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# Secure Network Release with Link Privacy

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Carl Yang<sup>1</sup>, Haonan Wang<sup>2</sup>, Lichao Sun<sup>3</sup>, Bo Li<sup>2</sup>

<sup>1</sup>Emory University, <sup>2</sup>University of Illinois at Urbana Champaign, <sup>3</sup>University of Illinois at Chicago  
<sup>1</sup>j.carlyang@emory.edu, <sup>2</sup>{haonan3, lbo}@illinois.edu, <sup>3</sup>lsun29@uic.edu

## Abstract

Many data mining and analytical tasks rely on the abstraction of networks (graphs) to summarize relational structures among individuals (nodes). Since relational data are often sensitive, we aim to seek effective approaches to release utility-preserved yet privacy-protected structured data. In this paper, we leverage the differential privacy (DP) framework, to formulate and enforce rigorous privacy constraints on deep graph generation models, with a focus on edge-DP to guarantee individual link privacy. In particular, we enforce edge-DP by injecting Gaussian noise to the gradients of a link reconstruction based graph generation model, and ensure data utility by improving structure learning with structure-oriented graph comparison. Extensive experiments on two real-world network datasets show that our proposed DPGGAN model is able to generate networks with effectively preserved global structure and rigorously protected individual link privacy.

## 1 Introduction

Nowadays, open data of networks play a pivotal role in data mining and data analytics [49, 44, 5, 34]. By releasing and sharing structured relational data with research facilities and enterprise partners, data companies are harvesting the enormous potential value from their data, which benefits decision making on various aspects including social, financial, environmental, through collectively improved ads, recommendation, retention and so on [56, 57, 47, 30]. However, network data usually encode sensitive information not only about individuals but also their interactions, which makes direct release and exploitation rather unsafe. More importantly, even with careful anonymization, individual privacy is still at stake under collective attack models facilitated by the underlying network structure [63, 8]. Can we find a way to securely release network data without drastic sanitization that essentially renders the released data useless?

To deal with such tension between the need to release utilizable data and the concern of data owners' privacy, quite a few models have been proposed recently, focusing on grid-based data like images, texts and gene sequences [19, 42, 50, 39, 38, 54, 9, 7, 15, 33, 62]. However, none of the existing models can be directly applied to the network (graph) setting. While a secure generative model on grid-based data apparently aims to preserve high-level semantics (*e.g.*, class distributions) and protect detailed training data (*e.g.*, exact images or sentences), it remains obtuse what to be preserved and what to be protected for network data, due to its modeling of complex interactive objects.

**Motivating scenario.** In Figure 1, a bank aims to encourage public studies on the community structures of its customers. It does so by firstly anonymizing all customers in the network and then sharing the anonymized network (*i.e.*, network (a) in Figure 1) to the public. However, an attacker interested in knowing the financial interactions (*e.g.*, money transfer) between particular customers can try to get access to another public social network and locate a group of users that likely overlap with the customers in network (a) (*e.g.*, by leveraging public user profiles like location). Simple graph properties like node degree distribution and triangle count can then be used to identify specific customers with high accuracy (*e.g.*, customer *A* as the only node with degree 5 and within 1 triangle,

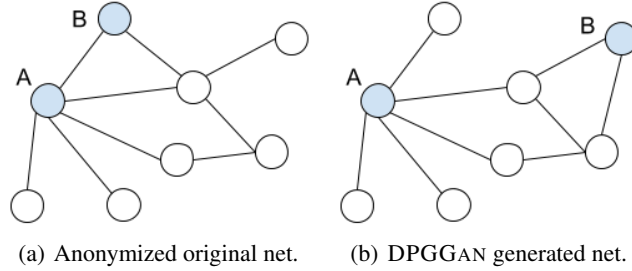


Figure 1: A toy pair of anonymized and generated networks.

and customer  $B$  as the only node with degree 2 and within 1 triangle). Thus, the attacker confidently knows the identities of  $A$  and  $B$  and the fact that they have financial interactions, which seriously harms customers' privacy and poses potential crises.

