

Detecting Byzantine Attacks for Gaussian Two-Way Relay System

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Abstract

This paper focuses on Byzantine attack detection for Gaussian two-way relay network. In this network, two source nodes communicate with each other with the help of an amplify-and-forward relay which may perform Byzantine attacks by forwarding altered symbols to the sources. For simple investigating the detectability of attacks conducted in Gaussian channels, we focus on the MA channel of the network, while assuming the BC channel is noiseless. Upon such model, we propose a attack detection scheme implemented in the sources. Specifically, we consider a open wireless propagation environment that allows the symbols, forwarded by the relay, to go through a continuous channel and arrive to the sources. With the observations of the source, we develop a detection scheme for the source by comparing the joint empirical distribution of its received and transmitted signals with the known channel statistics. The main contribution of this paper is to prove that if and only if the Gaussian relay network satisfies a non-manipulable channel condition, the proposed detection scheme can detect arbitrary attacks that allows the stochastic distributions of altered symbols to vary arbitrarily and depend on each other. No pre-shared secret or secret transmission is needed for the detection. Furthermore, we also prove that for the considered Gaussian two-way relay networks, the non-manipulable channel condition is always satisfied. This result indicates that arbitrary attacks conducted in MA Gaussian channels are detectable by only using observations, while providing a base for attack detection in more general Gaussian networks.

I. INTRODUCTION

Relay nodes are widely employed in modern communication networks to enhance coverage and connectivity of the networks. This dependence on the relaying infrastructure may increase the risk on security as malicious relays may forward false information in order to deceive the intended participants into accepting counterfeit information. These attacks, referred to as Byzantine attacks, impose significant ramifications on the design of network protocols [1][2]. With the presence of Byzantine attacks, the attack detection technique, which determines whether Byzantine attacks are conducted or not, is one of the key steps supporting secure communication.

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The work on attack detection starts above physical-layer, where each link is treated as a unit-capacity bit-pipe, while specific physical-layer characteristics are shielded. Based on this setting, cryptography keys are often used to make attacks detectable [3], [4], while requiring the cryptographic keys, to which the relays are not privy, to be shared between the source and destination before the communication takes place. Without using cryptography keys, information theoretic detection schemes are proposed for multicast system or Caterpillar Network [5], [6]. These schemes are able to achieve errorless performance in probability, yet assuming that at least one relay or link is absolutely trustworthy.

Besides these schemes treating channels as noiseless bit-pipes, there are also many other attack detection schemes designed according to specific characteristics of physical-layer channel for varying application scenarios. These schemes are mainly enabled by utilizing tracing symbols, or self-information provided by network topology structure. In particular, source node inserts tracing symbols into a sequence of information bits, and then sends them together to the destination. After tracing symbols go through the relay channel, degraded by channel fading, noise, and possible attacks, they are observed by the destination. Relying on the *priori* knowledge of tracing symbols, the destination could get the stochastic probability distributions of its observation on the tracing symbols, respectively under conditions that the relay is reliable or malicious. On the basis of the two conditional statistical distributions, applying theory of hypothesis test, attack detection schemes can be derived with perfect CSI [7], [8] or no need of CSI [9], [10] for varying network scenarios. The tracing-symbol based schemes commonly assume that the value and insertion location of the tracing symbols are known only at the source and the destination, which indeed requires a additional tracing-symbol distribution mechanism implemented between communication parties. Besides that, since the explicit conditional statistical distributions are needed, these schemes assume that each malicious relay garbles its received symbols according to independent and identically distributed (i.i.d.) stochastic distributions. This model of i.i.d. attacks may not always be valid in practice, although it makes analysis simple. The Byzantine attack detection methods presented in [7]-[10] may no longer be provably unbreakable for non-i.i.d. attacks.

Notice that all the above-mentioned schemes detect attacks by inserting redundancy, which increases the overhead cost. In contrast, the schemes, which utilize side information (SI), do not need to insert any redundancy or just insert negligible redundancy. To be more specific, in [11], for a wireless OFDMA network, source node detects attacks from the correlation between its overheard signals from the relay and its transmitted signals. In [12], for the detection implemented in the destination of wireless one-way network, perfect error correction codes (ECCs) are assumed to be used in the direct link between source and destination and the relay link. With the help of using ECC in the direct link and relay link, the destination has opportunity to observe the exact information that are transmitted by the source or the relay. Then, the detection can be done by comparing the observations from the two links. The detection performance of [11] and [12] are impacted by channel fading, especially,

the performance of [12] depends on the the quality of direct link, such that it may not work well in the network where direct link does not exist or suffers deep fading.

On the contrary, [13]-[17] could probabilistically detect attacks. To be more specific, [13]-[16] consider two-hop communication for the typical three-node network composing of an pair of communication parties and an signal one potential malicious relay, which is possible to conduct arbitrary attacks. To elaborate a little further, the scheme of [13] requires the communication parties to transmit signal to relay simultaneously so as to support secret transmission, with which some key components of algebraic manipulation detection (AMD) code could be shared between communication parties, while keeping confidential to the relay. Then, Byzantine attacks are detectable by applying AMD code to encrypt informations bits. Even the AMD encryption cost negligible redundancy, the secret transmission increases the overhead cost of the system. This scheme is difficult to extend this scheme to non-Gaussian channels. In our previous work [14]-[16], focusing on two-way relay system, we show that for discrete memoryless channels (DMCs), it is possible to detect potential Byzantine attacks dispensing any AMD code or cryptographic keys. The basic idea is that each node utilizes its own transmitted symbols as clean reference for statistically checking against the other node's symbols forwarded by the relay. This scheme is difficult to extend beyond DMC channels. We also extend the scheme of [14]-[16] to the DMC two-hop relay system composing of a pair of communication parties and two potential malicious relays, where the all observations of the destination are prone to be attacked [17]. No clean reference is available for the destination. This work relaxed the restriction imposed on the relay's misbehavior beyond i.i.d attacks for the DMC two-hop relay system. However, due to the lack of clean reference in DMC two-hop relay system, we cannot properly protect communication parties against arbitrary attacks. It indicates if all the signals observed by the destination are unsecured, the malicious relay have opportunity to fool the destination, which advocates the necessary of clean reference in attack detection.

This paper focuses on Byzantine attack detection for Gaussian two-way relay network. In two hops, two source nodes communicate with each other with the help of an amplify-and-forward relay which may perform Byzantine attacks by forwarding altered symbols to the sources. For simple investigating the detectability of attacks conducted in Gaussian channels, we focus on the MA channel of the network, while assuming the BC channel is noiseless. The goal of this paper is to make communication parties probabilistically detect arbitrary attacks, despite of i.i.d or non-i.i.d attacks, without using any AMD code or secret transmission. In particular, notice that the source nodes' transmitted signals are not attacked and statistic related to its observation in the BC phase, then we propose a detection method that use the transmitted signals as clean reference to statistically checking against the signals observed from the relay. This model and method may seem a little bit similar to our previous work [14]-[16]. It is worth noting that in the Gaussian channel, the observations are continuous signals, the possible attacks are also continuous

in the sense that they are likely to be conducted within continuous alphabet. It is considerable difference with our previous work that are applicable for discrete alphabets [14]-[16]. For the continuous attacks and channel model, the core contribution of this manuscript is to prove that the detectability of continuous attacks is equivalent to non-manipulability of continuous channels. Also, we prove the considered Gaussian channel satisfies the non-manipulable condition. This result indicates that arbitrary attacks conducted in MA Gaussian channels are detectable by only using observations, while providing a base for attack detection in more general Gaussian networks.

II. SYSTEM MODEL

Let us focus on the two-way relay example, where the two source nodes are time and phase synchronized. The two source nodes are termed as source 1 and source 2, respectively. The two sources exchange information with each other during two stages. Each stage includes n instant. In the first n instants, source 1 and source 2 respectively sends n -length sequences X_1^n and X_2^n to the relay node. X_1 and X_2 are equiprobability binary symbols generated from alphabet $(+1, -1)$. The MAC channel from the two sources to the relay is specified by $U = X_1 + X_2 + N_r$, where U is the received signal of the relay in each instant, N_r is AWGN existed in the MAC channel. Secondly, in the instants $n + 1, n + 2, \dots, 2n$, the the relay forwarded V^n to the two sources. Through a noiseless broadcast channel, source 1 receives sequence $Y_1^n = V^n$. N_r is random variable following continuous stochastic distribution. Without loss of generalization, we assume the PDF of N_r is $f_{N_r}(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2)$. Upon this assumption, the pdf of U conditioned on $X_1 = 1$ and $X_1 = -1$ can be given as $f_{U|+1}(x) = \frac{0.5}{\sqrt{2\pi}} \exp(-(x+2)^2) + \frac{0.5}{\sqrt{2\pi}} \exp(-x^2)$ and $f_{U|-1}(x) = \frac{0.5}{\sqrt{2\pi}} \exp(-(x-2)^2) + \frac{0.5}{\sqrt{2\pi}} \exp(-x^2)$. Then, the CDF of U given $X_1 = x_1$ is $F_{U|x_1}(t|x_1) = \int_{-\infty}^t f_{U|x_1}(u|x_1) du$.

Let us assume X_1^n and X_2^n are i.i.d sequences and both MAC channel and BC channel are memoryless. Hence, U^n , N_r^n are both i.i.d sequences. V_n is the forwarded sequences. In this paper, the relay is allowed to conduct arbitrary attack, Then, the stochastic distribution of V_n depends on the relay's behavior. In order to define the relay's behavior in mathematic sense, let us choose one n' -length sequence $\tilde{u}_1, \tilde{u}_2, \dots, \tilde{u}_{n'}$. Correspondingly, $\mathcal{B}(\tilde{u}_j)$ are domain consisting of \tilde{u}_j . They satisfy the constraints as follows.

$$\alpha_1 = \tilde{u}_1 < \tilde{u}_2 < \tilde{u}_3 \dots < \tilde{u}_{n'-1} = \beta_1, \beta_1 < \tilde{u}_{n'}$$

$$\tilde{u}_j - \tilde{u}_{j-1} = \frac{\beta_1 - \alpha_1}{n' - 2}, j = 2, 3, \dots, n' - 1$$

$$\mathcal{B}(\tilde{u}_j) = \begin{cases} (\tilde{u}_{j-1}, \tilde{u}_j], & j = 2, 3, \dots, n' - 1 \\ (-\infty, \alpha_1], & j = 1 \\ (\beta_1, +\infty), & j = n' \end{cases}$$

where α_1 and β_1 are assumed to depend on n' . We will prove later that there exist a setup method to make α_1 , β_1 and n' have the following properties.

$$\lim_{n' \rightarrow \infty} F_{U|X_1}(\alpha_1 | x_1) = 0, \quad \lim_{n' \rightarrow \infty} F_{U|X_1}(\beta_1 | x_1) = 1, \quad \lim_{n' \rightarrow \infty} \frac{\beta_1 - \alpha_1}{n' - 2} = 0. \quad (1)$$

