Practical Coding Schemes for Cognitive Overlay Radios

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***Abstract*—We develop practical coding schemes for the cognitive overlay radios as modeled by the cognitive interference channel, a variation of the classical two user interference channel where one of the transmitters has knowledge of both messages. Inspired by information theoretical results, we develop a coding strategy for each of the three parameter regimes where capacity is known. A key feature of the capacity achieving schemes in these regimes is the joint decoding of both users' codewords, which we accomplish by performing a posteriori probability calculation over a combined trellis. The schemes are shown to perform close to the capacity limit with low error rate.**

I. Introduction

Cognitive radio is widely considered as a key enabling technology to increase the spectral efficiency of wireless networks [1]. The underlying principle of cognitive radio is to allow a set of cognitive users to access the spectrum that belongs to the primary users (also known as the licensed users), without compromising the primary users' link quality. For the cognitive overlay radios, this is achieved through cooperation, whereby the primary user is willing to grant access to the medium under the conditions that the cognitive user assists its transmission, allowing for faster and more reliable communication.

We focus on a simple canonical model for cognitive overlay radios: the cognitive interference channel (CIC). This channel is comprised of two transmitter-receiver pairs (one for each primary and cognitive users), in which the cognitive transmitter has non-causal knowledge of the primary message. In practice, this non-causal knowledge can be obtained when the primary user's message is public, such as the TV program in a broadcasting networks [2]. When good communication link is available between the primary and secondary transmitters, this information can also be learned, as the secondary user is likely to be able to decode the primary message earlier than the primary receiver.

The capacity of the CIC is known only in a subset of the parameter region, namely the "weak interference" regime [3]-[4], the "very strong interference" regime [5]-[6], and the "primary decodes cognitive" regime [7]. In the "weak interference" regime, capacity is achieved by treating the interference as noise at the primary decoder and pre-canceling the interference at the cognitive decoder. In the "very strong interference" regime, instead, capacity is achieved by having both decoders decode both messages. In the "primary decodes cognitive" regime, capacity is achieved with a mixture of the previous two strategies: the cognitive messages is decoded at the primary receiver but also pre-coded against the interference experienced at the cognitive receiver. In all the other parameter regimes, capacity is known to within one bit/s/Hz and to within a factor two [7]. The motivation of this work is to develop practical coding schemes that approach the capacity limit of the CIC. We attempt to do so by designing multi-terminal communication schemes that implement the three fundamental random coding techniques used in the achievability proof: binning, superposition coding, and joint decoding. Although each of these techniques has different possible implementations in the coding community, it is yet not clear how to combine them for a multi-terminal network such as the CIC. Our contribution is to show how codes designed for single user systems can be efficiently combined for multi-terminal systems.

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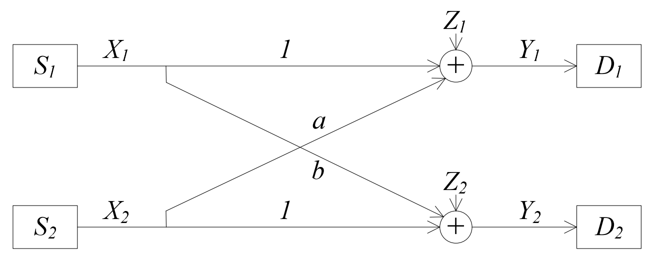

Fig. 1. The Gaussian Cognitive Interference Channel (CIC) in Standard Form.

The remainder of this paper is organized as follows. Section II describes the system and channel model in consideration. Practical coding schemes are then proposed in Section III for the regimes in which capacity is known. In Section IV we compare the performance of the different schemes against the theoretical capacity for a given bit error rate. Section V concludes the paper.

II. System and Channel Model

The Gaussian CIC as depicted in Fig. 1 is comprised of a cognitive user source and destination (S_1 and D_1) and a primary user source and destination (S_2 and D_2), sharing the same time-frequency resources to transmit their messages. In contrast to the conventional interference channel, the cognitive transmitter in CIC has non-causal access to the primary user's message, giving it the ability to assist the primary user.

Without loss of generality, we consider a CIC channel in the standard form where the direct channels have unitary gain [7, Appendix. A]. The received signals at D_1 and D_2 in this case are given by the following discrete time signal model

$$Y_1 = X_1 + aX_2 + Z_1 \tag{1a}$$
$$Y_2 = bX_1 + X_2 + Z_2, \tag{1b}$$

where X_j is the transmitted codeword from S_j, which satisfies the power constraint $E\left[|X_j|^2\right] \leq P_j$, and Z_i denotes the additive Gaussian noise at D_i with variance N_i. The interference channel gains[1] $a \in \mathcal{R}$ and $b \in \mathcal{R}^+$ at D_1 and D_2, respectively, are assumed to be known by all nodes in the network.

