

Controlling Privacy Loss in Survey Sampling (Working Paper)

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Abstract

Social science and economics research is often based on data collected in surveys. Due to time and budgetary constraints, this data is often collected using complex sampling schemes designed to increase accuracy while reducing the costs of data collection. A commonly held belief is that the sampling process affords the data subjects some additional privacy. This intuition has been formalized in the differential privacy literature for simple random sampling: a differentially private mechanism run on a simple random subsample of a population provides higher privacy guarantees than when run on the entire population. In this work we initiate the study of the privacy implications of more complicated sampling schemes including cluster sampling and stratified sampling. We find that not only do these schemes often not amplify privacy, but that they can result in privacy degradation.

1 Introduction

In social science and economics research, surveying an entire population is typically infeasible due to time and budgetary constraints. As a result, much of the data collected by statistical agencies is based on surveys performed on a random sample of the target population. While the main motivation for sampling is often financial, a commonly held belief is that sampling provides additional privacy. Intuitively, data subjects are afforded some plausible deniability by the fact that they may, or may not, have been sampled. This intuition of increased privacy from sampling has been formalized for some types of sampling schemes (such as simple random sampling with and without replacement or Poisson sampling) in a series of papers in the differential privacy literature [10, 14, 4, 13]. These types of *privacy amplification by sampling* results are useful for a variety of reasons including producing more accurate results at a target privacy level, and for providing additional incentive for participation in a survey study.

However, simple random sampling schemes like sampling with or without replacement and Poisson sampling are rarely used in practice. Statistical agencies typically use more complex sampling designs to increase the accuracy of the results and/or to reduce the costs of data collection. Unfortunately, these more complicated sampling schemes can result in privacy degradation. That is, sampling can actually make privacy guarantees worse. It is often the case that the design of a survey is based on sensitive historic or auxiliary data. Thus, the survey design itself can leak additional information about the population. We find that this degradation of the privacy guarantee occurs even for reasonably simple sampling designs when the sampling design is data dependent (see Lemma 2.1).

We focus on two common sampling schemes: cluster sampling and stratified sampling. We will discuss the implication of these sample designs on the effective privacy loss, in terms of the ϵ parameter from differential privacy. For stratified sampling, we will discuss how the data dependent nature of this sampling design results in privacy degradation. We'll then discuss a potential method for controlling the privacy loss for proportional sampling,

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a common sampling design for stratified sampling. For cluster sampling, we’ll discuss how the lack of independence between the inclusion of different data subjects affects the privacy guarantee. We’ll show that even for relaxed notions of privacy and “ideal” data, while the privacy does not degrade, privacy amplification is negligible.

1.1 Privacy Amplification by Sampling

Several works, e.g. [10, 14, 4, 13], have shown that by applying a differentially private data analysis to a random sample of the population, rather than to the population itself, one can achieve an amplified privacy guarantee. That is, the algorithm obtained by composing a differentially private algorithm with a random sampling process provides a quantitatively better privacy protection than the original algorithm. These works have considered different notions of privacy, e.g. bounded and unbounded differential privacy, and different sampling methods, e.g. Poisson sampling, simple random sampling with replacement and without replacement. Balle et al. [2] gave a unified framework for analyzing these privacy amplification effects via statistical divergences and probabilistic coupling between samples. However, all of these works assume that the sampling mechanism is data independent, which is not often the case in practical statistical surveys.

Let us now introduce some notation we will use in the remainder of the paper. Let \mathcal{U} be a data universe. For any $n \in \mathbb{N}$, let \mathcal{U}^n be the set of all datasets of size n and $\mathcal{U}^* = \cup_{n \in \mathbb{N}} \mathcal{U}^n$. Let $P \in \mathcal{U}^*$ be the population that is sampled from. We will refer to $x \in \mathcal{U}$ as the data of a particular *data subject*. We’ll use $|P|$ to denote the size of a dataset. We will not distinguish between “historic” or “auxiliary” data and “current” data. That is, P will be both the dataset used in designing the sampling scheme, and the dataset that is sampled from to get the final sample. This is for ease of notation, as well as to highlight the fact that we will treat historic and auxiliary data as sensitive. In future work, we would like to explore different models for the relationship between these datasets, for example by modeling temporal correlation between historic and current data.