As the first contribution in this work, we define and formulate the goals of secure network release as *preserving global network structure* while *protecting individual link privacy*. Continue with the toy example, the solution we propose is to train a graph neural network model on the original network and release the generated networks (e.g., (b) in Figure 1). Towards the utility of generated networks, we require them to be similar to the original networks from a global perspective, which can be measured by various graph global properties (e.g., network (b) has very similar degree distribution and the same triangle count as (a)). In this way, we expect many downstream data mining and analytical tasks on them to produce similar results as on the original networks. As for privacy protection, we require that the information in the generated networks cannot confidently reveal the existence or absence of any individual links in the original networks (e.g., the attacker may still identify customers  $A$  and  $B$  in network (b), but their individual link structure has changed).

Subsequently, there are two unique challenges in learning such structure-preserved and privacy-protected graph generation models, which have not been explored by existing literature so far.

**Challenge 1: Rigorous protection of individual link privacy.** The rich relational structures in graph data often allow attackers to recover private information through various ways of collective inference [61, 40, 2]. Moreover, graph structure can always be converted to numerical features such as spectral embedding, after which most attacks on grid-based data like model inversion [18] and membership inference [46] can be directly applied for link identification. How can we design an effective mechanism with rigorous privacy protection on links in networks against various attacks?

**Challenge 2: Effective preservation of global network structure.** In order to capture global network structure, the model has to constantly compare the structures of the input graphs and currently generated graphs during training. However, unlike images and other grid-based data, graphs have flexible structures, and thus lack efficient universal representations [13]. How can we allow a network generation model to effectively learn from the structural difference between two graphs, without conducting very time-costly operations like isomorphism tests all the time?

**Present work.** In this work, for the first time, we draw attention to the secure release of network data with deep generative models. Technically, towards the aforementioned two challenges, we develop Differentially Private Graph Generative Nets (DPGGAN), which imposes DP training over a link reconstruction based network generation model for rigorous individual link privacy protection, and further ensures structure-oriented graph comparison for effective global network structure preservation. In particular, we first formulate and enforce edge-DP via Gaussian gradient distortion by injecting designed noise into the sensitive modules during model training. Then we leverage graph convolutional networks [29] through a variational generative adversarial network architecture [22, 32] to enable structure-oriented network comparison.

To evaluate the effectiveness of DPGGAN, we conduct extensive experiments on two real-world network datasets. On one hand, we evaluate the utility of generated networks by computing a suite of commonly concerned graph properties to compare the global structure of generated networks with the original ones. On the other hand, we validate the privacy of individual links by evaluating links predicted from the generated networks on the original networks. Consistent experimental results show that DPGGAN is able to effectively generate networks that are similar to the original ones regarding global network structure, while at the same time useless towards individual link prediction.

## 2 Related Work

**Differential Privacy (DP).** With graph structured data, two types of privacy constraint can be applied, *i.e.*, node-DP [26] and edge-DP [4], which define two neighboring graphs to differ by at most one node or edge. In this work, we aim at the secure release of network data, and particularly, we focus on edge privacy, because it is essential for the protection of object interactions unique for network data in comparison with other types of data. Several existing works have studied the protection of edge-DP. For example, [43] generates graphs based on the statistical representations extracted from the original graphs blurred by designed noise, whereas [51] enforces the parameters of dK-graph models to be private. However, based on shallow graph generation models, they do not flexibly capture global network structure that can support various unknown downstream analytical tasks [63, 52].

Recent advances in deep learning has led to the rapid development of DP-oriented learning schemes. For example, [1] refines the analysis of privacy costs, which provides tighter estimation on the overall privacy loss by tracking detailed information of the stochastic gradient descent process. DP learning has also been widely adapted to generative models [19, 42, 50, 39, 38, 54, 9, 7, 15, 33, 62]. For example, [19, 9, 7, 62] share the same spirit by enforcing DP on the discriminators, and thus inductively on the generators, in a generative adversarial network (GAN) scheme. However, none of them can be directly applied to graph data due to the lack of consideration on structure generation.

**Graph Generation (GGen).** GGen has been studied for decades and widely used to synthesize network data used towards the development of various collective analysis and mining models [17, 23]. Earlier works mainly use probabilistic models to generate graphs with certain properties [16, 53, 3, 41], which are manually designed based on sheer observations and prior assumptions.

