEME ACHIEVING $(\frac{1}{2}, \frac{1}{2})$ SDOF PAIR IN PROPOSITION 5

The transmission scheme in this case is similar to the one in Proposition 2 and Proposition 4, so we outline it briefly. The communication takes place in four phases, each consisting of only one time slot. In this scheme the transmitter alternate between different states and uses $\lambda_{1\alpha}$ state at $t = 1, 2$, and $\lambda_{\alpha 1}$ state at $t = 3, 4$, respectively. The transmitter wants to send three symbols (v_1, v_2, v_3) to receiver 1 that are meant to be kept secret from receiver 1 and three symbols (w_1, w_2, w_3) to receiver 2 that are meant to be kept secret from receiver 1. In the first phase, by utilizing the leverage due to the topology of the network, transmitter injects structured noise (see Proposition 4 for details) and a confidential symbol v_1

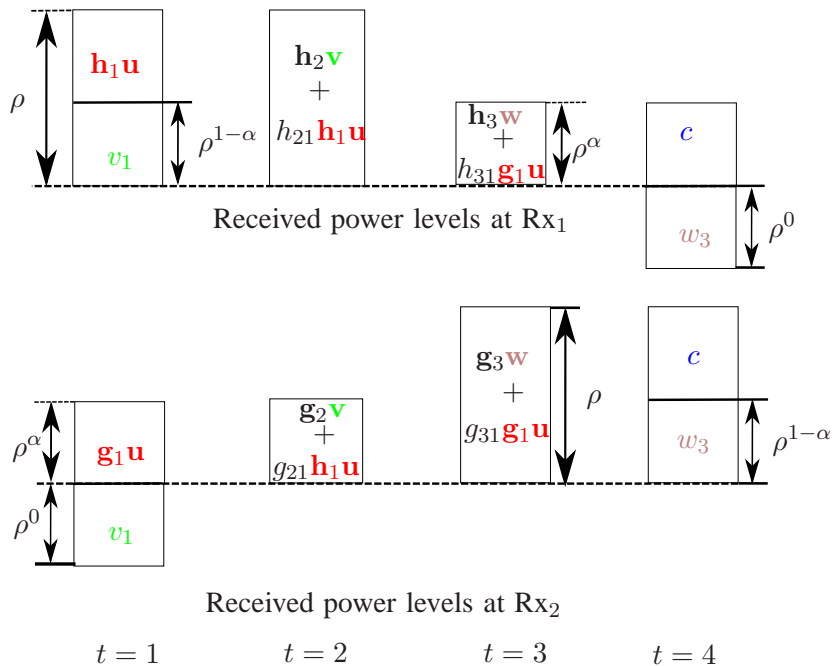


Fig. 11. Received power levels at receiver 1 and receiver 2 with alternating topology $(\lambda_{1\alpha}, \lambda_{\alpha 1}) := (\frac{1}{2}, \frac{1}{2})$.

to receiver 1. Figure 11 illustrates the power levels at both receivers. The untended receiver obtains v_1 below noise floor level and is thus unable to decode it. The rest of the steps in the coding scheme are similar to in Appendix IV and is omitted. Thus, at the end of four timeslot $(1 - \alpha) \log(\rho)$ bits are securely send by v_1 and $(1 + \alpha) \log(\rho)$ bits are securely conveyed by $\mathbf{v} := [v_2, v_3]^T$ over a total of four time slots yielding $d_1 = \frac{1}{2}$. Due to the symmetry of the problem the receiver 2 yields a GSDoF of $d_2 = \frac{1}{2}$.

This concludes the proof.

APPENDIX VII

CODING SCHEME ACHIEVING $(1 - \frac{\alpha}{3}, \frac{2\alpha}{3})$ DOF PAIR IN THEOREM 3

In this scheme transmitter wants to send five symbols $(v_1, v_2, v_3, v_4, v_5)$ to receiver 1 and two symbols (w_1, w_2) to receiver 2, respectively. The transmission scheme consists of four timeslot. In the first time slot the transmitter sends three symbols to receiver 1 as follows. By utilizing the topology of the network, the transmitter sends two symbols $\mathbf{v} := [v_1, v_2]^T$ at the power level of $(\mathcal{O}(\rho^\alpha))$ and uses the remaining power to send a new symbol v_3 to receiver 1. As said before in Proposition 4, the two symbols \mathbf{v} are chosen from a lattice codebook while symbol v_3 is selected from a Gaussian codebook. More specifically,

