On the Secrecy Rate Region of a Fading Multiple-Antenna Gaussian Broadcast Channel with Confidential Messages and Partial CSIT

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Abstract—In this paper we consider the secure transmission over the fast fading multiple antenna Gaussian broadcast channels with confidential messages (FMGBC-CM), where a multiple-antenna transmitter sends independent confidential messages to two users with information theoretic secrecy and only the statistics of the receivers' channel state information are known at the transmitter. We first use the same marginal property of the FMGBC-CM to classify the non-trivial cases, i.e., those not degraded to the common wiretap channels. We then derive the achievable rate region for the FMGBC-CM by solving the channel input covariance matrices and the inflation factor. Due to the complicated rate region formulae, we resort to low SNR analysis to investigate the characteristics of the channel. Finally, the numerical examples show that under the information-theoretic secrecy requirement both users can achieve positive rates simultaneously.

I. INTRODUCTION

Traditionally, the security of data transmission has been ensured by the key-based enciphering. However, for secure communication in large-scale wireless networks, the key distributions and managements may be challenging tasks [1] [2]. The physical-layer security introduced in [3] [4] is appealing due to its keyless nature. One of the fundamental setting for the physical-layer security is the wiretap channel. In this channel, the transmitter wishes to send messages securely to a legitimate receiver and to keep the eavesdropper as ignorant of the message as possible. Wyner first characterized the secrecy capacity of the discrete memoryless wiretap channel [3]. The secrecy capacity is the largest rate communicated between the source and legitimate receiver with the eavesdropper knowing no information of the messages. Motivated by the demand of high data rate transmission, the multiple antenna systems with security concern were considered by several works. In [5], Shafiee and Ulukus first proved the secrecy capacity of a Gaussian channel with two-input, two-output, single-antenna-eavesdropper. Then the authors of [6]–[8] extended the secrecy capacity result to the Gaussian multiple-input multiple-output, multiple-antenna-eavesdropper channel. On the other hand, the impacts of fading channels on the secure transmission were considered in [9]. Note that [5]–[9] require full channel state information at the transmitter (CSIT). When there is only partial CSIT, several works considered the secure transmission under this condition [10]–[14]. The artificial noise

(AN) assisted secure beamforming is a promising technique for the partial CSIT cases, where in addition to the message-bearing signal, an AN is intentionally transmitted to disrupt the eavesdropper's reception [10] [11]. Indeed, adding AN in transmission is crucial in increasing the secrecy rate in fading wiretap channels. However, the covariance matrices of AN in [11] [10] is heuristically selected without optimization, and the resulting secrecy rate is not optimal. In [12], the secure transmission under fast fading channels with only statistical CSIT and without AN is considered. Although the secrecy capacity for single antenna system with partial CSIT was found in [14], the decoding latency of the transmission scheme proposed in [14] is much longer than the common fast fading channels, e.g., [10]–[13], and may be unacceptable in practice.

However, the assumptions of wiretap channels with full or partial CSIT may not be practical. That is, the eavesdroppers needs to feedback the perfect/statistical CSI to transmitter or the transmitter needs to know this CSI by some means. On the contrary, the eavesdroppers may not be motivated to feedback this information. Furthermore, the eavesdroppers may feedback the wrong CSI to destroy the secure transmission. Thus in this paper, we consider the multiple antenna Gaussian broadcast channel with confidential messages (MGBC-CM) [15] under fast fading channels (abbreviated as *FMGBC-CM*). In the FMGBC-CM, both receivers are legitimate users such that they both are willing to feedback accurate CSI to maintain their secure transmission, and not to be eavesdropped by the other user. In the considered FMGBC-CM, we assume that the transmitter only has the statistics of the channels from both receivers. This is to taking the practical issues into account, such as the limited bandwidth of the feedback channels or the speed of the channel estimation at the receivers. And to the best knowledge of the authors, this problem has not been considered in the literature.

The main contribution of this paper is to provide an achievable rate region with explicit channel input covariance matrices of both users. An iterative algorithm is proposed to solve the inflation factor of the linear assignment Gelfand-Pinsker coding (LA-GPC) [16] used in the adopted transmission scheme. To accomplish these, we first classify the non-trivial cases such that the FMGBC-CM is not degraded as the conventional wiretap channel, i.e., both users have positive

secure transmission rates, by the same marginal property. We then prove that the MISO GBC-CM with identical and independently distributed (i.i.d.) Rayleigh fading channels is degraded as the MISO Gaussian wiretap channel. Thus in this paper we consider the non-i.i.d. Rayleigh fading MISO and Rician fading MISO BC-CM, respectively.

The rest of the paper is organized as follows. In Section II we introduce the considered system model. We then provide the necessary conditions for the FMGBC-CM to be not degraded as a conventional wiretap channel in Section III. In Section IV, we derive the achievable secrecy rate region of the FMGBC-CM. An achievable selection of the channel input covariance matrices and an iterative scheme for solving the inflation factor are also provided to calculate the explicit rate region. In Section V, we demonstrate the secrecy rate region in low SNR regime and the optimal signaling. In Section VI we illustrate the numerical results. Finally, Section VII concludes this paper.

