

Secret Writing on Dirty Paper: A Deterministic View

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Abstract—Recently there has been a lot of success in using deterministic approach to provide approximate characterization of capacity for Gaussian networks. In this paper, we take a deterministic view and revisit the problem of wiretap channel with side information. A precise characterization of the secrecy capacity is obtained for a linear deterministic model, which naturally suggests a coding scheme which we show to achieve the secrecy capacity of the Gaussian model (dubbed as “secret writing on dirty paper”) to within $(1/2) \log 3$ bits.

I. INTRODUCTION

In information theory, an interesting and useful communication scenario is a state-dependent channel where the channel states are noncausally known at the transmitter as side information. Particularly important is a discrete-time channel with real input and additive white Gaussian noise and interference, where the interference is noncausally known at the transmitter as side information.

Costa [1] was the first to study this communication scenario, which he whimsically coined as “writing on dirty paper.” Based on an earlier result of Gel’fand and Pinsker [2], Costa [1] proved the surprising result that the capacity of writing on dirty paper is the *same* as that of writing on clean paper without interference. Since then, dirty-paper coding has found a wide range of applications in digital watermarking and network information theory particularly involving broadcast scenarios.

Recent works [3] and [4] studied the problem of dirty-paper coding in the presence of an eavesdropper, which is a natural extension of Costa’s dirty-paper coding to secrecy communications. In this scenario (which we dub as “secret writing on dirty paper”), the legitimate receiver channel is a traditional dirty-paper channel of Costa. The signals received at the eavesdropper, on the other hand, are assumed to be a *degraded* version of the signals received at the legitimate receiver. An achievable secrecy rate was established based on a double-binning scheme and was shown to be the secrecy capacity of the channel under some channel parameter configurations [3], [4]. The secrecy capacity of the channel under a *general* channel parameter configuration, however, remains *unknown*.

In facing challenging Gaussian network communication problems, recent advances [5], [6] in network information

theory advocate a *deterministic* approach and seeks *approximate* characterization of channel capacity to within *finite* bits (per channel use). Motivated by the success of [5] and [6], in this paper we take a deterministic view and revisit the problem of wiretap channel with side information. A precise characterization of the secrecy capacity is obtained for a linear deterministic model, which naturally suggests a coding scheme which we show to achieve the secrecy capacity of the Gaussian model to within $(1/2) \log 3$ bits.

The rest of the paper is organized as follows. In Sec. II, we first take a deterministic view at the problem of Costa’s dirty-paper coding and provide an approximate characterization of the channel capacity to within half bit. Note that even though a precise characterization of Costa’s dirty-paper channel is well-known [1], such an approximate characterization lays the foundation for studying side-information problems via a deterministic approach. Building on the success of Sec. II, in Sec. III we extend the deterministic approach to the problem of secret writing on dirty paper and provide an approximate characterization of the secrecy capacity to within $(1/2) \log 3$ bits. Finally, in Sec. IV we conclude the paper with some remarks.

II. WRITING ON DIRTY PAPER

A. Gaussian Model

Consider the dirty-paper channel of Costa [1]. The received signal Y (at a given time index) can be written as

$$Y = hX + gS + N \quad (1)$$

where X is the channel input which is subject to a *unit* average power constraint, N and S are *standard* Gaussian noise and interference and are independent of each other, and h and g are the channel coefficients corresponding to the channel input and the interference, respectively. The interference S is assumed to be noncausally known at the transmitter as side information. The channel coefficients h and g are fixed during communication and are known at both transmitter and receiver.

The channel capacity, as shown by Costa [1], is given by

$$C = I(U; Y) - I(U; S)$$

where

$$U = hX + \frac{h^2}{h^2 + 1}gS \quad (2)$$

is an auxiliary random variable, and X is standard Gaussian and independent of S . For such a choice of auxiliary-input variable pair (U, X) ,

$$I(U; Y) - I(U; S) = \frac{1}{2} \log(1 + h^2)$$

which equals the capacity of channel (1) when the interference S is also known at the receiver.

B. Linear Deterministic Model

Consider the linear deterministic model [6] for Costa's dirty-paper channel (1), where the received signal Y is given by

$$Y = D^{q-n}X + D^{q-m}S. \quad (3)$$

Here, X is the binary input vector of length $q = \max\{n, m\}$, S is the vector interference whose elements are i.i.d. Bernoulli-1/2, D is the $q \times q$ down-shift matrix, and n and m are the integer channel gains corresponding to the channel input and the interference, respectively. The vector interference S is assumed to noncausally known at the transmitter as side information. The channel gains n and m are fixed during communication and are known at both transmitter and receiver.