We will focus on unbounded differential privacy, also known as add/remove differential privacy, in this paper. That is, the presence or removal of a data subject in the dataset is protected. This implies that the size of the dataset is not fixed, and is considered a sensitive attribute. The specific technical results may be slightly different for bounded differential privacy, where the size of the dataset is fixed and public, although we expect the general intuition to hold for both notions. Given the privacy degradation results shown in this paper, it would be interesting to consider other, possibly weaker, notions of privacy.

2 Privacy Degradation for Data Dependent Survey Design

It is often the case that the design of a survey is based on historic or auxiliary data. A common example is stratified sampling, where characteristics of the target units are used to assign each data subject in the population to a single stratum. Often knowledge about the within-stratum variability of the target variable is then used to further improve the accuracy of the collected data. Nonresponse adjustment is another example, where response rates from previous rounds of the survey are used to adjust the initial sampling rates for different subgroups of the data to ensure that the final number of successful interviews matches the desired sample size. In addition to this data dependence, many survey designs introduce dependencies between the inclusion of the different data subjects. An extreme example is snow-ball sampling where data subjects are asked to recruit other data subjects to the study. This phenomenon also occurs in simpler survey designs like cluster sampling. In survey sampling designs with these properties, it is possible for the privacy guarantee to actually degrade.

Many common survey sampling designs result in privacy degradation. A simple, but important, example is sampling without replacement when the number of samples is data-dependent. For a function $f : \mathcal{U}^* \rightarrow \mathbb{Z}$, let

$$GS_f = \max_{P \in \mathcal{U}^*, x \in \mathcal{U}} |f(P) - f(P \cup \{x\})|$$

be the global sensitivity of f .

Lemma 2.1. *Let $f : \mathcal{U}^* \rightarrow \mathbb{Z}$ be a deterministic function and let $F : \mathcal{U}^* \rightarrow \mathcal{U}^* \times \mathbb{Z}$ be defined by $F(P) = (P, f(P))$. Let $\mathcal{C}_{\text{WOR}} : \mathcal{U}^* \times \mathbb{Z} \rightarrow \mathcal{U}^*$ be the randomised sampling function that maps (P, m) to a dataset drawn from the uniform distribution on $\{Z \mid Z \subset P, |Z| = m\}$. If \mathcal{M} is ϵ -DP then $\mathcal{M} \circ \mathcal{C}_{\text{WOR}} \circ F$ is $(\epsilon \cdot \text{GS}_f)$ -DP. Moreover, there exists an ϵ -DP algorithm \mathcal{M} such that $\mathcal{M} \circ \mathcal{C}_{\text{WOR}} \circ F$ is not ϵ' -DP for any $\epsilon' < \text{GS}_f \cdot \epsilon$.*

The following is a simple example of a ϵ -DP mechanism \mathcal{M} that suffers from the worst case privacy degradation. Let f and \mathcal{C}_{WOR} be as in Lemma 2.1. Suppose that $\mathcal{M}(Z) = |Z| + \text{Lap}(1/\epsilon)$, which provides an ϵ -DP estimate of the size of the dataset. The resulting output of $\mathcal{M} \circ \mathcal{C}_{\text{WOR}} \circ F(P)$ is $f(P) + \text{Lap}(1/\epsilon)$. Then, a simple calculation shows that $\mathcal{M} \circ \mathcal{C}_{\text{WOR}} \circ f$ does not satisfy ϵ' -DP for any $\epsilon' < \text{GS}_f \cdot \epsilon$.

In the following section, we will focus on *stratified sampling*, where we'll see natural examples of functions f with sensitivity higher than 1. We will see that this exact type of data dependence is present in one of the most common stratified sampling schemes, *proportional sampling*. We will also suggest a slight alteration on these common sampling schemes that will result in privacy amplification rather than privacy degradation.