II. SYSTEM MODEL

In this paper we consider the FMGBC-CM system as shown in Fig. 1, where the transmitter has n_T antennas and the receiver 1 and 2 each has single antenna. The received signals at the two receivers can be respectively represented as [*]

$$Y_{1,k} = \boldsymbol{H}_{1,k}{}^H \boldsymbol{X}_k + N_{1,k}, \qquad (1)$$
$$Y_{2,k} = \boldsymbol{H}_{2,k}{}^H \boldsymbol{X}_k + N_{2,k}, \qquad (2)$$

where $\boldsymbol{X}_k \in \mathbb{C}^{n_T \times 1}$ is the transmit vector, k is the time index, respectively, which denote the fading vector channels from transmitter to the two receivers, and $N_{1,k}$ and $N_{2,k}$ are circularly symmetric complex additive white Gaussian noises with variances one at receiver 1 and receiver 2, respectively. In this system, we assume that only the statistics of both channels are known at transmitter, to take the practical issues of system design into account, such as the limited bandwidth of the feedback channels or the speed of the channel estimation at the receivers. We also assume that the receiver 1 and 2 perfectly know their channel vectors $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$, respectively. Without loss of generality, in the following we omit the time index to simplify the notation. We consider the power constraint as

$$\mathrm{tr}(E[\boldsymbol{X}\boldsymbol{X}^H]) \leq P_T.$$

The perfect secrecy and secrecy capacity are defined as follows. Consider a $(2^{nR_1}, 2^{nR_2}, n)$-code with an encoder that maps the message $W_1 \in \mathcal{W}_1 = \{1, 2, \ldots, 2^{nR_1}\}$ and $W_2 \in \mathcal{W}_2 = \{1, 2, \ldots, 2^{nR_2}\}$ into a length-n codeword, and receiver 1 and receiver 2 map the received sequence Y_1^n and Y_2^n (the collections of Y_1 and Y_2, respectively, over code length n) from

[*]In this paper, lower and upper case bold alphabets denote vectors and matrices, respectively. Italic upper alphabets with and without boldface denote random vectors and variables, respectively. The ith element of vector $\mathbf{a}$ is denoted by a_i. The superscript $(.)^H$ denotes the transpose complex conjugate. $|\mathbf{A}|$ and $|a|$ represent the determinant of the square matrix $\mathbf{A}$ and the absolute value of the scalar variable a, respectively. A diagonal matrix whose diagonal entries are $a_1 \ldots a_k$ is denoted by $diag(a_1 \ldots a_k)$. The trace of $\mathbf{A}$ is denoted by $\mathrm{tr}(\mathbf{A})$. We define $C(x) \triangleq \log(1 + x)$ and $(x)^+ \triangleq \max\{0, x\}$. The mutual information between two random variables is denoted by $I(;)$. $\mathbf{I}_n$ denotes the n by n identity matrix. $\mathbf{A} \succ 0$ and $\mathbf{A} \succeq 0$ denote that $\mathbf{A}$ is a positive definite and positive semi-definite matrix, respectively.

the MISO channel to the estimated message $\hat{W}_1 \in \mathcal{W}_1$ and $\hat{W}_2 \in \mathcal{W}_2$, respectively. Since $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ are both known at receiver 1 and 2, respectively, we can treat them as the channel outputs similar to [17]. We then have the following definition of secrecy capacity.

Definition 1 (Secrecy capacity region): Perfect secrecy with rate pair (R_1, R_2) is achievable if, for any positive ε and ε', there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n)$-codes and an integer n_0 such that for any $n > n_0$

$$I(W_1; Y_2^n, \boldsymbol{H}_2^n)/n < \varepsilon, \text{and } \Pr(\hat{W}_1 \neq W_1) \leq \varepsilon', \qquad (3)$$
$$I(W_2; Y_1^n, \boldsymbol{H}_1^n)/n < \varepsilon, \text{and } \Pr(\hat{W}_2 \neq W_2) \leq \varepsilon', \qquad (4)$$

where $\boldsymbol{H}_1^n$ and $\boldsymbol{H}_2^n$ are the collections of $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ over code length n, respectively. The secrecy capacity region *is the closure of the set of all achievable rate pairs (R_1, R_2).*

Note that as shown in the footnote, italic upper alphabets with and without boldface denote random vectors and variables, respectively. By treating $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ as the channel outputs, we can extend the achievable rate region of the discrete memoryless MBC-CM from [15] as

$$(R_1, R_2) \in \mathrm{co}\left\{ \bigcup_{\varpi \in \Omega} \mathcal{R}_l(\varpi) \right\},$$

where $\mathrm{co}\{.\}$ denotes the convex closure; $\mathcal{R}_l(\varpi)$ denotes the union of all (R_1, R_2) satisfying

$$R_1 \leq (I(\boldsymbol{V}_1; Y_1, \boldsymbol{H}_1) - I(\boldsymbol{V}_1; Y_2, \boldsymbol{H}_2, \boldsymbol{V}_2))^+, \qquad (5)$$
$$R_2 \leq (I(\boldsymbol{V}_2; Y_2, \boldsymbol{H}_2) - I(\boldsymbol{V}_2; Y_1, \boldsymbol{H}_1, \boldsymbol{V}_1))^+, \qquad (6)$$

for any given joint probability density ϖ belonging to the class of joint probability densities $p(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{x}, y_1, y_2, \boldsymbol{h}_1, \boldsymbol{h}_2)$, denoted by Ω, that factor as $p(\boldsymbol{v}_1, \boldsymbol{v}_2)p(\boldsymbol{x}|\boldsymbol{v}_1, \boldsymbol{v}_2)p(y_1, y_2, \boldsymbol{h}_1, \boldsymbol{h}_2|\boldsymbol{x})$; $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ are the auxiliary random vectors for user 1 and 2, respectively.