Based on the definition of $\mathcal{B}(\tilde{u}_j)$, the continuous variable U can be quantized to discrete \tilde{U} . In particular, if $U \in \mathcal{B}(\tilde{u}_j)$, then $\tilde{U} = \tilde{u}_j$. In other words, $\tilde{U} \triangleq \sum_{j=1}^{n'} 1(U \in \mathcal{B}(\tilde{u}_j)) \tilde{u}_j$. For the random variable \tilde{U} , we use $\tilde{\mathcal{U}}$ to denote its alphabet. Also, we use \tilde{u}_i denote the generic symbol over $\tilde{\mathcal{U}}$ in the i -th instant. From these notations, the function $F_{V^n|\tilde{U}^n}^{(n')}(t|u)$ for sequence pair (U^n, V^n) is defined as

$$F_{V^n|\tilde{U}^n}^{(n')}(t|u) = \begin{cases} \frac{\sum_{i=1}^n 1_i(v_i \leq t) 1_i(\tilde{u}_i = \tilde{u}_j)}{N(\tilde{u}_j|\tilde{U}^n)}, & N(\tilde{u}_j|\tilde{U}^n) \neq 0, u \in \mathcal{B}(\tilde{u}_j) \\ 0, & otherwise \end{cases} \quad (2)$$

By replacing U and V with their lower cases, the similar definition can be applied to $F_{v^n|\tilde{u}^n}^{(n')}(t|u)$, which is definite function associated with particular sequence (u^n, v^n) . If the relay is absolutely reliable, we must always have $\sum_{j=1}^{n'-1} \sum_{i=1}^{n'} |F_{V^n|\tilde{U}^n}^{(n')}(\tilde{u}_j|\tilde{u}_i) - \Phi(\tilde{u}_j - \tilde{u}_i)| = 0$, where

$$\Phi(t) = \begin{cases} 1, & t \geq 0 \\ 0, & otherwise \end{cases}. \quad (3)$$

From this intuitively understanding, the malicious relay is defined as follows.

Definition 1. (Malicious Relay) *The relay is said to be non-malicious if $\sum_{j=1}^{n'-1} \sum_{i=1}^{n'} |F_{V^n|\tilde{U}^n}^{(n')}(\tilde{u}_j|\tilde{u}_i) - \Phi(\tilde{u}_j - \tilde{u}_i)| \rightarrow 0$ in probability as n and n' approach to infinity. Otherwise, the relay is considered malicious.*

Note that Definition 1 tolerates manipulating only a negligible fraction of symbols by the relays. This relaxation has essentially no effect on the information rate from the source to the destination across the relays. Definition 1 also tolerates modification conducted by the relay within a negligible extent. This negligible extent is specified by $\frac{\beta_1 - \alpha_1}{n' - 2}$, as $\frac{\beta_1 - \alpha_1}{n' - 2} \rightarrow 0$, this modification has essentially no effect on the information reliability from the source to the destination across the relay.

For easy description of the main results on detectability, we give the expression of standard the empirical CDF of v^n conditioned on x_1 as

$$F_{v^n|x_1^n}^n(t|x_1) = \frac{\sum_{i=1}^n 1(v_i \leq t) 1(x_{1,i} = x_1)}{N(x_1|x_1^n)}, t \in (-\infty, +\infty). \quad (4)$$

By replacing v and x_1 with their upper cases, random $F_{V^n|X_1^n}^n(t|x_1)$ is similarly defined.

III. MAIN RESULTS

We first point that the wireless two-way relay channel is non-manipulable.

Proposition 1. Non-manipulability of Wireless Channel *The aforementioned channel is non-manipulable, which indicates there does not exist continuous functions $\Psi(v|u)$ that satisfies the following three conditions*

- 1) Fixing u , $\Psi(v|u)$ is a pdf.
- 2) For arbitrary value of u , $\int_{-\infty}^{+\infty} \left| \int_{-\infty}^t \Psi(v|u) dv - \Phi(t-u) \right|^2 dt$ has a strictly positive lower bound.
- 3) $\int_{-\infty}^{+\infty} f_{U|x_1}(u) \Psi(v|u) du = f_{U|x_1}(v)$.

Theorem 1. (Maliciousness detectability of wireless relay network) *As a beneficial result of the fact that wireless MAC channel is non-manipulable, there exist a sequence of decision statistics $\{D^n\}$ simultaneously having the following two properties:*

Fix any sufficiently small $\delta > 0$, $\epsilon > 0$, there has sufficiently large n' ,

- 1) $\lim_{n \rightarrow \infty} \Pr \left(D^n > \varepsilon(n', \delta) \mid \sum_{j=1}^{n'-1} \sum_{i=1}^{n'} |F_{V^n|\tilde{U}^n}^{(n')}(\tilde{u}_j|\tilde{u}_i) - \Phi(\tilde{u}_j - \tilde{u}_i)| > \delta \right) \geq 1 - \epsilon$ whenever $\Pr \left(\sum_{j=1}^{n'-1} \sum_{i=1}^{n'} |F_{V^n|\tilde{U}^n}^{(n')}(\tilde{u}_j|\tilde{u}_i) - \Phi(\tilde{u}_j - \tilde{u}_i)| > \delta \right) > 0$, where $\varepsilon(n', \delta)$ is strictly positive and can be arbitrary small.
- 2) $\lim_{n \rightarrow \infty} \Pr \left(D^n > \mu'(n', \delta) \mid \sum_{j=1}^{n'-1} \sum_{i=1}^{n'} |F_{V^n|\tilde{U}^n}^{(n')}(\tilde{u}_j|\tilde{u}_i) - \Phi(\tilde{u}_j - \tilde{u}_i)| \leq \delta \right) \leq \epsilon$ whenever $\Pr \left(\sum_{j=1}^{n'-1} \sum_{i=1}^{n'} |F_{V^n|\tilde{U}^n}^{(n')}(\tilde{u}_j|\tilde{u}_i) - \Phi(\tilde{u}_j - \tilde{u}_i)| \leq \delta \right) > 0$, where $\mu'(n', \delta) \rightarrow 0$ as $n' \rightarrow \infty$, $\delta \rightarrow 0$.

IV. PROOF OF PROPOSITION 1

Proof: Let us assume the manipulable wireless channel exists, which indicates there at least one i.i.d attack making the statistical distribution of U conditioned on X_1 is equivalent to the statistical distribution of V conditioned on X_1 . Hence, we have $I(X_1; U) = I(X_1; V)$, where $I(\cdot; \cdot)$ denotes mutual information between the two input variables. On the other hand, (X_1, U, V) forms a Markov chain as $X_1 \rightarrow U \rightarrow V$. From Data-Processing Inequality, $I(X_1; U) = I(X_1; V)$ implies the Markov chain $X_1 \rightarrow V \rightarrow U$ is also established. Then, we have

$$\Pr(a < U < b | V = v, X_1 = -1) = \Pr(a < U < b | V = v, X_1 = +1) \quad (5)$$

It is worth noting that due to the continuity of noise, for arbitrary value of $v \in (-\infty, +\infty)$, both $\Pr(a < U < b | V = v, X_1 = -1)$ and $\Pr(a < U < b | V = v, X_1 = +1)$ are well-defined. Furthermore, in this example $U = X_1 + X_2 + N$, $\Pr(a < U < b | V = v, X_1 = -1)$ can be reshaped as

$$\Pr(a < U < b | V = v, X_1 = -1) = \Pr(a + 1 < X_2 + N < b + 1 | V = v, X_1 = -1). \quad (6)$$

Since X_1 is independent with X_2 and N , therefore we obtain

$$\Pr(a + 1 < X_2 + N < b + 1 | V = v, X_1 = -1) = \Pr(a + 1 < X_2 + N < b + 1 | V = v) \quad (7)$$

which indicates

$$\Pr(a < U < b | V = v, X_1 = -1) = \Pr(a + 1 < X_2 + N < b + 1 | V = v). \quad (8)$$

Similarly, we can have

$$\Pr(a < U < b | V = v, X_1 = +1) = \Pr(a - 1 < X_2 + N < b - 1 | V = v). \quad (9)$$

Submitting (8) and (9) into (5), we get

$$\Pr(a + 1 < X_2 + N < b + 1 | V = v) = \Pr(a - 1 < X_2 + N < b - 1 | V = v). \quad (10)$$

then, there has

$$\int_{-\infty}^{+\infty} \Pr(a + 1 < X_2 + N < b + 1 | V = v) f(v) dv = \int_{-\infty}^{+\infty} \Pr(a - 1 < X_2 + N < b - 1 | V = v) f(v) dv \quad (11)$$

It is not hard to find the two sides of (11) are equivalent to $\Pr(a + 1 < X_2 + N < b + 1)$ and $\Pr(a - 1 < X_2 + N < b - 1)$, respectively. Finally, we have

$$\Pr(a + 1 < X_2 + N < b + 1) = \Pr(a - 1 < X_2 + N < b - 1). \quad (12)$$

In summary, under the assumption that the the channel is manipulable, the equation (12) should be established for arbitrary a and b . In other words, if we can find a pair (a, b) for a wireless channel that $\Pr(a + 1 < X_2 + N < b + 1) \neq \Pr(a - 1 < X_2 + N < b - 1)$, then the channel is non-manipulable. ■

Notice that the system model does not consider the noise of the BC channel. Actually, even consider the noisy BC channel, the wireless network is till non-manipulable. This assertion is proved as follows.

Proposition 2. For random variables Z_1, Z_2, Z_3, Z_4, Z_5 , where $Z_3 = Z_1 + Z_2$, $Z_5 = Z_4 + Z_2$, Z_1 and Z_4 are both stochastic independent with Z_2 , if pdf $f_{Z_3|X_1}(z_3|x_1) = f_{Z_5|X_1}(z_5|x_1)$, then there must have $f_{Z_1|X_1}(z_1|x_1) = f_{Z_4|X_1}(z_4|x_1)$.