Thanks to the surge of deep learning, many advanced GGen models have been developed recently, which leverage different powerful neural networks in a learn-to-generate manner [28, 6, 60, 48, 35, 59, 25, 21, 12, 64, 36]. For example, NetGAN [6] converts graphs into biased random walks, learns the generation of walks with GAN, and assembles the generated walks into graphs; GraphRNN [60] regards the generation of graphs as node-and-edge addition sequences, and models it with a heuristic breadth-first-search scheme and hierarchical RNN. These neural network based models can often generate graphs with much richer properties and flexible structures learned from real-world graphs.

To the best of our knowledge, no existing work on deep GGen has looked into the potential privacy threats laid during the learning and releasing of the powerful models. In fact, such concerns are rather urgent in the network setting, where sensitive information can often be more easily compromised in a collective manner [11, 2, 61] and privacy leakage can easily further propagate [40, 65].

## 3 DPGGAN

In this work, we propose DPGGAN for the secure release of generated networks, whose global graph structures are similar to the original sensitive networks, but the individual links (edges) between objects (nodes) are safely protected.

To provide robust privacy guarantees towards various graph attacks, we propose to leverage the well-studied technique of differential privacy (DP) [14] by enforcing the edge-DP defined as follows.

**Definition 1 (Edge Differential Privacy [4])** *A randomized mechanism  $\mathcal{M}$  satisfies  $(\varepsilon, \delta)$ -edge-DP if for any two neighboring graphs  $\mathbf{G}_1, \mathbf{G}_2 \in \mathcal{G}$ , which differ by at most one edge,  $\Pr[\mathcal{M}(\mathbf{G}_1) \in S] \leq \exp(\varepsilon) \times \Pr[\mathcal{M}(\mathbf{G}_2) \in S] + \delta$ , where  $S \subset \text{range}(\mathcal{M})$ .*

Our key insight is, *a graph generation model  $\mathcal{M}$  satisfying the above edge-DP should learn to generate similar graphs given the input of two neighboring graphs that differ by at most one edge; as a consequence, information in the generated graph does not confidently reveal the existence or absence of any one particular edge in the original graph, thus protecting individual link privacy.*

To ensure DP on individual links, we exploit the existing link reconstruction based graph generation model GVAE [28], and design a training algorithm to dynamically distort the gradients of its sensitive model parameters by injecting proper amounts of Gaussian noise based on the framework of DPSGD [1]. Moreover, to improve the capturing of global graph structures, we replace the direct binary cross-entropy (BCE) loss on graph adjacency matrices in GVAE with a structure-oriented graph discriminator based on GCN [29] and the framework of VAEGAN [22, 32].

**Backbone GVAE.** Recent research on graph models has been largely focused around GCN [29], which is shown to be promising in calculating universal graph representations [37, 55, 10, 27]. In this work, we harness the power of GCN under the consideration of edge-DP by adapting the link reconstruction based graph variational autoencoder (GVAE) [28] as our backbone graph generation model.

Particularly, we are given a graph  $\mathbf{G} = \{\mathbf{V}, \mathbf{E}\}$ , where  $\mathbf{V}$  is the set of  $N$  nodes (vertices), and  $\mathbf{E}$  is the set of  $M$  links (edges), which can be further modeled by a binary adjacency matrix  $\mathbf{A}$ . As a common practice [24], we set the node features  $\mathbf{X}$  simply as the one-hot node identity matrix. The autoencoder architecture of GVAE consists of a GCN-based graph encoder to guide the learning of a fully connected feedforward neural network (FNN) based adjacency matrix decoder, which can be trained to directly reconstruct graphs with similar links as in the input graphs. A stochastic latent variable  $\mathbf{Z}$  is further introduced as the latent representation of  $\mathbf{A}$  as

$$q(\mathbf{Z}|\mathbf{X}, \mathbf{A}) = \prod_{i=1}^N q(\mathbf{z}_i|\mathbf{X}, \mathbf{A}) = \prod_{i=1}^N \mathcal{N}(\mathbf{z}_i|\mu_i, \text{diag}(\sigma_i^2)), \quad (1)$$

