

Piecewise Latent Variables for Neural Variational Text Processing

Iulian V. Serban^{1*} and Alexander G. Ororbia II^{2*} and Joelle Pineau³ and Aaron Courville¹

¹ Department of Computer Science and Operations Research, Universite de Montreal

² College of Information Sciences & Technology, Penn State University

³ School of Computer Science, McGill University

iulian [DOT] vlad [DOT] serban [AT] umontreal [DOT] ca

ago109 [AT] psu [DOT] edu

jpineau [AT] cs [DOT] mcgill [DOT] ca

aaron [DOT] courville [AT] umontreal [DOT] ca

Abstract

Advances in neural variational inference have facilitated the learning of powerful directed graphical models with continuous latent variables, such as variational autoencoders. The hope is that such models will learn to represent rich, multi-modal latent factors in real-world data, such as natural language text. However, current models often assume simplistic priors on the latent variables — such as the uni-modal Gaussian distribution — which are incapable of representing complex latent factors efficiently. To overcome this restriction, we propose the simple, but highly flexible, piecewise constant distribution. This distribution has the capacity to represent an exponential number of modes of a latent target distribution, while remaining mathematically tractable. Our results demonstrate that incorporating this new latent distribution into different models yields substantial improvements in natural language processing tasks such as document modeling and natural language generation for dialogue.

1 Introduction

The development of the variational autoencoder framework (Kingma and Welling, 2014; Rezende et al., 2014) has paved the way for learning large-scale, directed latent variable models. This has led to significant progress in a diverse set of machine learning applications, ranging from computer vision (Gregor et al., 2015; Larsen et al., 2016) to natural language processing tasks (Mnih and Gregor, 2014; Miao et al., 2016; Bowman et al., 2015;

Serban et al., 2017b). It is hoped that this framework will enable the learning of generative processes of real-world data — including text, audio and images — by disentangling and representing the underlying latent factors in the data. However, latent factors in real-world data are often highly complex. For example, topics in newswire text and responses in conversational dialogue often possess latent factors that follow non-linear (non-smooth), multi-modal distributions (i.e. distributions with multiple local maxima).

Nevertheless, the majority of current models assume a simple prior in the form of a multivariate Gaussian distribution in order to maintain mathematical and computational tractability. This is often a highly restrictive and unrealistic assumption to impose on the structure of the latent variables. First, it imposes a strong uni-modal structure on the latent variable space; latent variable samples from the generating model (prior distribution) all cluster around a single mean. Second, it forces the latent variables to follow a perfectly symmetric distribution with constant kurtosis; this makes it difficult to represent asymmetric or rarely occurring factors. Such constraints on the latent variables increase pressure on the down-stream generative model, which in turn is forced to carefully partition the probability mass for each latent factor throughout its intermediate layers. For complex, multi-modal distributions — such as the distribution over topics in a text corpus, or natural language responses in a dialogue system — the uni-modal Gaussian prior inhibits the model’s ability to extract and represent important latent structure in the data. In order to learn more expressive latent variable models, we therefore need more flexible, yet tractable, priors.

In this paper, we introduce a simple, flexible

* The first two authors contributed equally.

prior distribution based on the piecewise constant distribution. We derive an analytical, tractable form that is applicable to the variational autoencoder framework and propose a differentiable parametrization for it. We then evaluate the effectiveness of the distribution when utilized both as a prior and as approximate posterior across variational architectures in two natural language processing tasks: document modeling and natural language generation for dialogue. We show that the piecewise constant distribution is able to capture elements of a target distribution that cannot be captured by simpler priors — such as the uni-modal Gaussian. We demonstrate state-of-the-art results on three document modeling tasks, and show improvements on a dialogue natural language generation. Finally, we illustrate qualitatively how the piecewise constant distribution represents multi-modal latent structure in the data.

2 Related Work

The idea of using an artificial neural network to approximate an inference model dates back to the early work of Hinton and colleagues (Hinton and Zemel, 1994; Hinton et al., 1995; Dayan and Hinton, 1996). Researchers later proposed Markov chain Monte Carlo methods (MCMC) (Neal, 1992), which do not scale well and mix slowly, as well as variational approaches which require a tractable, factored distribution to approximate the true posterior distribution (Jordan et al., 1999). Others have since proposed using feed-forward inference models to initialize the mean-field inference algorithm for training Boltzmann architectures (Salakhutdinov and Larochelle, 2010; Ororibia II et al., 2015). Recently, the variational autoencoder framework (VAE) was proposed by Kingma and Welling (2014) and Rezende et al. (2014), closely related to the method proposed by Mnih and Gregor (2014). This framework allows the joint training of an inference network and a directed generative model, maximizing a variational lower-bound on the data log-likelihood and facilitating exact sampling of the variational posterior. Our work extends this framework.

With respect to document modeling, neural architectures have been shown to outperform well-established topic models such as Latent Dirichlet Allocation (LDA) (Hofmann, 1999; Blei et al., 2003). Researchers have successfully proposed several models involving discrete latent vari-

ables (Salakhutdinov and Hinton, 2009; Hinton and Salakhutdinov, 2009; Srivastava et al., 2013; Larochelle and Lauly, 2012; Uria et al., 2014; Lauly et al., 2016; Bornschein and Bengio, 2015; Mnih and Gregor, 2014). The success of such discrete latent variable models — which are able to partition probability mass into separate regions — serves as one of our main motivations for investigating models with more flexible continuous latent variables for document modeling. More recently, Miao et al. (2016) proposed to use continuous latent variables for document modeling.

Researchers have also investigated latent variable models for dialogue modeling and dialogue natural language generation (Bangalore et al., 2008; Crook et al., 2009; Zhai and Williams, 2014). The success of discrete latent variable models in this task also motivates our investigation of more flexible continuous latent variables. Closely related to our proposed approach is the Variational Hierarchical Recurrent Encoder-Decoder (VHRED, described below) (Serban et al., 2017b), a neural architecture with latent multivariate Gaussian variables. In parallel with our work, Zhao et al. (2017) has also proposed a latent variable model for dialogue modeling with the specific goal of generating diverse natural language responses.

Researchers have explored more flexible distributions for the latent variables in VAEs, such as autoregressive distributions, hierarchical probabilistic models and approximations based on MCMC sampling (Rezende et al., 2014; Rezende and Mohamed, 2015; Kingma et al., 2016; Ranganath et al., 2016; Maaløe et al., 2016; Salimans et al., 2015; Burda et al., 2016; Chen et al., 2017; Ruiz et al., 2016). These are all complimentary to our approach; it is possible to combine them with the piecewise constant latent variables. In parallel to our work, multiple research groups have also proposed VAEs with discrete latent variables (Maddison et al., 2017; Jang et al., 2017; Rolfe, 2017; Johnson et al., 2016). This is a promising line of research, however these approaches often require approximations which may be inaccurate when applied to larger scale tasks, such as document modeling or natural language generation. Finally, discrete latent variables may be inappropriate for certain natural language processing tasks.

3 Neural Variational Models

We start by introducing the neural variational learning framework. We focus on modeling discrete output variables (e.g. words) in the context of natural language processing applications. However, the framework can easily be adapted to handle continuous output variables.

3.1 Neural Variational Learning

Let w_1, \dots, w_N be a sequence of N tokens (words) conditioned on a continuous latent variable z . Further, let c be an additional observed variable which conditions both z and w_1, \dots, w_N . Then, the distribution over words is:

$$P_\theta(w_1, \dots, w_N | c) = \int \prod_{n=1}^N P_\theta(w_n | w_{<n}, z, c) P_\theta(z | c) dz,$$

where θ are the model parameters. The model first generates the higher-level, continuous latent variable z conditioned on c . Given z and c , it then generates the word sequence w_1, \dots, w_N . For unsupervised modeling of documents, the c is excluded and the words are assumed to be independent of each other, when conditioned on z :

$$P_\theta(w_1, \dots, w_N) = \int \prod_{n=1}^N P_\theta(w_n | z) P_\theta(z) dz.$$

Model parameters can be learned using the variational lower-bound (Kingma and Welling, 2014):

$$\begin{aligned} & \log P_\theta(w_1, \dots, w_N | c) \\ & \geq \mathbb{E}_{z \sim Q_\psi(z | w_1, \dots, w_N, c)} [\log P_\theta(w_1, \dots, w_N | z, c)] \\ & \quad - \text{KL} [Q_\psi(z | w_1, \dots, w_N, c) || P_\theta(z | c)], \quad (1) \end{aligned}$$

where we note that $Q_\psi(z | w_1, \dots, w_N, c)$ is the approximation to the intractable, true posterior $P_\theta(z | w_1, \dots, w_N, c)$. Q is called the *encoder*, or sometimes the *recognition model* or *inference model*, and it is parametrized by ψ . The distribution $P_\theta(z | c)$ is the prior model for z , where the only available information is c . The VAE framework further employs the re-parametrization trick, which allows one to move the derivative of the lower-bound inside the expectation. To accomplish this, z is parametrized as a transformation of a fixed, parameter-free random distribution $z = f_\theta(\epsilon)$, where ϵ is drawn from a random distribution. Here, f is a transformation of ϵ , parametrized by θ , such that $f_\theta(\epsilon) \sim P_\theta(z | c)$. For example, ϵ might be drawn from a standard

Gaussian distribution and f might be defined as $f_\theta(\epsilon) = \mu + \sigma\epsilon$, where μ and σ are in the parameter set θ . In this case, z is able to represent any Gaussian with mean μ and variance σ^2 .