Note that we can further rearrange the right hand side (RHS) of (5) as

$$R_1 \overset{(a)}{\leq} (I(\boldsymbol{V}_1; Y_1, \boldsymbol{H}_1) - I(\boldsymbol{V}_1; \boldsymbol{V}_2, \boldsymbol{H}_2) - I(\boldsymbol{V}_1; Y_2|\boldsymbol{V}_2, \boldsymbol{H}_2))^+$$

$$\overset{(b)}{=} (I(\boldsymbol{V}_1; Y_1, \boldsymbol{H}_1) - I(\boldsymbol{V}_1; \boldsymbol{V}_2) - I(\boldsymbol{V}_1; Y_2|\boldsymbol{V}_2, \boldsymbol{H}_2))^+$$

$$\overset{(c)}{=} (I(\boldsymbol{V}_1; Y_1|\boldsymbol{H}_1) + I(\boldsymbol{V}_1; \boldsymbol{H}_1) - I(\boldsymbol{V}_1; Y_2|\boldsymbol{V}_2, \boldsymbol{H}_2) - I(\boldsymbol{V}_1; \boldsymbol{V}_2))^+$$

$$\overset{(d)}{=} (I(\boldsymbol{V}_1; Y_1|\boldsymbol{H}_1) - I(\boldsymbol{V}_1; Y_2|\boldsymbol{V}_2, \boldsymbol{H}_2) - I(\boldsymbol{V}_1; \boldsymbol{V}_2))^+, \qquad (7)$$

where (a) is by applying the chain rule of mutual information to the second term on the RHS of (5); (b) is due to $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ are independent of $\boldsymbol{H}_2$; (c) is again applying the chain rule to the first term; (d) is due to the fact that there is only statistical CSIT, and $\boldsymbol{V}_1$ is independent of $\boldsymbol{H}_1$. Thus $I(\boldsymbol{V}_1; \boldsymbol{H}_1) = 0$. Similarly, we can can rearrange R_2 as

$$R_2 \leq (I(\boldsymbol{V}_2; Y_2|\boldsymbol{H}_2) - I(\boldsymbol{V}_2; Y_1|\boldsymbol{V}_1, \boldsymbol{H}_1) - I(\boldsymbol{V}_1; \boldsymbol{V}_2))^+. \qquad (8)$$

III. CONDITIONS FOR NON-DEGRADED FMGBC-CM

Before investigating the rate region of the FMGBC-CM, we need to exclude the cases that FMGBC-CM are degraded as fast fading wiretap channels, for which one of the two

receivers always has zero rate. The capacity result can be concluded according to the results in [18]. Furthermore, from [18] we know that for such channel the optimal eigenvectors and eigenvalues of the channel input covariance matrix are arbitrarily orthonormal basis and uniform allocated powers, respectively.

Our first result is as following.

Lemma 1: A necessary condition for the two users in the fast Rayleigh FMGBC-CM both having positive rates is that $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ are not i.i.d..

Note that the wiretap channel is a special case of the GBC-CM which can be easily derived by letting W_2 in the GBC-CM as null. To prove this result, we need to introduce the following lemma first, which is extended from [15, Lemma 4]

Lemma 2: Let $\mathcal{P}$ denote the set of channels $p(\tilde{y}_1, \tilde{y}_2, \tilde{\boldsymbol{h}}_1, \tilde{\boldsymbol{h}}_2 | \boldsymbol{x})$ whose marginal distributions satisfy

$$p_{\tilde{Y}_1, \tilde{\boldsymbol{H}}_1 | \boldsymbol{X}}(\tilde{y}_1, \tilde{\boldsymbol{h}}_1 | \boldsymbol{x}) = p_{Y_1, \boldsymbol{H}_1 | \boldsymbol{X}}(y_1, \boldsymbol{h}_1 | \boldsymbol{x}), \quad (9)$$

$$p_{\tilde{Y}_2, \tilde{\boldsymbol{H}}_2 | \boldsymbol{X}}(\tilde{y}_2, \tilde{\boldsymbol{h}}_2 | \boldsymbol{x}) = p_{Y_2, \boldsymbol{H}_2 | \boldsymbol{X}}(y_2, \boldsymbol{h}_2 | \boldsymbol{x}), \quad (10)$$

for all y_1, y_2, and $\boldsymbol{x}$. The secrecy capacity region is the same for all channels $p(\tilde{y}_1, \tilde{y}_2, \tilde{\boldsymbol{h}}_1, \tilde{\boldsymbol{h}}_2 | \boldsymbol{x}) \in \mathcal{P}$.

Note that $p(\tilde{y}_1, \tilde{y}_2, \tilde{\boldsymbol{h}}_1, \tilde{\boldsymbol{h}}_2 | \boldsymbol{x})$ is from the factorization below (6). Due to limited space, we give a sketch of the proof of Lemma 1 in the following. Assume both channels are i.i.d., i.e., $\boldsymbol{H}_1 \sim CN(0, \sigma_1^2 \mathbf{I})$ and $\boldsymbol{H}_2 \sim CN(0, \sigma_2^2 \mathbf{I})$. With the same marginal property in Definition 2, we can replace $\boldsymbol{H}_1$ in (1) by $(\sigma_1/\sigma_2)\boldsymbol{H}_2$ without affecting the capacity. Thus we have a new pair of channels with the same capacity as (1) and (2)

$$Y_1' = (\sigma_1/\sigma_2)\boldsymbol{H}_2^H \boldsymbol{X} + N_1,$$
$$Y_2 = \boldsymbol{H}_2^H \boldsymbol{X} + N_2,$$

which can be further represented as

$$Y_1'' = \boldsymbol{H}_2^H \boldsymbol{X} + (\sigma_2/\sigma_1)N_1,$$
$$Y_2 = \boldsymbol{H}_2^H \boldsymbol{X} + N_2.$$