3 Stratified Sampling

In stratified sampling, the population is partitioned into disjoint subsets, called strata. Random sampling is then performed within each stratum. Stratified sampling is a method of variance reduction if the data collecting agency expects the existence of mutually heterogenous but internally homogeneous subpopulations. It also ensures that each stratum is represented in the final sample. An informative example is the IAB Job Vacancy Survey [3], a business survey by the Institute for Employment Research (IAB) in Germany that stratifies the population by industry, region, and size of establishment. Other examples include stratification by race or gender. A crucial feature of this sampling design is that the sampling rate may differ from stratum to stratum, and may be data dependent. We will consider strata membership to be a sensitive attribute that is determined before sampling begins.

While the sampling within strata can be performed in a variety of ways, simple random sampling without replacement is often used for business surveys. Unfortunately, sampling without replacement falls exactly into the type of data dependent sampling that was discussed in Lemma 2.1. Let $\mathbf{f} : \mathcal{U}^* \rightarrow \mathbb{Z}^k$ be the function that computes the number of cases that will be sampled per stratum. The global sensitivity of \mathbf{f} is

$$\text{GS}_{\mathbf{f}} := \max_{P \in \mathcal{U}^*, x \in \mathcal{U}} \|\mathbf{f}(P) - \mathbf{f}(P \cup \{x\})\|_1,$$

the sum of the changes in each stratum. The following is an extension of Lemma 2.1.

Lemma 3.1. *Let $\mathbf{f} : \mathcal{U}^* \rightarrow \mathbb{Z}^k$ be a deterministic function. Let $\mathbf{F} : \mathcal{U}^* \rightarrow \mathcal{U}^* \times \mathbb{Z}^k$ be defined by $\mathbf{F}(P) = (P, \mathbf{f}(P))$ and \mathcal{C} be the function that randomly samples a dataset of size $\mathbf{f}(P)_i$ from stratum i for all $i \in [k]$. For any ϵ -DP algorithm \mathcal{M} , $\mathcal{M} \circ \mathcal{C} \circ \mathbf{F}$ is $\text{GS}_{\mathbf{f}} \cdot \epsilon$ -DP. Moreover, there exists a differentially private mechanism \mathcal{M} such that $\mathcal{M} \circ \mathcal{C} \circ \mathbf{F}$ is not ϵ' -DP for any $\epsilon' < \text{GS}_{\mathbf{f}} \cdot \epsilon$.*

As a concrete example, let us consider *proportional sampling*. Let S_1, \dots, S_k be the strata so $P = S_1 \cup \dots \cup S_k$. In proportional sampling a total number of samples, n , is decided in advance. Then $n \cdot \frac{|S_i|}{|P|}$ data subjects are randomly sampled from stratum S_i . Since $n \cdot \frac{|S_i|}{|P|}$ can be fractional, one must determine a method of apportioning the samples. If this process is not done carefully then privacy degradation can be significant, for example if deterministic rounding is used, then \mathbf{f} can be unstable and $\text{GS}_{\mathbf{f}}$ can be as large as k .

An important feature of stratified sampling is that the parameters of the sampling design can vary between the strata. This means that data subjects in strata with low sampling rates should expect a higher level of privacy than data subjects in strata with high sampling rates. This leads us to a form of differential privacy that allows the privacy guarantee to vary between the strata. Suppose there exist k strata, and that each data point is a pair (s, x) where $s \in [k]$ denotes which stratum the data subject belongs to, and x denotes their data.

Definition 3.2. Suppose there are k strata. A mechanism \mathcal{A} satisfies $(\epsilon_1, \dots, \epsilon_k)$ -stratified differential privacy if for all datasets P , data points (s, x) , and sets of outcomes E ,

$$e^{-\epsilon_s} \mathbb{P}(\mathcal{A}(P) \in E) \leq \mathbb{P}(\mathcal{A}(P \cup \{(s, x)\}) \in E) \leq e^{\epsilon_s} \mathbb{P}(\mathcal{A}(P) \in E)$$

This definition is an adaptation of personalized differential privacy [9, 7, 1]. Notice that this definition protects whether or not the data subject is included in the dataset. It also implies that the *value* of the data subject’s data is also protected, although one has to pay for the privacy loss of the true stratum, and the privacy loss of the “fake” stratum: for any dataset P , and data points (s, x) and (s', x') , and any set of outcomes E ,

$$\mathbb{P}(\mathcal{A}(P \cup \{(s, x)\}) \in E) \leq e^{\epsilon s + \epsilon s'} \mathbb{P}(\mathcal{A}(P \cup \{(s', x')\}) \in E).$$

If the sampling method within each stratum is Poisson sampling, then not only does the privacy guarantee not degrade; it actually improves! The following is an immediate consequence of the results for traditional differential privacy [10, 14, 4, 13].