The non-causal knowledge of the primary user's message allows the cognitive transmitter to allocate a fraction of its power to assist the primary user transmission. With this cooperation strategy and denoting $0 \leq \alpha \leq 1$ as the fraction of transmit power used to transmit its own message, the received signal model can be expressed as

$$Y_1 = \sqrt{\alpha}X_1 + (a + \sqrt{\overline{\alpha}P_1/P_2})X_2 + Z_1 \tag{2a}$$
$$Y_2 = b\sqrt{\alpha}X_1 + (1 + b\sqrt{\overline{\alpha}P_1/P_2})X_2 + Z_2, \tag{2b}$$

where $\overline{\alpha} \triangleq 1 - \alpha$. Capacity for the CIC is known in three subsets of parameter regimes, namely the "weak interference" regime, the "very strong interference" regime, and the "primary decodes cognitive" regime.

III. Practical Coding Schemes

Guided by information theoretic capacity results, we propose practical coding schemes for the Gaussian CIC in the three parameter regimes where capacity is known.

A. The Weak Interference Regime

A CIC is said to be in the "weak interference" regime whenever the direct link from S_1 to D_1 is "more capable" than the interference link from S_1 to D_2 [4], which corresponds to

$$|b|/\sqrt{N_2} \leq 1/\sqrt{N_1}. \tag{3}$$

In [4], the authors prove that the capacity achieving strategy in this regime is for the cognitive transmitter S_1 to apply Dirty Paper Coding (DPC) against the total interference caused by the primary user's message X_2 at D_1, while the primary receiver D_2 treats the interference as noise. The decoding at the cognitive receiver D_1 is then performed using a DPC decoder, which achieves the interference-free performance.

Several practical DPC techniques have been proposed in the literature [8]-[10]. In this work, we follow the state of the art DPC implementation of [10] for the cognitive transmitter, which combines Trellis Coded Quantization (TCQ) and Irregular Repeat Accumulate (IRA) codes [11] to achieve substantial shaping as well as coding gain. The primary transmitter, which has no knowledge of the interference, employs an IRA code.

Given the interference sequence $S = (a + \sqrt{\overline{\alpha}P_1/P_2})X_2$ which is known non-causally at the cognitive transmitter, the

[1]In this work we assume a real channel for simplicity. However, the results presented can be easily extended to complex channels.

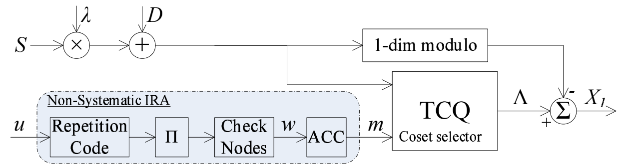

Fig. 2. Dirty Paper Coding (DPC) Encoder Block Diagram

DPC encoder generates $\lambda S + D$ using a multiplication factor λ and a pseudo random dither sequence D, and forwards it to the input of a TCQ. The K-bits message u to be transmitted is first encoded using a non-systematic rate-$1/4$ IRA code into m, and used to determine the coset of the TCQ codebook. The TCQ is performed using the Viterbi algorithm over a rate-$1/2$ convolutional code with modulo distance metric and output alphabet $\{0, 1, 2, 3\}$ corresponding to the two-bits output. The coset codebook is obtained by applying a shift of $0 < \Delta < 1$ (a tunable parameter) to the convolutional code output. The TCQ codeword obtained in this manner, Λ, satisfies $\Lambda \subset \mathcal{A}^{4K}$, where the set $\mathcal{A} = \{0, \Delta, 1, 1+\Delta, 2, 2+\Delta, 3, 3+\Delta\}$. The actual transmitted sequence X_1 is then the quantization error between $(\lambda S + D)_{\text{mod } 4}$ and the TCQ output codeword Λ. Following the lattice property of TCQ, the sequence X_1 is approximately Gaussian [12]. A schematic representation of the proposed DPC encoder can be found in Fig. 2.

Denoting $[\lambda S + D, m]_{TCQ}$ as the quantization operation of $\lambda S + D$ over the TCQ coset codebook specified by m, the transmitted sequence X_1 can be expressed as

$$\begin{aligned} X_1 &= [\lambda S + D, m]_{TCQ} - (\lambda S + D)_{\text{mod } 4} \\ &= \Lambda - (\lambda S + D)_{\text{mod } 4}. \end{aligned} \tag{4}$$

Upon receiving $Y_1 = X_1 + S + Z_1$, the receiver calculates

$$\begin{aligned} Y_1' &= (\lambda Y_1 + D)_{\text{mod } 4} \\ &= \Bigg(\underbrace{X_1 + (\lambda S + D)_{\text{mod } 4}}_{\Lambda} + \underbrace{\lambda Z_1 + (\lambda - 1)X_1}_{Z'}\Bigg)_{\text{mod } 4} \end{aligned} \tag{5}$$

which is equivalent to a transmission of the TCQ coset codeword Λ in the presence of additive Gaussian noise Z' over a modulo channel. To minimize the effective noise variance N', λ is set to the minimum mean square error scaling $\lambda_C = \frac{\alpha P_1}{\alpha P_1 + N_1}$, which coincides with Costa's DPC scaling [13].