where  $\mu = \mathbf{g}_\mu(\mathbf{X}, \mathbf{A})$  is the matrix of mean vectors  $\mu_i$ , and  $\sigma = \mathbf{g}_\sigma(\mathbf{X}, \mathbf{A})$  is the matrix of standard deviation vectors  $\sigma_i$ .  $\mathbf{g}_\bullet(\mathbf{X}, \mathbf{A}) = \tilde{\mathbf{A}}\text{ReLU}(\tilde{\mathbf{A}}\mathbf{X}\mathbf{W}_0)\mathbf{W}_1$  is a two-layer GCN model.  $\mathbf{g}_\mu$  and  $\mathbf{g}_\sigma$  share the first-layer parameters  $\mathbf{W}_0$ .  $\tilde{\mathbf{A}} = \mathbf{D}^{-\frac{1}{2}}\mathbf{A}\mathbf{D}^{-\frac{1}{2}}$  is the symmetrically normalized adjacency matrix of  $\mathbf{G}$ , with degree matrix  $\mathbf{D}_{ii} = \sum_{j=1}^N \mathbf{A}_{ij}$ .  $\mathbf{g}_\mu$  and  $\mathbf{g}_\sigma$  form the encoder network.

To generate a graph  $\mathbf{G}'$ , a reconstructed adjacency matrix  $\mathbf{A}'$  is computed from  $\mathbf{Z}$  by a decoder network as

$$p(\mathbf{A}|\mathbf{Z}) = \prod_{i=1}^N \prod_{j=1}^N p(\mathbf{A}_{ij}|\mathbf{z}_i, \mathbf{z}_j) = \prod_{i=1}^N \prod_{j=1}^N \sigma(\mathbf{f}(\mathbf{z}_i)^T \mathbf{f}(\mathbf{z}_j)), \quad (2)$$

where  $\sigma(z) = 1/(1 + e^{-z})$ ,  $\mathbf{f}$  is a two-layer FNN appended to  $\mathbf{Z}$  before the logistic sigmoid function. It aims to generate individual links to be compared with those in the input graph.

The whole model is trained through standard variational inference by optimizing the following variational lower bound

$$\begin{aligned} \mathcal{L}_{vae} &= \mathcal{L}_{\text{rec}} + \mathcal{L}_{\text{prior}} \\ &= \mathbb{E}_{q(\mathbf{Z}|\mathbf{X}, \mathbf{A})}[\log p(\mathbf{A}|\mathbf{Z})] - D_{\text{KL}}(q(\mathbf{Z}|\mathbf{X}, \mathbf{A})||p(\mathbf{Z})), \end{aligned} \quad (3)$$

where  $\mathcal{L}_{\text{rec}}$  is implemented as the sum of an element-wise binary cross entropy (BCE) loss between the adjacency matrices of the input and generated graphs, and  $\mathcal{L}_{\text{prior}}$  is a prior loss based on the Kullback-Leibler divergence towards the Gaussian prior  $p(\mathbf{Z}) = \prod_{i=1}^N p(\mathbf{z}_i) = \prod_{i=1}^N \mathcal{N}(\mathbf{z}_i|\mathbf{0}, \mathbf{I})$ .

**Enforcing DP.** The probabilistic nature of  $\mathbf{Z}$  allows the model to be generative, meaning that after training the model with an input graph  $\mathbf{G}$ , we can detach and disregard the encoder, and then freely generate an unlimited amount of graphs  $\mathbf{G}'$  with similar links to  $\mathbf{G}$ , by solely drawing random samples of  $\mathbf{Z}$  from the prior distribution  $\mathcal{N}(\mathbf{0}, \mathbf{I})$  and computing  $\mathbf{A}'$  with the learned decoder network *w.r.t.* Eq. 2. However, as shown in [31, 20], powerful neural network models like VAE can easily overfit training data, so directly releasing a trained GVAE model poses potential privacy threats, as links in its generated graphs may be highly indicative towards links in the training graphs.

