

On the Security of an Unconditionally Secure, Universally Composable Inner Product Protocol

Babak Siabi, Mehdi Berenjkoub

Abstract

In this paper we discuss the security of a distributed inner product (DIP) protocol [*IEEE TIFS*, 11(1), (2016), 59-73]. We show information leakage in this protocol that does not happen in an ideal execution of DIP functionality. In some scenarios, this information leakage enables one of the parties to completely learn the other party's input. We will give examples of such scenarios.

Keywords: inner product, preprocessing model, secure computation, security analysis

1. Preliminaries

1.1. Notations

In accordance with [1], in the following, we denote by \mathbb{F}_q the finite field of order q , by \mathbb{F}_q^n the space of all n -vector with elements in \mathbb{F}_q and by $\mathbb{F}_q^{m \times n}$ the space of all $m \times n$ matrices with elements belonging to \mathbb{F}_q . We use overbar lowercase letters to represent vectors in \mathbb{F}_q^n (e.g. \bar{a} represents a vector), and bold uppercase letters to represent matrices in $\mathbb{F}_q^{m \times n}$ (e.g. \mathbf{A} represents a matrix). We denote by $x \in_R D$ the process of uniformly random sampling of element x from domain D .

1.2. Distributed inner product functionality

The functionality considered in [1], is the distributed version of two-party inner product (we refer to this functionality as DIP). In contrast to the conventional inner product, in DIP the result is shared between parties. More precisely, it is assumed that P_1 and P_2 hold private vectors $\bar{x}_1 \in \mathbb{F}_q^k$ and $\bar{x}_2 \in \mathbb{F}_q^k$, respectively, and intend to calculate $w = \langle \bar{x}_1 \cdot \bar{x}_2 \rangle$ such that for $i = 1, 2$ party P_i receives an additive random share $w_i \in \mathbb{F}_q$ satisfying $w_1 + w_2 = w$.

In an ideal world, this functionality is handled by a TTP as illustrated in Fig. 1. Since the TTP honestly follows the illustrated procedure, the output shares are uniformly distributed on \mathbb{F}_q and parties learn nothing about $w = \langle \bar{x}_1 \cdot \bar{x}_2 \rangle$ or the input of the other party.

Email addresses: b.siabi@ec.iut.ac.ir (Babak Siabi), brnjkbc@cc.iut.ac.ir (Mehdi Berenjkoub)

Functionality \mathcal{F}_{DIP}
<hr/> <p>Inputs: P_1 and P_2 hold $\bar{x}_1 \in \mathbb{F}_q^k$ and $\bar{x}_2 \in \mathbb{F}_q^k$ respectively.</p> <p>Output: P_1 learns $w_1 \in_R \mathbb{F}_q$ and P_2 learns $w_2 = \langle \bar{x}_1 \cdot \bar{x}_2 \rangle - w_1$.</p> <hr/>
<p>P_1 : Sends \bar{x}_1 to TTP.</p> <p>P_1 : Sends \bar{x}_2 to TTP.</p> <p>TTP: Upon receiving \bar{x}_1 and \bar{x}_2,</p> <ul style="list-style-type: none"> • if $\bar{x}_1 \notin \mathbb{F}_q^k$ or $\bar{x}_2 \notin \mathbb{F}_q^k$ sets $w_1 = w_2 = \lambda$, • otherwise, chooses $u \in_R \mathbb{F}_q$, sets $w_1 = u$ and $w_2 = \langle \bar{x}_1 \cdot \bar{x}_2 \rangle - u$. <p>TTP: Sends w_1 to P_1 and w_2 to P_2.</p>

Figure 1: The ideal functionality of distributed inner product (\mathcal{F}_{DIP})

Remark 1. It is of vital importance to note the difference between DIP and conventional inner product. In the latter, since the parties learn the product, they always (even in the ideal world) can derive an equation for the other party's input using their own input and output. But, derivation of such an equation is not possible in the case of DIP ideal functionality.

2. DIP Protocol of [1]

In [1], David *et al.* propose a protocol to realize \mathcal{F}_{DIP} . Fig. 2 illustrates this protocol (Protocol π_{DDG+}). Protocol π_{DDG+} is designed in the preprocessing model and planned to be universally composable and unconditionally secure. In the preprocessing model, it is assumed that an initiator (denoted by *Init.* in Fig. 2) distributes a set of correlated randomness between parties before they decide (or fix) their inputs (preprocessing phase). After deciding the inputs, parties compute the functionality with the aid of preprocessed data (computation phase).