Thus as long as $\sigma_1 > \sigma_2$, we can have the Markov chain $\boldsymbol{X} \to Y_1'' \to Y_2$. On the other hand, by extending the outer bound of [15, Theorem 3], we know that *less noisy* [19, Ch. 5] makes one of the FMGBC-CM user have zero rate. Since Y_2 is degraded of Y_1'', and degradedness is more strict than the less noisy, thus we know $R_2 = 0$. Similarly, when $\sigma_2 > \sigma_1$, we know that $R_1 = 0$.

An intuitive explanation is that, if a message can be successfully decoded by the inferior user, then the superior user is also ensured of decoding it. Thus the secrecy rate of the degraded user is zero. Based on the concept mentioned above, we can extend Lemma 1 to the following.

Corollary 1: A necessary condition for the two users in the fast Rayleigh FMGBC-CM both having positive rates is that the covariance matrices of $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ should not be scaled of each other.

Therefore to avoid the investigation of such cases, in the following we assume $\boldsymbol{H}_1 \sim CN(\boldsymbol{\mu}_1, \mathbf{K}_{\boldsymbol{H}_1})$ and $\boldsymbol{H}_2 \sim CN(\boldsymbol{\mu}_2, \mathbf{K}_{\boldsymbol{H}_2})$, where $\mathbf{K}_{\boldsymbol{H}_1}$ and $\mathbf{K}_{\boldsymbol{H}_2}$ may not be scaled of each other.

Two special cases with single input single output (SISO) antenna GBC-CM are also summarized as follows.

Corollary 2: All SISO Rayleigh fading GBC-CMs with only statistical CSIT degrade as wiretap channels.

Corollary 3: All SISO Rician fading GBC-CMs with only statistical CSIT degrade as wiretap channels if the channels have the same K-factor.

Remark: Directly verifying the *less noisy* [19] property, i.e., $I(\boldsymbol{V}, Y_1) \lessgtr I(\boldsymbol{V}, Y_2)$ from extending the upper bound derivation in [20, Theorem 3, Example 1] to multiple antenna case involves manipulations of two rates similar to (15) and (16), which is intractable. $\boldsymbol{V}$ here can be either $\boldsymbol{V}_1$ or $\boldsymbol{V}_2$.

IV. THE ACHIEVABLE SECRECY RATE REGION OF FMGBC-CM

Due to the fact that there is only statistical CSIT, we can not use the original minimum mean square error (MMSE) inflation factor as Costa [21], where the exact channel state information is required. Thus we need to re-derive the achievable rate region of the FMGBC-CM instead of directly using Liu's result in [15, Lemma 3]. To derive the new achievable rate region, we resort to the linear assignment Gel'fand Pinsker coding [16], which is the generalized case of DPC, to deal with the fading channels, similar to our previous work [22]. For the FMGBC-CM, we consider the secret LA-GPC with Gaussian codebooks. First, separate the channel input $\boldsymbol{X}$ into two random vectors $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ so that $\boldsymbol{X} = \boldsymbol{U}_1 + \boldsymbol{U}_2$. Then $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ are chosen as follows:

$$\boldsymbol{U}_1 \sim CN(\mathbf{0}, \mathbf{K}_{\boldsymbol{U}_1}), \quad (11)$$
$$\boldsymbol{U}_2 \sim CN(\mathbf{0}, \mathbf{K}_{\boldsymbol{U}_2}), \quad (12)$$

where $\boldsymbol{U}_2$ is independent of $\boldsymbol{U}_1$, $\mathbf{K}_{\boldsymbol{U}_1} \succeq 0$ and $\mathbf{K}_{\boldsymbol{U}_2} \succeq 0$ are the covariance matrices of $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$, respectively. After that, we do the decomposition $\mathbf{K}_{\boldsymbol{U}_1} = \mathbf{T}_1 \mathbf{T}_1^H$, and define $\boldsymbol{U}_1' \sim CN(\mathbf{0}, \mathbf{I}_N)$ so that $\boldsymbol{U}_1 = \mathbf{T}_1 \boldsymbol{U}_1'$, where $\mathbf{T}_1 \in \mathbb{C}^{n_T \times N}$ and N is the rank of $\mathbf{K}_{\boldsymbol{U}_1}$. The auxiliary random variables are then defined as:

$$\boldsymbol{V}_1 = \boldsymbol{U}_1' + \mathbf{a}\boldsymbol{H}_1^H \boldsymbol{U}_2, \quad (13)$$
$$\boldsymbol{V}_2 = \boldsymbol{U}_2, \quad (14)$$

where $\mathbf{a}$ is the inflation factor in LA-GPC. The reason to choose (13) is that if we do LA-GPC for $\boldsymbol{U}_1$ directly, i.e., $\boldsymbol{V}_1 = \boldsymbol{U}_1 + \mathbf{a}\boldsymbol{H}_1^H \boldsymbol{U}_2$, after substituting it into the RHS of (5) and (8), we can find that the rate formula includes $\log|\mathbf{K}_{\boldsymbol{U}_1}|$ when calculating $I(\boldsymbol{V}_1; \boldsymbol{V}_2)$, which requires $\mathbf{K}_{\boldsymbol{U}_1} \succ 0$. However, the expression of (13) would bypass this constraint. Note that in the rest of this paper, for convenience of

computation, we combine $\mathbf{a}\boldsymbol{H}_1^H$ as $\mathbf{b}$. To present the rate regions compactly, recall that the permutation π specifies the encoding order, i.e., the message of user π_1 is encoded first while the message of user π_2 is encoded second.