Following the result of Gel'fand and Pinsker [2], the capacity of the linear deterministic dirty-paper channel (3) is given by

$$C = I(U; Y) - I(U; S)$$

where the auxiliary random variable

$$U = Y = D^{q-n}X + D^{q-m}S \quad (4)$$

(we may choose $U = Y$ as Y here is a deterministic function of X and S) and X is an i.i.d. Bernoulli-1/2 random vector and independent of S . For such a choice of auxiliary-input variable pair (U, X) ,

$$\begin{aligned} I(U; Y) - I(U; S) &= H(Y) - I(Y; S) \\ &= H(Y|S) \\ &= H(D^{q-n}X) \\ &= \text{rank}(D^{q-n}) \\ &= n \end{aligned}$$

which equals the capacity of channel (3) when the interference S is also known at the receiver.

C. Connections between the Gaussian and the Linear Deterministic Models

A quick comparison between the Gaussian model (1) and the linear deterministic model (3) reveals the following equivalence relationship between these two models:

$$h \longleftrightarrow D^{q-n} \quad \text{and} \quad g \longleftrightarrow D^{q-m}. \quad (5)$$

Given this equivalence relationship, the optimal choice (4) of auxiliary random variable U for the linear deterministic model (3) suggests the following choice of auxiliary random variable U for the Gaussian model (1):

$$U = hX + gS \quad (6)$$

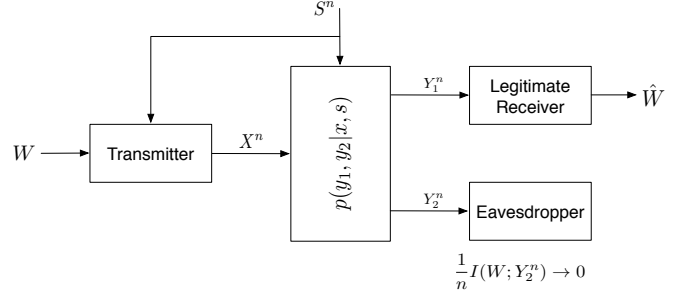


Fig. 1. Wiretap channel with side information.

where X is standard Gaussian and independent of S . Compared with the optimal choice (2), the choice (6) of auxiliary random variable U is *suboptimal*. However, for this suboptimal choice of auxiliary-input variable pair (U, X) , we have

$$\begin{aligned} I(U; S) &= \frac{1}{2} \log \left(1 + \frac{g^2}{h^2} \right) \\ \text{and } I(U; Y) &= \frac{1}{2} \log(1 + h^2 + g^2) \end{aligned}$$

giving an achievable rate

$$I(U; Y) - I(U; S) = \frac{1}{2} \log \frac{(1 + h^2 + g^2)h^2}{h^2 + g^2} \geq \frac{1}{2} \log(h^2)$$

which is within half bit of the channel capacity $\frac{1}{2} \log(1 + h^2)$.

The fact that the choice (6) of auxiliary random variable U leads to an achievable rate that is within half bit of the dirty-paper channel capacity is well-known [7]. However, it is interesting to see that such a choice comes up *naturally* in the context of deterministic approach.

III. SECRET WRITING ON DIRTY PAPER

As illustrated in Fig. 1, consider a discrete-time memoryless wiretap channel with transition probability $p(y_1, y_2 | x, s)$, where X is the channel input, S is the channel state, and Y_1 and Y_2 are the received signals at the legitimate receiver and eavesdropper, respectively. The channel states S are i.i.d. across time and are noncausally known at the transmitter as side information. The transmitter has a message W , which is intended for the legitimate receiver but needs to be kept asymptotically perfectly secret from the eavesdropper. Following the classical works [8] and [9], it is required that

$$\frac{1}{n} I(W; Y_2^n) \rightarrow 0 \quad (7)$$

in the limit as the block length $n \rightarrow \infty$, where $Y_2^n := (Y_2[1], \dots, Y_2[n])$. The secrecy capacity C_s of the channel is defined as the largest secrecy rate that can be achieved by any coding scheme.