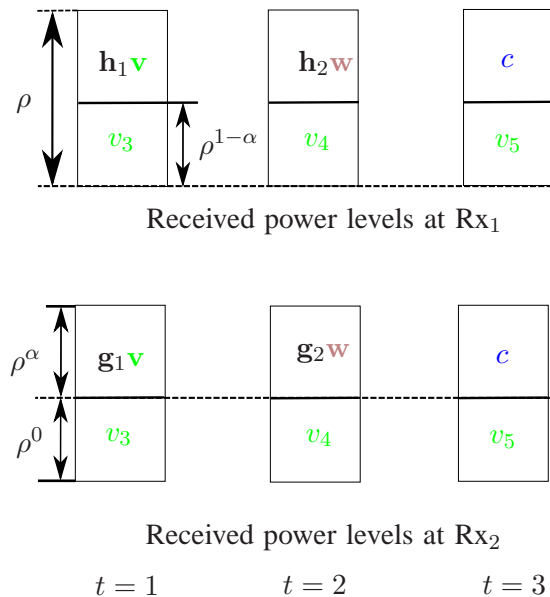


Fig. 12. Received power levels at receiver 1 and receiver 2.

the transmitter sends

$$\mathbf{x}_1 = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} v_3 \rho^{-\alpha/2} \\ \phi \end{bmatrix}. \quad (75)$$

The channel input-output relationship in terms of power levels is illustrated in Figure 12. In the second timeslot, the transmitter sends fresh symbols $\mathbf{w} := [w_1, w_2]^T$ to receiver 2 chosen from a lattice codebook along with a new symbol v_4 to receiver 1 — selected from a Gaussian codebook, as follows

$$\mathbf{x}_2 = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} v_4 \rho^{-\alpha/2} \\ \phi \end{bmatrix}. \quad (76)$$

At the end of the second time slot, the receiver 1 can easily construct $\mathbf{h}_1 \mathbf{v}$ and $\mathbf{h}_2 \mathbf{w}$, respectively; and, subsequently decodes v_3 and v_4 , where each symbol contains $(1 - \alpha) \log(\rho)$ bits. At the end of second timeslot, receiver 1 requires the channel output at receiver 1 in time slot 1 and receiver 2 requires the part of channel output at receiver 1 in timeslot 2 to decode the intended symbols. Due to the availability of delayed CSI, the transmitter can learn the side informations $(\mathbf{g}_1 \mathbf{v}, \mathbf{h}_2 \mathbf{w})$ and in the third timeslot sends a linear combination of them along with a fresh symbol for receiver 1. Note that, since linear combination

of $\mathbf{g}_1\mathbf{v}$ and $\mathbf{h}_2\mathbf{w}$ is also a lattice point, so the transmitter first constructs $c := \mathbf{g}_1\mathbf{v} + \mathbf{h}_2\mathbf{w}$ and sends

$$\mathbf{x}_3 = \begin{bmatrix} c \\ \phi \end{bmatrix} + \begin{bmatrix} v_5\rho^{-\alpha/2} \\ \phi \end{bmatrix}. \quad (77)$$

At the end of third timeslot, receiver 1 can first compute c and then by subtracting the contribution of c from y_3 decodes v_5 . Afterwards, from c it recovers $\mathbf{g}_1\mathbf{v}$, and with the help of $(\mathbf{g}_1\mathbf{v}, \mathbf{h}_1\mathbf{v})$ receiver 1 decodes \mathbf{v} through channel inversion. Due to the symmetry of problem receiver 2 can also perform similar operations to decode \mathbf{w} . At the end of transmission, $(2\alpha)\log(\rho)$ bits are sent by $\mathbf{v} := [v_1, v_2]$ and $(1 - \alpha)\log(\rho)$ bits are sent by each symbol v_3, v_4 and v_5 to receiver 1, respectively, over a total of three time slots yielding a DoF of $1 - \frac{\alpha}{3}$ at receiver 1. Similarly, $(2\alpha)\log(\rho)$ bits are sent to receiver 2 via $\mathbf{w} := [w_1, w_2]$ over a total of 3 timeslots yielding a DoF of $2\alpha/3$ at receiver 2.

This concludes the proof.