3. Security Analysis

A careful inspection of protocol π_{DDG+} reveals that it does not simulate the ideal \mathcal{F}_{DIP} completely. It is easy to check this for $k = 1$, where the input vectors reduce to scalar values. In this case, at the end of a run of protocol π_{DDG+} , P_2 learns P_1 's input. In [2], authors ignore this case and reason that DIP functionality inherently is not private for $k = 1$. As intuitively mentioned earlier in Remark 1, it is the case for conventional inner product and not for DIP. In the following proposition, we formally state an almost trivial security property of \mathcal{F}_{DIP} which flatly contradicts the reasons given in [2].

Proposition 1. *If π is a protocol that securely computes \mathcal{F}_{DIP} for input vectors of length k , then for every $1 \leq k' \leq k$, there is a protocol $\pi_{k'}$ constructed with black-box use of π which securely computes \mathcal{F}_{DIP} for input vectors of length k' .*

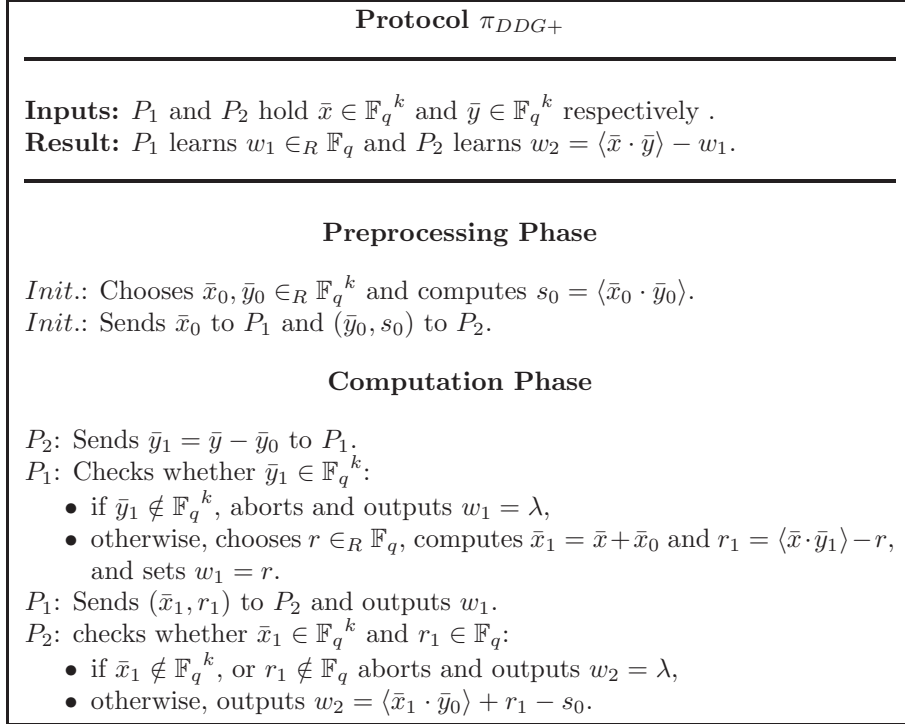


Figure 2: The DIP protocol of [1]

Proof. P_1 and P_2 perform protocol $\pi_{k'}$ on inputs $\bar{x} = (x_1, x_2, \dots, x_{k'})$ and $\bar{y} = (y_1, y_2, \dots, y_{k'})$ as follows:

1. They append a zero vector of length $k - k'$ to their inputs and form extended inputs as $\bar{x}_e = (x_1, \dots, x_{k'}, 0, \dots, 0)$ and $\bar{y}_e = (y_1, \dots, y_{k'}, 0, \dots, 0)$.
2. Then, they run π on the extended inputs \bar{x}_e, \bar{y}_e , and receive w_1, w_2 .
3. They output w_1, w_2 .