Proof: According to the fact that $Z_3 = Z_1 + Z_2$, where Z_1 and Z_2 are stochastic independent with each other, then the characteristic function of Z_3 conditioned on $X_1 = x_1$ is expressed by

$$\varphi_{Z_3|X_1}(t|x_1) = \varphi_{Z_1|X_1}(t|x_1) \varphi_{Z_2|X_1}(t|x_1), \quad (13)$$

where $\varphi_{Z_3|X_1}(t|x_1)$, $\varphi_{Z_1|X_1}(t|x_1)$ and $\varphi_{Z_2|X_1}(t|x_1)$ denote the characteristic functions of Z_3 , Z_2 and Z_1 conditioned on $X_1 = x_1$, respectively. Similarly, according to the fact that $Z_5 = Z_4 + Z_2$, where Z_4 and Z_2 are stochastic independent with each other, then the characteristic function of Z_5 conditioned on $X_1 = x_1$ is expressed by

$$\varphi_{Z_5|X_1}(t|x_1) = \varphi_{Z_4|X_1}(t|x_1) \varphi_{Z_2|X_1}(t|x_1), \quad (14)$$

where $\varphi_{Z_5|X_1}(t|\mathbf{x}_1)$ and $\varphi_{Z_4|X_1}(t|\mathbf{x}_1)$ denote the characteristic functions of Z_4 and Z_2 conditioned on $X_1 = \mathbf{x}_1$, respectively. Since $f_{Z_3|X_1}(z_3|\mathbf{x}_1) = f_{Z_5|X_1}(z_5|\mathbf{x}_1)$, we have

$$\varphi_{Z_3|X_1}(t|\mathbf{x}_1) = \varphi_{Z_5|X_1}(t|\mathbf{x}_1) \quad (15)$$

Substituting (13) and (14) into (13), we get

$$\varphi_{Z_4|X_1}(t|\mathbf{x}_1)\varphi_{Z_2|X_1}(t|\mathbf{x}_1) = \varphi_{Z_1|X_1}(t|\mathbf{x}_1)\varphi_{Z_2|X_1}(t|\mathbf{x}_1). \quad (16)$$

Since $\varphi_{Z_2|X_1}(t|\mathbf{x}_1)$ is characteristic function which always attains strictly non-zero value across $t \in (-\infty, +\infty)$, then we have

$$\varphi_{Z_4|X_1}(t|\mathbf{x}_1) = \varphi_{Z_1|X_1}(t|\mathbf{x}_1). \quad (17)$$

From the knowledge that pdf can be uniquely determined by characteristic function, (17) indicates

$$f_{Z_1|X_1}(z_1|\mathbf{x}_1) = f_{Z_4|X_1}(z_4|\mathbf{x}_1). \quad (18)$$

■

Let us back to the proof of non-manipulability for the system with noisy BC channels. In such case, the source observes $Y = V + N_s$ in the BC phase, where N_s denotes the noise of the BC channel. Revisiting $Y = V + N_s$, based on proof firstly given in the section, if and only if the relay is absolutely reliable, i.e., $U = V$, we can get $f_{U|X_1}(u|\mathbf{x}_1) = f_{V|X_1}(v|\mathbf{x}_1)$. Then, consider that Proposition 2 indicates if and only if $f_{U|X_1}(u|\mathbf{x}_1) = f_{V|X_1}(v|\mathbf{x}_1)$, there exists $f_{U+N_s|X_1}(y|\mathbf{x}_1) = f_{V+N_s|X_1}(y|\mathbf{x}_1)$. We finally get that if and only if $U = V$, $f_{U+N_s|X_1}(y|\mathbf{x}_1) = f_{V+N_s|X_1}(y|\mathbf{x}_1)$ holds true. Hence, we have proved the non-manipulability of the wireless network having noisy BC channel. The above-mentioned proof indicates our work could be applicable for the wireless network where MA and BC channels are both noisy. For simplicity, we give the detailed proof for the wireless network where only MA channel is noisy.

V. PROOF OF THEOREM 1: PREPARATIONS

In order to prove one convergence property of $F_{V^n|\bar{U}^n}^{(n')}(t|u)$ given later, we also define

$$\begin{aligned} \Delta F_{i,i',j,j'} &= P_{X_i, X_{i'}|V_i, V_{i'}, \bar{U}_i, \bar{U}_{i'}} \{ \mathbf{x}_1, \mathbf{x}_1 | v_i \leq t, v_{i'} \leq t', \bar{u}_j, \bar{u}_{j'} \} - P_{X_1|\bar{U}}(\mathbf{x}_1|\bar{u}_j) P_{X_{i'}|V_i, V_{i'}, \bar{U}_i, \bar{U}_{i'}} \{ \mathbf{x}_1 | v_i \leq t, v_{i'} \leq t, \bar{u}_j, \bar{u}_{j'} \} \\ &- P_{X_i|V_i, V_{i'}, \bar{U}_i, \bar{U}_{i'}} \{ \mathbf{x}_1 | v_i \leq t, v_{i'} \leq t, \bar{u}_j, \bar{u}_{j'} \} P_{X_1|\bar{U}}(\mathbf{x}_1|\bar{u}_{j'}) + P_{X_1|\bar{U}}(\mathbf{x}_1|\bar{u}_j) P_{X_1|\bar{U}}(\mathbf{x}_1|\bar{u}_{j'}) \end{aligned}$$

where $i \neq i', j, j' = 1, 2, \dots, n', i, i' = 1, 2, \dots, n$. Then, we have the following lemma.

Lemma 1. *If we choose $\alpha_1 = -\beta_1$, and*

$$\frac{1}{n' - 2} = \max \left\{ \underbrace{P_{X_1|U}(1|-\beta_1)}_{\xi_1(\beta_1)}, \underbrace{1 - P_{X_1|U}(1|\beta_1)}_{\xi_2(\beta_1)}, \underbrace{P_{X_1|U}(-1|\beta_1)}_{\xi_3(\beta_1)}, \underbrace{1 - P_{X_1|U}(-1|-\beta_1)}_{\xi_4(\beta_1)} \right\}. \quad (19)$$

then upon this setup, besides the statement of (1) is true, there also exist a upper bound for $F_{i,i',j,j'}$ across $j, j' = 1, 2, \dots, n'$, $i, i' = 1, 2, \dots, n$, $i \neq i'$. This upper bound only depends on n' rather than n . Hence, we denote the upper bound as $\Delta F_{max}(n')$.

$\Delta F_{max}(n')$ has property that

$$\lim_{n' \rightarrow \infty} (\beta_1 - \alpha_1)^k n' F_{max}(n') \rightarrow 0 \quad (20)$$

where k is a bounded integer.

Proof: To that end, we first reshape $F_{i,i',j,j'}$ as

$$\begin{aligned} \Delta F_{i,i',j,j'} &= P_{X_{1,i'}|V_i, V_{i'}, \tilde{v}_i, \tilde{v}_{i'}} \{x_1 | v_i \leq t, v_{i'} \leq t, \tilde{u}_j, \tilde{u}_{j'}\} \left(P_{X_{1,i}|V_i, V_{i'}, \tilde{v}_i, \tilde{v}_{i'}} \{x_1 | v_i \leq t, v_{i'} \leq t, \tilde{u}_j, \tilde{u}_{j'}\} - P_{X_1|\tilde{U}}(x_1 | \tilde{u}_j) \right) \\ &- P_{X_1|\tilde{U}}(x_1 | \tilde{u}_{j'}) \left(P_{X_{1,i}|V_i, V_{i'}, \tilde{v}_i, \tilde{v}_{i'}} \{x_1 | v_i \leq t, v_{i'} \leq t, \tilde{u}_j, \tilde{u}_{j'}\} - P_{X_1|\tilde{U}}(x_1 | \tilde{u}_j) \right) \\ &= \left(P_{X_{1,i}|V_i, V_{i'}, \tilde{v}_i, \tilde{v}_{i'}} \{x_1 | v_i \leq t, v_{i'} \leq t, \tilde{u}_j, \tilde{u}_{j'}\} - P_{X_1|\tilde{U}}(x_1 | \tilde{u}_j) \right) \left(P_{X_{1,i'}|V_i, V_{i'}, \tilde{v}_i, \tilde{v}_{i'}} \{x_1 | v_i \leq t, v_{i'} \leq t, \tilde{u}_j, \tilde{u}_{j'}\} - P_{X_1|\tilde{U}}(x_1 | \tilde{u}_{j'}) \right), \end{aligned} \quad (21)$$

where the first equality follows X_1^n is i.i.d sequence, $X_{1,i'}$ is independent on $X_{1,i}$ due to $i \neq i'$. To bound $\Delta F_{i,i',j,j'}$, we have

$$P_{X_1|\tilde{U}}(x_1 | \tilde{u}_j) = \frac{\int_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1 | u) f_U(u) du}{\int_{u \in \mathcal{B}(\tilde{u}_j)} f_U(u) du} \quad (22)$$

which indicates

$$\min_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1 | u) \leq P_{X_1|\tilde{U}}(x_1 | \tilde{u}_j) \leq \max_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1 | u). \quad (23)$$

On the other hand, since

$$\begin{aligned} &P_{X_{1,i}|V_i, V_{i'}, \tilde{v}_i, \tilde{v}_{i'}} \{x_1 | v_i \leq t, v_{i'} \leq t, \tilde{u}_j, \tilde{u}_{j'}\} = \\ &\frac{\int_{-\infty}^t \int_{-\infty}^t \int_{u \in \mathcal{B}(\tilde{u}_j)} \int_{u' \in \mathcal{B}(\tilde{u}_{j'})} P_{X_1|U}(x_1 | u) f_{U_i, U_{i'}|V_i, V_{i'}}(u, u' | v, v') f_{V_i, V_{i'}}(v_i, v_{i'}) du du' dv dv'}{\int_{-\infty}^t \int_{-\infty}^t \int_{u \in \mathcal{B}(\tilde{u}_j)} \int_{u' \in \mathcal{B}(\tilde{u}_{j'})} f_{U_i, U_{i'}|V_i, V_{i'}}(u, u' | v, v') f_{V_i, V_{i'}}(v_i, v_{i'}) du du' dv dv'} \end{aligned}$$

we have

$$\min_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1 | u) \leq P_{X_{1,i}|V_i, V_{i'}, \tilde{v}_i, \tilde{v}_{i'}} \{x_1 | v_i \leq t, v_{i'} \leq t, \tilde{u}_j, \tilde{u}_{j'}\} \leq \max_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1 | u) \quad (24)$$

Jointly considering (23) and (24), we have

$$\left| P_{X_1|\tilde{U}}(x_1 | \tilde{u}_j) - P_{X_{1,i}|V_i, V_{i'}, \tilde{v}_i, \tilde{v}_{i'}} \{x_1 | v_i \leq t, v_{i'} \leq t, \tilde{u}_j, \tilde{u}_{j'}\} \right| < \max_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1 | u) - \min_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1 | u) \quad (25)$$

Following the similar logic from (23) to (25), we also have

$$\left| P_{X_{1,i'}|V_i, V_{i'}, \tilde{v}_i, \tilde{v}_{i'}} \{x_1 | v_i \leq t, v_{i'} \leq t, \tilde{u}_j, \tilde{u}_{j'}\} - P_{X_1|\tilde{U}}(x_1 | \tilde{u}_{j'}) \right| \leq \max_{u \in \mathcal{B}(\tilde{u}_{j'})} P_{X_1|U}(x_1 | u) - \min_{u \in \mathcal{B}(\tilde{u}_{j'})} P_{X_1|U}(x_1 | u) \quad (26)$$