Lemma 3: Let $\mathcal{R}(\mathbf{K}_{\boldsymbol{U}_{\pi_1}}, \mathbf{K}_{\boldsymbol{U}_{\pi_2}})$ denote the union of all (R_{π_1}, R_{π_2}) satisfying

$$R_{\pi_1} \leq \left(E_{\boldsymbol{H}_{\pi_1}}[\log(1 + \boldsymbol{H}_{\pi_1}{}^H(\mathbf{K}_{\boldsymbol{U}_{\pi_1}} + \mathbf{K}_{\boldsymbol{U}_{\pi_2}})\boldsymbol{H}_{\pi_1})] - \triangle \right)^+, \tag{15}$$

$$R_{\pi_2} \leq \left(E_{\boldsymbol{H}_{\pi_2}}[\log(1 + \boldsymbol{H}_{\pi_2}{}^H(\mathbf{K}_{\boldsymbol{U}_{\pi_1}} + \mathbf{K}_{\boldsymbol{U}_{\pi_2}})\boldsymbol{H}_{\pi_2})] - \triangle \right)^+, \tag{16}$$

where

$$\triangle \triangleq E_{\boldsymbol{H}_{\pi_2}}[\log(1 + \boldsymbol{H}_{\pi_2}{}^H \mathbf{K}_{\boldsymbol{U}_{\pi_1}}\boldsymbol{H}_{\pi_2})] +$$
$$E_{\boldsymbol{H}_{\pi_1}}\left[\log \left| \begin{matrix} \mathbf{I} + \mathbf{b}\mathbf{K}_{\mathrm{U}_{\pi_2}}\mathbf{b}^H & (\mathbf{T}_1^H + \mathbf{b}\mathbf{K}_{\mathrm{U}_{\pi_2}})\boldsymbol{H}_{\pi_1} \\ \boldsymbol{H}_{\pi_1}{}^H(\mathbf{T}_1 + \mathbf{K}_{\mathrm{U}_{\pi_2}}\mathbf{b}^H) & 1 + \boldsymbol{H}_{\pi_1}{}^H(\mathbf{K}_{\boldsymbol{U}_{\pi_1}} + \mathbf{K}_{\boldsymbol{U}_{\pi_2}})\boldsymbol{H}_{\pi_1} \end{matrix} \right| \right]. \tag{17}$$

Then any rate pair

$$(R_1, R_2) \in \mathrm{co} \left\{ \bigcup_{\mathrm{tr}(\mathbf{K}_{\boldsymbol{U}_{\pi_1}} + \mathbf{K}_{\boldsymbol{U}_{\pi_2}}) \leq P_T} \mathcal{R}(\mathbf{K}_{\boldsymbol{U}_{\pi_1}}, \mathbf{K}_{\boldsymbol{U}_{\pi_2}}) \right\}$$

is achievable for the FMGBC-CM.

Due to the limited space, we do not provide the derivation in detail. In the following, we provide an achievable scheme to approximately achieve the above two bounds in (15) and (16).

Lemma 4: With the selection $\mathbf{K}_{\boldsymbol{U}_{\pi_1}}^* = \alpha P_T \mathbf{e}_{\pi_1}^*(\mathbf{e}_{\pi_1}^*)^H$ and $\mathbf{K}_{\boldsymbol{U}_{\pi_2}}^* = (1 - \alpha) P_T \mathbf{e}_{\pi_2}^*(\mathbf{e}_{\pi_2}^*)^H$, where $\|\mathbf{e}_{\pi_1}^*\|^2 = 1$, $\|\mathbf{e}_{\pi_2}^*\|^2 = 1$, and

$$\mathbf{e}_{\pi_1}^* = \max_{\mathbf{e}_{\pi_1}} \frac{\mathbf{e}_{\pi_1}^H \left(\mathbf{I} + \alpha P_T \left(\mathbf{K}_{\boldsymbol{H}_{\pi_1}} + \boldsymbol{\mu}_{\pi_1}\boldsymbol{\mu}_{\pi_1}^H \right) \right) \mathbf{e}_{\pi_1}}{\mathbf{e}_{\pi_1}^H \left(\mathbf{I} + \alpha P_T \left(\mathbf{K}_{\boldsymbol{H}_{\pi_1}} + \boldsymbol{\mu}_{\pi_2}\boldsymbol{\mu}_{\pi_2}^H \right) \right) \mathbf{e}_{\pi_1}}, \tag{18}$$