Model parameters are learned by maximizing the variational lower-bound in eq. (1) using gradient descent, where the expectation is computed using samples from the approximate posterior.

The majority of work on VAEs propose to parametrize z as multivariate Gaussian distributions. However, this unrealistic assumption may critically hurt the expressiveness of the latent variable model. See Appendix A for a detailed discussion. This motivates the proposed piecewise constant latent variable distribution.

3.2 Piecewise Constant Distribution

We propose to learn latent variables by parametrizing z using a piecewise constant probability density function (PDF). This should allow z to represent complex aspects of the data distribution in latent variable space, such as non-smooth regions of probability mass and multiple modes.

Let $n \in \mathbb{N}$ be the number of piecewise constant components. We assume z is drawn from PDF:

$$P(z) = \frac{1}{K} \sum_{i=1}^n 1_{\left(\frac{i-1}{n} \leq z \leq \frac{i}{n}\right)}^{a_i}, \quad (2)$$

where $1_{(x)}$ is the indicator function, which is one when x is true and otherwise zero. The distribution parameters are $a_i > 0$, for $i = 1, \dots, n$. The normalization constant is:

$$K = \sum_{i=1}^n K_i, \text{ where } K_0 = 0, K_i = \frac{a_i}{n}, \text{ for } i = 1, \dots, n.$$

It is straightforward to show that a piecewise constant distribution with more than $n > 2$ pieces is capable of representing a bi-modal distribution. When $n > 2$, a vector z of piecewise constant variables can represent a probability density with $2^{|z|}$ modes. Figure 1 illustrates how these variables help model complex, multi-modal distributions.

In order to compute the variational bound, we need to draw samples from the piecewise constant distribution using its inverse cumulative distribution function (CDF). Further, we need to compute the KL divergence between the prior and posterior. The inverse CDF and KL divergence quantities are both derived in Appendix B. During training we must compute derivatives of the variational bound in eq. (1). These expressions involve derivatives

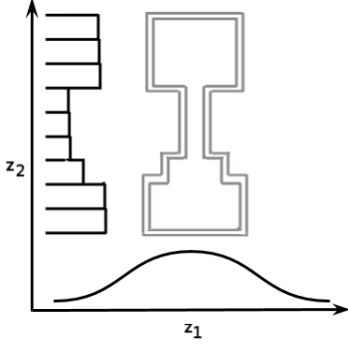


Figure 1: Joint density plot of a pair of Gaussian and piecewise constant variables. The horizontal axis corresponds to z_1 , which is a univariate Gaussian variable. The vertical axis corresponds to z_2 , which is a piecewise constant variable.

of indicator functions, which have derivatives zero everywhere except for the changing points where the derivative is undefined. However, the probability of sampling the value exactly at its changing point is effectively zero. Thus, we fix these derivatives to zero. Similar approximations are used in training networks with rectified linear units.

4 Latent Variable Parametrizations

In this section, we develop the parametrization of both the Gaussian variable and our proposed piecewise constant latent variable.

Let x be the current output sequence, which the model must generate (e.g. w_1, \dots, w_N). Let c be the observed conditioning information. If the task contains additional conditioning information this will be embedded by c . For example, for dialogue natural language generation c represents an embedding of the dialogue history, while for document modeling $c = \emptyset$.

4.1 Gaussian Parametrization

Let μ^{prior} and $\sigma^{2,\text{prior}}$ be the prior mean and variance, and let μ^{post} and $\sigma^{2,\text{post}}$ be the approximate posterior mean and variance. For Gaussian latent variables, the prior distribution mean and variances are encoded using linear transformations of a hidden state. In particular, the prior distribution covariance is encoded as a diagonal covariance matrix using a softplus function:

$$\begin{aligned}\mu^{\text{prior}} &= H_{\mu}^{\text{prior}} \text{Enc}(c) + b_{\mu}^{\text{prior}}, \\ \sigma^{2,\text{prior}} &= \text{diag}(\log(1 + \exp(H_{\sigma}^{\text{prior}} \text{Enc}(c) + b_{\sigma}^{\text{prior}}))),\end{aligned}$$

where $\text{Enc}(c)$ is an embedding of the conditioning information c (e.g. for dialogue natural language

generation this might, for example, be produced by an LSTM encoder applied to the dialogue history), which is shared across all latent variable dimensions. The matrices $H_{\mu}^{\text{prior}}, H_{\sigma}^{\text{prior}}$ and vectors $b_{\mu}^{\text{prior}}, b_{\sigma}^{\text{prior}}$ are learnable parameters. For the posterior distribution, previous work has shown it is better to parametrize the posterior distribution as a linear interpolation of the prior distribution mean and variance and a new estimate of the mean and variance based on the observation x (Fraccaro et al., 2016). The interpolation is controlled by a gating mechanism, allowing the model to turn on/off latent dimensions:

$$\begin{aligned}\mu^{\text{post}} &= (1 - \alpha_{\mu})\mu^{\text{prior}} + \alpha_{\mu} (H_{\mu}^{\text{post}} \text{Enc}(c, x) + b_{\mu}^{\text{post}}), \\ \sigma^{2,\text{post}} &= (1 - \alpha_{\sigma})\sigma^{2,\text{prior}} \\ &\quad + \alpha_{\sigma} \text{diag}(\log(1 + \exp(H_{\sigma}^{\text{post}} \text{Enc}(c, x) + b_{\sigma}^{\text{post}}))),\end{aligned}$$

where $\text{Enc}(c, x)$ is an embedding of both c and x . The matrices $H_{\mu}^{\text{post}}, H_{\sigma}^{\text{post}}$ and the vectors $b_{\mu}^{\text{post}}, b_{\sigma}^{\text{post}}, \alpha_{\mu}, \alpha_{\sigma}$ are parameters to be learned. The interpolation mechanism is controlled by α_{μ} and α_{σ} , which are initialized to zero (i.e. initialized such that the posterior is equal to the prior).

4.2 Piecewise Constant Parametrization

We parametrize the piecewise prior parameters using an exponential function applied to a linear transformation of the conditioning information:

$$a_i^{\text{prior}} = \exp(H_{a,i}^{\text{prior}} \text{Enc}(c) + b_{a,i}^{\text{prior}}), \quad i = 1, \dots, n,$$

where matrix H_a^{prior} and vector b_a^{prior} are learnable. As before, we define the posterior parameters as a function of both c and x :

$$a_i^{\text{post}} = \exp(H_{a,i}^{\text{post}} \text{Enc}(c, x) + b_{a,i}^{\text{post}}), \quad i = 1, \dots, n,$$

where H_a^{post} and b_a^{post} are parameters.

5 Variational Text Modeling

We now introduce two classes of VAEs. The models are extended by incorporating the Gaussian and piecewise latent variable parametrizations.

5.1 Document Model

The neural variational document model (*NVDM*) model has previously been proposed for document modeling (Mnih and Gregor, 2014; Miao et al., 2016), where the latent variables are Gaussian. Since the original *NVDM* uses Gaussian latent variables, we will refer to it as *G-NVDM*. We propose two novel models building on *G-NVDM*. The

first model we propose uses piecewise constant latent variables instead of Gaussian latent variables. We refer to this model as *P-NVDM*. The second model we propose uses a combination of Gaussian and piecewise constant latent variables. The models sample the Gaussian and piecewise constant latent variables independently and then concatenate them together into one vector. We refer to this model as *H-NVDM*.