To recover the message u, we need to identify the coset of Λ as specified by m. From equation (5), the likelihood value that the i^{th} symbol of the TCQ codeword $\Lambda[i]$ takes on a specific value $q \in \mathcal{A}$ given the i^{th} observed symbol $Y'[i]$ is

$$\Pr\left(\Lambda[i] = q | Y'[i]\right) = \frac{p\left(Y'[i] | \Lambda[i] = q\right) \Pr\left(\Lambda[i] = q\right)}{p\left(Y'[i]\right)}. \tag{6}$$

Due to the properties of the modulo channel, the first numerator term $p\left(Y'[i] | \Lambda[i] = q\right)$ above can be calculated as follows:

$$\begin{aligned} &\sum_{j \in \mathcal{Z}} \frac{1}{\sqrt{2\pi N'}} \exp\left(\frac{-(Y'[i] - (q + j4))^2}{2N'}\right) \\ &\approx \frac{1}{\sqrt{2\pi N'}} \exp\left(\frac{-\min(|Y'[i] - q|, 4 - |Y'[i] - q|)^2}{2N'}\right), \end{aligned} \tag{7}$$

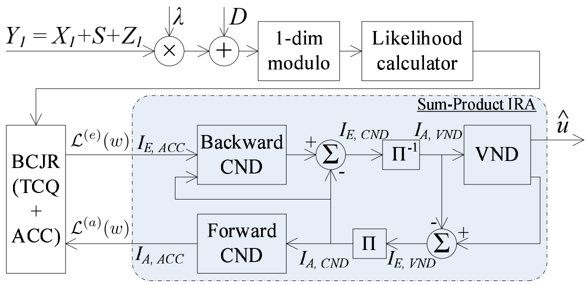

Fig. 3. Dirty Paper Coding (DPC) Decoder Block Diagram

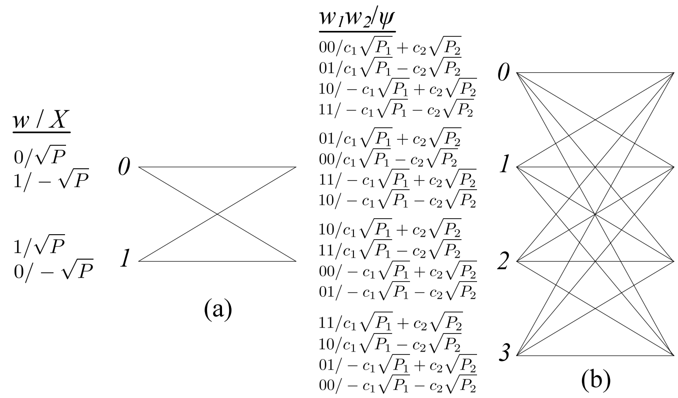

Fig. 4. Trellis diagram of (a) single ACC for the decoding of a single IRA codeword and (b) combination of two ACCs for decoding of a superposition of two IRA codewords.

where the approximation is performed by taking only the most significant element in the summation. The second term $\Pr(\Lambda[i]=q)$ represents the a priori probability of $\Lambda[i]=q$ which (for a given trellis state) is directly related to the a priori of the bit at the accumulator (ACC) input, while the normalization term $p(Y'[i])$ ensures that $\sum_{q\in\mathcal{A}}\Pr(\Lambda[i]=q|Y'[i])=1$.

The likelihood $\Pr(\Lambda[i]=q|Y'[i])$ is then used to calculate the state transition probability in the BCJR algorithm [14], which is executed over the combined trellis of TCQ and the ACC trellis of the IRA code, producing the A Posteriori Probability (APP) of the ACC input sequence w. The decoding of the message sequence u can then be performed using the IRA sum-product algorithm through several iterations of extrinsic information exchange with the BCJR decoder. The block diagram of the DPC receiver is depicted in Fig. 3, resembling that given in [10, Fig. 9].

B. The Very Strong Interference Regime

In multi-terminal information theory, the "very strong interference" regime corresponds to the regime where the capacity of the channel reduces to capacity of the compound multiple access channel where each decoder decodes all the messages in the network. For the CIC, this parameter regime is expressed by the inequalities [5]

$$|b|/\sqrt{N_2}\geq 1/\sqrt{N_1}\text{ , and} \tag{8a}$$

$$(P_1+P_2a^2+2a\sqrt{\overline{\alpha}P_1P_2})/N_1\geq(b^2P_1+P_2+2b\sqrt{\overline{\alpha}P_1P_2})/N_2. \tag{8b}$$

For the above condition to hold irrespective of the value of α, the channel parameters should satisfy

$$\left(\frac{P_1}{N_1}-\frac{P_1b^2}{N_2}\right)+\left(\frac{P_2a^2}{N_1}-\frac{P_2}{N_2}\right)-\left|\frac{a}{N_1}-\frac{b}{N_2}\right|2\sqrt{P_1P_2}\geq 0. \tag{9}$$

According to [5], the capacity achieving strategy in this regime is to use superposition coding at the cognitive transmitter, and let both the primary and cognitive receivers decode both messages. In this case, X_1 and X_2 are generated according to a channel code such as an IRA code. Correspondingly, the cognitive transmitter S_1 sends a weighted sum of the two codewords according to the power splitting parameter α.