Chen and Vinck [4] derived a single-letter lower bound on the secrecy capacity, which can be written as

$$C_s \geq \max_{p(u, x|s)} \min \left\{ I(U; Y_1) - I(U; S), I(U; Y_1) - I(U; Y_2) \right\}. \quad (8)$$

We also have the following simple upper bound on the secrecy capacity.

Lemma 1: The secrecy capacity C_s of a discrete memoryless wiretap channel $p(y_1, y_2|x, s)$ with channel states S noncausally known at the transmitter as side information can be bounded from above as

$$C_s \leq \max_{p(v,x,s)} \min \{I(X; Y_1|S), I(V; Y_1) - I(V; Y_2)\} \quad (9)$$

where V is an auxiliary random variable satisfying the Markov chain $V \rightarrow (X, S) \rightarrow (Y_1, Y_2)$, and the marginal distribution of S is fixed to be $p(s)$.

Note that

$$\max_{p(v,x,s)} [I(V; Y_1) - I(V; Y_2)]$$

is the secrecy capacity of a discrete memoryless wiretap channel with input X , fully *action-dependent* state S [10], legitimate receiver output Y_1 , and eavesdropper output Y_2 . Also note that the upper bound (9) is stronger than the following upper bound derived in [4, Theorem 2]:

$$C_s \leq \min \left\{ \max_{p(x|s)} I(X; Y_1|S), \max_{p(v,x,s)} [I(V; Y_1) - I(V; Y_2)] \right\}.$$

Here, a simple single-letterization technique of Williams [11] allows us to move the max's outside the min. The details of the proof are omitted from the paper due to the space limit.

The following upper bound on the secrecy capacity is (potentially) weaker than (9), but is much easier to evaluate for specific channels as it does not involve any auxiliary random variables. The result follows directly from Lemma 1 and standard information-theoretic argument.

Lemma 2: The secrecy capacity C_s of a discrete memoryless wiretap channel $p(y_1, y_2|x, s)$ with channel states S noncausally known at the transmitter as side information can be bounded from above as

$$C_s \leq \max_{p(x|s)} \min \{I(X; Y_1|S), I(X, S; Y_1|Y_2)\}. \quad (10)$$

For *semi-deterministic* channels where the channel output at the legitimate receiver is a deterministic (bivariate) function of the channel input and the channel state, the lower bound (8) and the upper bound (10) coincide, leading to a *precise* characterization of the secrecy capacity.

Theorem 1: Consider a discrete memoryless wiretap channel $p(y_1, y_2|x, s)$ with channel states S noncausally known at the transmitter as side information. If the received signal Y_1 at the legitimate receiver is a *deterministic* function of the channel input X and the state S , i.e., $Y_1 = f(X, S)$ for some bivariate function f , the secrecy capacity C_s of the channel is given by

$$C_s = \max_{p(x|s)} \min \{H(Y_1|S), H(Y_1|Y_2)\}. \quad (11)$$

Proof: The fact that

$$C_s \geq \max_{p(x|s)} \min \{H(Y_1|S), H(Y_1|Y_2)\}$$

follows from the lower bound (8) by setting $U = Y_1$ (we may do so only because by assumption, Y_1 is a deterministic function of (X, S)), which gives

$$I(U; Y_1) - I(U; S) = H(Y_1) - I(Y_1; S) = H(Y_1|S)$$

and similarly

$$I(U; Y_1) - I(U; Y_2) = H(Y_1) - H(Y_1|Y_2) = H(Y_1|Y_2).$$

The converse part of the theorem follows from the upper bound (10) and the fact that Y_1 is a deterministic function of (X, S) so we have

$$I(X; Y_1|S) = H(Y_1|S) - H(Y_1|X, S) = H(Y_1|S)$$

and similarly

$$I(X, S; Y_1|Y_2) = H(Y_1|Y_2) - H(Y_1|X, S, Y_2) = H(Y_1|Y_2).$$

This completes the proof of the theorem. \blacksquare

A. Linear Deterministic Model

Next, we use the results of Theorem 1 to determine the secrecy capacity of a linear deterministic wiretap channel with side information. In this model, the received signals at the legitimate receiver and the eavesdropper are given by

$$\begin{aligned} Y_1 &= D^{q-n_1} X + D^{q-m_1} S \\ Y_2 &= D^{q-n_2} X + D^{q-m_2} S \end{aligned} \quad (12)$$

where X is the binary input vector of length $q = \max\{n_1, n_2, m_1, m_2\}$, S is the vector interference whose elements are i.i.d. Bernoulli-1/2, D is the $q \times q$ down-shift matrix, and n_1, n_2, m_1 and m_2 are the integer channel gains. The vector interference S is assumed to be noncausally known at the transmitter as side information. The channel gains n_1, n_2, m_1 and m_2 are fixed during communication and are known at all terminals.