Lemma 3.3 (Privacy Amplification for Stratified Sampling with Poisson Sampling). *Suppose there exist constants $r_1, \dots, r_k \in [0, 1]$ and let $\mathbf{r} : \mathcal{U}^* \rightarrow \mathcal{U}^* \times [0, 1]^k$ be the function $\mathbf{r}(P) = (P, r_1, \dots, r_k)$. Let $\mathcal{C}_P : \mathcal{U}^* \times [0, 1]^k \rightarrow \mathcal{U}^*$ be the sampling mechanism that maps $(P, (r_1, \dots, r_k))$ to subset where each data subject in stratum i is sampled with probability r_i . If \mathcal{M} is ϵ -DP then $\mathcal{M} \circ \mathcal{C}_P \circ \mathbf{r}$ is $(\log(1 + r_1(e^\epsilon - 1)), \dots, \log(1 + r_k(e^\epsilon - 1)))$ -stratified DP.*

Note that when ϵ is small then

$$\log(1 + r_i(e^\epsilon - 1)) \approx r_i \epsilon,$$

so the privacy parameter is scaled by the probability of sampling any individual data subject. When ϵ is large, $\log(1 + r_i(e^\epsilon - 1)) \approx \epsilon + \log r_i$, which is slightly smaller than ϵ , so we don’t gain a significant advantage in this regime.

3.1 Controlling privacy loss in stratified sampling

We discussed two results in the previous section: the sampling without replacement can results in privacy degradation and Poisson sampling can result in privacy amplification. However, sampling without replacement is the popular sampling method in practice for in-strata sampling. This method allows the data collecting agency to control the total sample size, an important property in practice due to budgetary constraints. In contrast, in Poisson sampling, the number of samples follows a binomial distribution, and so it can be difficult to budget appropriately for the number of samples. Thus, it is important to consider lightweight adaptations of these sampling schemes that control privacy, while also allowing the analyst to upper bound the number of samples.

One option to control privacy loss in sampling without replacement is to use more stable apportionment methods. Apportionment methods like Huntington-Hill (the algorithm for senate seat apportionment used by the US congress) produce more stable apportionment functions than deterministic rounding. Under some mild assumptions the Huntington-Hill algorithm has global sensitivity at most 2. These methods do have computational overhead, in particular because the number of samples in stratum i is a function of the sizes of all the strata, so the sample size can not be computed on a stratum-by-stratum basis. We conjecture that a simpler algorithm, randomized rounding, provides not only lower computational overhead, but actually results in privacy amplification for sampling without replacement.

Conjecture 3.4 (Privacy Amplification for Stratified Sampling without Replacement). *Let $\mathbf{r} : \mathcal{U}^* \rightarrow [0, 1]^k$ be the constant function $\mathbf{r}(P) = (r_1, \dots, r_k)$ for some constants $r_1, \dots, r_k \in [0, 1]$. Let $\mathcal{C}_P : \mathcal{U}^* \times [0, 1]^k \rightarrow \mathcal{U}^*$ be the sampling mechanism that maps $(P = S_1 \cup \dots \cup S_k, (r_1, \dots, r_k))$ to a sample $T_1 \cup \dots \cup T_k$ as follows. For each $i = 1, \dots, k$, let $p_i = r_i |S_i| - \lfloor r_i |S_i| \rfloor$. Choose $n_i = \lfloor r_i |S_i| \rfloor$ with probability $1 - p_i$, and $n_i = \lceil r_i |S_i| \rceil$ with probability p_i . Let T_i be a random sample from S_i of size n_i .*

If \mathcal{M} is ϵ -DP for $\epsilon < 1$, then $\mathcal{M} \circ \mathcal{C}_P \circ \mathbf{r}$ is $(\log(1 + O(\epsilon)r_1), \dots, \log(1 + O(\epsilon)r_k))$ -stratified DP.