Let V be the vocabulary of document words. Let W represent a document matrix, where row w_i is the 1-of- $|V|$ binary encoding of the i 'th word in the document. Each model has an encoder component $Enc(W)$, which compresses a document vector into a continuous distributed representation upon which the approximate posterior is built. For document modeling, word order information is not taken into account and no additional conditioning information is available. Therefore, each model uses a bag-of-words encoder, defined as a multi-layer perceptron (MLP) $Enc(c = \emptyset, x) = Enc(x)$. Based on preliminary experiments, we choose the encoder to be a two-layered MLP with parametrized rectified linear activation functions (we omit these parameters for simplicity). For the approximate posterior, each model has the parameter matrix W_a^{post} and vector b_a^{post} for the piecewise latent variables, and the parameter matrices $W_\mu^{\text{post}}, W_\sigma^{\text{post}}$ and vectors $b_\mu^{\text{post}}, b_\sigma^{\text{post}}$ for the Gaussian means and variances. For the prior, each model has parameter vector b_a^{prior} for the piecewise latent variables, and vectors $b_\mu^{\text{prior}}, b_\sigma^{\text{prior}}$ for the Gaussian means and variances. We initialize the bias parameters to zero in order to start with centered Gaussian and piecewise constant priors. The encoder will adapt these priors as learning progresses, using the gating mechanism to turn on/off latent dimensions.

Let z be the vector of latent variables sampled according to the approximate posterior distribution. Given z , the decoder $Dec(w, z)$ outputs a distribution over words in the document:

$$Dec(w, z) = \frac{\exp(-w^T R z + b_w)}{\sum_{w'} \exp(-w'^T R z + b_{w'})},$$

where R is a parameter matrix and b is a parameter vector corresponding to the bias for each word to be learned. This output probability distribution is combined with the KL divergences to compute the lower-bound in eq. (1). See Appendix C.

Our baseline model *G-NVDM* is an improvement over the original *NVDM* proposed by Mnih

and Gregor (2014) and Miao et al. (2016). We learn the prior mean and variance, while these were fixed to a standard Gaussian in previous work. This increases the flexibility of the model and makes optimization easier. In addition, we use a gating mechanism for the approximate posterior of the Gaussian variables. This gating mechanism allows the model to turn off latent variable (i.e. fix the approximate posterior to equal the prior for specific latent variables) when computing the final posterior parameters. Furthermore, Miao et al. (2016) alternated between optimizing the approximate posterior parameters and the generative model parameters, while we optimize all parameters simultaneously.

5.2 Dialogue Model

The variational hierarchical recurrent encoder-decoder (*VHRED*) model has previously been proposed for dialogue modeling and natural language generation (Serban et al., 2017b, 2016b). The model decomposes dialogues using a two-level hierarchy: sequences of utterances (e.g. sentences), and sub-sequences of tokens (e.g. words). Let \mathbf{w}_n be the n 'th utterance in a dialogue with N utterances. Let $w_{n,m}$ be the m 'th word in the n 'th utterance from vocabulary V given as a 1-of- $|V|$ binary encoding. Let M_n be the number of words in the n 'th utterance. For each utterance $n = 1, \dots, N$, the model generates a latent variable z_n . Conditioned on this latent variable, the model then generates the next utterance:

$$P_\theta(\mathbf{w}_1, z_1, \dots, \mathbf{w}_N, z_N) = \prod_{n=1}^N P_\theta(z_n | \mathbf{w}_{<n}) \\ \times \prod_{m=1}^{M_n} P_\theta(w_{n,m} | w_{n,<m}, \mathbf{w}_{<n}, z_n),$$

where θ are the model parameters. *VHRED* consists of three RNN modules: an *encoder* RNN, a *context* RNN and a *decoder* RNN. The *encoder* RNN computes an embedding for each utterance. This embedding is fed into the *context* RNN, which computes a hidden state summarizing the dialogue context before utterance n : h_{n-1}^{con} . This state represents the additional conditioning information, which is used to compute the prior distribution over z_n :

$$P_\theta(z_n | \mathbf{w}_{<n}) = f_\theta^{\text{prior}}(z_n; h_{n-1}^{\text{con}}),$$

where f_θ^{prior} is a PDF parametrized by both θ and h_{n-1}^{con} . A sample is drawn from this distribution:

$z_n \sim P_\theta(z_n | \mathbf{w}_{<n})$. This sample is given as input to the *decoder* RNN, which then computes the output probabilities of the words in the next utterance. The model is trained by maximizing the variational lower-bound, which factorizes into independent terms for each sub-sequence (utterance):

$$\begin{aligned} & \log P_\theta(\mathbf{w}_1, \dots, \mathbf{w}_N) \\ & \geq \sum_{n=1}^N -\text{KL}[Q_\psi(z_n | \mathbf{w}_1, \dots, \mathbf{w}_n) || P_\theta(z_n | \mathbf{w}_{<n})] \\ & \quad + \mathbb{E}_{Q_\psi(z_n | \mathbf{w}_1, \dots, \mathbf{w}_n)} [\log P_\theta(\mathbf{w}_n | z_n, \mathbf{w}_{<n})], \end{aligned}$$

where distribution Q_ψ is the approximate posterior distribution with parameters ψ , computed similarly as the prior distribution but further conditioned on the *encoder* RNN hidden state of the next utterance.

The original *VHRED* model (Serban et al., 2017b) used Gaussian latent variables. We refer to this model as *G-VHRED*. The first model we propose uses piecewise constant latent variables instead of Gaussian latent variables. We refer to this model as *P-VHRED*. The second model we propose takes advantage of the representation power of both Gaussian and piecewise constant latent variables. This model samples both a Gaussian latent variable z_n^{gaussian} and a piecewise latent variable $z_n^{\text{piecewise}}$ independently conditioned on the *context* RNN hidden state:

$$\begin{aligned} P_\theta(z_n^{\text{gaussian}} | \mathbf{w}_{<n}) &= f_\theta^{\text{prior, gaussian}}(z_n^{\text{gaussian}}; h_{n-1}^{\text{con}}), \\ P_\theta(z_n^{\text{piecewise}} | \mathbf{w}_{<n}) &= f_\theta^{\text{prior, piecewise}}(z_n^{\text{piecewise}}; h_{n-1}^{\text{con}}), \end{aligned}$$

where $f^{\text{prior, gaussian}}$ and $f^{\text{prior, piecewise}}$ are PDFs parametrized by independent subsets of parameters θ . We refer to this model as *H-VHRED*.

6 Experiments

We evaluate the proposed models on two types of natural language processing tasks: document modeling and dialogue natural language generation. All models are trained with back-propagation using the variational lower-bound on the log-likelihood or the exact log-likelihood. We use the first-order gradient descent optimizer Adam (Kingma and Ba, 2015) with gradient clipping (Pascanu et al., 2012)¹

Model	20-NG	RCV1	CADE
<i>LDA</i>	1058	--	--
<i>docNADE</i>	896	--	--
<i>NVDM</i>	836	--	--
<i>G-NVDM</i>	651	905	339
<i>H-NVDM-3</i>	607	865	258
<i>H-NVDM-5</i>	566	833	294

Table 1: Test perplexities on three document modeling tasks: 20-NewGroup (20-NG), Reuters corpus (RCV1) and CADE12 (CADE). Perplexities were calculated using 10 samples to estimate the variational lower-bound. The *H-NVDM* models perform best across all three datasets.

6.1 Document Modeling

Tasks We use three different datasets for document modeling experiments. First, we use the 20 News-Groups (20-NG) dataset (Hinton and Salakhutdinov, 2009). Second, we use the Reuters corpus (RCV1-V2), using a version that contained a selected 5,000 term vocabulary. As in previous work (Hinton and Salakhutdinov, 2009; Larochelle and Lauly, 2012), we transform the original word frequencies using the equation $\log(1 + \text{TF})$, where TF is the original word frequency. Third, to test our document models on text from a non-English language, we use the Brazilian Portuguese CADE12 dataset (Cardoso-Cachopo, 2007). For all datasets, we track the validation bound on a subset of 100 vectors randomly drawn from each training corpus.

