An Efficient Signature Verification Method based on an Interval Symbolic Representation and a Fuzzy Similarity Measure

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***Abstract*— In this paper an efficient off-line signature verification method based on an interval symbolic representation and a fuzzy similarity measure is proposed. In the feature extraction step, a set of Local Binary Pattern (LBP) based features is computed from both the signature image and its under-sampled bitmap. Interval-valued symbolic data is then created for each feature in every signature class. As a result, a signature model composed of a set of interval values (corresponding to the number of features) is obtained for each individual's handwritten signature class. A novel fuzzy similarity measure is further proposed to compute the similarity between a test sample signature and the corresponding interval-valued symbolic model for the verification of the test sample. To evaluate the proposed verification approach, a benchmark off-line English signature dataset (GPDS-300) and a large dataset (BHSig260) composed of Bangla and Hindi off-line signatures were used. A comparison of our results with some recent signature verification methods available in the literature was provided in terms of average error rate and we noted that the proposed method always outperforms when the number of training samples is eight or more.**

***Index Terms*— Off-line signature verification; Interval-valued symbolic representation; Fuzzy similarity measure; Texture feature extraction.**

I. Introduction

BIOMETRICS is defined as an automated use of physiological or behavioural characteristics of an individual for identification/authentication purposes. Many different biometric identification systems have been proposed as a means of determining or verifying personal identity using different behavioural characteristics. Signatures, as one of the behavioural human characteristics, are extensively used as a proof of identity for legal purposes on many documents such as bank cheques, credit cards, and wills in our daily lives. Considering the large number of signatures handled daily through visual inspection by authorized persons, construction of an efficient automatic system to handle such a huge volume of signatures has many potential benefits for signature authentication to reduce fraud and other crimes [1, 23-26].

A quick look at the literature of signature identification / verification indicates that handwritten signature identification / verification systems are well established. A wide range of algorithms have already been developed in the past few decades to automatically process handwritten signatures in various signature-based applications, such as person identification / verification, cheque fraud detection, bank transactions, and crime detection [1, 23-26]. Considering the way in which the proposed methods in the literature dealt with the handwritten signatures, the methods can be categorized into two groups: a) identification, and b) verification [4]. The identification methods decide the signature group among a number of groups that the claimed signature belongs to, and the verification methods decide acceptance or rejection of a person's claimed signature. Three different types of forgeries (random, simple and skilled forgeries) have commonly been used in the literature [1]. Random and simple forgery samples are generated by individuals without any knowledge about the signers and their signatures, whereas, samples of skilled forgeries are produced by people who have already seen an original instance of a signature and try to generate a copy of the original signature as close as possible to the original one. Indeed, the problem of signature verification considering skilled forgeries is a challenging task [1, 4].

Since data/signature collection can be performed using on-line and off-line mediums, the signature verification methods in the literature can consequently be grouped into on-line and off-line approaches [1, 4]. On-line signature verification systems generally have higher performance compared to off-line signature verification systems. This is because on-line systems take into account different dynamic information such as velocity, acceleration, pressure, stroke order, force, etc. that are not available in off-line systems. Moreover, as off-line signature verification systems use statistical information determined from the signature images, the problem becomes much more complicated. Nevertheless, off-line signature verification systems are more popular as most signatures are written on papers, documents, checks, etc.

In this research work, a new model-based writer-dependent technique employing interval-based symbolic representation and a fuzzy membership function for off-line signature verification is proposed. Interval-valued symbolic data is

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created using texture-based features. The proposed signature verification system is further evaluated using different datasets.

The remainder of this paper is organized as follows. Section II explains the related work. The main contributions of the work are discussed in Section III. In Section IV, the proposed method is detailed. Database details, experimental settings and evaluation metrics are presented in Section V. The experimental results are provided in Section VI. In Section VII, a comparative analysis of the proposed method with other existing methods is provided. Finally, conclusions and future work are presented in Section VIII.

II. Related work

In the off-line handwritten signature verification area, a significant volume of research has been undertaken for the development of different authentication methods [1-14, 19, 23-45, 51-54]. Reviews of different methods proposed for signature identification/verification at different periods of time can be found in [1, 23-26]. Recently, some competitions have also been organized to evaluate the technological achievements in terms of reliability and accuracy in the field of signature verification [2, 9].

The state-of-the-art methods for signature verification are commonly composed of the following steps: pre-processing, feature extraction, training a classifier or creating a knowledge-based model followed by the final verification criteria [1, 24]. In the pre-processing step, different tasks such as signature extraction, size normalization, binarization, skeletonization, skew/slant correction, and noise removal have generally been undertaken to prepare the signature images for further processing [24].

Various feature extraction techniques based on geometric or Connected Component (CC) analysis, directional and gradient of orientation, mathematical transformations, profiles and shadow-code, texture information, and interest points have been proposed in the literature to interpret different aspects of signature images for the purpose of verification [3-14, 19, 27-31]. Off-line signature verification methods can broadly be categorized into local [3, 6, 7, 8, 10-14, 19, 27-31, 34-37, 39, 41, 42, 55] and global [3-6, 11, 13, 32, 33, 36-38, 40] approaches considering the features extracted at the local or global level in the feature extraction stage. It is worth mentioning that local features have predominantly provided better verification accuracies than the global features.