Substituting (25) and (26) into (21), we have

$$|\Delta F_{i,i',j,j'}| < \left(\max_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1 | u) - \min_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1 | u) \right) \left(\max_{u \in \mathcal{B}(\tilde{u}_{j'})} P_{X_1|U}(x_1 | u) - \min_{u \in \mathcal{B}(\tilde{u}_{j'})} P_{X_1|U}(x_1 | u) \right), \quad (27)$$

and

$$\max_{j,j'=1,2,\dots,n', i,i'=1,2,\dots,n, i \neq i'} \Delta F_{i,i',j,j'} < \max_{j=1,2,\dots,n'} \left(\max_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1|u) - \min_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1|u) \right)^2 \triangleq \Delta F_{max}(n') \quad (28)$$

Then, we proceed to focus on the property of $F_{max}(n')$. Revisiting the system model, we have

$$P_{X_1|U}(1|u) = \frac{1}{2 + \exp(4u - 4) + \exp(-4u - 4)} + \frac{1}{2 \exp(4 - 4u) + 1 + \exp(-8u)} \quad (29)$$

and

$$P_{X_1|U}(-1|u) = \frac{1}{2 + \exp(4u - 4) + \exp(-4u - 4)} + \frac{1}{2 \exp(4 + 4u) + 1 + \exp(8u)} \quad (30)$$

According to $P_{X_1|U}(1|u)$, $P_{X_1|U}(-1|u)$ and (19), we get $\lim_{n' \rightarrow \infty} \frac{\beta_1 - \alpha_1}{n' - 2} = 0$. The statement of (1) is thus proved. Furthermore, in (19) we have $\lim_{\beta_1 \rightarrow \infty} (2\beta_1)^{k+2} \xi_i(\beta_1) = 0$ and $\lim_{\beta_1 \rightarrow \infty} (2\beta_1)^k \xi_i(\beta_1) = 0$ for $i = 1, 2, 3, 4$. With these properties on limitation, for $j = 2, 3, \dots, n' - 1$, we have

$$\left(\max_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1|u) - \min_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1|u) \right)^2 \leq P'^2_{X_1|U}(x_1|u'_j) \left(\frac{2\beta_1}{n' - 2} \right)^2 \quad (31)$$

where $u'_j \in \mathcal{B}(\tilde{u}_j)$, $P'_{X_1|U}(x_1|u)$ is derived function of $P_{X_1|U}(x_1|u)$. The maximum of $P'_{X_1|U}(x_1|u)$ in $(-\infty, +\infty)$ is bounded.

$$\begin{aligned} \lim_{n' \rightarrow \infty} (2\beta_1)^k n' \left(\max_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1|u) - \min_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1|u) \right)^2 &\leq \lim_{n' \rightarrow \infty} (2\beta_1)^k n' P'^2_{X_1|U}(x_1|u'_j) \left(\frac{2\beta_1}{n' - 2} \right)^2 \\ &= \lim_{\beta_1 \rightarrow \infty} P'^2_{X_1|U}(x_1|u'_j) \frac{(2\beta_1)^{k+2}}{n' - 2} = 0 \end{aligned} \quad (32)$$

where the last equality follows the fact that the maximum of $P'_{X_1|U}(x_1|u)$ in $(-\infty, +\infty)$ is bounded and in (19) we have $\lim_{\beta_1 \rightarrow \infty} (2\beta_1)^{k+2} \xi_i(\beta_1) = 0$ for $i = 1, 2, 3, 4$. Then, for $j = 1$ and $j = n'$,

$$\left(\max_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1|u) - \min_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1|u) \right)^2 \leq \left(\frac{1}{n' - 2} \right)^2 \quad (33)$$

Hence, for $j = 1$ and $j = n'$, we have

$$\lim_{n' \rightarrow \infty} (2\beta_1)^k n' \left(\max_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1|u) - \min_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1|u) \right)^2 \leq \lim_{n' \rightarrow \infty} (2\beta_1)^k n' \left(\frac{1}{n' - 2} \right)^2 = \lim_{\beta_1 \rightarrow \infty} \frac{(2\beta_1)^k}{n' - 2} = 0 \quad (34)$$

where the last equality follows in (19) we have $\lim_{\beta_1 \rightarrow \infty} (2\beta_1)^k \xi_i(\beta_1) = 0$ for $i = 1, 2, 3, 4$. Combining (32) and (34), we get for each $j = 1, 2, \dots, n'$,

$$\lim_{n' \rightarrow \infty} (2\beta_1)^k n' \left(\max_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1|u) - \min_{u \in \mathcal{B}(\tilde{u}_j)} P_{X_1|U}(x_1|u) \right)^2 = 0 \quad (35)$$

Finally, based on the definition of $F_{max}(n')$ in (28), the statement of this lemma is immediate. \blacksquare

Upon this lemma, we have the following convergence property.

Lemma 2. For arbitrary t , sufficiently small μ and $\varepsilon \leq \frac{\mu}{2n'}$,

$$\Pr \left\{ \left| F_{V^n|X_1}^n(t|x_1) - \int_{-\infty}^{+\infty} f_{U|X_1}(u|x_1) F_{V^n|\tilde{u}_n}^{(n')}(t|u) du \right| > \mu \right\}$$

$$< \frac{4}{\mu^2} \left(\frac{n'^2}{n} + \frac{1}{\Pr \left\{ (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\}} \frac{\Delta F_{max}(n')}{(P_{X_1}(\mathbf{x}) - \varepsilon)^2} \right) + \Pr \left\{ (X_1^n, \tilde{U}^n) \notin T_{[X_1, \tilde{U}]_\varepsilon}^n \right\} \quad (36)$$

where

$$\mathcal{A}_\varepsilon = \left\{ (x_1^n, \tilde{u}^n) : \left| P_{\tilde{U}|X_1}(\tilde{u}|\mathbf{x}_1) - \frac{N(\tilde{\mathbf{u}}|\tilde{U}^n) P_{X_1|\tilde{U}}(\mathbf{x}_1|\tilde{\mathbf{u}})}{N(\mathbf{x}_1|X_1^n)} \right| < \varepsilon, \left| P_{X_1}(\mathbf{x}_1) - \frac{N(\mathbf{x}_1|X_1^n)}{n} \right| < \varepsilon \right\}. \quad (37)$$

Proof: Notice that

$$\begin{aligned} & \Pr \left\{ \left| F_{V^n|X_1^n}^n(t|\mathbf{x}_1) - \int_{-\infty}^{+\infty} f_{U|X_1}(u|\mathbf{x}_1) F_{V^n|\tilde{U}^n}^{(n')}(t|u) du \right| > \mu \right\} \\ & < \Pr \left\{ \left| F_{V^n|X_1^n}^n(t|\mathbf{x}_1) - \int_{-\infty}^{+\infty} f_{U|X_1}(u|\mathbf{x}_1) F_{V^n|\tilde{U}^n}^{(n')}(t|u) du \right| > \mu \mid (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} + \Pr \left\{ (X_1^n, \tilde{U}^n) \notin \mathcal{A}_\varepsilon \right\} \end{aligned} \quad (38)$$

The proof of this lemma is equivalent to prove the two items in the right side of (38) both approach to 0. Firstly notice that after n' , α_1 and β_1 are chosen and fixed properly,

$$\Pr \left\{ (X_1^n, \tilde{U}^n) \notin \mathcal{A}_\varepsilon \right\} \rightarrow 0 \quad (39)$$

as n approaches to infinity. Then, focusing on the first item in the right side of (38), $(X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon$ indicates $N(\tilde{\mathbf{u}}_j|\tilde{U}^n) > 0$ for all $j = 1, 2, \dots, n'$. Hence, $\frac{\sum_{i=1}^n \mathbf{1}_i(v_i \leq t) \mathbf{1}_i(\tilde{u}_i = \tilde{u}_j)}{N(\tilde{\mathbf{u}}_j|\tilde{U}^n)}$ is well-defined for all $j = 1, 2, \dots, n'$. Under the condition $(X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon$, we have

$$\begin{aligned} & \left| F_{V^n|X_1^n}^n(t|\mathbf{x}_1) - \int_{-\infty}^{+\infty} f_{U|X_1}(u|\mathbf{x}_1) F_{V^n|\tilde{U}^n}^{(n')}(t|u) du \right| \\ & = \left| \frac{\sum_{i=1}^n \mathbf{1}_i(v_i \leq t) \mathbf{1}_i(x_i = \mathbf{x}_1)}{N(\mathbf{x}_1|X_1^n)} - \sum_{j=1}^{n'} \frac{\sum_{i=1}^n \mathbf{1}_i(v_i \leq t) \mathbf{1}_i(\tilde{u}_i = \tilde{u}_j)}{N(\tilde{\mathbf{u}}_j|\tilde{U}^n)} \int_{\mathcal{B}(\tilde{u}_j)} f_{U|X_1}(u|\mathbf{x}_1) du \right| \\ & = \left| \frac{\sum_{j=1}^{n'} \sum_{i=1}^n \mathbf{1}_i(v_i \leq t) \mathbf{1}_i(x_i = \mathbf{x}_1) \mathbf{1}_i(\tilde{u}_i = \tilde{u}_j)}{N(\mathbf{x}_1|X_1^n)} - \sum_{j=1}^{n'} \frac{\sum_{i=1}^n \mathbf{1}_i(v_i \leq t) \mathbf{1}_i(\tilde{u}_i = \tilde{u}_j)}{N(\tilde{\mathbf{u}}_j|\tilde{U}^n)} P_{\tilde{U}|X_1}(\tilde{u}_j|\mathbf{x}_1) \right| \end{aligned} \quad (40)$$

Substituting (40) into the first item in the right side of (38), it becomes

$$\Pr \left\{ \left| \frac{\sum_{j=1}^{n'} \sum_{i=1}^n \mathbf{1}_i(v_i \leq t) \mathbf{1}_i(x_i = \mathbf{x}_1) \mathbf{1}_i(\tilde{u}_i = \tilde{u}_j)}{N(\mathbf{x}_1|X_1^n)} - \sum_{j=1}^{n'} \frac{\sum_{i=1}^n \mathbf{1}_i(v_i \leq t) \mathbf{1}_i(\tilde{u}_i = \tilde{u}_j)}{N(\tilde{\mathbf{u}}_j|\tilde{U}^n)} P_{\tilde{U}|X_1}(\tilde{u}_j|\mathbf{x}_1) \right| > \mu \mid (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} \quad (41)$$

$$\begin{aligned} & < \Pr \left\{ \left| \underbrace{\sum_{j=1}^{n'} \left(\frac{\sum_{i=1}^n \mathbf{1}_i(v_i \leq t) \mathbf{1}_i(x_i = \mathbf{x}_1) \mathbf{1}_i(\tilde{u}_i = \tilde{u}_j)}{n(P_{X_1}(\mathbf{x}) - \varepsilon)} - \frac{\sum_{i=1}^n \mathbf{1}_i(v_i \leq t) \mathbf{1}_i(\tilde{u}_i = \tilde{u}_j) P_{X_1|\tilde{U}}(\mathbf{x}_1|\tilde{u}_j)}{n(P_{X_1}(\mathbf{x}) - \varepsilon)} \right)}_{H_j} \right| > \frac{\mu}{2} \mid (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} \\ & + \sum_{j=1}^{n'} \Pr \left\{ \left| \frac{\sum_{i=1}^n \mathbf{1}_i(v_i \leq t) \mathbf{1}_i(\tilde{u}_i = \tilde{u}_j)}{N(\tilde{\mathbf{u}}_j|\tilde{U}^n)} P_{\tilde{U}|X_1}(\tilde{u}_j|\mathbf{x}_1) - \frac{\sum_{i=1}^n \mathbf{1}_i(v_i \leq t) \mathbf{1}_i(\tilde{u}_i = \tilde{u}_j) P_{X_1|\tilde{U}}(\mathbf{x}_1|\tilde{u}_j)}{N(\mathbf{x}_1|X_1^n)} \right| > \frac{\mu}{2n'} \mid (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} \end{aligned}$$