We shall need the following simple lemma, which can be proved using standard counting argument.

Lemma 3: For any given matrices A and B in \mathbb{F}_2 that have the same number of columns,

$$\max H(AZ|BZ) = \text{rank} \left(\begin{bmatrix} A \\ B \end{bmatrix} \right) - \text{rank}(B) \quad (13)$$

where the maximization is over all possible binary random vector Z . The maximum is achieved when Z is an i.i.d. Bernoulli-1/2 random vector.

The following theorem provides an explicit characterization of the secrecy capacity of channel (12).

Theorem 2: The secrecy capacity C_s of channel (12) is given by

$$C_s = \min \left\{ n_1, \text{rank} \left(\begin{bmatrix} A \\ B \end{bmatrix} \right) - \text{rank}(B) \right\} \quad (14)$$

where

$$\begin{aligned} A &:= \begin{bmatrix} D^{q-n_1} & D^{q-m_1} \\ D^{q-n_2} & D^{q-m_2} \end{bmatrix} \\ \text{and } B &:= \begin{bmatrix} D^{q-n_2} & D^{q-m_2} \end{bmatrix}. \end{aligned}$$

Proof: We show that for the linear deterministic model (12), both $H(Y_1|S)$ and $H(Y_1|Y_2)$ are simultaneously maximized when X is an i.i.d. Bernoulli-1/2 random vector and independent of S .

First,

$$\begin{aligned} H(Y_1|S) &= H(D^{q-n_1}X|S) \\ &\leq H(D^{q-n_1}X) \\ &\leq \text{rank}(D^{q-n_1}) \\ &= n_1 \end{aligned} \quad (15)$$

where the equality holds when X is an i.i.d. Bernoulli-1/2 random vector and independent of S .

Next, let

$$Z := \begin{bmatrix} X \\ S \end{bmatrix}.$$

By Lemma 3,

$$\begin{aligned} H(Y_1|Y_2) &= H(AZ|BZ) \\ &\leq \text{rank} \left(\begin{bmatrix} A \\ B \end{bmatrix} \right) - \text{rank}(B) \end{aligned} \quad (16)$$

where the equality holds also when X is an i.i.d. Bernoulli-1/2 random vector and independent of S .

Substituting (15) and (16) into (11) completes the proof of the theorem. ■

B. Gaussian Model

Finally, let us consider the Gaussian wiretap channel where the received signals at the legitimate receiver and the eavesdropper are given by

$$\begin{aligned} Y_1 &= h_1X + g_1S + N_1 \\ Y_2 &= h_2X + g_2S + N_2. \end{aligned} \quad (17)$$

Here, X is the channel input which is subject to a *unit* average power constraint, N and S are *standard* Gaussian noise and interference and are independent of each other, and h_1 , h_2 , g_1 and g_2 are the channel coefficients. The interference S is assumed to be noncausally known at the transmitter as side information. The channel coefficients h_1 , h_2 , g_1 and g_2 are fixed during communication and are known at all terminals.

A single-letter expression for an achievable secrecy rate was given as the right-hand side of (8), which involves an auxiliary random variable U . It is, however, *not* clear what would be a reasonable choice of auxiliary random variable U , letting alone an optimal one to maximize the achievable secrecy rate. On the other hand, for the linear deterministic model (12), we known from Theorems 1 and 2 that the following choice of auxiliary random variable U is optimal:

$$U = Y_1 = D^{q-n_1}X + D^{q-m_1}S \quad (18)$$

where X is an i.i.d. Bernoulli-1/2 random vector and independent of S . Based on the equivalence relationship (5) between the Gaussian and the linear deterministic models and the success of Sec. II for Costa's dirty-paper coding, the optimal choice (18) for the linear deterministic model (12) suggests

the following choice of auxiliary random variable U for the Gaussian model (17):

$$U = h_1X + g_1S \quad (19)$$

where X is standard Gaussian and independent of S . For this choice of auxiliary-input variable pair (U, X) ,

$$\begin{aligned} I(U; S) &= \frac{1}{2} \log \left(1 + \frac{g_1^2}{h_1^2} \right) \\ I(U; Y_1) &= \frac{1}{2} \log(1 + h_1^2 + g_1^2) \\ \text{and } I(U; Y_2) &= \frac{1}{2} \log \frac{(h_1^2 + g_1^2)(1 + h_2^2 + g_2^2)}{h_1^2 + g_1^2 + (h_1g_2 - h_2g_1)^2}. \end{aligned}$$