Training All models were trained using mini-batches with 100 examples each. A learning rate of 0.002 was used. Model selection and early stopping were conducted using the validation lower-bound, estimated using five stochastic samples per validation example. Inference networks used 100 units in each hidden layer for 20-NG and CADE, and 100 for RCV1. We experimented with both 50 and 100 latent random variables for each class of models, and found that 50 latent variables performed best on the validation set. For *H-NVDM* we vary the number of components used in the PDF, investigating the effect that 3 and 5 pieces had on the final quality of the model. The number

¹Code and scripts are available at <https://github.com/ago109/piecewise-nvdm-emnlp-2017> and <https://github.com/julianser/hred-latent-piecewise>.

G-NVDM	H-NVDM-3	H-NVDM-5
environment	project	science
project	gov	built
flight	major	high
lab	based	technology
mission	earth	world
launch	include	form
field	science	scale
working	nasa	sun
build	systems	special
gov	technical	area

Table 2: Word query similarity test on 20 News-Groups: for the query ‘space’, we retrieve the top 10 nearest words in word embedding space based on Euclidean distance. *H-NVDM-5* associates multiple meanings to the query, while *G-NVDM* only associates the most frequent meaning.

of hidden units was chosen via preliminary experimentation with smaller models. On 20-NG, we use the same set-up as (Hinton and Salakhutdinov, 2009) and therefore report the perplexities of a topic model (*LDA*, (Hinton and Salakhutdinov, 2009)), the document neural auto-regressive estimator (*docNADE*, (Larochelle and Lauly, 2012)), and a neural variational document model with a fixed standard Gaussian prior (*NVDM*, lowest reported perplexity, (Miao et al., 2016)).

Results In Table 1, we report the test document perplexity: $\exp(-\frac{1}{D} \sum_n \frac{1}{L_n} \log P_\theta(x_n))$. We use the variational lower-bound as an approximation based on 10 samples, as was done in (Mnih and Gregor, 2014). First, we note that the best baseline model (i.e. the *NVDM*) is more competitive when both the prior and posterior models are learnt together (i.e. the *G-NVDM*), as opposed to the fixed prior of (Miao et al., 2016). Next, we observe that integrating our proposed piecewise variables yields even better results in our document modeling experiments, substantially improving over the baselines. More importantly, in the 20-NG and Reuters datasets, increasing the number of pieces from 3 to 5 further reduces perplexity. Thus, we have achieved a new state-of-the-art perplexity on 20 News-Groups task and — to the best of our knowledge — better perplexities on the CADE12 and RCV1 tasks compared to using a state-of-the-art model like the *G-NVDM*. We also evaluated the converged models using a non-parametric inference procedure, where a separate

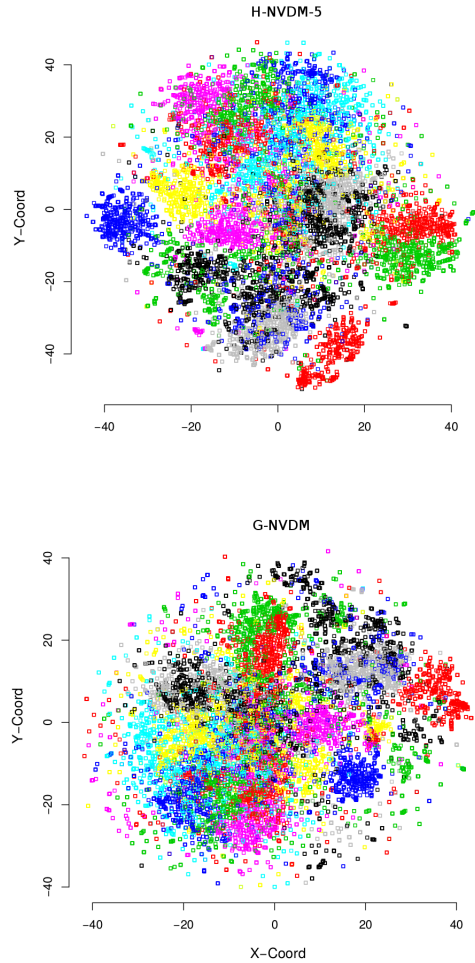


Figure 2: Latent variable approximate posterior means t-SNE visualization on 20-NG for *G-NVDM* and *H-NVDM-5*. Colors correspond to the topic labels assigned to each document.

approximate posterior is learned for each test example in order to tighten the variational lower-bound. *H-NVDM* also performed best in this evaluation across all three datasets, which confirms that the performance improvement is due to the piecewise components. See appendix for details.

In Table 2, we examine the top ten highest ranked words given the query term ‘space’, using the decoder parameter matrix. The piecewise variables appear to have a significant effect on what is uncovered by the model. In the case of ‘space’, the hybrid with 5 pieces seems to value two senses of the word—one related to ‘outer space’ (e.g., ‘sun’, ‘world’, etc.) and another related to the dimensions of depth, height, and width within which things may exist and move (e.g., ‘area’, ‘form’, ‘scale’, etc.). On the other hand, *G-NVDM* appears to only capture the ‘outer space’ sense of

Model	Activity	Entity
<i>HRED</i>	4.77	2.43
<i>G-VHRED</i>	9.24	2.49
<i>P-VHRED</i>	5	2.49
<i>H-VHRED</i>	8.41	3.72

Table 3: Ubuntu evaluation using F1 metrics w.r.t. activities and entities. *G-VHRED*, *P-VHRED* and *H-VHRED* all outperform the baseline *HRED*. *G-VHRED* performs best w.r.t. activities and *H-VHRED* performs best w.r.t. entities.

the word. More examples are in the appendix.

Finally, we visualized the means of the approximate posterior latent variables on 20-NG through a t-SNE projection. As shown in Figure 2, both *G-NVDM* and *H-NVDM-5* learn representations which disentangle the topic clusters on 20-NG. However, *G-NVDM* appears to have more dispersed clusters and more outliers (i.e. data points in the periphery) compared to *H-NVDM-5*. Although it is difficult to draw conclusions based on these plots, these findings could potentially be explained by the Gaussian latent variables fitting the latent factors poorly.

6.2 Dialogue Modeling

Task We evaluate *VHRED* on a natural language generation task, where the goal is to generate responses in a dialogue. This is a difficult problem, which has been extensively studied in the recent literature (Ritter et al., 2011; Lowe et al., 2015; Sordani et al., 2015; Li et al., 2016; Serban et al., 2016b,a). Dialogue response generation has recently gained a significant amount of attention from industry, with high-profile projects such as Google SmartReply (Kannan et al., 2016) and Microsoft Xiaoice (Markoff and Mozur, 2015). Even more recently, Amazon has announced the Alexa Prize Challenge for the research community with the goal of developing a natural and engaging chatbot system (Farber, 2016).

We evaluate on the technical support response generation task for the Ubuntu operating system. We use the well-known Ubuntu Dialogue Corpus (Lowe et al., 2015, 2017), which consists of about 1/2 million natural language dialogues extracted from the #Ubuntu Internet Relayed Chat (IRC) channel. The technical problems discussed span a wide range of software-related and hardware-related issues. Given a dialogue history — such

as a conversation between a user and a technical support assistant — the model must generate the next appropriate response in the dialogue. For example, when it is the turn of the technical support assistant, the model must generate an appropriate response helping the user resolve their problem.

We evaluate the models using the activity- and entity-based metrics designed specifically for the Ubuntu domain (Serban et al., 2017a). These metrics compare the *activities* and *entities* in the model generated responses with those of the reference responses; activities are verbs referring to high-level actions (e.g. *download*, *install*, *unzip*) and entities are nouns referring to technical objects (e.g. *Firefox*, *GNOME*). The more activities and entities a model response overlaps with the reference response (e.g. expert response) the more likely the response will lead to a solution.

Training The models were trained to maximize the log-likelihood of training examples using a learning rate of 0.0002 and mini-batches of size 80. We use a variant of truncated back-propagation. We terminate the training procedure for each model using early stopping, estimated using one stochastic sample per validation example. We evaluate the models by generating dialogue responses: conditioned on a dialogue context, we fix the model latent variables to their median values and then generate the response using a beam search with size 5. We select model hyperparameters based on the validation set using the F1 activity metric, as described earlier.