The second item in the right of (41) can be further bound as

$$\begin{aligned} & \Pr \left\{ \left| \frac{\sum_{i=1}^n \mathbf{1}_i(v_i \leq t) \mathbf{1}_i(\tilde{u}_i = \tilde{u}_j)}{N(\tilde{\mathbf{u}}_j|\tilde{U}^n)} P_{\tilde{U}|X_1}(\tilde{u}_j|\mathbf{x}_1) - \frac{\sum_{i=1}^n \mathbf{1}_i(v_i \leq t) \mathbf{1}_i(\tilde{u}_i = \tilde{u}_j) P_{X_1|\tilde{U}}(\mathbf{x}_1|\tilde{u}_j)}{N(\mathbf{x}_1|X_1^n)} \right| > \frac{\mu}{2n'} \mid (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} \\ & \leq \Pr \left\{ \left| P_{\tilde{U}|X_1}(\tilde{u}_j|\mathbf{x}_1) - \frac{N(\tilde{\mathbf{u}}|\tilde{U}^n) P_{X_1|\tilde{U}}(\mathbf{x}_1|\tilde{u}_j)}{N(\mathbf{x}_1|X_1^n)} \right| > \frac{\mu}{2n'} \mid (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} = 0 \end{aligned} \quad (42)$$

where the last equality follows the definition of \mathcal{A}_ε and set of $\varepsilon \leq \frac{\mu}{2n'}$. From (42), the second item in the right of (41) equals to 0. Then, we proceed to bound the first item in the right of (41) as

$$\Pr \left\{ \left| \sum_{j=1}^{n'} H_j \right| > \frac{\mu}{2} \mid (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} < \frac{4}{\mu^2} E_{\mathcal{A}_\varepsilon} \left| \sum_{j=1}^{n'} H_j \right|^2 = \frac{4}{\mu^2} \sum_{j=1}^{n'} \sum_{j'=1}^{n'} E_{\mathcal{A}_\varepsilon} (H_j H_{j'}) \quad (43)$$

which follows the Chebyshev theorem. $E_{\mathcal{A}_\varepsilon}(\cdot)$ indicates the expectation of its input conditioned on $(X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon$.

$$\begin{aligned} & E_{\mathcal{A}_\varepsilon} (H_j H_{j'}) = \\ & \frac{E_{\mathcal{A}_\varepsilon} \left(\sum_{i=1}^n \mathbf{1}_i (v_i \leq t) \mathbf{1}_i (\tilde{u}_i = \tilde{u}_j) \left(\mathbf{1}_i (x_{1,i} = x_1) - P_{X_1|\tilde{U}}(x_1|\tilde{u}_j) \right) \right) \left(\sum_{i=1}^n \mathbf{1}_i (v_i \leq t) \mathbf{1}_i (\tilde{u}_i = \tilde{u}_{j'}) \left(\mathbf{1}_i (x_{1,i} = x_1) - P_{X_1|\tilde{U}}(x_1|\tilde{u}_{j'}) \right) \right)}{n^2 (P_{X_1}(x) - \varepsilon)^2} \\ & \leq \frac{\sum_{i=1}^n E_{\mathcal{A}_\varepsilon} \left\{ \mathbf{1}_i (v_i \leq t) \mathbf{1}_i (\tilde{u}_i = \tilde{u}_j) \left(\mathbf{1}_i (x_{1,i} = x_1) - P_{X_1|\tilde{U}}(x_1|\tilde{u}_j) \right) \mathbf{1}_i (\tilde{u}_i = \tilde{u}_{j'}) \left(\mathbf{1}_i (x_{1,i} = x_1) - P_{X_1|\tilde{U}}(x_1|\tilde{u}_{j'}) \right) \right\}}{n^2 (P_{X_1}(x) - \varepsilon)^2} \\ & + \frac{E \sum_{i=1}^n \sum_{i=1, i' \neq i}^n \mathbf{1}_i (v_i \leq t) \mathbf{1}_i (\tilde{u}_i = \tilde{u}_j) \mathbf{1}_{i'} (v_{i'} \leq t) \mathbf{1}_{i'} (\tilde{u}_{i'} = \tilde{u}_{j'}) \left(\mathbf{1}_i (x_{1,i} = x_1) - P_{X_1|\tilde{U}}(x_1|\tilde{u}_j) \right) \left(\mathbf{1}_{i'} (x_{1,i'} = x_1) - P_{X_1|\tilde{U}}(x_1|\tilde{u}_{j'}) \right)}{\Pr \left\{ (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} n^2 (P_{X_1}(x) - \varepsilon)^2} \\ & \leq \frac{1}{n} + \frac{\sum_{i=1}^n \sum_{i=1, i' \neq i}^n E \{ \mathbf{1}_i (v_i \leq t) \mathbf{1}_i (\tilde{u}_i = \tilde{u}_j) \mathbf{1}_{i'} (v_{i'} \leq t) \mathbf{1}_{i'} (\tilde{u}_{i'} = \tilde{u}_{j'}) \mathbf{1}_i (x_{1,i} = x_1) \mathbf{1}_{i'} (x_{1,i'} = x_1) \}}{\Pr \left\{ (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} n^2 (P_{X_1}(x) - \varepsilon)^2} \\ & - \frac{\sum_{i=1}^n \sum_{i=1, i' \neq i}^n E \{ \mathbf{1}_i (v_i \leq t) \mathbf{1}_i (\tilde{u}_i = \tilde{u}_j) \mathbf{1}_{i'} (v_{i'} \leq t) \mathbf{1}_{i'} (\tilde{u}_{i'} = \tilde{u}_{j'}) \mathbf{1}_{i'} (x_{1,i'} = x_1) \} P_{X_1|\tilde{U}}(x_1|\tilde{u}_j)}{\Pr \left\{ (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} n^2 (P_{X_1}(x) - \varepsilon)^2} \\ & - \frac{\sum_{i=1}^n \sum_{i=1, i' \neq i}^n E \{ \mathbf{1}_i (v_i \leq t) \mathbf{1}_i (\tilde{u}_i = \tilde{u}_j) \mathbf{1}_{i'} (v_{i'} \leq t) \mathbf{1}_{i'} (\tilde{u}_{i'} = \tilde{u}_{j'}) \mathbf{1}_i (x_{1,i} = x_1) \} P_{X_1|\tilde{U}}(x_1|\tilde{u}_{j'})}{\Pr \left\{ (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} n^2 (P_{X_1}(x) - \varepsilon)^2} \\ & + \frac{\sum_{i=1}^n \sum_{i=1, i' \neq i}^n E \{ \mathbf{1}_i (v_i \leq t) \mathbf{1}_i (\tilde{u}_i = \tilde{u}_j) \mathbf{1}_{i'} (v_{i'} \leq t) \mathbf{1}_{i'} (\tilde{u}_{i'} = \tilde{u}_{j'}) \} P_{X_1|\tilde{U}}(x_1|\tilde{u}_j) P_{X_1|\tilde{U}}(x_1|\tilde{u}_{j'})}{\Pr \left\{ (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} n^2 (P_{X_1}(x) - \varepsilon)^2} \\ & = \frac{1}{n} + \frac{\sum_{i=1}^n \sum_{i=1, i' \neq i}^n P_{V_i, V_{i'}, \tilde{U}_i, \tilde{U}_{i'}} \{ v_i \leq t, v_{i'} \leq t, \tilde{u}_j, \tilde{u}_{j'} \} \Delta F_{i, i', j, j'}}{\Pr \left\{ (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} n^2 (P_{X_1}(x) - \varepsilon)^2} \quad (44) \end{aligned}$$

Substituting (44) into (43), we have

$$\begin{aligned} & \Pr \left\{ \left| \sum_{j=1}^{n'} H_j \right| > \frac{\mu}{2} \mid (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} < \frac{4}{\mu^2} \sum_{j=1}^{n'} \sum_{j'=1}^{n'} \left\{ \frac{1}{n} + \frac{\sum_{i=1}^n \sum_{i=1, i' \neq i}^n P_{V_i, V_{i'}, \tilde{U}_i, \tilde{U}_{i'}} \{ v_i \leq t, v_{i'} \leq t, \tilde{u}_j, \tilde{u}_{j'} \} \Delta F_{max}(n')}{\Pr \left\{ (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} n^2 (P_{X_1}(x) - \varepsilon)^2} \right\} \\ & \leq \frac{4}{\mu^2} \left(\frac{n'^2}{n} + \frac{\sum_{i=1}^n \sum_{i=1, i' \neq i}^n \sum_{j=1}^{n'} \sum_{j'=1}^{n'} P_{\tilde{U}} \{ \tilde{u}_j \} P_{\tilde{U}} \{ \tilde{u}_{j'} \} \Delta F_{max}(n')}{\Pr \left\{ (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} n^2 (P_{X_1}(x) - \varepsilon)^2} \right) \\ & \leq \frac{4}{\mu^2} \left(\frac{n'^2}{n} + \frac{1}{\Pr \left\{ (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} (P_{X_1}(x) - \varepsilon)^2} \Delta F_{max}(n') \right) \quad (45) \end{aligned}$$

From (45) (42) (41) and (38), we have

$$\begin{aligned} & \Pr \left\{ \left| F_{V^n|X_1^n}^n(t|x_1) - \int_{-\infty}^{+\infty} f_{U|X_1}(u|x_1) F_{V^n}^{(n')}(\tilde{U}^n)(t|u) du \right| > \mu \right\} \\ & < \frac{4}{\mu^2} \left(\frac{n'^2}{n} + \frac{1}{\Pr \left\{ (X_1^n, \tilde{U}^n) \in \mathcal{A}_\varepsilon \right\} (P_{X_1}(x) - \varepsilon)^2} \Delta F_{max}(n') \right) + \Pr \left\{ (X_1^n, \tilde{U}^n) \notin T_{[X_1, \tilde{U}]_\varepsilon}^n \right\} \quad (46) \end{aligned}$$

The proof is finished. ■

Upon the aforementioned lemmas, the following assertion is immediate.