In this work, we care about rigorously protecting the privacy of individual links in the training data, *i.e.*, ensuring edge-DP. Particularly, in Definition 1, the inequality guarantees that the distinguishability of any one edge in the graph will be restricted to the privacy leak level proportional to  $\epsilon$ ;  $\delta$  relaxes the outlier nodes existing in the graph. The two parameters together quantify the absolute value of privacy information possibly to be leaked by the mechanism  $\mathcal{M}$ , *i.e.*, a graph generation model.

According to Eq. 2, GVAE essentially takes a graph  $\mathbf{G}$ , in particular, the links  $\mathbf{E}$  among the nodes  $\mathbf{V}$  in  $\mathbf{G}$ , as input and generates a new graph  $\mathbf{G}'$  by reconstructing the links  $\mathbf{E}'$  among the same set of nodes  $\mathbf{V}$ . Therefore, if we regard GVAE as the mechanism  $\mathcal{M}$ , as long as its model parameters are properly randomized, the framework satisfies edge-DP. To be specific, any two input graphs  $\mathbf{G}_1$  and  $\mathbf{G}_2$  differing by at most one link in principle lead to similar generated graphs  $\mathbf{G}'$ , so information in  $\mathbf{G}'$  does not confidently reveal the existence or absence of any particular link in  $\mathbf{G}_1$  or  $\mathbf{G}_2$ . To exploit the well-structured graph generation framework of GVAE, we leverage the technique of Gaussian mechanism to enforce edge-DP on it.

**Theorem 1 (Gaussian Mechanism [14])** *If the  $\ell_2$ -norm sensitivity of a deterministic function  $f$  is  $\Delta_2 f$ , we have:*

$$\mathcal{M}_f(\mathbf{G}) \triangleq f(\mathbf{G}) + \mathcal{N}(0, \Delta_2 f^2 \sigma^2), \quad (4)$$

where  $\mathcal{N}(0, \Delta_2 f^2 \sigma^2)$  is a random variable obeying the Gaussian distribution with mean 0 and standard deviation  $\Delta_2 f \sigma$ . The randomized mechanism  $\mathcal{M}_f(\mathbf{G})$  is  $(\epsilon, \delta)$ -differentially private if  $\sigma \geq \sqrt{2 \ln(1.25/\delta)}/\epsilon$  and  $\epsilon < 1$ .

In our setting,  $\mathbf{G}$  is the original training graph. Then Eq. 4 tells us that a link reconstruction based graph generation model  $\mathcal{M}$  can be randomized to ensure  $(\epsilon, \delta)$ -edge-DP with properly parameterized Gaussian noise. Therefore, we leverage Theorem 1 by perturbing the gradient optimization of GVAE. In particular, we follow [19] to inject a designed Gaussian noise to the gradients of our decoder network clipped by a hyper-parameter  $C$  as follows

$$\tilde{g}_\theta = \frac{1}{N} \sum_{i=1}^N \left( \nabla_{v_i, \mathbf{f}} \mathcal{L}_{rec} / \max(1, \frac{\nabla_{v_i, \mathbf{f}} \mathcal{L}_{rec}}{C}) + \mathcal{N}(\mathbf{0}, \sigma^2 C^2 \mathbf{I}) \right), \quad (5)$$

where  $\nabla_{v_i, \mathbf{f}} \mathcal{L}_{rec}$  is the original gradient of decoder network on node  $v_i$ ,  $C$  is the clipping hyper-parameter required to bound the influence of each individual node, and  $\sigma$  is the noise scale hyper-parameter. The idea behind this method is called DPSGD [1]. According to Theorem 1 and the analysis in [1, 19], a model trained with such distorted gradients is guaranteed to be  $(\epsilon, \delta)$ -DP.

Since GVAE is trained in iterations, to guarantee  $(\epsilon, \delta)$ -DP in the whole training process, we leverage the moments accountant mechanism proposed in [1]. Particularly, according to the composability property of moments accountant, we can accurately bound the total privacy loss of GVAE by setting the degree of perturbation (noise scale) at each training iteration as

$$\sigma = \frac{q \sqrt{T \log(1/\delta)}}{\epsilon}, \quad (6)$$

where  $T$  is the number of training iterations, and  $q$  the sampling ratio. We term this model DPGVAE.