$$\mathbf{e}_{\pi_2}^* = \max_{\mathbf{e}_{\pi_2}} \frac{\mathbf{e}_{\pi_2}^H \left(\mathbf{I} + \frac{(1-\alpha)P_T\left(\mathbf{K}_{\boldsymbol{H}_{\pi_2}} + \boldsymbol{\mu}_{\pi_2}\boldsymbol{\mu}_{\pi_2}^H \right)}{1 + \alpha P_T (\mathbf{e}_{\pi_1}^*)^H \left(\mathbf{K}_{\boldsymbol{H}_{\pi_2}} + \boldsymbol{\mu}_{\pi_2}\boldsymbol{\mu}_{\pi_2}^H \right)\mathbf{e}_{\pi_1}^*} \right) \mathbf{e}_{\pi_2}}{\mathbf{e}_{\pi_2}^H \left(\mathbf{I} + \frac{(1-\alpha)P_T\left(\mathbf{K}_{\boldsymbol{H}_{\pi_2}} + \boldsymbol{\mu}_{\pi_1}\boldsymbol{\mu}_{\pi_1}^H \right)}{1 + \alpha P_T (\mathbf{e}_{\pi_1}^*)^H \left(\mathbf{K}_{\boldsymbol{H}_{\pi_1}} + \boldsymbol{\mu}_{\pi_1}\boldsymbol{\mu}_{\pi_1}^H \right)\mathbf{e}_{\pi_1}^*} \right) \mathbf{e}_{\pi_2}}, \tag{19}$$

where α is the ratio of power allocated to user π_1, we can get the non-trivial rate region for the FMGBC-CM as

$$(R_1, R_2) \in \mathrm{co} \left\{ \bigcup_{0 \leq \alpha \leq 1} \mathcal{R}(\mathbf{K}_{\boldsymbol{U}_{\pi_1}}^*, \mathbf{K}_{\boldsymbol{U}_{\pi_2}}^*) \right\}.$$

Due to the limited space, only the proof sketch is given as follows. Instead of solving $\mathbf{K}_{\boldsymbol{U}_{\pi_1}}$ and $\mathbf{K}_{\boldsymbol{U}_{\pi_2}}$ from (15) and (16) directly, which may be intractable, we resort to solving the upper bound of the rate region described by Lemma 3. That is, the transmitter can use full CSIT to design the inflation factor. Then it is clear that the optimal $\mathbf{b}$ is the MMSE estimator

$$\mathbf{b} = \mathbf{T}_{\pi_1}^H \boldsymbol{H}_{\pi_1} \boldsymbol{H}_{\pi_1}^H / (1 + \boldsymbol{H}_{\pi_1}^H \mathbf{K}_{\boldsymbol{U}_{\pi_1}} \boldsymbol{H}_{\pi_1}). \tag{20}$$

Then after some manipulations and applying the Jensen's inequality followed by the unit rank selection of $\mathbf{K}_{\boldsymbol{U}_{\pi_1}}$ and $\mathbf{K}_{\boldsymbol{U}_{\pi_2}}$, we can have the Rayleigh quotient form as (18) and (19). Note that with [23, Property 2, 3] it can be proved that when the number of transmit antenna is 2 with $\mathbf{K}_{\boldsymbol{H}_{\pi_1}} - \mathbf{K}_{\boldsymbol{H}_{\pi_2}} \not\succeq 0$, then unit rank $\mathbf{K}_{\boldsymbol{U}_{\pi_1}}$ and $\mathbf{K}_{\boldsymbol{U}_{\pi_2}}$ is optimal for the considered upper bound.

After deriving the covariance matrices, we then need to solve the inflation factor due to the fact that there is indeed no full CSIT. Here we resort to the following fixed point iteration to solve $\mathbf{b}$

$$\mathbf{b} = -(E_{\boldsymbol{H}_1}[\mathbf{A}_1^H])^{-1} E_{\boldsymbol{H}_1}[\mathbf{A}_2^H \boldsymbol{H}_1^H] \triangleq f(\mathbf{b}), \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix} \triangleq \mathbf{M}^{-1} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}, \tag{21}$$

where $\mathbf{M}$ is defined as the block matrix inside the determinant of the second term in (17). Note that (21) is derived by $\partial R_1 / \partial \mathbf{b} = 0$. Note also that the iteration stops when the maximum relative error of R_1 and R_2 in the successive iterations is less than a predefined value. The iteration steps are summarized in Table I.

TABLE I
THE ITERATIVE STEPS FOR SOLVING $\mathbf{b}$.

Step 1	Set $i = 0$ and initialize $\mathbf{b}^{(i)} = \mathbf{0}$. Also initialize $\mathbf{e}_{\pi_1}$ and $\mathbf{e}_{\pi_2}$ as (18) and (19), respectively.
Step 2	Evaluate $\mathbf{b}^{(i+1)} = f(\mathbf{b}^{(i)})$.
Step 3	Let $i = i + 1$ and repeat Step 2 until $\max\{R_1^{(i)} - R_1^{(i-1)}, R_2^{(i)} - R_2^{(i-1)}\} < \varepsilon$.

V. LOW SNR ANALYSIS

In this section we study the achievable secrecy rate region in the low-SNR regime. Note that operation at low SNRs is beneficial from a security perspective since it is generally difficult for a eavesdropper to detect the signal [24]. In addition, due to the rate region in Lemma 3 is complicated to analyze, we resort to the low SNR regime to get some insights.