Lemma 3. For sequence $t_1, t_2, \dots, t_{n'-1}$, we have

1) Fix μ to arbitrary small value, there has

$$\lim_{n \rightarrow \infty, n' \rightarrow \infty} \Pr \left\{ \frac{\beta_1 - \alpha_1}{n' - 2} \sum_{j=1}^{n'-1} \left| F_{V^n | X_1^n}^n(t_j | \mathbf{x}_1) - \int_{-\infty}^{+\infty} f_{U | X_1}(u | \mathbf{x}_1) F_{V^n | \tilde{U}^n}^{(n')}(t_j | u) du \right| > \mu \right\} = 0. \quad (47)$$

2) Fix n' to arbitrary large value, and ϵ to arbitrary small value, there has

$$\Pr \left\{ \frac{\beta_1 - \alpha_1}{n' - 2} \sum_{j=1}^{n'-1} \left| F_{V^n | X_1^n}^n(t_j | \mathbf{x}_1) - \int_{-\infty}^{+\infty} f_{U | X_1}(u | \mathbf{x}_1) F_{V^n | \tilde{U}^n}^{(n')}(t_j | u) du \right| > \mu_{n'} \right\} \leq \epsilon \quad (48)$$

where n approaches to infinity, $\lim_{n' \rightarrow \infty} \mu_{n'} = 0$.

Proof: For arbitrary small μ , we have

$$\begin{aligned} & \Pr \left\{ \frac{\beta_1 - \alpha_1}{n' - 2} \sum_{j=1}^{n'-1} \left| F_{V^n | X_1^n}^n(t_j | \mathbf{x}_1) - \int_{-\infty}^{+\infty} f_{U | X_1}(u | \mathbf{x}_1) F_{V^n | \tilde{U}^n}^{(n')}(t_j | u) du \right| > \mu \right\} \\ & \leq \sum_{j=1}^{n'-1} \Pr \left\{ \left| F_{V^n | X_1^n}^n(t_j | \mathbf{x}_1) - \int_{-\infty}^{+\infty} f_{U | X_1}(u | \mathbf{x}_1) F_{V^n | \tilde{U}^n}^{(n')}(t_j | u) du \right| > \frac{\mu(n' - 2)}{(\beta_1 - \alpha_1)(n' - 1)} \right\} \\ & \leq \underbrace{\frac{4(n' - 1)^2}{\mu^2(n' - 2)^2} \left(\frac{n'^3 (\beta_1 - \alpha_1)^2}{n} + \frac{(\beta_1 - \alpha_1)^2 n'}{\Pr \left\{ (X_1^n, \tilde{U}^n) \in \mathcal{A}_\epsilon \right\} (P_{X_1}(\mathbf{x}) - \epsilon)^2} \Delta F_{max}(n') \right)}_{\Theta(\mu, n, n')} + n' \Pr \left\{ (X_1^n, \tilde{U}^n) \notin T_{[X_1, \tilde{U}]_\epsilon}^n \right\} \end{aligned}$$

where the last inequality follows lemma 2. From lemma 1, $\lim_{n' \rightarrow \infty} (\beta_1 - \alpha_1)^2 n' \Delta F_{max}(n') = 0$, then we have

$$\lim_{n \rightarrow \infty, n' \rightarrow \infty} \Theta(\mu, n, n') = 0 \quad (49)$$

for arbitrary value of μ . The first statement of this lemma is proved. Furthermore, according to the expression of $\Theta(\mu, n, n')$,

$\Theta(\mu, n, n') < \epsilon$ would be yielded by

$$\frac{4(n' - 1)^2 (\beta_1 - \alpha_1)^2 n' \Delta F_{max}(n')}{(n' - 2)^2 (P_{X_1}(\mathbf{x}) - \epsilon)^2 \epsilon_1} \leq \mu^2 \quad (50)$$

$n \rightarrow \infty$, $\epsilon_1 < \epsilon$. Hence, by setting

$$\frac{4(n' - 1)^2 (\beta_1 - \alpha_1)^2 n' \Delta F_{max}(n')}{(n' - 2)^2 (P_{X_1}(\mathbf{x}) - \epsilon)^2 \epsilon_1} = \mu_{n'}^2 \quad (51)$$

the second statement can be proved. ■

Lemma 4. There exist n_0 , by which for arbitrary $n' > n_0$ we have sequence $\alpha_1 = t_1 < t_2 < \dots < t_{n'-1} = \beta_1$ making equation

$$\frac{\beta_1 - \alpha_1}{n' - 2} \sum_{j=1}^{n'-1} \left| \int_{-\infty}^{t_j} f_{U | X_1}(u | \mathbf{x}_1) du - \sum_i^{n'} P_{\tilde{U} | X_1}(\tilde{u}_i) w_{i,j} \right|^2 = 0$$

have single solution that $w_{i,j} = \Phi(t_j - \tilde{u}_i)$, $i = 1, \dots, n'$, $j = 1, \dots, n' - 1$ in the domain $\mathcal{D} = \{w_{i,j} : 0 \leq w_{i,1} \leq w_{i,2} \leq \dots \leq w_{i,\tilde{n}} \leq 1, i = 1, \dots, n', j = 1, \dots, n' - 1\}$.

Proof: Let us choose $t_i = \tilde{u}_j$, for $j = 1, 2, \dots, n' - 1$. Then, the proof follows the logic of the proof that wireless channel is non-manipulable. ■

VI. PROOF OF THEOREM 1

Let us go back to the proof of theorem 1. With the aforementioned lemmas, we will show the decision statistic $D^n = \frac{1}{n'-2} \sum_{j=1}^{n'-1} \left| F_{V^n|X_1^n}^n(t_j|x_1) - \int_{-\infty}^{t_j} f_{U|X_1}(u|x_1) du \right|$ simultaneously satisfies the properties stated by theorem 1.

From aforementioned work, it is not hard to find many variables, such as α_1, β_1 and \tilde{u} , depend on n' . For easy description, n' does not appear in these notations. However, the dependency between n' and these variables will be utilized in the proof given below. Hence, these notations are written with a superscript n' or possible value of n' so as to highlight the dependency on n' . To be more specific, n' takes value from sequence $n'_1, n'_2, \dots, \infty$, there has $\frac{\Delta_{n'_k}}{\Delta_{n'_{k-1}}} = k$ and $\frac{\Delta_{n'_k}}{\Delta_{n'_1}} = s_k$, where $\Delta_{n'_k} = \frac{\alpha_1^{(n'_k)} - \beta_1^{(n'_k)}}{n'_k - 2}$. Upon n' , we define function $M^{(n')} \left(W^{(n')} \right) = \frac{\beta_1^{(n')} - \alpha_1^{(n')}}{n'-2} \sum_{j=1}^{n'-1} \left| \int_{-\infty}^{t_j} f_{U|X_1}(u|x_1) du - \sum_i^{n'} P_{\tilde{U}|X_1} \left(\tilde{u}_i^{(n')} \right) w_{i,j}^{(n')} \right|^2$ where $W^{(n')}$ is a matrix variable $\left[W^{(n')} \right]_{i,j} = w_{i,j}^{(n')}, i = 1, 2, \dots, n', j = 1, 2, \dots, n'-1$. As stated by lemma 4, $M^{(n')} \left(W^{(n')} \right) = 0$ has single solution in the point that $W_0^{(n')}$ defined as $\left[W_0^{(n')} \right]_{i,j} = \Phi \left(t_j^{(n')} - \tilde{u}_i^{(n')} \right), i = 1, \dots, n', j = 1, \dots, n'-1$ in the domain $\mathcal{D}^{(n')} = \{ W^{(n')} : 0 \leq w_{i,1}^{(n')} \leq w_{i,2}^{(n')} \leq \dots \leq w_{i,n'-1}^{(n')} \leq 1, i = 1, \dots, n' \}$.

Lemma 5. *If n' is sufficient large and $W^{(n')} \in \mathcal{D}_s^{(n')}$, where $\mathcal{D}_s^{(n')} = \{ W^{(n')} : |W^{(n')} - W_0^{(n')}| \geq \delta, W^{(n')} \in \mathcal{D}^{(n')} \}$, then $M^{(n')} \left(W^{(n')} \right)$ has positive infimum across $\mathcal{D}_s^{(n')}$, denoted as $\lambda^{(n')}(\delta)$. Moreover, $\lambda^{(n')}(\delta) \rightarrow 0, \delta \rightarrow 0$.*

Proof: Using the assertion given by lemma 4, the proof of this lemma follows our previous work. ■

Lemma 6. *If the wireless channel is non-manipulable, then for arbitrary small δ , there exist sufficient large n_0 , such that for any $n' > n_0$, $\lambda^{(n')}(\delta) \geq \mu_{n'}$.*

Proof: First notice that for arbitrary $W_f^{(n')} \in \mathcal{D}^{(n')}$, there exist a CDF function $F(t|u)$ which satisfies

$$\left[W_f^{(n')} \right]_{i,j} = \frac{\int_{u \in \mathcal{B}(\tilde{u}_i^{(n')})} F(t_j^{(n')}|u) f_{U|X}(u) du}{\int_{u \in \mathcal{B}(\tilde{u}_i^{(n')})} f_{U|X}(u) du}. \quad (52)$$

Then, according to the condition that $\frac{\Delta_{n'_k}}{\Delta_{n'_{k-1}}} = k$ and $\frac{\Delta_{n'_k}}{\Delta_{n'_1}} = s_k$, fixing $F(t|u)$, $|W_f^{(n')} - W_0^{(n')}| \geq \delta$ implies $|W_f^{(n'_k)} - W_0^{(n'_k)}| \geq \delta$ for $n'_k > n'$. For the sake of proof, we define a function set

$$\mathcal{F} = \left\{ F(t|u) : \lim_{n' \rightarrow \infty} \sum_{j=1}^{n'-1} \sum_{i=1}^{n'} \left| \frac{\int_{u \in \mathcal{B}(\tilde{u}_i^{(n')})} F(t_j^{(n')}|u) f_{U|X}(u) du}{\int_{u \in \mathcal{B}(\tilde{u}_i^{(n')})} f_{U|X}(u) du} - \Phi \left(t_j^{(n')} - \tilde{u}_i^{(n')} \right) \right| \geq \delta \right\}. \quad (53)$$

Then, we define $\tilde{\mathcal{D}}_s^{(n')}$ as

$$\tilde{\mathcal{D}}_s^{(n')} = \left\{ W^{(n')} : \left[W^{(n')} \right]_{i,j} = \frac{\int_{u \in \mathcal{B}(\tilde{u}_i^{(n')})} F(t_j^{(n')}|u) f_{U|X}(u) du}{\int_{u \in \mathcal{B}(\tilde{u}_i^{(n')})} f_{U|X}(u) du}, F(t|u) \in \mathcal{F} \right\} \quad (54)$$

Obviously, $\mathcal{D}_s^{(n')} \subseteq \tilde{\mathcal{D}}_s^{(n')}$, hence, we get $\lambda^{(n')}(\delta) \geq \tilde{\lambda}^{(n')}(\delta)$ where $\tilde{\lambda}^{(n')}(\delta)$ is infimum of $M^{(n')} \left(W^{(n')} \right)$ across $\tilde{\mathcal{D}}_s^{(n')}$.