In the generation stage, we can disregard the encoder and only use the decoder to generate an unlimited amount of graphs from randomly sampled vectors from the prior distribution  $\mathcal{N}(\mathbf{0}, \mathbf{I})$ . Since the normal Gaussian distribution is privacy irrelevant, it can be regarded as  $(0, 0)$ -DP. By the composability property of DP [14], graphs generated by DPGVAE satisfy  $(\epsilon, \delta)$ -DP. In particular, according to Eq. 2, since GVAE is essentially reconstructing links in the generated networks based on the input of links in the original networks, the system is  $(\epsilon, \delta)$ -edge-DP, whose release in principle does not disclose sensitive information regarding individual links in the original sensitive networks.

Note that, although the encoder network also directly touches sensitive data, according to Eq. 1, the gradients are already mixed with randomness of samples from the Gaussian prior before reaching the decoder network, so we do not need to add noise to it. Through this design, we can improve training of the decoder network with limited privacy gradient budget, with minimum interruptions to the encoder network, while guaranteeing the whole generation process to be edge-DP.

**Improving structure learning.** Besides individual link privacy, we also aim to preserve global network structure so as to ensure the utility of released data. As we discuss before, original GVAE computes the reconstruction loss between input and generated graphs based on the element-wise BCE between their adjacency matrices. Such a computation is specified on each individual link, rather than the structure of the graph as a whole. To improve the learning of global graph structure, we leverage GCN again, which has been shown universally powerful in capturing graph-level structures [37, 55, 10, 27]. In particular, we borrow the framework of VAEGAN from recent research [22, 32, 58], and compute a structure-oriented generative adversarial network (GAN) loss as

$$\begin{aligned} \mathcal{L}_{gan} &= \log(\mathcal{D}(\mathbf{A})) + \log(1 - \mathcal{D}(\mathbf{A}')) \\ &\text{with } \mathcal{D}(\mathbf{A}) = \mathbf{f}'(\mathbf{g}'(\mathbf{X}, \mathbf{A})), \end{aligned} \quad (7)$$

where  $\mathbf{g}'$  and  $\mathbf{f}'$  are GCN and FNN networks similarly as defined before, besides in the end of  $\mathbf{g}'$  the node-level representations are element-wise summed up as the graph-level representation, which resembles the recently proposed GIN model for graph-level representation learning [55]. In the VAEGAN framework, the decoder also serves as the generator, while  $\mathcal{D} = \mathbf{f}' \cdot \mathbf{g}'$  is the discriminator.



To facilitate better understanding towards how the graph statistics reflect the global network structure captured by the models, we also provide results of two recent state-of-the-art network generation methods, *i.e.*, NetGAN [6] and GraphRNN [60], with default parameter settings and no DP constraints at all. In this experiment, we expect to see the more effective *structure-preserving* models generate networks that are *more similar* to the original ones regarding various graph properties, thus maintaining high network data utility. Besides similarity on graph statistics, we further evaluate the utilities of generated graphs against the original ones on the downstream task of graph classification, of which the details and results are put in the Appendix.

To show that DPGGAN effectively guarantees individual link privacy, we train all models for another  $K$  times on each dataset. Differently from the previous setting where the complete networks are used, we randomly sample 80% of the links from the original networks to train the models. After generating the full networks from the trained models, we use degree distribution to align the nodes in the generated networks with those in the original networks. Then we evaluate the standard AUC metric on the task of individual link prediction<sup>2</sup> by comparing links predicted in the generated networks and links hidden during training in the original networks. In this experiment, we expect to see the more effective *privacy-protecting* models generate networks that are *less useful* when used to predict individual links in the original networks, thus rigorously guaranteeing network data privacy.