REFERENCES

- [1] Z. H. Awan and A. Sezgin, "On MISO wiretap channel with delayed CSIT and alternating topology," in *10th International ITG Conference on Systems, Communication and Coding*, Feb 2015.
- [2] S. A. Jafar, "Interference alignment — A new look at signal dimensions in a communication network," *Foundations and Trends in Communications and Information Theory*, vol. 7, no. 1, pp. 1–134, 2010.
- [3] M. A. Maddah-Ali and D. Tse, "Completely stale transmitter channel state information is still very useful," *IEEE Trans. Inf. Theory*, vol. 58, no. 7, pp. 4418–4431, Jul. 2012.
- [4] C. S. Vaze and M. K. Varanasi, "The degrees of freedom region of the two-user and certain three-user MIMO broadcast channel with delayed CSI," 2011. [Online]. Available: <http://arxiv.org/abs/1101.0306>
- [5] M. J. Abdoli, A. Ghasemi, and A. K. Khandani, "On the degrees of freedom of three-user MIMO broadcast channel with delayed CSIT," in *IEEE International Symposium on Information Theory*, St. Petersburg, Russia, Aug. 2011, pp. 209–213.
- [6] C. S. Vaze and M. K. Varanasi, "The degrees of freedom region and interference alignment for the MIMO interference channel with delayed CSIT," *IEEE Trans. Inf. Theory*, vol. 58, no. 7, pp. 4396–4417, Jul. 2012.
- [7] A. Ghasemi, A. S. Motahari, and A. K. Khandani, "Interference alignment for the MIMO interference channel with delayed local CSIT," 2011. [Online]. Available: <http://arxiv.org/abs/1102.5673>
- [8] R. Tandon, S. Mohajer, H. V. Poor, and S. Shamai (Shitz), "On X-channels with feedback and delayed CSI," in *IEEE International Symposium on Information Theory*, Boston, USA, Jul. 2012, pp. 1887–1891.
- [9] A. Ghasemi, M. J. Abdoli, and A. K. Khandani, "On the degrees of freedom of MIMO X channel with delayed CSIT," in *IEEE International Symposium on Information Theory*, Boston, USA, Jul. 2012, pp. 1902–1906.
- [10] S. Jafar and S. Vishwanath, "Generalized degrees of freedom of the symmetric Gaussian K user interference channel," *IEEE Trans. Inf. Theory*, vol. 56, no. 7, pp. 3297–3303, Jul. 2010.
- [11] S. A. Jafar, "Topological interference management through index coding," *IEEE Trans. Inf. Theory*, vol. 60, no. 1, pp. 529–568, Jan. 2014.
- [12] J. Chen, P. Elia, and S. A. Jafar, "On the vector broadcast channel with alternating CSIT: A topological perspective," 2014. [Online]. Available: <http://arxiv.org/abs/1402.5912>