In order to prove $\tilde{\lambda}^{(n')}(\delta) > \mu_{n'}$, for arbitrary $F(t|u) \in \mathcal{F}$, we assume for arbitrary large n'_0 , there exist $n' > n'_0$, such that $M^{(n')} \left(W_f^{(n')} \right) \leq \mu_{n'}$. In other words, there exist a sequence denoted as $\hat{n}_1 < \hat{n}_2 < \dots, \infty$ by which

$$M^{(\hat{n}_k)} \left(W_f^{(\hat{n}_k)} \right) \leq \mu_{\hat{n}_k} \quad (55)$$

Then, we have

$$\lim_{k \rightarrow \infty} M^{(\hat{n}_k)} \left(W_f^{(\hat{n}_k)} \right) \leq \lim_{k \rightarrow \infty} \mu_{\hat{n}_k} = 0 \quad (56)$$

From the expressions of $W_f^{(n')}$ and $M^{(n')} \left(W_f^{(n')} \right)$, we get there is a division manner for $t \in (-\infty, +\infty)$ characterized by $\hat{n}_1 < \hat{n}_2 < \dots, \infty$ such that

$$\lim_{k \rightarrow \infty} \frac{\beta_1^{(\hat{n}_k)} - \alpha_1^{(\hat{n}_k)}}{\hat{n}_k - 2} \sum_{j=1}^{\hat{n}_k - 2} \left| \int_{-\infty}^{t_j} f_{U|X_1}(u|x_1) du - \int_{-\infty}^{t_j} f_{U|X_1}(u) F(t|u) du \right|^2 = 0 \quad (57)$$

On the other hand, from the definition of \mathcal{F} and the condition that the wireless channel is non-manipulable, we get

$$\int_{-\infty}^t f_{U|X_1}(u|x_1) du \neq \int_{-\infty}^t f_{U|X_1}(u) F(t|u) du \quad (58)$$

Hence, if $\int_{-\infty}^{\infty} \left| \int_{-\infty}^t f_{U|X_1}(u|x_1) du - \int_{-\infty}^t f_{U|X_1}(u) F(t|u) du \right|^2 dt$ can be integrated, we must have

$$\int_{-\infty}^{\infty} \left| \int_{-\infty}^t f_{U|X_1}(u|x_1) du - \int_{-\infty}^t f_{U|X_1}(u) F(t|u) du \right|^2 dt > 0, \quad (59)$$

which indicates there is no division manner for $t \in (-\infty, +\infty)$ making (57) be true. It contradicts with the meaning of (57).

We proceed to examine another case that if $\int_{-\infty}^{\infty} \left| \int_{-\infty}^t f_{U|X_1}(u|x_1) du - \int_{-\infty}^t f_{U|X_1}(u) F(t|u) du \right|^2 dt$ cannot be integrated, since $\left| \int_{-\infty}^t f_{U|X_1}(u|x_1) du - \int_{-\infty}^t f_{U|X_1}(u) F(t|u) du \right| > 0$, we have

$$\lim_{\beta \rightarrow \infty, \alpha \rightarrow -\infty} \int_{\alpha}^{\beta} \left| \int_{-\infty}^t f_{U|X_1}(u|x_1) du - \int_{-\infty}^t f_{U|X_1}(u) F(t|u) du \right|^2 dt = \infty \quad (60)$$

Hence, there has α' and β' by which

$$\int_{\alpha'}^{\beta'} \left| \int_{-\infty}^t f_{U|X_1}(u|x_1) du - \int_{-\infty}^t f_{U|X_1}(u) F(t|u) du \right|^2 dt > 0 \quad (61)$$

Meanwhile, (57) indicates

$$\lim_{k \rightarrow \infty} \frac{\beta_1^{(\hat{n}_k)} - \alpha_1^{(\hat{n}_k)}}{\hat{n}_k - 2} \sum_{t_j \in [\alpha', \beta']} \left| \int_{-\infty}^{t_j} f_{U|X_1}(u|x_1) du - \int_{-\infty}^{t_j} f_{U|X_1}(u) F(t|u) du \right|^2 = 0. \quad (62)$$

However, (61) indicates there is no division manner for $t \in (\alpha', \beta')$ making (62) be true. Hence, the contradiction happens. Due

to these contradictions, we attain the assumption that for arbitrary large n'_0 , there exist $n' > n'_0$, such that $M^{(n')} \left(W_f^{(n')} \right) \leq \mu_{n'}$ is not right. Therefore, we have there exist n'_0 , for any $n' > n'_0$, there has $M^{(n')} \left(W_f^{(n')} \right) > \mu_{n'}$. Applying the aforementioned derivation to each function belonging to \mathcal{F} , we get there exist n_0 , for any $n' > n_0$, $M^{(n')} \left(W_f^{(n')} \right) > \mu_{n'}$ is available for all possible functions of \mathcal{F} . Since $\tilde{\lambda}^{(n')}(\delta)$ is infimum of $M^{(n')} \left(W_f^{(n')} \right)$ across $\tilde{\mathcal{D}}_s^{(n')}$, we thus have

$$\tilde{\lambda}^{(n')} > \mu_{n'} \quad (63)$$

Revisiting $\lambda^{(n')}(\delta) \geq \tilde{\lambda}^{(n')}(\delta)$, we get

$$\lambda^{(n')}(\delta) > \mu_{n'} \quad (64)$$

Finally, the proof is completed. ■

Lemma 7. Fixing arbitrary small ϵ and δ , if there exist n'_0 such that

$$\Pr\left\{ \underbrace{\sum_{j=1}^{n'_0-1} \sum_{i=1}^{n'_0} \left| F_{V^n|\tilde{U}^n}^{(n'_0)}(t_j^{(n'_0)} | \tilde{u}_i^{(n'_0)}) - \Phi(t_j^{(n'_0)} - \tilde{u}_i^{(n'_0)}) \right|}_{R(U^n, V^n, n'_0)} > \delta \right\} > 0$$

then, we have for $n' > n'_0$, $\Pr\{D^n < \epsilon(n', \delta) | R(U^n, V^n, n') > \delta\}$ is well-defined and

$$\Pr\{D^n < \epsilon(n', \delta) | R(U^n, V^n, n') > \delta\} \leq \epsilon$$

where $n \rightarrow \infty$, n' is sufficient large so as to satisfy the properties given by lemma 4 and lemma 6. $\epsilon(n', \delta)$ is strictly positive and can be arbitrary small value.

Proof: According to lemma 3, there exist $\mu_{n'}$ such that

$$\begin{aligned} & \Pr\left\{ \left| \frac{\beta_1^{(n')} - \alpha_1^{(n')}}{n' - 2} \sum_{j=1}^{n'-1} \left| F_{V^n|X_1^n}^{(n')} (t_j^{(n')} | x_1) - \sum_i^{n'} P_{\tilde{U}|X_1} (\tilde{u}_i^{(n')}) F_{V^n|\tilde{U}^n}^{(n')} (t_j^{(n')} | \tilde{u}_i^{(n')}) \right| \right| > \mu_{n'} \right\} \\ & \leq \epsilon \Pr\{R(U^n, V^n, n'_0) > \delta\} \leq \epsilon \Pr\{R(U^n, V^n, n') > \delta\}, \end{aligned} \quad (65)$$

where $\mu_{n'} \rightarrow 0$ as $n' \rightarrow \infty$, $n \rightarrow \infty$. The last inequality follows the fact that $R(U^n, V^n, n'_0) > \delta$ implies $R(U^n, V^n, n') > \delta$ according to the definition of $F_{V^n|\tilde{U}^n}^{(n')} (t_j^{(n')} | \tilde{u}_i^{(n')})$.

Then, if

$$\underbrace{\left| \frac{\beta_1^{(n')} - \alpha_1^{(n')}}{n' - 2} \sum_{j=1}^{n'-1} \left| F_{V^n|X_1^n}^{(n')} (t_j^{(n')} | x_1) - \sum_i^{n'} P_{\tilde{U}|X_1} (\tilde{u}_i^{(n')}) F_{V^n|\tilde{U}^n}^{(n')} (t_j^{(n')} | \tilde{u}_i^{(n')}) \right| \right|}_{G(V^n, U^n, X_1^n)} < \mu_{n'},$$

we must have

$$\begin{aligned} (\beta_1^{(n')} - \alpha_1^{(n')}) D^n & \geq \frac{\beta_1^{(n')} - \alpha_1^{(n')}}{n' - 2} \sum_{j=1}^{n'-1} \left| \int_{-\infty}^{t_j^{(n')}} f_{U|X_1}(u | x_1) du - \sum_i^{n'} P_{\tilde{U}|X_1} (\tilde{u}_i^{(n')}) F_{V^n|\tilde{U}^n}^{(n')} (t_j^{(n')} | \tilde{u}_i^{(n')}) \right| - \mu_{n'} \quad (66) \\ & \geq \frac{\beta_1^{(n')} - \alpha_1^{(n')}}{n' - 2} \sum_{j=1}^{n'-1} \left| \int_{-\infty}^{t_j^{(n')}} f_{U|X_1}(u | x_1) du - \sum_i^{n'} P_{\tilde{U}|X_1} (\tilde{u}_i^{(n')}) F_{V^n|\tilde{U}^n}^{(n')} (t_j^{(n')} | \tilde{u}_i^{(n')}) \right|^2 - \mu_{n'} \end{aligned}$$

On the other hand, if $\underbrace{\sum_{j=1}^{n'-1} \sum_{i=1}^{n'} \left| F_{V^n|\tilde{U}^n}^{(n')} (t_j^{(n')} | \tilde{u}_i^{(n')}) - \Phi(t_j^{(n')} - \tilde{u}_i^{(n')}) \right|}_{R(U^n, V^n, n')} > \delta$, according to lemma 5, the right side of (66)

becomes

$$(\beta_1^{(n')} - \alpha_1^{(n')}) D^n \geq \lambda^{(n')}(\delta) - \mu_{n'} \quad (67)$$

which can be reshaped as

$$D^n \geq \frac{\lambda^{(n')}(\delta) - \mu_{n'}}{(\beta_1^{(n')} - \alpha_1^{(n')})} \quad (68)$$

Define $\varepsilon(n', \delta) = \frac{\lambda^{(n')}(\delta) - \mu_{n'}}{(\beta_1^{(n')} - \alpha_1^{(n')})}$, according to lemma 6, $\varepsilon(n', \delta) > 0$ as n' is sufficient large. From the properties of $\mu_{n'}$ and $\lambda^{(n')}(\delta)$, $\varepsilon(n', \delta)$ can be arbitrarily small.