For GVAE and our models, we use two-layer GCNs with sizes  $32 \rightarrow 16$  for both  $\mathbf{g}_\mu$  and  $\mathbf{g}_\sigma$  of the encoder network, where the first layer is shared, and we use two-layer FNNs with sizes  $16 \rightarrow 32$  for  $\mathbf{f}$  of the decoder (generator) network. For DPGGAN, we use another two-layer GCN with the same sizes for  $\mathbf{g}'$  and a three-layer FNN with sizes  $16 \rightarrow 32 \rightarrow 1$  for  $\mathbf{f}'$ . For DP-related hyper-parameters, we follow existing works [14, 1, 45] to fix  $\delta$  to  $10^{-5}$ , noise scale  $\sigma$  to 5, and sampling ratio  $q$  to 0.01 (which determines the batch size  $B$  as  $B = qN$  with  $N$  as the graph size). Then we vary  $\varepsilon$  from 0.1 to 10 to see how much graph-level utilities are preserved under different privacy budgets. According to Eq. 6, we terminate the training of DPGGAN at  $T$  iterations when  $\varepsilon$  is depleted. Other than the essential parameters in Eq. 6, we empirically set the clipping parameter  $C$  to 5, decay ratio  $\gamma$  to 0.99, learning rate  $\eta$  to  $10^{-3}$ , and the loss weighing parameters  $\lambda_1$  and  $\lambda_2$  both to 0.1. We do not observe the model to be very sensitive to the setting of these non-essential parameters.

All experiments are done on a server with four GeForce GTX 1080 GPUs and a 12-core 2.2GHz CPU. The training time of DP-enforced models is often slightly shorter due to early stops when the privacy budget runs out, (*e.g.*, a typical train of GVAE, DPGVAE, and DPGGAN takes 60, 42 and 53 seconds on average on DBLP, respectively). After training, the generation times of the three models are roughly the same (*e.g.*, 0.02 second on average on DBLP). As a direct comparison, the state-of-the-art deep network generation models of NetGAN and GraphRNN take longer times under the same settings especially for generation (*e.g.*, 89 and 4.5 seconds for NetGAN to train and generate on DBLP, and 75 and 2.4 seconds for GraphRNN). Note that, although efficiency is not our major concern in this work, short runtimes (especially for generation) are favorable for efficient data share.

Models	DBLP Networks					IMDB Networks				
	LCC	TC	CPL	GINI	REDE	LCC	TC	CPL	GINI	REDE
Original	107.5	59.90	3.6943	0.3248	0.9385	13.001	305.9	1.2275	0.1222	0.9894
GVAE(no DP)	<b>7.51</b>	66.93	<b>0.1330</b>	<b>0.0213</b>	<b>0.0084</b>	0.0145	25.83	<b>0.0121</b>	0.0030	<b>0.0016</b>
NetGAN(no DP)	<b>9.66</b>	<b>39.87</b>	<b>0.1943</b>	<b>0.0105</b>	<b>0.0022</b>	0.0083	27.54	0.0192	0.0042	<b>0.0011</b>
GraphRNN(no DP)	<b>10.27</b>	<b>57.43</b>	0.2043	0.0415	<b>0.0052</b>	0.0594	27.26	0.0214	0.0155	0.0094
DPGVAE( $\varepsilon=10$ )	21.96	175.29	0.2471	0.0339	0.0153	0.0147	43.63	0.0367	0.0036	0.0030
DPGVAE( $\varepsilon=1$ )	23.80	187.20	0.3059	0.0343	0.0156	0.0253	43.73	0.0373	0.0038	0.0031
DPGVAE( $\varepsilon=0.1$ )	26.07	215.13	0.3342	0.0344	0.0158	0.0320	44.12	0.0392	0.0042	0.0032
DPGGAN( $\varepsilon=10$ )	10.61	<b>64.75</b>	<b>0.2035</b>	<b>0.0224</b>	0.0093	<b>0.0040</b>	<b>22.89</b>	<b>0.0164</b>	<b>0.0010</b>	<b>0.0017</b>
DPGGAN( $\varepsilon=1$ )	12.38	70.97	0.2643	0.0353	0.0117	<b>0.0053</b>	<b>23.81</b>	<b>0.0168</b>	<b>0.0029</b>	0.0023
DPGGAN( $\varepsilon=0.1$ )	24.62	77.41	0.2713	0.0485	0.0191	<b>0.0113</b>	<b>24.91</b>	<b>0.0168</b>	<b>0.0029</b>	0.0025

Table 1: Performance evaluation over compared models regarding a suite of important graph structural statistics. The Original rows include the values of original networks, while the rest are the average absolute difference between generated networks by different models and the original networks. Therefore, *smaller values* indicate better capturing of global network structure and thus *better global data utility*. Bold font is used for values ranked top-3.