In general, the received signal at the two receivers as expressed in equations (2a) and (2b) take the form of

$$Y=c_1X_1+c_2X_2+Z,$$

where c_1 and c_2 are the effective gain of X_1 and X_2, respectively. Since both X_1 and X_2 are IRA codewords, we propose a joint decoding technique which combines the trellises of the ACC from the IRA code used at both the cognitive and the primary encoders.

The trellis diagram of a single ACC and the combined trellis of two ACCs are illustrated in Fig. 4. Given that both IRA codewords employ binary phase shift keying (BPSK), the term $c_1X_1+c_2X_2$ can be regarded as a super-symbol ψ which is drawn from a size-4 alphabet $\mathcal{B}=\{\pm c_1\sqrt{P_1}\pm c_2\sqrt{P_2}\}$. The likelihood value is then calculated with respect to this super-symbol rather than the individual codeword bit. Given the i^{th} observed symbol $Y[i]$, the likelihood that $\psi[i]$ takes on a particular value $q\in\mathcal{B}$ is

$$\Pr(\psi[i]=q|Y[i])=\frac{p(Y[i]|\psi[i]=q)\Pr(\psi[i]=q)}{p(Y[i])}. \tag{10}$$

In a similar manner, the normalization term $p(Y[i])$ ensures that $\sum_{q\in\mathcal{B}}\Pr(\psi[i]=q|Y[i])=1$. Since Z is Gaussian with variance N, the term $p(Y[i]|\psi[i]=q)$ can be calculated as

$$\frac{1}{\sqrt{2\pi N}}\exp(-(Y[i]-q)^2/2N). \tag{11}$$

For a given trellis state, instead of one bit as in the case of decoding a single IRA code, the a priori probability $\Pr(\psi[i]=q)$ is now determined by the a priori of the bits at the ACC input of both IRA encoders. As an example consider the combined trellis in Fig. 4b, when we are at state 0, the a priori probability $\Pr\left(\psi[i]=c_1\sqrt{P_1}+c_2\sqrt{P_2}\right)$ is given by the product of two a priori probabilities: $\Pr(w_1=0)\Pr(w_2=0)$. The calculation of the APP of the bits at the ACC inputs can then be performed following the general Soft Input Soft Output (SISO) processing as proposed in [15], which generalizes the BCJR algorithm. For a given edge ϵ on the trellis, denote $s^S(\epsilon)$ and $s^E(\epsilon)$ as its starting and ending state; and $w_1(\epsilon)$, $w_2(\epsilon)$, and $\psi(\epsilon)$ as the corresponding w_1, w_2, and ψ values, respectively. Define the edge transition probability at the i^{th} stage as

$$\begin{aligned}\gamma_i(\epsilon)\;=\;&\xi\;p(Y[i]|\psi[i]=\psi(\epsilon))\\&\Pr(W_1[i]=w_1(\epsilon);I)\Pr(W_2[i]=w_2(\epsilon);I)\,,\end{aligned} \tag{12}$$

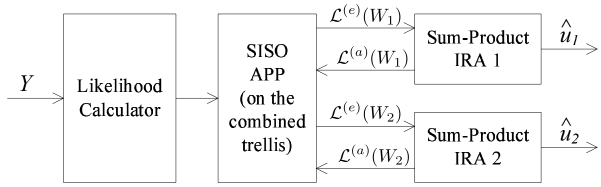

Fig. 5. Multiuser Joint Decoder Block Diagram

where ξ is the normalization factor. The letter I indicates that the probabilities are the input to the SISO processor. Note that for a given edge, (10) and (12) are equivalent. We also define

$$A_i(s) = \sum_{\epsilon: s^E(\epsilon)=s} A_{i-1}(s^S(\epsilon))\,\gamma_i(\epsilon), \tag{13}$$

$$B_i(s) = \sum_{\epsilon: s^S(\epsilon)=s} \gamma_i(\epsilon)\,B_{i+1}(s^E(\epsilon)). \tag{14}$$

The initial values of 1 is set on $A_0(s)$ and $B_K(s)$ for the initial and final state, respectively; and value 0 is used for other states. The APP of $W_t[i]$ for $t=\{1,2\}$ can be calculated as

$$\Pr(W_t[i]=\delta;O) = \xi' \sum_{\epsilon: w_t(\epsilon)=\delta} A_{i-1}(s^S(\epsilon))\,\gamma_i(\epsilon)\,B_i(s^E(\epsilon)), \tag{15}$$

where ξ' is another normalization factor, and the letter O signifies it as an output. The corresponding extrinsic Log Likelihood Ratio (LLR) of $W_t[i]$ can then be calculated as

$$\begin{aligned}\mathcal{L}^{(e)}(W_t[i]) &= \ln\frac{\Pr(W_t[i]=0;O)}{\Pr(W_t[i]=1;O)} - \ln\frac{\Pr(W_t[i]=0;I)}{\Pr(W_t[i]=1;I)} \\ &= \ln\frac{\Pr(W_t[i]=0;O)}{\Pr(W_t[i]=1;O)} - \mathcal{L}^{(a)}(W_t[i]).\end{aligned} \tag{16}$$

The obtained extrinsic LLR is then passed to the corresponding sum-product decoder of its respective IRA, and several iterations of extrinsic information exchange are performed before a hard decision is made. The block diagram of the proposed joint decoder is illustrated in Fig. 5.