Lemma 5: In the low SNR regime, the optimal input co-variance matrices $\mathbf{K}_{\boldsymbol{U}_1}$ and $\mathbf{K}_{\boldsymbol{U}_2}$ are both unit rank, with the direction aligned to the eigenvector corresponding to the max-imum eigenvalue of $\mathbf{K}_{\boldsymbol{H}_1} - \mathbf{K}_{\boldsymbol{H}_2}$ and $\mathbf{K}_{\boldsymbol{H}_2} - \mathbf{K}_{\boldsymbol{H}_1}$, respectively. And the asymptote of the secrecy rate region is

$$R_1 \leq \left(\frac{\alpha P_T}{\ln 2} \lambda_{max}(\mathbf{K}_{\boldsymbol{H}_1} - \mathbf{K}_{\boldsymbol{H}_2}) \right)^+, \tag{22}$$

$$R_2 \leq \left(\frac{(1-\alpha)P_T}{\ln 2} \lambda_{max}(\mathbf{K}_{\boldsymbol{H}_2} - \mathbf{K}_{\boldsymbol{H}_1}) \right)^+. \tag{23}$$

Note that the unit rank result is consistent to that of MGBC-CM with perfect CSIT, also our selection of the $\mathbf{K}_{\boldsymbol{U}_{\pi_1}}^*$ and $\mathbf{K}_{\boldsymbol{U}_{\pi_2}}^*$ in Lemma 4. From the rate region described in (22) and (23), we have

Corollary 4: In the low SNR regime, both users can have positive rates simultaneously if and only if $\mathbf{K}_{\boldsymbol{H}_1} - \mathbf{K}_{\boldsymbol{H}_2}$ is

indefinite.

Remark: Note that it can be easily seen that Corollary 4 includes Lemma 1 in the low SNR regime. That is, if both H_1 and H_2 are i.i.d., respectively, then $\mathbf{K}_{H_1} - \mathbf{K}_{H_2}$ has all eigenvalues positive or negative. Thus the two results coincide.

VI. NUMERICAL RESULTS

In this section, we compare the rate regions of our proposed achievable scheme under both Rayleigh (with at least one channel having non-i.i.d. distribution) and Rician fading channels to that of full CSIT MGBC-CM, respectively. We set $n_T = 2$ and the power constraint $P_T = 10$, respectively. We also set the stopping criteria of the iterative algorithm as $\varepsilon = 10^{-3}$. In the simulation of Rayleigh fading case, we set the covariance matrices of the two channels as

$$\mathbf{K}_{H_1} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.04 \end{bmatrix}, \mathbf{K}_{H_2} = \begin{bmatrix} 0.1 & 0.08 \\ 0.08 & 0.1 \end{bmatrix}, \quad (24)$$

which satisfy Lemma 1. Since the selection of $\mathbf{K}^*_{U_{\pi_1}}$ in Lemma 4 is rank 1, we know that $\mathbf{b} \in \mathbb{C}^{2\times 1}$ by definition. For the full CSIT case, we consider the rate region which is the convex closure of the following rate pair

$$R_{\pi_1} \leq E\left[\left(\log_2 \frac{1 + H^H_{\pi_1} \mathbf{K}_{U_{\pi_1}} H_{\pi_1}}{1 + H^H_{\pi_2} \mathbf{K}_{U_{\pi_1}} H_{\pi_2}}\right)^+\right], \quad (25)$$

$$R_{\pi_2} \leq$$
$$E\left[\left(\log_2 \frac{[1 + H^H_{\pi_2}(\mathbf{K}_{U_{\pi_1}} + \mathbf{K}_{U_{\pi_2}})H_{\pi_2}](1 + H^H_{\pi_1}\mathbf{K}_{U_{\pi_1}}H_{\pi_1})}{[1 + H^H_{\pi_1}(\mathbf{K}_{U_{\pi_1}} + \mathbf{K}_{U_{\pi_2}})H_{\pi_1}](1 + H^H_{\pi_2}\mathbf{K}_{U_{\pi_1}}H_{\pi_2})}\right)^+\right], \quad (26)$$

with the power constraint $\mathrm{tr}(\mathbf{K}_{U_{\pi_1}} + \mathbf{K}_{U_{\pi_2}}) \leq 10$, where the optimal $\mathbf{K}_{U_{\pi_1}}$ and $\mathbf{K}_{U_{\pi_2}}$ are described in [15, (16)] and the optimal $\mathbf{b}$ is as (20). Note that (25) and (26) are the straightforward extension of [15] to the fast fading channels with full CSIT. From Fig. 2 we can easily see that the proposed transmission scheme for the fast FMGBC-CM with partial CSIT apparently outperforms the time sharing scheme. Time sharing means that the transmitter sends the two messages with different powers during a fraction of time where these powers satisfy the average power constraint. And in each fraction of time, the fast FMGBC-CM reduces to a fading Gaussian MISO wiretap channel. We also consider the case that the inflation factor $\mathbf{b}$ uses the MMSE estimator with channel mean, i.e., $\mathbf{b} = [0\,0]$, which is the same as treating interference as noise. From Fig. 2 it can be seen that the performance of treating interference as noise is slightly better than the time sharing. On the other hand, by comparing the regions of full and partial CSIT cases, we can easily find the impact of the CSIT to the rate performance.