- [13] A. D. Wyner, "The wiretap channel," *Bell System Technical Journal*, vol. 54, pp. 1355–1387, Oct. 1975.
- [14] I. Csiszár and J. Körner, "Broadcast channels with confidential messages," *IEEE Trans. Inf. Theory*, vol. 24, no. 3, pp. 339–348, May. 1978.
- [15] R. Liu, T. Liu, H. V. Poor, and S. Shamai (Shitz), "New results on multiple-input multiple-output broadcast channels with confidential messages," *IEEE Trans. Inf. Theory*, vol. 59, no. 3, pp. 1346–1359, Mar. 2013.
- [16] E. Tekin and A. Yener, "The Gaussian multiple access wire-tap channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5747–5755, Dec. 2008.
- [17] Y. Liang and H. V. Poor, "Multiple access channels with confidential messages," *IEEE Trans. Inf. Theory*, vol. 54, no. 3, pp. 976–1002, Mar. 2008.
- [18] E. Tekin and A. Yener, "The general Gaussian multiple access and two-way wire-tap channels: Achievable rates and cooperative jamming," *IEEE Trans. Inf. Theory*, vol. 54, no. 6, pp. 2735–2751, Jun. 2008.
- [19] Z. H. Awan, A. Zaidi, and L. Vandendorpe, "On multiaccess channel with unidirectional cooperation and security constraints," in *50th Annual Allerton Conference Communication, Control and Computing*, Monticello, IL, USA, Oct. 2012, pp. 982–987.
- [20] —, "Multiaccess channel with partially cooperating encoders and security constraints," *IEEE Trans. Inf. Forensics Security*, vol. 8, no. 7, pp. 1243–1254, Jul. 2013.
- [21] L. Lai and H. E. Gamal, "The relay eavesdropper channel: Cooperation for secrecy," *IEEE Trans. Inf. Theory*, vol. 54, no. 9, pp. 4005–4019, Sept. 2008.
- [22] Z. H. Awan, A. Zaidi, and L. Vandendorpe, "On secure transmission over parallel relay eavesdropper channel," in *48th Annual Allerton Conference Communication, Control and Computing*, Monticello, IL, USA, Sept. 2010, pp. 859–866.
- [23] —, "Secure communication over parallel relay channel," *IEEE Trans. Inf. Forensics Security*, vol. 7, no. 2, pp. 359–371, Apr. 2012.
- [24] O. O. Koyluoglu and H. E. Gamal, "Cooperative encoding for secrecy in interference channels," *IEEE Trans. Inf. Theory*, vol. 57, no. 9, pp. 5682–5694, Sept. 2011.
- [25] Z. Li, R. Yates, and W. Trappe, "Secrecy capacity of a class of one-sided interference channel," in *IEEE International Symposium on Information Theory*, Toronto, ON, Jul. 2008, pp. 379–383.
- [26] A. Khisti and G. W. Wornell, "Secure transmission with multiple antennas II: The MIMOME wiretap channel," *IEEE Trans. Inf. Theory*, vol. 56, no. 11, pp. 5515–5532, Nov. 2010.
- [27] F. Oggier and B. Hassibi, "The secrecy capacity of the MIMO wiretap channel," *IEEE Trans. Inf. Theory*, vol. 57, no. 8, pp. 4961–4972, Aug. 2011.
- [28] T. Liu and S. Shamai (Shitz), "A note on the secrecy capacity of the multi-antenna wiretap channel," *IEEE Trans. Inf. Theory*, vol. 55, no. 6, pp. 2547–2553, Jun. 2009.
- [29] R. Bustin, R. Liu, H. V. Poor, and S. Shamai (Shitz), "MMSE approach to the secrecy capacity of the MIMO Gaussian wiretap channel," *EURASIP Journal on Wireless Communications and Networking. Special issue on physical layer security*, Nov. 2009.
- [30] Y. Liang, H. V. Poor, and S. Shamai (Shitz), "Information theoretic security," *Foundations and Trends in Communications and Information Theory*, vol. 5, no. 4-5, pp. 355–580, 2009.
- [31] O. O. Koyluoglu, H. E. Gamal, L. Lai, and H. V. Poor, "Interference alignment for secrecy," *IEEE Trans. Inf. Theory*, vol. 57, no. 6, pp. 3323–3332, Jun. 2011.

- [32] S. Yang, M. Kobayashi, P. Piantanida, and S. Shamai, "Secrecy degrees of freedom of MIMO broadcast channels with delayed CSIT," *IEEE Trans. Inf. Theory*, vol. 59, no. 9, pp. 5244–5256, 2013.
- [33] A. Zaidi, Z. H. Awan, S. Shamai (Shitz), and L. Vandendorpe, "Secure degrees of freedom of MIMO X-channels with output feedback and delayed CSI," in *IEEE Information Theory Workshop*, Seville, Spain, Sept. 2013.
- [34] —, "Secure degrees of freedom of MIMO X-channels with output feedback and delayed CSIT," *IEEE Trans. Inf. Forensics Security*, vol. 8, no. 11, pp. 1760–1774, 2013.
- [35] Z. H. Awan, A. Zaidi, and A. Sezgin, "Achievable secure degrees of freedom of MISO broadcast channel with alternating CSIT," in *IEEE International Symposium on Information Theory*, Honolulu, USA, Jun. 2014, pp. 31–35.
- [36] B. Nazer and M. Gastpar, "Compute-and-forward: Harnessing interference through structured codes," *IEEE Trans. Inf. Theory*, vol. 57, no. 10, pp. 6463–6486, Oct. 2011.
- [37] U. Niesen and P. Whiting, "The degrees of freedom of compute-and-forward," *IEEE Trans. Inf. Theory*, vol. 58, no. 8, pp. 5214–5232, Aug. 2012.
- [38] U. Erez and R. Zamir, "Achieving $1/2 \log(1+\text{SNR})$ on the AWGN channel with lattice encoding and decoding," *IEEE Trans. Inf. Theory*, vol. 50, no. 10, pp. 2293–2314, Oct. 2004.