C. The Primary Decodes Cognitive Regime

The last regime where capacity is known is the "primary decodes cognitive" regime. This regime partially overlaps with the "very strong interference" regime, and in such intersection, capacity can be achieved using either of the approaches for the two regimes. The set of channel parameters in this regime satisfy the condition given in [7, Theorem V.1], namely

$$|b|/\sqrt{N_2} \geq 1/\sqrt{N_1}, \tag{17}$$

and $f(\alpha)\geq 0, \forall 0\leq\alpha\leq 1$ where $f(\alpha)=$

$$\frac{\alpha P_1 N_2 b^{-2}}{\alpha P_1 + N_2 b^{-2}} - \frac{\alpha P_1 N_1}{\alpha P_1 + N_1} + P_2\left(\left(\frac{\alpha P_1 b^{-1}}{\alpha P_1 + N_2 b^{-1}} - \frac{\alpha P_1 a}{\alpha P_1 + N_1}\right) + \sqrt{\frac{(1-\alpha)P_1}{P_2}}\left(\frac{\alpha P_1}{\alpha P_1 + N_2 b^{-2}} - \frac{\alpha P_1}{\alpha P_1 + N_1}\right)\right)^2. \tag{18}$$

The optimal transmission strategy for this regime is similar to that in the "weak interference" regime for the cognitive user, whereby the cognitive transmitter S_1 applies DPC against the total interference caused by X_2 at D_1, while the primary transmitter S_2 uses conventional channel coding to generate X_2. The decoding at D_1 is also performed using the DPC decoder to achieve interference-free capacity. For this reason, the cognitive decoder is as described in Sec. III.A, and we only need to detail the decoding process at the primary receiver D_2.

To simplify some of the mathematical expressions, denote

$$Q = a + \sqrt{\alpha P_1/P_2} \tag{19}$$

$$R = 1 + b\sqrt{\alpha P_1/P_2}. \tag{20}$$

As pointed out in [7], joint decoding is necessary in this sub-regime. We now present a possible implementation of the joint decoder using a similar approach as for the joint decoding in the "very strong interference" regime.

The main idea here is to calculate the soft values of both TCQ codeword and X_2 for SISO processing. Firstly, knowing that $\sqrt{\alpha}X_1$ is a DPC codeword designed to cancel the total interference $S=QX_2$ at D_1, we can use (2b) to compute

$$\left(\frac{Q\lambda_C}{R}Y_2 + D\right)_{\text{mod } 4} = \Big(\underbrace{\sqrt{\alpha}X_1 + (\lambda_C S + D)_{\text{mod } 4}}_{\Lambda} + Z^{(1)}\Big)_{\text{mod } 4}, \tag{21}$$

where $Z^{(1)} = (\lambda_C Q/R)Z_2 + ((Qb/R)-1)\sqrt{\alpha}X_1$ is the effective noise which is approximately Gaussian with variance $N^{(1)} = (\lambda_C Q/R)^2 N_2 + ((Qb/R)-1)^2\alpha P_1$. The likelihood value of the i^{th} TCQ symbol $\Pr(\Lambda[i]=q|Y_2[i])$ for $q\in\mathcal{A}$ can then be calculated using equation (6), with the noise variance N' replaced by $N^{(1)}$.

Secondly, exploiting the fact that X_1 is approximately Gaussian, the LLR of X_2 can be calculated from (2b) as

$$\mathcal{L}(X_2[i]) = \ln\frac{\Pr(X_2[i]=\sqrt{P_2}|Y_2[i])}{\Pr(X_2[i]=-\sqrt{P_2}|Y_2[i])} = \frac{2Y_2[i]}{\sqrt{P_2}RN^{(2)}}, \tag{22}$$

where $N^{(2)} = (b/(R\sqrt{P_2}))^2\alpha P_1 + N_2/(R^2P_2)$ is the variance of the effective noise.