<sup>2</sup>[https://github.com/graph-star-team/graph\\_star](https://github.com/graph-star-team/graph_star)

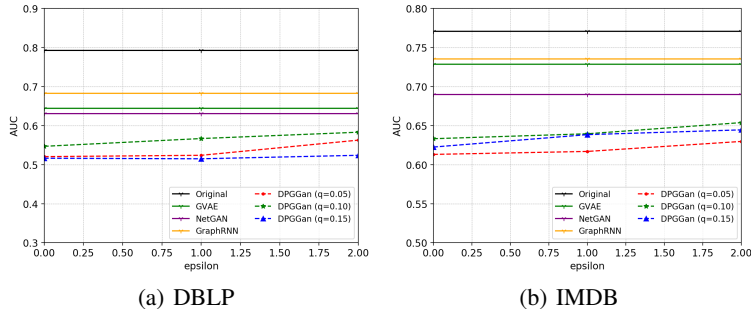


Figure 3: Accuracy of links predicted based on networks generated by DPGGAN with varying hyper-parameters and evaluated on the original networks. *Lower AUC* means the information in the generated networks is less useful in revealing the true existence or absence of links in the original networks, thus *better individual data privacy*.

**Performances.** In Table 1, our strictly DP-constrained models constantly yield highly competitive and even better results compared with the strongest DP-free baselines regarding global network structural similarity between generated and original networks on both datasets, clearly showing the effectiveness of our models on global network structure preservation. As we gradually increase the privacy budget  $\epsilon$ , our two models (especially DPGGAN) apparently perform better, showing the effectiveness of our privacy constraints and a clear trade-off between privacy and utility.

Looking deeper into the numbers, we observe that DPGGAN constantly achieves significantly better performance over DPGVAE under the same privacy budgets on both datasets (scores all passed t-tests with p-value 0.01), which corroborates our novel design of the structure-oriented graph discriminator. Moreover, the suite of statistics measure global network structure from different perspectives. As can be inferred from TC, CPL and GINI, the IMDB networks are in general smaller, tighter and likely more structurally complex than the DBLP networks, which favors link generation models (*e.g.*, GVAE) over sequence generation models (*e.g.*, NetGAN and GraphRNN). Consequently, DPGGAN also performs better on the IMDB networks, indicating its advantages on modeling complex link structures as a whole.

As shown in Figure 3, for both datasets, links predicted on the networks generated by DPGGAN are much *less accurate* than those predicted on the original networks (26%-35% and 15%-20% AUC drops on DBLP and IMDB, respectively) as well as the networks generated by all baselines. This means that even if the attackers somehow identify nodes in the generated (released) networks of DPGGAN, they cannot leverage the information there to accurately infer the existence or absence of links between particular pairs of nodes on the original networks. This directly corroborates our claim that DPGGAN is effective in protecting individual link privacy.

To conduct more detailed inspections, we vary two of the major hyper-parameters, *i.e.*, the privacy budget  $\epsilon$  and sampling ratio  $q$ . Consistently with the results in Table 1, larger privacy budgets lead to more privacy leakage, which allow attackers to infer individual links in the original networks with higher accuracy. While some DP-constrained deep learning models are observed to be sensitive to the sampling ratio during training [1, 45], the privacy protection of DPGGAN is robust when  $q$  is changed in large ranges in practice.

## 5 Conclusion

Due to the recent development of deep graph generation models, synthetic networks are generated and released for granted, without the concern about possible privacy leakage over the original networks used for model training. In this work, for the first time, we pay attention to the task of secure network release and formulate its goals as *preserving global network structure* while *protecting individual link privacy*. Subsequently, we adopt the well-studied DP framework and develop DPGGAN, which protects individual link privacy by enforcing edge-DP over the link prediction based graph generation model of GVAE while preserving global network structure through adversarial learning with a structure-oriented graph discriminator. Comprehensive experiments show that DPGGAN is advantageous in generating networks that are globally similar to the original ones (thus effectively maintaining network data utility), and at the same time useless for predicting individual links in the original network (thus rigorously protecting network data privacy).



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