It is often difficult to train generative models for language with stochastic latent variables (Bowman et al., 2015; Serban et al., 2017b). For the latent variable models, we therefore experiment with reweighing the KL divergence terms in the variational lower-bound with values 0.25, 0.50, 0.75 and 1.0. In addition to this, we linearly increase the KL divergence weights starting from zero to their final value over the first 75000 training batches. Finally, we weaken the *decoder* RNN by randomly replacing words inputted to the decoder RNN with the unknown token with 25% probability. These steps are important for effectively training the models, and the latter two have been used in previous work by Bowman et al. (2015) and Serban et al. (2017b).

HRED (Baseline): We compare to the *HRED* model (Serban et al., 2016b): a sequence-to-sequence model, shown to outperform other es-

tablished models on this task, such as the LSTM RNN language model (Serban et al., 2017a). The *HRED* model’s *encoder* RNN uses a bidirectional GRU RNN encoder, where the forward and backward RNNs each have 1000 hidden units. The context RNN is a GRU encoder with 1000 hidden units, and the decoder RNN is an LSTM decoder with 2000 hidden units.² The encoder and context RNNs both use layer normalization (Ba et al., 2016).³ We also experiment with an additional rectified linear layer applied on the inputs to the decoder RNN. As with other hyper-parameters, we choose whether to include this additional layer based on the validation set performance. *HRED*, as well as all other models, use a word embedding dimensionality of size 400.

G-HRED: We compare to *G-VHRED*, which is *VHRED* with Gaussian latent variables (Serban et al., 2017b). *G-VHRED* uses the same hyper-parameters for the encoder, context and decoder RNNs as the *HRED* model. The model has 100 Gaussian latent variables per utterance.

P-HRED: The first model we propose is *P-VHRED*, which is *VHRED* model with piecewise constant latent variables. We use $n = 3$ number of pieces for each latent variable. *P-VHRED* also uses the same hyper parameters for the encoder, context and decoder RNNs as the *HRED* model. Similar to *G-VHRED*, *P-VHRED* has 100 piecewise constant latent variables per utterance.

H-HRED: The second model we propose is *H-VHRED*, which has 100 piecewise constant (with $n = 3$ pieces per variable) and 100 Gaussian latent variables per utterance. *H-VHRED* also uses the same hyper-parameters for the encoder, context and decoder RNNs as *HRED*.

Results: The results are given in Table 3. All latent variable models outperform *HRED* w.r.t. both activities and entities. This strongly suggests that the high-level concepts represented by the latent variables help generate meaningful, goal-directed responses. Furthermore, each type of latent variable appears to help with a different aspects of the generation task. *G-VHRED* performs best w.r.t. activities (e.g. *download*, *install* and so on), which occur frequently in the dataset.

²Since training lasted between 1-3 weeks for each model, we had to fix the number of hidden units during preliminary experiments on the training and validation datasets.

³We did not apply layer normalization to the decoder RNN, because several of our colleagues have found that this may hurt the performance of generative language models.

This suggests that the Gaussian latent variables learn useful latent representations for frequent actions. On the other hand, *H-VHRED* performs best w.r.t. entities (e.g. *Firefox*, *GNOME*), which are often much rarer and mutually exclusive in the dataset. This suggests that the combination of Gaussian and piecewise latent variables help learn useful representations for entities, which could not be learned by Gaussian latent variables alone. We further conducted a qualitative analysis of the model responses, which supports these conclusions. See Appendix G.⁴

7 Conclusions

In this paper, we have sought to learn rich and flexible multi-modal representations of latent variables for complex natural language processing tasks. We have proposed the piecewise constant distribution for the variational autoencoder framework. We have derived closed-form expressions for the necessary quantities required for in the autoencoder framework, and proposed an efficient, differentiable implementation of it. We have incorporated the proposed piecewise constant distribution into two model classes — *NVDM* and *VHRED* — and evaluated the proposed models on document modeling and dialogue modeling tasks. We have achieved state-of-the-art results on three document modeling tasks, and have demonstrated substantial improvements on a dialogue modeling task. Overall, the results highlight the benefits of incorporating the flexible, multi-modal piecewise constant distribution into variational autoencoders. Future work should explore other natural language processing tasks, where the data is likely to arise from complex, multi-modal latent factors.

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⁴Results on a Twitter dataset are given in the appendix.

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A Appendix: Inappropriate Gaussian Priors

The majority of work on VAEs propose to parametrize z — both the prior and approximate posterior (encoder) — as a multivariate Gaussian variable. However, the multivariate Gaussian is a uni-modal distribution and can therefore only represent one mode in latent space. Furthermore, the multivariate Gaussian is perfectly symmetric with a constant kurtosis. These properties are problematic if the latent variables we aim to represent are inherently multi-modal, or if the latent variables follow complex, non-linear probability manifolds (e.g. asymmetric distributions or heavy-tailed distributions). For example, the frequency of topics in news articles could be represented by a continuous probability distribution, where each topic has its own island of probability mass; *sports* and *politics* topics might each be clustered on their own separate island of probability mass with zero or little mass in between them. Due to its uni-modal nature, the Gaussian distribution can never represent such probability distributions. As another example, ambiguity and uncertainty in natural language conversations could similarly be represented by islands of probability mass; given the question *How do I install Ubuntu on my laptop?*, a model might assign positive probability mass to specific, unambiguous entities like *Ubuntu 4.10* and to well-defined procedures like *installation using a DVD*. In particular, certain entities like *Ubuntu 4.10* are now outdated — these entities occur rarely in practice and should be considered rare events. When modeling such complex, multi-modal latent distributions, the mapping from multivariate Gaussian latent variables to outputs — i.e. the conditional distribution $P_\theta(w_n|z)$ — has to be highly non-linear in order to compensate for the simplistic Gaussian distribution and capture the natural latent factors in an intermediate layer of the model. However, it is difficult to learn such non-linear mappings when using the variational bound in eq. (1), as it incurs additional variance from sampling the latent variable z . Consequently, such models are likely to converge on solutions that do not capture salient aspects of the latent variables, which in turn leads to a poor fit of the output distribution.

B Appendix: Piecewise Constant Variable Derivations

To train the model using the re-parametrization trick, we need to generate $z = f(\epsilon)$ where $\epsilon \sim \text{Uniform}(0, 1)$. To do so, we employ inverse transform sampling (Devroye, 1986), which requires finding the inverse of the cumulative distribution function (CDF). We derive the CDF of eq. (2):

$$\phi(z) = \frac{1}{K} \sum_{i=1}^n \mathbb{1}\left(\frac{i}{n} \leq z\right) K_i + 1 \left(\frac{i-1}{n} \leq z \leq \frac{i}{n}\right) * \left(z - \frac{i-1}{n}\right) a_i. \quad (3)$$

Next, we derive its inverse:

$$\phi^{-1}(\epsilon) = \sum_{i=1}^n \mathbb{1}\left(\frac{1}{K} \sum_{j=0}^{i-1} K_j \leq \epsilon \leq \frac{1}{K} \sum_{j=0}^i K_j\right) * \left(\frac{i-1}{n} + \frac{K}{a_i} \left(\epsilon - \frac{1}{K} \sum_{j=0}^{i-1} K_j\right)\right) \quad (4)$$

Armed with the inverse CDF, we can now draw a sample z :

$$z = \phi^{-1}(\epsilon), \quad \text{where } \epsilon \sim \text{Uniform}(0, 1). \quad (5)$$

In addition to sampling, we need to compute the Kullback-Leibler (KL) divergence between the prior and approximate posterior distributions of the piecewise constant variables. We assume both the prior and the posterior are piecewise constant distributions. We use the *prior* superscript to denote prior parameters and the *post* superscript to denote posterior parameters (encoder model parameters). The KL divergence between the prior and posterior can be computed using a sum of integrals, where each integral inside the sum corresponds to one constant segment:

$$\begin{aligned} & \text{KL}[Q_\psi(z|w_1, \dots, w_N) || P_\theta(z)] \\ &= \int_0^1 Q_\psi(z|w_1, \dots, w_N) \log\left(\frac{Q_\psi(z|w_1, \dots, w_N)}{P_\theta(z)}\right) dz \end{aligned} \quad (6)$$

$$= \sum_{i=1}^n \int_0^{1/n} \frac{a_i^{\text{post}}}{K^{\text{post}}} \log\left(\frac{a_i^{\text{post}}/K^{\text{post}}}{a_i^{\text{prior}}/K^{\text{prior}}}\right) dz \quad (7)$$

$$= \frac{1}{n} \sum_{i=1}^n \frac{a_i^{\text{post}}}{K^{\text{post}}} \log\left(\frac{a_i^{\text{post}}/K^{\text{post}}}{a_i^{\text{prior}}/K^{\text{prior}}}\right) \quad (8)$$

$$\begin{aligned} &= \frac{1}{n} \frac{1}{K^{\text{post}}} \sum_{i=1}^n a_i^{\text{post}} \left(\log(a_i^{\text{post}}) - \log(a_i^{\text{prior}})\right) \\ &+ \log(K^{\text{prior}}) - \log(K^{\text{post}}) \end{aligned} \quad (9)$$

In order to improve training, we further transform the piecewise constant latent variables to lie within the interval $[-1, 1]$ after sampling: $z' = 2z - 1$. This ensures the input to the decoder RNN has mean zero initially.