4 Clustered Sampling

In this section, we switch our attention to another feature of many survey sampling designs: dependencies between the inclusion of different data subjects. That is, sampling schemes where the inclusion of data subject i is

correlated with the inclusion of data subject j . We will focus on cluster sampling, a common survey sampling design. In cluster sampling, the population is partitioned into disjoint subsets, called clusters. A fraction of the clusters are chosen, and then random sampling is performed within the chosen clusters. This type of sampling scheme produces accurate results when the clusters are mutually homogeneous yet internally heterogeneous. That is, when the distributions within each cluster are similar to the distribution over the entire population. Cluster sampling is often performed due to time or budgetary restrictions that make sampling many units from a few clusters cheaper and/or faster than sampling a few units from each cluster. A typical example is when clusters are chosen to be geographic regions where traveling between clusters is both time consuming and expensive. Note that this will typically result in clusters that are internally homogeneous reducing the statistical efficiency compared to simple random sample. This loss in efficiency is accepted to save costs.

We will assume that cluster membership is a sensitive attribute that is determined before sampling begins. There are many sampling designs that analysts use to choose which clusters to sample from. We focus on a simple cluster sampling design that might naively look like a good candidate for privacy amplification. We show that even under a weaker notion of privacy (random differential privacy), this sampling design achieves less amplification than might be expected. Note that more complicated cluster sampling designs may result in privacy degradation.

For a data subject i , let χ_i be a random variable that is 1 if i is sampled from P , and 0 if i is not sampled from P . A crucial feature of a cluster sampling scheme, is that for two data subjects i and j , the random variables χ_i and χ_j are correlated. For example, if two data subjects i and j belong in the same cluster, then the probability of i being sampled conditioned on j being sampled is higher than the probability of i being sampled conditioned on j not being sampled. While some level of dependence can be handled by the traditional amplification by sampling results discussed in Section 1.1, e.g. sampling without replacement, these dependences are a lot milder than those present in cluster sampling.

The traditional privacy amplification by subsampling results are often called “secrecy of the sample” since the additional privacy is derived from the fact that whether or not any data subject is included in the sample is hidden. This intuition fails us for cluster sampling, a few simple examples show that there is no general amplification result for cluster sampling. To see this consider the following simple cluster sampling scheme: there are two clusters C_1 and C_2 so $P = C_1 \cup C_2$. A single cluster C_i is chosen uniformly at random and a ϵ -differentially private mechanism \mathcal{M} is performed on C_i . Let \mathcal{C} be the sampling algorithm. Let C'_1 differ from C_1 on a single data point. Then

$$\frac{\mathbb{P}(\mathcal{M} \circ \mathcal{C}(P) = a)}{\mathbb{P}(\mathcal{M} \circ \mathcal{C}(P') = a)} = \frac{\frac{1}{2}\mathbb{P}(\mathcal{M}(C_1) = a) + \frac{1}{2}\mathbb{P}(\mathcal{M}(C_2) = a)}{\frac{1}{2}\mathbb{P}(\mathcal{M}(C'_1) = a) + \frac{1}{2}\mathbb{P}(\mathcal{M}(C_2) = a)} \quad (1)$$

Now we can see that if the distribution on outputs of $\mathcal{M}(C_1)$ and $\mathcal{M}(C_2)$ is very different then we don't expect amplification (for intuition suppose $\mathbb{P}(\mathcal{M}(C_2) = a) = 0$ and $\mathbb{P}(\mathcal{M}(C_1) = a) \neq 0$). For a concrete example, suppose $\mathcal{M}(Z) = |Z| + \text{Lap}(1/\epsilon)$ for some mechanism \mathcal{M} with sensitivity 1. Suppose $|C_1| = 0$ and $|C_2| = b$ and $|C'_1| = 1$ then the privacy guarantee of $\mathcal{M} \circ \mathcal{C}$ is

$$\frac{\mathbb{P}(\mathcal{M} \circ \mathcal{C}(P) = 0)}{\mathbb{P}(\mathcal{M} \circ \mathcal{C}(P') = 0)} = \frac{1 + e^{-\epsilon b}}{e^{-\epsilon} + e^{-\epsilon b}}, \quad (2)$$

which quickly approaches e^ϵ as b increases. Intuitively, the problem is that from the output, it is easy to guess which cluster was chosen, so there is no *secrecy of the sample*.