Thirdly, using a different scaling factor $\lambda\neq\lambda_C$, it is possible to transform the received signal into

$$\begin{aligned}(\lambda Y_2 + \Gamma D)_{\text{mod } 4\Gamma} &= (\lambda b\sqrt{\alpha}X_1 + \lambda R X_2 + \Gamma D + \lambda Z_2)_{\text{mod } 4\Gamma} \\ &= \left(\Gamma\Lambda + \mu X_2 + Z^{(3)}\right)_{\text{mod } 4\Gamma},\end{aligned} \tag{23}$$

with

$$\begin{aligned}\Gamma\Lambda &= \Gamma\sqrt{\alpha}X_1 + (\Gamma(\lambda_C S + D))_{\text{mod } 4\Gamma} \\ \mu &= \lambda R - \Gamma\lambda_C Q \\ Z^{(3)} &= \lambda Z_2 + (\lambda b - \Gamma)\sqrt{\alpha}X_1.\end{aligned}$$

It is apparent that the received signal model in (23) represents a transmission of a TCQ coset codeword and a primary user codeword (which are scaled by a factor of Γ and μ, respectively) over a modulo additive Gaussian channel. Considering $\Psi = \Gamma\Lambda + \mu X_2$ as the super-symbol which is drawn from the concatenated alphabet $\mathcal{C} = \{\Gamma\mathcal{A} + \mu\sqrt{P_2},\, \Gamma\mathcal{A} - \mu\sqrt{P_2}\}$, the likelihood value of $\Pr(\Psi[i]=v|Y_2[i])$ for any $v\in\mathcal{C}$ can be calculated using the same technique as (6).

In general, it is possible to optimize the scaling factor λ

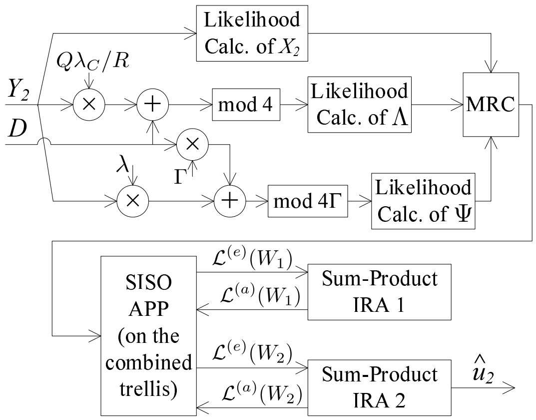

Fig. 6. DPC and IRA Joint Decoder Block Diagram

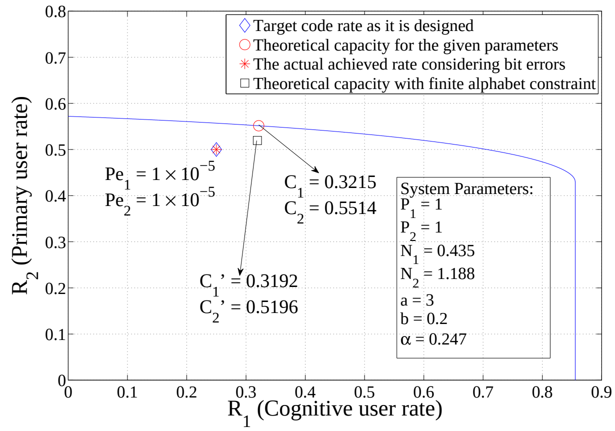

Fig. 7. Designed Code Performance in the "Weak Interference" Regime

and Γ to improve the decoding performance. A good choice of λ and Γ should minimize the effective noise variance and should maximize the minimum modulo-4Γ-distance $d^*_{\min}(\mathcal{C})$ between two elements in $\mathcal{C}$. In this work, we do not perform any optimization to either λ or Γ, and simply select their values to approximate the noise minimizing criterion

$$\frac{\lambda}{\Gamma} \approx \frac{\alpha P_1 b}{N_2 + b^2 \alpha P_1},$$

and at the same time maximizing the following metric[2]

$$\frac{(d^*_{\min}(\mathcal{C}))^2}{2(\lambda^2 N_2 + (\lambda b - \Gamma)^2 \alpha P_1)},$$

which is found to produce good error performance.

To combine the three soft values obtained so far, we apply maximum ratio combining (MRC) and calculate the following

$$\Pr\left(\Lambda[i] = q | Y_2[i]\right)\ \Pr(X_2[i] = \rho | Y_2[i])\ \Pr\left(\Psi[i] = v | Y_2[i]\right), \tag{24}$$

where $q \in \mathcal{A}$, $\rho \in \{\pm\sqrt{P_2}\}$, $v \in \mathcal{C}$, and $v = \Gamma q + \mu\rho$. Note that although all of the soft values above are derived from the same $Y_2[i]$, only two of them involve a modulo operation (with different modulus), therefore the MRC computation will generally produce a better likelihood value.

The final step is to use the likelihood value obtained in (24) to calculate the state transition probability required for SISO processing, which is to be executed on the combined trellis of the ACC of the primary user IRA code and the cognitive user DPC (which is itself a combined trellis of the TCQ and the ACC of cognitive user IRA code). The APP of the bits at the ACC input of the IRA codes and their extrinsic LLR can then be calculated in a similar way as (15) and (16), respectively. The only distinction from the previous scenario is that the number of edges to be considered is larger, as the combined trellis includes the TCQ trellis from DPC. The final estimate of u_2 is obtained after several iterations of extrinsic information exchange with the sum-product decoder of both IRA codes. The block diagram of the joint decoder is illustrated in Fig. 6. It is worth noting that even though we are only interested in decoding u_2, the sum-product decoder of u_1 is also needed to refine its a priori probability at the SISO processor at every decoding iteration.