It is easy to check that correctness and security of protocol $\pi_{k'}$ reduces to correctness and security of protocol π . ■

Now, it can be concluded that protocol π_{DDG+} does not realize \mathcal{F}_{DIP} for $k = 1$ and therefore it is not secure. Protocol π_{DDG+} is insecure for $k > 1$ as well. Particularly, P_2 can always deduce an equation on P_1 's input vector, \bar{x} , after receiving $\bar{x}_1 = \bar{x} + \bar{x}_0$. That is,

$$\begin{aligned}
 \bar{x}_1 &= \bar{x} + \bar{x}_0 \\
 \xrightarrow{\cdot \bar{y}_0} \langle \bar{x}_1 \cdot \bar{y}_0 \rangle &= \langle \bar{x} \cdot \bar{y}_0 \rangle + \langle \bar{x}_0 \cdot \bar{y}_0 \rangle \\
 \implies \langle \bar{x} \cdot \bar{y}_0 \rangle &= \langle \bar{x}_1 \cdot \bar{y}_0 \rangle - s_0
 \end{aligned}$$

Note that P_2 is not able to deduce such an equation in the ideal run of DIP

functionality. Therefore, Protocol π_{DDG+} leaks some information about P_1 's input beyond what is available in the ideal world. Technically, the ideal world simulator fails to construct the view of a real world adversary that controls P_2 . It is important to note that the simulation failure happens considering both semi-honest and malicious adversaries. Hence Protocol π_{DDG+} is insecure in both adversarial models.

The information leakage explained above, may seem harmless for large vectors because the adversary learns virtually nothing about P_1 's input. But, regarding composition, this little information leakage will be a serious problem. We demonstrate it in an example scenario.

3.1. Example scenario: multiplication of a vector by a matrix

Suppose that P_1 has a $1 \times k$ vector $\bar{x} \in \mathbb{F}_q^k$ and P_2 has a $k \times k$ matrix $\mathbf{Y} \in \mathbb{F}_q^{k \times k}$ and they intend to compute $\bar{w} = \bar{x} \times \mathbf{Y}$ in a shared manner. That is, they want to receive random shares \bar{w}_1 and \bar{w}_2 , correspondingly, such that $\bar{w} = \bar{w}_1 + \bar{w}_2$. This computation will be trivial provided that P_1 and P_2 can run an arbitrary number of instances of a universally composable protocol for computing \mathcal{F}_{DIP} .

It is easy to check that protocol π_{DDG+} is completely insecure to be the underlying DIP protocol for this scenario. Assume that P_1 and P_2 run protocol π_{DDG+} to compute each element of \bar{w} . We denote the i th element of \bar{w} by w_i and the i th column of \mathbf{Y} by Y_i . To compute w_i , P_1 inputs \bar{x} and P_2 inputs Y_i in protocol π_{DDG+} . The output of each party is a share of w_i . In addition to its output share, P_2 derives an equation on \bar{x} from each run of protocol π_{DDG+} as discussed above. After k runs, the parties receive their desired outputs. But, in this state P_2 has k equations on \bar{x} that will be enough to extract \bar{x} if they are linearly independent. Specifically, if we denote by \bar{x}_0^i, \bar{y}_0^i and s_0^i the randomness received by the corresponding parties in the preprocessing phase of the i th run of protocol π_{DDG+} , then the equation that P_2 can deduce from the i th run will be of the form

$$\langle \bar{x} \cdot \bar{y}_0^i \rangle = \langle \bar{x}_1^i \cdot \bar{y}_0^i \rangle + s_0^i,$$

where $\bar{x}_1^i = \bar{x} + \bar{x}_0^i$ is the message that P_2 receives from P_1 in the i th run. Let \mathbf{Y}_0 be the matrix with \bar{y}_0^i as its i th column and \bar{q}_0 be the vector with $\langle \bar{x}_1^i \cdot \bar{y}_0^i \rangle + s_0^i$ as its i th element. Now, we can write the set of these equations in the form of $\bar{x} \times \mathbf{Y}_0 = \bar{q}_0$ which is a system of linear equations. Therefore, vector \bar{x} will be found uniquely if \mathbf{Y}_0 is a non-singular matrix.

It is easy to extend above argument to matrix multiplication scenario and therefore all linear algebra protocols of [1].

4. Conclusion

Inspection of the proof given in [1] and [2] for security of protocol π_{DDG+} reveals that, in the simulation process, the case of corrupted P_2 is not discussed in detail due to the apparent similarity to the case of corrupted P_1 . Considering

the presented security flaw in this protocol, we can conclude a general recommendation to avoid any frugality in the process of security proofs, especially when tasks of parties are not exactly identical in the protocol.

In [1] and [2], various higher level privacy preserving linear algebra protocols are proposed that the DIP protocol is their fundamental building block. Fortunately, these higher level protocols use the DIP protocol as a black-box and, thus, they can be implemented using any universally composable secure DIP protocol.

References

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