Conversely, if the output distributions of $\mathcal{M}(C_1)$ and $\mathcal{M}(C_2)$ really are similar then equation 1 shows that we do obtain amplification. For example if the distributions are identical then the privacy guarantee of $\mathcal{M} \circ \mathcal{C}$ is $\log((1 + e^\epsilon)/2) < \epsilon$. Unfortunately, in practice as pointed out above, we rarely have homogeneity between clusters.

Random differential privacy [8] gives a way to formalize the notion that we *expect* the datasets C_1 and C_2 to look similar. Given a distribution \mathcal{D} over \mathcal{U} , random differential privacy requires that the differential privacy guarantee holds with high probability over neighbouring datasets P and P' drawn from \mathcal{D}^* . Unfortunately, even under this significantly weaker notion of the privacy guarantee and the strong assumption that both clusters are drawn i.i.d. from the same distribution, we obtain negligible amplification. Suppose that the clusters are mutually homogeneous such that each data point (in both clusters) x_i is draw from the uniform distribution on $\{0, 1\}$. Let

$\mathcal{M}(Z) = g(Z) + \text{Lap}(1/\epsilon)$ where $g(Z) = \sum_{i=1}^{|Z|} z_i$, and $|C_1| = |C_2| = n$. Both $g(C_1)$ and $g(C_2)$ are drawn from the Binomial distribution $\text{Bin}(n, 1/2)$ which has variance $\frac{1}{4}n$. Therefore, we expect $|g(C_1) - g(C_2)| \approx \frac{1}{4}\sqrt{n}$ with constant probability. This results in a privacy guarantee of $\frac{e^\epsilon + e^{-\frac{1}{4}\epsilon\sqrt{n}}}{1 + e^{-\frac{1}{4}\epsilon\sqrt{n}}}$, which quickly approaches e^ϵ as n increases. The problem again is that there is no *secrecy of the sample* since the noise due to privacy is smaller than the sampling error.

5 Conclusions and Future Work

Data-dependent sampling rates and correlations between individuals’ sampling events necessitate caution in using the intuition that subsampling improves privacy in real sampling designs. Indeed, our negative results provide scenarios where sampling degrades privacy.

We are working to extend our positive result for stratified sampling with Poisson sampling within each stratum to sampling designs with non-constant sampling rates. In proportional sampling, the number of samples drawn from each stratum i is proportional to the size of stratum i to ensure constant sampling rates in each stratum. A more complex sampling design is the Neyman (or “optimal”) allocation which accounts for both the size of each stratum and the variability of a statistic within each stratum to allocate a fixed number of samples in a fashion that minimizes the total variance of the statistic.

Our predominant goal has been to analyse the privacy implications of sampling schemes *as they are currently deployed*. However, we are also working to identify lightweight changes current sampling schemes that would result in improved privacy guarantees. We are working to circumvent our lower bounds for sampling without replacement when working with datasets for which \mathbf{f} has lower local sensitivity than the worst case. Techniques such as the propose-test-release framework [6] would let us privately verify that \mathbf{f} has low local sensitivity and, in such cases, give a guaranteed amplified privacy parameter. The downside to such an approach is that testing for non-worst-case sensitivity requires changing the algorithm and consuming additional privacy budget.

As our ultimate objective is to understand what kinds of amplified privacy guarantees are afforded by real sampling designs *themselves*, a more appealing option is to take a definitional perspective: Could there be a variant of differential privacy (with similar semantics) that does enjoy amplification under data-dependent sampling without replacement?

Finally, an additional challenge is to model sampling designs that involve complex-decision making. In particular, sampling designs where rates depend on human judgment or opaque analyses of past data releases are difficult to encode when conducting an end-to-end privacy analysis.

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