[2]This metric is equal to the logarithmic of the smallest likelihood ratio of two points separated by $d^*_{\min}(\mathcal{C})$ apart when the noise variance is $N^{(3)}$.

IV. Performance Analysis

In this section, we evaluate the performance of the proposed coding scheme via numerical simulations. We select a set of channel parameters in each of the regimes in Sec. III and evaluate their theoretical capacity region. We then evaluate the performance of each of the proposed practical coding schemes in terms of the achieved Bit Error Probability (BER) and the proximity to the theoretical capacity.

For the DPC encoder, the memory size of the TCQ is chosen to be 8, and the generator polynomial is set to $[625, 242]$ (in octal form), whose circuitry is given by [16, Fig. 4]. The coset shift of $\Delta = 0.75$ is used following [10]. As far as the IRA code is concerned, two code rates are considered, namely a rate-$1/2$ systematic IRA code with check node and variable node degree profile in [11, Table 3], and a rate-$1/4$ non-systematic IRA code with check node and variable node degree profile in [10, Table III]. The message block size is set to $K = 10,000$, and the transmission of 10 blocks are simulated for each regime, with the number of decoding iteration set to 100.

A. The Weak Interference Regime

Fig. 7 shows the performance of the designed code as compared to the theoretical limit in the "weak interference" regime. Here, DPC is used at the cognitive transmitter with rate-$1/4$ non-systematic IRA code, while rate-$1/2$ systematic IRA code is used at the primary transmitter. To achieve a BER in the order of 10^{-5}, the gap from the theoretical capacity is approximately 0.0514 b/s/Hz (10.82%) for the primary user and 0.0715 b/s/Hz (28.60%) for the cognitive user. These gaps are due to the limitation of the underlying channel code used, which include finite block length, limited decoding iteration, as well as finite alphabet constellation. When the last limitation is dropped by calculating the theoretical capacity using a finite alphabet constraint, the gap reduces to 0.0196 b/s/Hz (3.92%) for the primary user and 0.0692 b/s/Hz (27.68%) for the cognitive user.

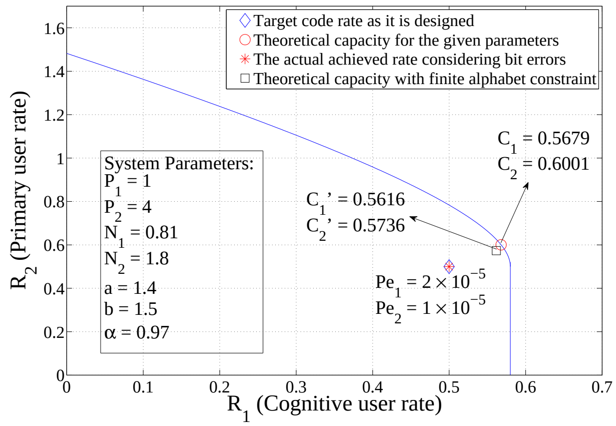

Fig. 8. Designed Code Performance in the "Very Strong Interference" Regime

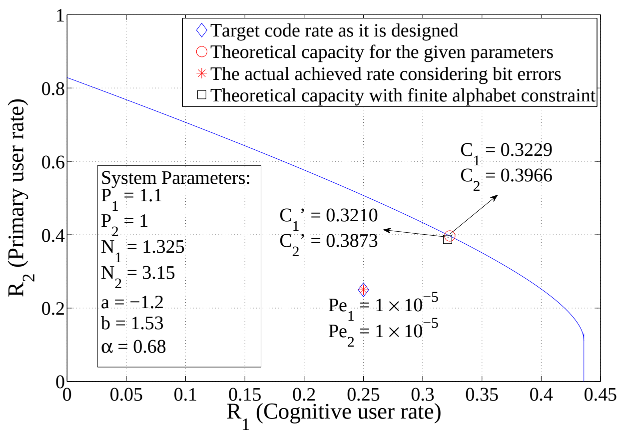

Fig. 9. Designed Code Performance in the "Primary Decodes Cognitive" Regime

B. The Very Strong Interference Regime

For the "very strong interference" regime, the performance of the designed code as compared to the theoretical limit is shown in Fig. 8. Here, both primary and cognitive transmitters employ rate-$1/2$ systematic IRA code. It is apparent from the figure that to achieve the same BER order of 10^{-5}, the gap from the capacity is 0.1001 b/s/Hz (20.02%) for the primary user and 0.0679 b/s/Hz (13.58%) for the cognitive user. These gaps reduce to 0.0736 b/s/Hz (14.72%) and 0.0616 b/s/Hz (12.32%) for the primary and cognitive user, respectively, when the finite alphabet constraint is taken into consideration.