We thus have

$$I(U; Y_1) - I(U; S) = \frac{1}{2} \log \frac{(1 + h_1^2 + g_1^2)h_1^2}{h_1^2 + g_1^2} \geq \frac{1}{2} \log(h_1^2)$$

and

$$\begin{aligned} I(U; Y_1) - I(U; Y_2) &= \frac{1}{2} \log \left(\frac{1 + h_1^2 + g_1^2}{h_1^2 + g_1^2} \cdot \frac{h_1^2 + g_1^2 + (h_1g_2 - h_2g_1)^2}{1 + h_2^2 + g_2^2} \right) \\ &\geq \frac{1}{2} \log \frac{h_1^2 + g_1^2 + (h_1g_2 - h_2g_1)^2}{1 + h_2^2 + g_2^2}. \end{aligned}$$

By (8),

$$\min \left\{ \frac{1}{2} \log(h_1^2), \frac{1}{2} \log \frac{h_1^2 + g_1^2 + (h_1g_2 - h_2g_1)^2}{1 + h_2^2 + g_2^2} \right\} \quad (20)$$

is an achievable secrecy rate for the Gaussian wiretap channel (17) with side information.

Following the works [3] and [4], below we focus on the case where

$$h_2 = \beta h_1 \quad \text{and} \quad g_2 = \beta g_1 \quad (21)$$

for some $0 < \beta \leq 1$ and show that the achievable secrecy rate (20) is always within $(1/2) \log 3$ bits of the secrecy capacity.

Note that the secrecy capacity of channel (17) does *not* depend on the correlation between the additive Gaussian noise N_1 and N_2 , so we may write

$$N_2 = \beta N_1 + N$$

where N is Gaussian with zero mean and variance $1 - \beta^2$ and is independent of N_1 . Thus, for case (21), channel (17) can be equivalently written as

$$\begin{aligned} Y_1 &= h_1X + g_1S + N_1 \\ Y_2 &= \beta Y_1 + N \end{aligned} \quad (22)$$

i.e., the received signal Y_2 at the eavesdropper is *degraded* with respect to the the received signal Y_1 at the legitimate receiver.

Theorem 3: For the degraded Gaussian wiretap channel (22) with side information, the achievable secrecy rate (20) is always within $(1/2) \log 3$ bits of the secrecy capacity.

Proof: To show that the achievable secrecy rate (20) is always within $(1/2) \log 3$ bits of the secrecy capacity, we shall consider the single-letter upper bound (10) and show that:

- 1) $\frac{1}{2} \log(h_1^2)$ is always within $1/2$ (and hence $(1/2) \log 3$) bits of $\max_{p(x|s)} I(X; Y_1|S)$; and
- 2) $\frac{1}{2} \log \frac{h_1^2 + g_1^2 + (h_1 g_2 - h_2 g_1)^2}{1 + h_2^2 + g_2^2}$ is always within $(1/2) \log 3$ bits of $\max_{p(x|s)} I(X, S; Y_1|Y_2)$.

To prove statement 1), note that for any input variable X such that $E[X^2] \leq 1$

$$\begin{aligned}
I(X; Y_1|S) &= h(Y_1|S) - h(Y_1|X, S) \\
&= h(h_1 X + N_1|S) - h(N_1) \\
&\leq h(h_1 X + N_1) - h(N_1) \\
&\leq \frac{1}{2} \log(1 + h_1^2 \text{Var}(X)) \\
&\leq \frac{1}{2} \log(1 + h_1^2)
\end{aligned}$$

It is well-known that $\frac{1}{2} \log(h_1^2)$ is within $1/2$ bits of $\frac{1}{2} \log(1 + h_1^2)$.