C Appendix: NVDM Implementation

The complete *NVDM* architecture is defined as:

$$\begin{aligned}\pi(W) &= f^0(E^0W + b^0), \\ \text{Enc}(W) &= f^1(E^1\pi(W) + b^1), \\ z_{\text{Gaussian}} &= \mu^{\text{post}} + \sqrt{\sigma^{2,\text{post}}} \otimes \epsilon_0, \\ z_{\text{Piecewise}} &= \phi^{-1,\text{post}}(\epsilon_1), \\ z &= \langle z_{\text{Gaussian}}, z_{\text{Piecewise}} \rangle, \\ \text{Dec}(w, z) &= g(-w^T Rz),\end{aligned}$$

where \otimes is the Hadamard product, $\langle \circ, \circ \rangle$ is an operator that combines the Gaussian and the Piecewise variables and $\text{Dec}(w, z)$ is the decoder model.⁵ As a result of using the re-parametrization trick and choice of prior, we calculate the latent variable z through the two samples, ϵ_0 and ϵ_1 . $f(\circ)$ is a non-linear activation function, which was the parametrized linear rectifier (with a learnable ‘‘leak’’ parameters) for the 20 News-Groups experiments and the softsign function, or $f(v) = v/(1 + |v|)$, for Reuters and CADE. The decoder model $\text{Dec}(z)$ outputs a probability distribution over words conditioned on z . In this case, we define $g(\circ)$ as the softmax function (omitting the bias term c for clarity) computed as:

$$\text{Dec}(w, z) = P_\theta(w|z) = \frac{\exp(-w^T Rz)}{\sum_{w'} \exp(-w'^T Rz)},$$

The decoder’s output is used to calculate the first term in the variational lower-bound: $\log P_\theta(W|z)$. The prior and posterior distributions are used to compute the KL term in the variational lower-bound. The lower-bound is:

$$\begin{aligned}\mathcal{L} &= \mathbb{E}_{Q_\psi(z|W)} \left[\sum_{i=1}^N \log P_\theta(w_i|z) \right] \\ &\quad - \text{KL}[Q_\psi(z|W) || P_\theta(z)],\end{aligned}$$

where the KL term is the sum of the Gaussian and piecewise KL-divergence measures:

$$\begin{aligned}\text{KL}[Q(z|W) || P(z)] & \\ &= \text{KL}_{\text{Gaussian}}[Q(z|W) || P(z)] \\ &\quad + \text{KL}_{\text{Piecewise}}[Q(z|W) || P(z)].\end{aligned}$$

⁵Operations include vector concatenation, summation, or averaging.

The KL-terms may be interpreted as regularizers of the parameter updates for the encoder model (Kingma and Welling, 2014). These terms encourage the posterior distributions to be similar to their corresponding prior distributions, by limiting the amount of information the encoder model transmits regarding the output.

D Appendix: VHRED Implementation

As described in the model section, the probability distribution of the generative model factorizes as:

$$\begin{aligned}P_\theta(\mathbf{w}_1, \dots, \mathbf{w}_N) & \\ &= \prod_{n=1}^N P_\theta(\mathbf{w}_n | \mathbf{w}_{<n}, z_n) P_\theta(z_n | \mathbf{w}_{<n}), \\ &= \prod_{n=1}^N \prod_{m=1}^{M_n} P_\theta(w_{n,m} | w_{n,<m}, \mathbf{w}_{<n}, z_n) P_\theta(z_n | \mathbf{w}_{<n}),\end{aligned}\tag{10}$$

where θ are the model parameters. VHRED uses three RNN modules: an *encoder* RNN, a *context* RNN and a *decoder* RNN. First, each utterance is encoded into a vector by the *encoder* RNN:

$$\begin{aligned}h_{n,0}^{\text{enc}} &= \mathbf{0}, \quad h_{n,m}^{\text{enc}} = f_\theta^{\text{enc}}(h_{n,m-1}^{\text{enc}}, w_{n,m}) \\ &\quad \forall m = 1, \dots, M_n,\end{aligned}$$

where f_θ^{enc} is either a GRU or a bidirectional GRU function. The last hidden state of the *encoder* RNN is given as input to the *context* RNN. The *context* RNN uses this state to update its internal hidden state:

$$h_0^{\text{con}} = \mathbf{0}, \quad h_n^{\text{con}} = f_\theta^{\text{con}}(h_{n-1}^{\text{con}}, h_{n,M_n}^{\text{enc}}),$$

where f_θ^{con} is a GRU function taking as input two vectors. This state conditions the prior distribution over z_n :

$$P_\theta(z_n | \mathbf{w}_{<n}) = f_\theta^{\text{prior}}(z_n; h_{n-1}^{\text{con}}),\tag{11}$$

where f_θ^{prior} is a PDF parametrized by both θ and h_{n-1}^{con} . Next, a sample is drawn from this distribution: $z_n \sim P_\theta(z_n | \mathbf{w}_{<n})$. The sample and *context* state are given as input to the *decoder* RNN:

$$\begin{aligned}h_{n,0}^{\text{dec}} &= \mathbf{0}, \quad h_{n,m}^{\text{dec}} = f_\theta^{\text{dec}}(h_{n,m-1}^{\text{dec}}, h_{n-1}^{\text{con}}, z_n, w_{n,m}) \\ &\quad \forall m = 1, \dots, M_n,\end{aligned}$$

where f_θ^{dec} is the LSTM gating function taking as input four vectors. The output distribution is computed by passing $h_{n,m}^{\text{dec}}$ through an MLP f_θ^{mlp} , an

affine transformation and a softmax function:

$$\begin{aligned} P_\theta(w_{n,m+1} | w_{n,\leq m}, \mathbf{w}_{<n}, z_n) \\ = \frac{e^{(Ow_{n,m+1})^\top f_\theta^{\text{mlp}}(h_{n,m}^{\text{dec}})}}{\sum_{w'} e^{(Ow')^\top f_\theta^{\text{mlp}}(h_{n,m}^{\text{dec}})}}, \end{aligned} \quad (12)$$

where $O \in \mathbb{R}^{|V| \times d}$ is the word embedding matrix for the output distribution with embedding dimensionality $d \in \mathbb{N}$.

As mentioned in the model section, the approximate posterior is conditioned on the *encoder* RNN state of the next utterance:

$$Q_\psi(z_n | \mathbf{w}_{\leq n}) = f_\psi^{\text{post}}(z_n; h_{n-1}^{\text{con}}, h_{n,M_n}^{\text{enc}}), \quad (13)$$

where f_ψ^{post} is a PDF parametrized by ψ and h_{n,M_n}^{enc} (i.e. the future state of the *encoder* RNN after processing \mathbf{w}_n).

For the Gaussian latent variables, we use the interpolation gating mechanism described in the main text for the approximate posterior. We experimented with other mechanisms for controlling the gating variables, such as defining α_μ and α_σ to be a linear function of the encoder. However, this did not improve performance in our preliminary experiments.

E Appendix: Training Details

Piecewise Constant Variable Interpolation We conducted initial experiments with the interpolation gating mechanism for the approximate posterior of the piecewise constant latent variables. However, we found that this did not improve performance.

Dialogue Modeling We use the Ubuntu Dialogue Corpus v2.0 extracted January, 2016: <http://cs.mcgill.ca/~jpineau/datasets/ubuntu-corpus-1.0/>.