C. The Primary Decodes Cognitive Regime

Lastly, the performance of the designed code in the "primary decodes cognitive" regime is shown in Fig. 9. In this case, DPC is used at the cognitive user, while a rate-$1/4$ non-systematic IRA code is employed at the primary transmitter. The gap of 0.1466 b/s/Hz (58.64%) for the primary user and 0.0729 b/s/Hz (29.16%) for the cognitive user are observed for the same BER order of 10^{-5}. Similarly, these gaps decrease when the finite alphabet constraint is taken into consideration in calculating the theoretical capacity, resulting in a 0.1373 b/s/Hz (54.92%) gap for the primary user and 0.0710 b/s/Hz (28.40%) gap for the cognitive user.

V. Conclusions

In this paper we propose three novel coding schemes for the cognitive overlay radios. Each coding scheme is motivated by the capacity results available for a subset of the parameter region and implements a combination of multi-terminal coding startegies such as dirty paper coding, superposition coding, and joint decoding. In each regime it is shown that the codes we design achieve low BER in the order of 10^{-5} while maintaining good proximity to the capacity point. Our future research will focus on the optimization of the code parameters such as the TCQ scaling factor Γ, coset shift Δ, and DPC scaling λ, which together will result in better performance.

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References

[1] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, "Breaking Spectrum Gridlock with Cognitive Radios: An Information Theoretic Perspective," *Proceedings of the IEEE*, vol. 97, no. 5, pp. 894-914, May 2009

[2] J. Sachs, I. Maric, and A. Goldsmith, "Cognitive Cellular Systems within the TV Spectrum," *IEEE Symp. on New Frontiers in Dynamic Spectrum Access Networks*, Singapore, pp. 1-12, April 2010

[3] A. Jovicic and P. Viswanath, "Cognitive Radio: An Information-Theoretic Perspective," *IEEE Trans. on Inform. Theory*, vol. 55, no. 9, pp. 3945-3958, Sept. 2009

[4] W. Wu, S. Vishwanath, and A. Arapostathis, "Capacity of a Class of Cognitive Radio Channels: Interference Channels with Degraded Message Sets," *IEEE Trans. on Inform. Theory*, vol. 53, no. 11, pp. 4391-4399, Nov. 2007

[5] I. Maric, R. Yates, and G. Kramer, "Capacity of Interference Channels with Partial Transmitter Cooperation," *IEEE Trans. on Inform. Theory*, vol. 53, no. 10, pp. 3536-3548, Oct. 2007

[6] I. Maric, A. Goldsmith, G. Kramer, and S. Shamai, "On the Capacity of Interference Channels with One Cooperating Transmitter," *European Trans. on Telecomm.*, vol. 19, pp. 405-420, April 2008

[7] S. Rini, D. Tunietti, and N. Devroye, "Inner and Outer Bounds for the Gaussian Cognitive Interference Channel and New Capacity Results," *IEEE Trans. on Inform. Theory*, vol. 58, no. 2, pp. 820-848, Jan. 2012

[8] U. Erez and S. T. Brink, "A Close-to-Capacity Dirty Paper Coding Scheme," *IEEE Trans. on Inform. Theory*, vol. 51, no. 10, pp. 3417-3432, Oct. 2005

[9] A. Bennatan, D. Burshtein, G. Caire, and S. Shamai, "Superposition Coding for Side-Information Channels," *IEEE Trans. on Inform. Theory*, vol. 52, no. 5, pp. 1872-1889, May 2006

[10] Y. Sun, Y. Yang, A. D. Liveris, V. Stankovic, and Z. Xiong, "Near-Capacity Dirty-Paper Code Design: A Source-Channel Coding Approach," *IEEE Trans. on Inform. Theory*, vol. 55, no. 7, pp. 3013-3031, July 2009

[11] H. Jin, A. Khandekar, and R. McEliece, "Irregular Repeat-Accumulate Codes," *Second Intl. Conf. on Turbo Codes*, Brest, France, pp. 1-8, Sept. 2000

[12] R. Zamir and M. Feder, "On Lattice Quantization Noise," *IEEE Trans. on Inform. Theory*, vol. 42, no. 4, pp. 1152-1159, July 1996

[13] M. Costa, "Writing on Dirty Paper (Corresp.)," *IEEE Trans. on Inform. Theory*, vol. 29, no. 3, pp. 439-441, May 1983

[14] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal Decoding of Linear Codes for Minimizing Symbol Error Rate," *IEEE Trans. on Inform. Theory*, vol. 20, pp. 284-287, March 1974

[15] S. Benedetto, D. Divsalar, G. Montorsi, and F. Pollara, "A Soft-Input Soft-Output APP Module for Iterative Decoding of Concatenated Codes," *IEEE Commun. Letter*, vol. 1, no. 1, pp. 22-24, Jan. 1997

[16] M. W. Marcellin and T. R. Fischer, "Trellis Coded Quantization of Memoryless and Gauss-Markov Sources," *IEEE Trans. on Commun.*, vol.38, no. 1, pp. 82-93, Jan. 1990