To prove statement 2), note that

$$\begin{aligned}
I(X, S; Y_1|Y_2) &= h(Y_1|Y_2) - h(Y_1|X, S, Y_2) \\
&= h(Y_1|\beta Y_1 + N) - h(N_1|\beta N_1 + N) \\
&= h(Y_1|\beta Y_1 + N) - \frac{1}{2} \log [2\pi e(1 - \beta^2)].
\end{aligned}$$

Following an inequality of Thomas [12, Lemma 1] and the independence between Y_1 and N , we have

$$h(Y_1|\beta Y_1 + N) \leq \frac{1}{2} \log \frac{2\pi e \text{Var}(Y_1)(1 - \beta^2)}{\beta^2 \text{Var}(Y_1) + (1 - \beta^2)}. \quad (23)$$

Further note that the right-hand side of (23) is a monotone increasing function of $\text{Var}(Y_1)$, and

$$\begin{aligned}
\text{Var}(Y_1) &= \text{Var}(h_1 X + g_1 S + N_1) \\
&= \text{Var}(h_1 X + g_1 S) + 1 \\
&\leq 2 [\text{Var}(h_1 X) + \text{Var}(g_1 S)] + 1 \\
&= 2 [h_1^2 \text{Var}(X) + g_1^2 \text{Var}(S)] + 1 \\
&\leq 2h_1^2 + 2g_1^2 + 1.
\end{aligned}$$

We thus have

$$\begin{aligned}
h(Y_1|\beta Y_1 + N) &\leq \frac{1}{2} \log \frac{2\pi e(2h_1^2 + 2g_1^2 + 1)(1 - \beta^2)}{\beta^2(2h_1^2 + 2g_1^2 + 1) + (1 - \beta^2)} \\
&= \frac{1}{2} \log \frac{2\pi e(2h_1^2 + 2g_1^2 + 1)(1 - \beta^2)}{2\beta^2(h_1^2 + g_1^2) + 1}
\end{aligned}$$

giving

$$\max_{p(x|s)} I(X, S; Y_1|Y_2) \leq \frac{1}{2} \log \frac{2(h_1^2 + g_1^2) + 1}{2\beta^2(h_1^2 + g_1^2) + 1}. \quad (24)$$

For case (21),

$$\frac{1}{2} \log \frac{h_1^2 + g_1^2 + (h_1 g_2 - h_2 g_1)^2}{1 + h_2^2 + g_2^2} = \frac{1}{2} \log \frac{h_1^2 + g_1^2}{1 + \beta^2(h_1^2 + g_1^2)}. \quad (25)$$

When $h_1^2 + g_1^2 \leq 1$, the upper bound

$$\max_{p(x|s)} I(X, S; Y_1|Y_2) \leq \frac{1}{2} \log[2(h_1^2 + g_1^2) + 1] \leq \frac{1}{2} \log 3$$

so there is nothing to prove. When $h_1^2 + g_1^2 \geq 1$, the difference between the right-hand sides of (24) and (25) can be calculated as

$$\begin{aligned}
&\frac{1}{2} \log \frac{2(h_1^2 + g_1^2) + 1}{2\beta^2(h_1^2 + g_1^2) + 1} - \frac{1}{2} \log \frac{h_1^2 + g_1^2}{1 + \beta^2(h_1^2 + g_1^2)} \\
&\leq \frac{1}{2} \log \left(2 + \frac{1}{h_1^2 + g_1^2} \right) \leq \frac{1}{2} \log 3.
\end{aligned}$$

We thus conclude that $\frac{1}{2} \log \frac{h_1^2 + g_1^2 + (h_1 g_2 - h_2 g_1)^2}{1 + h_2^2 + g_2^2}$ is always within $(1/2) \log 3$ bits of $\max_{p(x|s)} I(X, S; Y_1|Y_2)$.

Combining statements 1) and 2) completes the proof of the theorem. \blacksquare

IV. CONCLUDING REMARKS

In this paper, we took a deterministic view and revisited the problem of wiretap channel with side information. A precise characterization of the secrecy capacity was obtained for a linear deterministic model, which naturally suggests a coding scheme which we showed to achieve the secrecy capacity of the Gaussian model to within $(1/2) \log 3$ bits. This success suggested a new way of using the linear deterministic model of [6]: to use it to find *approximately* optimal choice of auxiliary random variable for the corresponding Gaussian model in side-information problems.

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