For the *HRED* model we found that an additional rectified linear units layer decreased performance on the validation set according to the activity F1 metric. Hence we test *HRED* without the rectified linear units layer. On the other hand, for all *VHRED* models we found that the additional rectified linear units layer improved performance on the validation set. For *P-VHRED*, we found that a final weight of one for the KL divergence terms performed best on the validation set. For *G-VHRED* and *H-VHRED*, reweighing the KL divergence terms with a final value 0.25 performed best on the validation set. We conducted preliminary experiments with $n = 3$ and $n = 5$ pieces,

and found that models with $n = 3$ were easier to train. Therefore, we use $n = 3$ pieces for both *P-VHRED* and *H-VHRED*.

For all models, we compute the log-likelihood and variational lower-bound costs starting from the second utterance in each dialogue.

F Appendix: Additional Document Modeling Experiments

Iterative Inference For the document modeling experiments, our results and conclusions depend on how tight the variational lower-bound is. As such, it is in theory possible that some of our models are performing much better than reported by the variational lower-bound on the test set. Therefore, we use a non-parametric iterative inference procedure to tighten the variational lower-bound, which aims to learn a separate approximate posterior for each test example. The iterative inference procedure consists of simple stochastic gradient descent (no more than 100 steps), with a learning rate of 0.1 and the same gradient rescaling used in training. For 20 News-Groups, the iterative inference procedure is stopped on a test example if the bound does not improve over 10 iterations. For Reuters and CADE, the iterative inference procedure is stopped if the bound does not improve over 5 iterations. During iterative inference the parameters of the model, as well as the generated prior, are all fixed. Only the gradients of the variational lower-bound with respect to generated posterior model parameters (i.e. the mean and variance of the Gaussian variables, and the piecewise components, a_i) are used to update the posterior model for each document (using a freshly drawn sample for each inference iteration step).

Note, this form of inference is expensive and requires additional meta-parameters (e.g. a step-size and an early-stopping criterion). We remark that a simpler, and more accurate, approach to inference might perhaps be to use importance sampling.

The results based on iterative inference are reported in Table 5. As Section 6.1, we find that *H-NVDM* outperforms the *G-NVDM* model. This confirms our previous conclusions.

In our current examples, it appears that the *H-NVDM* with 5 pieces returns more general words. For example, as evidenced in Table 4, in the case of “government”, the baseline seems to value the plural form of the word (which is largely based on morphology) while the hybrid model actually

G-NVDM	H-NVDM-3	H-NVDM-5
governments	citizens	arms
citizens	rights	rights
country	governments	federal
threat	civil	country
private	freedom	policy
rights	legitimate	administration
individuals	constitution	protect
military	private	private
freedom	court	citizens
foreign	states	military

Table 4: Word query similarity test on 20 News-Groups: for the query ‘government’.

pulls out meaningful terms such as “federal”, “policy”, and “administration”.

Approximate Posterior Analysis We present an additional analysis of the approximate posterior on 20 News-Groups, in order to understand what the models are capturing. For a test example, we calculate the squared norm of the gradient of the KL terms w.r.t. the word embedding inputted to the approximate posterior model. The higher the squared norm of the gradients of a word is, the more influence it will have on the posterior approximation (encoder model). For every test example, we count the top 5 words with highest squared gradients separately for the multivariate Gaussian and piecewise constant latent variables.⁶

The results shown in Table 6, illustrate how the piecewise variables capture different aspects of the document data. The Gaussian variables were originally were sensitive to some of the words in the table. However, in the hybrid model, nearly all of the temporal words that the Gaussian variables were once more sensitive to now more strongly affect the piecewise variables, which themselves also capture all of the words that were originally missed. This shift in responsibility indicates that the piecewise constant variables are better equipped to handle certain latent factors. This effect appears to be particularly strong in the case of certain nationality-based adjectives (e.g., “american”, “israeli”, etc.). While the *G-NVDM* could model multi-modality in the data to some degree, this work would be primarily done in the model’s decoder. In the *H-NVDM*, the piecewise variables provide an explicit mechanism for capturing modes in the unknown target distribution, so it makes sense that the model would learn to use the piecewise variables instead, thus freeing up the

⁶Our approach is equivalent to counting the top 5 words with the highest L2 gradient norms.

Gaussian variables to capture other aspects of the data, as we found was the case with names (e.g., “jesus”, “kent”, etc.).

G Appendix: Additional Dialogue Modeling Experiments

Ubuntu Experiments We present test examples — dialogue context and model responses generated using beam search — for the Ubuntu models in Table 7. The examples qualitatively illustrate the differences between models. First, we observe that *HRED* tends to generate highly generic responses compared to all the latent variable models. This supports the quantitative results reported in the main text, and suggests that modeling the latent factors through latent variables is critical for this task. Next, we observe that *H-VHRED* tends to generate relevant entities and commands — such as *mount command*, *xserver-xorg*, *static ip address* and *pulseaudio* in examples 1-4. On the other hand, *G-VHRED* tends to be better at generating appropriate verbs — such as *list*, *install*, *pastebin* and *reboot* in examples 1-3 and example 5. Qualitatively, *P-VHRED* model appears to perform somewhat worse than both *G-VHRED* and *H-VHRED*. This suggests that the Gaussian latent variables are important for the Ubuntu task, and therefore that the best performance may be obtained by combining both Gaussian and piecewise latent variables together in the *H-VHRED* model.

Twitter Experiments We also conducted a dialogue modeling experiment on a Twitter corpus, extracted from based on public Twitter conversations (Ritter et al., 2011). The dataset is split into training, validation, and test sets, containing respectively 749,060, 93,633 and 9,399 dialogues each. On average, each dialogue contains about 6 utterances (dialogue turns) and about 94 words. We pre-processed the tweets using byte-pair encoding (Sennrich et al., 2016) with a vocabulary consisting of 5000 sub-words.

We trained our models with a learning rate of 0.0002 and mini-batches of size 40 or 80.⁷ As for the Ubuntu experiments, we used a variant of truncated back-propagation and apply gradient clipping. We experiment with *G-VHRED* and *H-VHRED*. Similar to (Serban et al., 2017b), we use a bidirectional GRU RNN *encoder*, where the forward and backward RNNs each have 1000 hid-

⁷We had to vary the mini-batch size to make the training fit on GPU architectures with low memory.

20-NG	Sampled	SGD-Inf	RCV1	Sampled	SGD-Inf
<i>LDA</i>	1058	--	<i>G-NVDM</i>	905	837
<i>RSM</i>	953	--	<i>H-NVDM-3</i>	865	807
<i>docNADE</i>	896	--	<i>H-NVDM-5</i>	833	781
<i>SBN</i>	909	--			
<i>fDARN</i>	917	--			
<i>NVDM</i>	836	--	CADE	Sampled	SGD-Inf
<i>G-NVDM</i>	651	588	<i>G-NVDM</i>	339	230
<i>H-NVDM-3</i>	607	546	<i>H-NVDM-3</i>	258	193
<i>H-NVDM-5</i>	566	496	<i>H-NVDM-5</i>	294	209

Table 5: Comparative test perplexities on various document datasets (50 latent variables). Note that document probabilities were calculated using 10 samples to estimate the variational lower-bound.

den units. We experiment with *context* RNN encoders with 500 and 1000 hidden units, and find that that 1000 hidden units reach better performance w.r.t. the variational lower-bound on the validation set. The *encoder* and *context* RNNs use layer normalization (Ba et al., 2016). We experiment with *decoder* RNNs with 1000, 2000 and 4000 hidden units (LSTM cells), and find that 2000 hidden units reach better performance. For the *G-VHRED* model, we experiment with latent multivariate Gaussian variables with 100 and 300 dimensions, and find that 100 dimensions reach better performance. For the *H-VHRED* model, we experiment with latent multivariate Gaussian and piecewise constant variables each with 100 and 300 dimensions, and find that 100 dimensions reach better performance. We drop words in the decoder with a fixed drop rate of 25% and multiply the KL terms in the variational lower-bound by a scalar, which starts at zero and linearly increases to 1 over the first 60,000 training batches. Note, unlike the Ubuntu experiments, the final weight of the KL divergence is exactly one (hence the bound is tight).