For the Rician fading case, in addition to (24), we let the mean vectors of H_1 and H_2 as

$$\boldsymbol{\mu}_1 = \begin{bmatrix} 0.7 \\ 0.1 \end{bmatrix}, \boldsymbol{\mu}_2 = \begin{bmatrix} 0.1 \\ 0.6 \end{bmatrix},$$

respectively. From Fig. 3, we also can easily see that the CSIT plays an important role in improving the rate region. And time
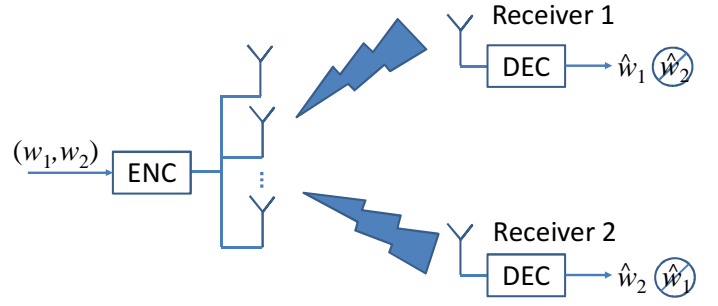

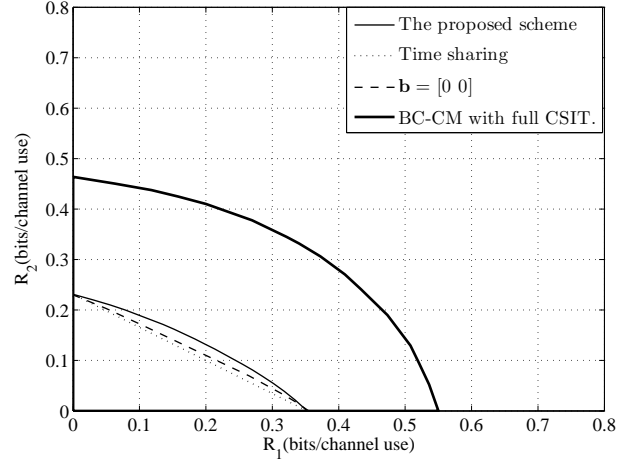Fig. 1. The system model of FMGBC-CM.

Fig. 2. The comparison of rate regions under fast Rayleigh fading channel with full and statistical CSIT.

sharing is still the worst. With the aid of line of sight, the performances of all schemes under Rician fading are much better than the corresponding ones under Rayleigh fading. We also compare the case where $\mathbf{b}$ is derived from substituting $H_1 = \boldsymbol{\mu}_1$ into (20). It can easily be seen that the proposed $\mathbf{b}$ outperforms this selection of $\mathbf{b}$. On the other hand, due to the gap is small, when low complexity is an important issue, the transmitter can choose such $\mathbf{b}$ to implement the secure LA-GPC. Furthermore, we also show the rate region derived by $\mathbf{b} = [0\,0]$. Similar to the Rayleigh fading case, this method is still worse than the proposed method, but slightly better than the time sharing. In Fig. 4 and Fig. 5 we also compare the rate regions with different transmit SNRs under both Rayleigh and Rician fading channels. It can be seen that the rate regions of both cases enlarge with increasing transmit SNR.

VII. CONCLUSION

In this paper we considered the secure transmission over the fast fading multiple antenna Gaussian broadcast channels with confidential messages (FMGBC-CM), where a multiple-antenna transmitter sends independent confidential messages to two users with information theoretic secrecy and only the statistics of the receiver's channel state information are known at the transmitter. We first used the same marginal property of the FMGBC-CM to classify the non-trivial cases, i.e., not degraded to the common wiretap channels. We then
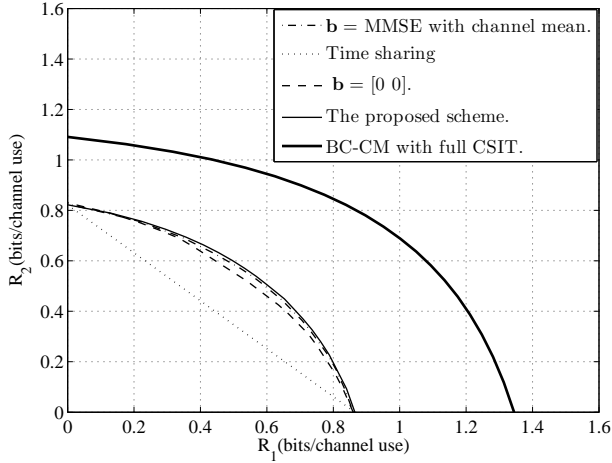
derive the achievable rate region for the FMGBC-CM by solving the channel input covariance matrices and the inflation factor. We also provided a low SNR analysis for finding the asymptotic property of the channel due to the complicated rate region formulae. Numerical examples demonstrated that both users can achieve positive rates simultaneously under the information-theoretic secrecy requirement.

Fig. 3. The comparison of rate regions under fast Rician fading channel with full and statistical CSIT.
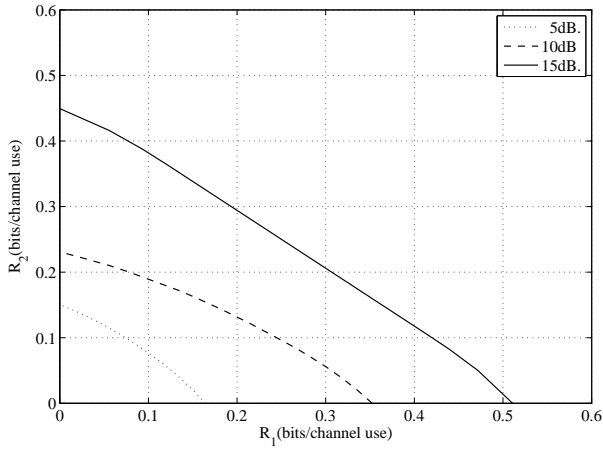
Fig. 4. The comparison of rate regions under fast Rayleigh fading channel with statistical CSIT and different transmit SNRs.

Fig. 5. The comparison of rate regions under fast Rician fading channel with statistical CSIT and different transmit SNRs.

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