Upon (66), (67) and (68), we have

$$\begin{aligned} & \Pr \{D^n \geq \varepsilon(n', \delta), R(U^n, V^n, n') \geq \delta\} \\ & \geq \Pr \{D^n \geq \varepsilon(n', \delta), R(U^n, V^n, n') \geq \delta, G(V^n, U^n, X_1^n) \leq \mu'\} \\ & = \Pr \{R(U^n, V^n, n') \geq \delta, G(V^n, U^n, X_1^n) \leq \mu_{n'}\} \\ & \geq \Pr \{R(U^n, V^n, n') \geq \delta\} - \Pr \{G(V^n, U^n, X_1^n) \geq \mu_{n'}\} \end{aligned}$$

where the equation follows the logic from (66), (67) to (68). Then, we have

$$\Pr \{D^n \geq \varepsilon(n', \delta) | R(U^n, V^n, n') \geq \delta\} = \frac{\Pr \{D^n \geq \varepsilon(n', \delta), R(U^n, V^n, n') \geq \delta\}}{\Pr \{R(U^n, V^n, n') \geq \delta\}} > 1 - \frac{\Pr \{G(V^n, U^n, X_1^n) \geq \mu_{n'}\}}{\Pr \{R(U^n, V^n, n') \geq \delta\}} \geq 1 - \epsilon \quad (69)$$

where the last inequality follows (65). The proof is finished. ■

The first property of theorem 1 is direct result from lemma 7.

We proceed to prove the second property of theorem 1. For arbitrary small δ , μ and $\mu'(n', \delta) = \mu + \frac{\delta}{n'-2}$, we have

$$\begin{aligned} & \Pr \left\{ D^n \leq \mu'(n', \delta) \cap \sum_{j=1}^{n'-1} \sum_{i=1}^{n'} \left| F_{V^n | \tilde{U}^n}^{(n')} (t_j^{(n')} | \tilde{u}_i^{(n')}) - \Phi(t_j^{(n')} - \tilde{u}_i^{(n')}) \right| \leq \delta \right\} \geq \\ & \Pr \left\{ D^n \leq \mu'(n', \delta) \cap \frac{1}{n'-2} \sum_{j=1}^{n'-1} \left| F_{V^n | X_1^n}^{(n')} (t_j^{(n')} | x_1) - \int_{-\infty}^{+\infty} f_{U|X_1}(u | x_1) F_{V^n | \tilde{U}^n}^{(n')} (t_j^{(n')} | u) du \right| \leq \mu \right. \\ & \left. \cap \sum_{j=1}^{n'-1} \sum_{i=1}^{n'} \left| F_{V^n | \tilde{U}^n}^{(n')} (t_j^{(n')} | \tilde{u}_i^{(n')}) - \Phi(t_j^{(n')} - \tilde{u}_i^{(n')}) \right| \leq \delta \right\} \\ & = \Pr \left\{ \frac{1}{n'-2} \sum_{j=1}^{n'-1} \left| F_{V^n | X_1^n}^{(n')} (t_j^{(n')} | x_1) - \int_{-\infty}^{+\infty} f_{U|X_1}(u | x_1) F_{V^n | \tilde{U}^n}^{(n')} (t_j^{(n')} | u) du \right| \leq \mu \right. \\ & \left. \cap \sum_{j=1}^{n'-1} \sum_{i=1}^{n'} \left| F_{V^n | \tilde{U}^n}^{(n')} (t_j^{(n')} | \tilde{u}_i^{(n')}) - \Phi(t_j^{(n')} - \tilde{u}_i^{(n')}) \right| \leq \delta \right\} \\ & \geq \Pr \left\{ \sum_{j=1}^{n'-1} \sum_{i=1}^{n'} \left| F_{V^n | \tilde{U}^n}^{(n')} (t_j^{(n')} | \tilde{u}_i^{(n')}) - \Phi(t_j^{(n')} - \tilde{u}_i^{(n')}) \right| \leq \delta \right\} - \\ & \Pr \left\{ \frac{1}{n'-2} \sum_{j=1}^{n'-1} \left| F_{V^n | X_1^n}^{(n')} (t_j^{(n')} | x_1) - \int_{-\infty}^{+\infty} f_{U|X_1}(u | x_1) F_{V^n | \tilde{U}^n}^{(n')} (t_j^{(n')} | u) du \right| > \mu \right\} \end{aligned}$$

where the equality firstly follows the fact that according to $t_j^{(n')} \in \tilde{\mathcal{U}}$ and the definition of $F_{V^n | \tilde{U}^n}^{(n')} (t | u)$,

$$\sum_{j=1}^{n'-1} \sum_{i=1}^{n'} \left| F_{V^n | \tilde{U}^n}^{(n')} (t_j^{(n')} | \tilde{u}_i^{(n')}) - \Phi(t_j^{(n')} - \tilde{u}_i^{(n')}) \right| \leq \delta$$

indicates $\sum_{j=1}^{n'-1} \sup_u \left| F_{V^n | \tilde{U}^n}^{(n')} (t_j^{(n')} | u) - \Phi(t_j^{(n')} - u) \right| \leq \delta$, combining with another event

$$\frac{1}{n'-2} \sum_{j=1}^{n'-1} \left| F_{V^n | X_1^n}^{(n')} (t_j^{(n')} | x_1) - \int_{-\infty}^{+\infty} f_{U|X_1}(u | x_1) F_{V^n | \tilde{U}^n}^{(n')} (t_j^{(n')} | u) du \right| \leq \mu,$$

we have

$$\begin{aligned}
D^n &\leq \frac{1}{n'-2} \sum_{j=1}^{n'-1} \left| F_{V^n|X_1^n}^{(n')} (t_j^{(n')} | x_1) - \int_{-\infty}^{+\infty} f_{U|X_1} (u | x_1) F_{V^n|\bar{U}^n}^{(n')} (t_j^{(n')} | u) du \right| \\
&+ \frac{1}{n'-2} \sum_{j=1}^{n'-1} \left| \int_{-\infty}^{+\infty} f_{U|X_1} (u | x_1) \left(F_{V^n|\bar{U}^n}^{(n')} (t_j | u) - \Phi (t_j^{(n')} - u) \right) du \right| \\
&< \mu + \frac{\delta}{n'-2} = \mu'(n', \delta).
\end{aligned} \tag{70}$$

Hence, the equality in (70) is established. Upon (70) and lemma 3, the property 2 in theorem 1 is direct.

REFERENCES

- [1] L. Buttyan and J-P. Hubaux, *Security and Cooperation in Wireless Networks*, Cambridge University Press, 2007.
- [2] M. Bloch and J. Barros, *Physical-Layer Security: From Information Theory to Security Engineering*, Cambridge University Press, 2011.
- [3] P. Papadimitratos and Z. J. Haas, "Secure data communication in mobile ad hoc networks," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 2, pp. 343–356, Feb. 2006.
- [4] Y. C. Hu, A. Perrig, and D. B. Johnson, "Ariadne: A secure on-demand routing protocol for ad hoc networks," *Wirel. Netw.*, vol. 11, no. 1-2, pp. 21–38, Jan. 2005.
- [5] T. Ho, B. Leong, R. Koetter, M. Médard, M. Effros, and D. R. Karger, "Byzantine modification detection in multicast networks with random network coding," *IEEE Trans. Inf. Theory*, vol. 54, no. 6, pp. 2798–2803, Jun. 2008.
- [6] O. Kosut, L. Tong, and D. Tse, "Nonlinear network coding is necessary to combat general Byzantine attacks," in *Proc. 47th Annu. Allerton Conf. on Commun., Control, and Compu.*, Monticello, IL, Oct. 2009, pp. 593–599.
- [7] Y. Mao and M. Wu, "Tracing malicious relays in cooperative wireless communications," *IEEE Trans. Inf. Forens. Security*, vol. 2, no. 2, pp.198–212, Jun. 2007.
- [8] T. Khalaf, S. Kim, and A. Abdel-Hakim, "Tradeoff between reliability and security in multiple access relay networks under falsified data injection attack," *IEEE Trans. Inf. Forens. Security*, vol. 9, no. 3, pp. 386–396, Mar. 2014.
- [9] L.-C. Lo, Z.-J. Wang, and W. J. Huang, "Noncoherent misbehavior detection in space-time coded cooperative networks," in *Proc. IEEE Intl. Conf. Acoustics, Speech and Signal Processing (ICASSP)*, Kyoto, Mar. 2012, pp. 3061–3064.
- [10] L.-C. Lo, W. J. Huang, R. Y. Chang, and W.-H. Chung, "Noncoherent detection of misbehaving relays in decode-and-forward cooperative networks," *IEEE Comm. Letters*, vol. 19, no. 9, pp.1536–1539, Sept. 2015.
- [11] W. Hou, X. Wang, and A. Refaey, "Misbehavior detection in amplify-and-forward cooperative OFDM systems," in *Proc. IEEE Intl. Conf. Commun. (ICC)*, Budapest, 2013, Jun. 2013, pp. 5345–5349.
- [12] S. W. Kim, "Physical integrity check in cooperative relay communications," *IEEE Trans. on Wirel. commun.*, vol. 14, no.11, pp. 6401–6413, Nov. 2015.
- [13] X. He, and A. Yener, "Strong secrecy and reliable Byzantine detection in the presence of an untrusted relay," *IEEE Trans. Inf. Theory*, vol. 59, no. 1, pp. 177–192, Jan. 2013.
- [14] E. Graves and T. F. Wong, "Detection of channel degradation attack by intermediary node in linear networks," in *Proc. IEEE Intl. Conf. Comput. Commun. (INFOCOM)*, Orlando, FL, Mar. 2012, pp. 747–755.
- [15] E. Graves and T. F. Wong, "Detectability of symbol manipulation by an amplify-and-forward relay," in *arXiv preprint arXiv:1205.2681*, 2012.
- [16] E. Graves and T. F. Wong, "A coding approach to guarantee information integrity against a Byzantine relay," in *Proc. IEEE Intl. Symp. Inf. Theory (ISIT)*, Istanbul, Jul. 2013, pp. 2780–2784.

- [17] R. Cao, T. F. Wong, T. Lv, H. Gao and S. Yang, "Detecting Byzantine Attacks Without Clean Reference," *IEEE Trans. Inf. Forens. Security*, vol. 11, no. 12, pp. 2717-2731, Dec. 2016.