Our hypothesis is that the piecewise constant latent variables are able to capture multi-modal aspects of the dialogue. Therefore, we evaluate the models by analyzing what information they have learned to represent in the latent variables. For each test dialogue with n utterances, we condition each model on the first $n - 1$ utterances and compute the latent posterior distributions using all n utterances. We then compute the gradients of the KL terms of the multivariate Gaussian and piecewise constant latent variables w.r.t. each word in the dialogue. Since the words vectors are discrete, we compute the sum of the squared gradi-

ents w.r.t. each word embedding. The higher the sum of the squared gradients of a word is, the more influence it will have on the posterior approximation (encoder model). For every test dialogue, we count the top 5 words with highest squared gradients separately for the multivariate Gaussian and piecewise constant latent variables.⁸

The results are shown in Table 8. The piecewise constant latent variables clearly capture different aspects of the dialogue compared to the Gaussian latent variables. The piecewise constant variable approximate posterior encodes words related to time (e.g. weekdays and times of day) and events (e.g. parties, concerts, Easter). On the other hand, the Gaussian variable approximate posterior encodes words related to sentiment (e.g. laughter and appreciation) and acronyms, punctuation marks and emoticons (i.e. smilies). We also conduct a similar analysis on the document models evaluated in Sub-section 6.1, the results of which may be found in the Appendix.

⁸Our approach is equivalent to counting the top 5 words with the highest L2 gradient norms. We also did some experiments using L1 gradient norms, which showed similar patterns.

Word	G-NVDM			H-NVDM-5			
	Time-related	G-KL	G-KL	P-KL	Word Names	G-KL	G-KL
months	23	33	40	henry	33	47	39
day	28	32	35	tim	32	27	11
time	55	22	40	mary	26	51	30
century	28	13	19	james	40	72	30
past	30	18	28	jesus	28	87	39
days	37	14	19	george	26	56	29
ahead	33	20	33	keith	65	94	61
years	44	16	38	kent	51	56	15
today	46	27	71	chris	38	55	28
back	31	30	47	thomas	19	35	19
future	20	15	20	hitler	10	14	9
order	42	14	26	paul	25	52	18
minute	15	34	40	mike	38	76	40
began	16	5	13	bush	21	20	14
night	49	12	18				
hour	18	17	16	Adjectives	G-KL	G-KL	P-KL
early	42	42	69	american	50	12	40
yesterday	25	26	36	german	25	21	22
year	60	17	21	european	20	17	27
week	28	54	58	muslim	19	7	23
hours	20	26	31	french	11	17	17
minutes	40	34	38	canadian	18	10	16
months	23	33	40	japanese	16	9	24
history	32	18	28	jewish	56	37	54
late	41	45	31	english	19	16	26
moment	23	17	16	islamic	14	18	28
season	45	29	37	israeli	24	14	18
summer	29	28	31	british	35	15	17
start	30	14	38	russian	14	19	20
continue	21	32	34				
happened	22	27	35				

Table 6: Approximate posterior word encodings (20-NG). For P-KL, we bold every case where piecewise variables showed greater word sensitivity than Gaussian variables w/in the same hybrid model.

Dialogue Context (History)	Response
Hi . I am installing ubuntu now in my new laptop . In " something else " partitioning , what mount point should I set for a drive which is not root or not home ... → It 's up to you , just choose a directory that will remind you of the contents of that partition . E.G . : if it 's the Windows partition , use /windows . → it 's a new harddrive with full free space . I bought it without windows preinstalled . I want to create drives in which I will only store files .. I mean , not root or not home . What mount point do I set for it ? "/mount " is not shown in drop down menu sorry . I mean /mount I mean , in my desktop , extra drives are mounted in /media do you understand my problem ? Sorry , English is not my native language . → I do :) Just use a directory with a meaningful name and preferably in the root , for example "/files " . Choose something like "/files " . → / for root , /home for home , but what mount point for a file storage partition . wait , let me see if there is a "/files " , thank you .	HRED: No problem . G-VHRED: It should be in the list of the drive . If you want to be able to mount it . P-VHRED: If you want to mount it , you can use the mount command . H-VHRED: As far as I know , you can use the " mount " command to mount the partitions .
I have a problem installing Ubuntu 12.04 desktop . Tried desktop with USB -i hangs . alternate with usb -i hangs . alternate with cd -i hangs (even faster) . I tried using different usb drives . Could someone please help me ? → how are you making the usb ? and whats the pc specs . → I used Universal usb installer → what exactly does it do when it ' hangs ' → Specs : CPU core i7 @ 3.4 Ghz , Sapphire Radeon HD 7800 , 8GB RAM → ATi video ? tried the NOMODESET option yet ? → nothing at all , I also can't witch to console using Ctrl + alt + F not yet Will try that , thanks	HRED: you could try the alternate install cd . G-VHRED: it might be a problem with your graphics card . You could also try the alternate install cd and see if it works . P-VHRED: try ' nomodeset ' or ' nomodeset '
anyone know how to add a route in ubuntu ? getting an error on adding a route , and I'm not sure why → pastebin your command and proplem → <url >	HRED: can you pastebin the output of " route -n " please G-VHRED: if you want to use pastebin to pastebin the output of " sudo route -n " please pastebin the output of " route "
Any idea why empathy 's not playing notification sounds ? Even though I have ' em ticked in preferences → restarted it yet ? → yar → check pulse to see if the application is muted for some reason ? well Sound settings . → Had sound effects turned off in sound settings , didn't realize that controlled other applications → Ah yea , ive done it a few time it 's annoying My favorite though is recently pulse has been freezing on my desktop and audio will just not be adjustable for like ... 30 seconds or so	HRED: I don't think it 's a problem with pulseaudio , but I'm not sure how to fix it G-VHRED: Thank you for your help . I will try that . Thanks for the suggestion . P-VHRED: I think it 's a bug with pulseaudio , but I don't know how to fix it .
Question about setting up drives : Running 12.04 LTS . I have two drives . An SSD that holds the OS for booting , and a second drive " data " for all the data , home directories , etc . The data drive is encrypted with LUKS . With my current setup , whenever I reboot , I get a password entry screen to decrypt the data drive , where my home directory is . This works fine . The problem is that if I reboot remotely , I'm stuck as I can't enter password . If I keep my home directory on the SSD , there won't be enough room . If I use a symbolic link , and the encrypted drive isn't mounted , that creates errors and issues . Suggestions ?? → yes , you can redirect the → Message was cut off ...	HRED: if you want to see the data on the drive , you can mount it in /etc/fstab G-VHRED: it should be a good place to do that , but if you want to do a clean install , you will need to reboot P-VHRED: it 's not a problem , it 's just a bug H-VHRED: if you want to see the data on the drive , you can mount it in /etc/fstab
hey guys , does anyone know why i cannot connect to ssh via my external IP ? (connection refused) but ssh -v localhost works fine ? → Have you forwarded the port on your router to the machine you're trying to access ? → It is a work machine , so am not sure of the rules	HRED: You need to forward port 22 to your router to forward port 22 to the server . G-VHRED: That 's odd . What are you trying to do ? Can you pastebin the output of " sudo netstat " to " pastebin " please ? P-VHRED: Can you pastebin the output of " sudo apt-get install openssh-server " ? H-VHRED: Even if it 's not working , then you need to set the port forward to your router .

Table 7: Ubuntu model examples. The → token indicates a change of turn.

Word	G-VHRED			H-VHRED			
	G-KL	G-KL	P-KL	Word	G-VHRED	H-VHRED	
Time-related				Event-related			
monday	3	5	10	school	9	16	50
tuesday	2	3	7	class	11	16	27
wednesday	4	11	13	game	20	26	41
thursday	2	3	9	movie	12	20	41
friday	9	18	26	club	13	22	28
saturday	6	6	13	party	8	10	32
sunday	2	2	9	wedding	7	13	23
weekend	8	16	32	birthday	12	20	23
today	18	28	56	easter	15	15	23
night	16	31	68	concert	7	16	20
tonight	32	36	47	dance	11	12	21
Word	G-VHRED	H-VHRED		Word	G-VHRED	H-VHRED	
Sentiment-related	G-KL	G-KL	P-KL	Acronyms, Punctuation Marks & Emoticons	G-KL	G-KL	P-KL
good	72	73	44	lol	394	358	312
love	102	101	38	omg	52	45	19
awesome	26	44	39	.	386	558	1009
cool	14	28	29	!	648	951	525
haha	132	101	75	?	507	851	221
hahaha	60	48	24	*	108	54	19
amazing	14	38	33	xd	28	42	26
thank	137	153	29	♡	56	42	24

Table 8: Approximate posterior word encoding on Twitter. The numbers are computed by counting the number of times each word is among the 5 words with the largest sum of squared gradients of the Gaussian KL divergence (G-KL) and piecewise constant KL divergence (P-KL)