Different learning strategies, such as machine learning and similarity-based approaches, have been proposed in the literature to solve the problem of signature verification. Machine learning approaches include Neural Networks (NNs) [4, 7, 19, 42], Bayes classifier [3], Hidden Markov Models (HMMs) [30, 34, 36, 38, 41], Support Vector Machines (SVMs) [5, 10-14, 27-28, 31, 37, 38, 40-42], Gaussian Mixture Models (GMMs) [6], Gentle AdaBoost algorithm [39], and Ensembles of classifiers [41]. Similarity-based approaches comprise the following: k-Nearest Neighbour (kNN), Dynamic Time Warping, and point matching [3, 6, 8, 12, 19, 29, 32-36]. These models can further be categorized into writer-dependent and writer-independent models. In a writer-dependent approach, a specific model is trained for each individual's signatures (class) using some genuine signatures of the individual and a few random forgeries. In the testing phase, using the trained model, a test signature is classified as a genuine or forged one. For the writer-independent approach, on the other hand, a single model is trained for all the signature classes to be used for the verification task. Hybrid models have also been used for signature verification [1, 23-26].

Signature verification methods based on the concept of fuzzy sets and different fuzzy membership functions have also been developed in the literature [43-46]. In [43], geometric features, as a family of shape factors, and the coding of information related to the dynamics of the signature, have been used to characterize signature images. A fuzzy technique has then been adapted to combine these two types of information for off-line signature verification. In the studies presented in [44, 45], a signature image has initially been pre-processed using binarization, size normalization, and thinning methods. The thinned image has then been partitioned into a number of sub-images to compute features consisting of angle information. A fuzzy system based on the Takagi–Sugeno (TS) model and an exponential membership function has further been used for the signature verification task [44, 45]. The TS model with structural parameters takes into account the local variations in the characteristics of the signature [45]. The membership functions constitute weights in the proposed modification of the TS model to provide better results [45]. A method based on the spectral analysis of a directional gradient density function and a weighted fuzzy classifier has been proposed for off-line signature verification in [46]. The outline of a signature image was initially extracted and the frequency spectrum was then computed using a directional gradient density function as the feature set. A weighted fuzzy classifier based on a triangular membership function was adapted for the verification of forgeries [46].

An overview of the signature verification methods, which exist in the literature, is provided in Table I. From the literature reviewed, it is noted that there has been significant progress in the off-line signature verification domain. However, despite the progress in the area over the past decades, it remains an open research problem [1, 2, 23, 24]. In addition to the limitations mentioned in Table 1, some general challenges [24] in the area of off-line signature verification that still attract many researchers for further investigation in this field are indicated as follows: i) high intra-class variability in handwritten signatures of every individual compared to the physiological biometrics, such as fingerprints or iris of the individual, ii) low inter-class variability between genuine signatures and skilled forgeries of every individual, iii) the existence of only genuine signatures as partial knowledge for training off-line signature verification systems, iv) limitations in the amount of signature data available for training off-line signature verification systems in real scenarios, as during the enrolment process users often provide only a few samples of their signatures, and v) the presence of

TABLE I
OVERVIEW OF DIFFERENT SIGNATURE VERIFICATION METHODS.

Type / approach	Methods	Limitations
HMM-based	30, 34, 36, 38, 41	Poor performance when few signature samples are available for training, needs reconstruction whenever a new writer is added to the system.
NN-based	4, 7, 19, 42	Needs enough data for training and convergence, needs to retrain the neural networks in the case of changing the number of signature classes.
SVM-based	5, 10-14, 27-28, 31, 37, 38, 40-42	Needs to find an appropriate kernel and then tuning its parameters, has high algorithmic complexity and extensive memory requirements in large-scale tasks.
GMM-based	6	Needs to estimate an appropriate number for the Gaussian components, lacks the generalisation ability to make accurate predictions for new data.
Bayesian-based	3	There is no precise way to obtain prior knowledge, posterior distributions are heavily influenced by the prior knowledge.
Similarity-based	3, 8, 12, 29, 32, 33, 35, 36	Needs to choose an appropriate distance, it is very sensitive to irrelevant features as all features contribute to the similarity and thus to the classification

signatures written in different scripts. Furthermore, with particular reference to fuzzy-based signature verification methods, we noted that only a few related papers have been reported in the literature despite the progress achieved in the signature verification field. Moreover, the membership functions and also the fuzzification processes used for signature verification are mostly based on the exponential and Gaussian membership functions, which provide a probability very close to zero for a sample that deviates much from the mean in the Gaussian as well as in the log-space models. This would mean a very large negative number is added to the accumulation probability result. That is a big penalty for the cases where handwritten genuine signatures are roughly written by genuine authors. In the present research work, we mainly focus on the problem of inter-/intra-class variability of handwritten signatures.

III. MAIN CONTRIBUTIONS

In the past, the concept of symbolic data analysis and its representation [16] has been used in various applications of document image analysis, such as document classification and signature verification [17-19]. In this research work, a new model-based writer-dependent technique is proposed to create a fuzzy set for each feature by means of an interval-based symbolic representation. This process provides only one representative model for each individual's handwritten signature instead of many feature vectors, which represent different signatures of an individual. The proposed interval-valued symbolic representation can take care of intra-class variability of the signatures in each class. A fuzzy membership function as a fuzzy similarity measure is also proposed to obtain a similarity measure between a test feature vector and the interval-valued symbolic data. The proposed fuzzy similarity measure can handle the problem of inter-class variability between genuine signatures and skilled forgeries providing low membership values for the forged signature features compared to the genuine signature features. For feature extraction, we consider texture features, as they have demonstrated their strength on different applications of biometrics and texture analysis [27, 47, 48]. A texture feature extraction technique based on under-sampled bitmaps of the signatures is further proposed. The applicability of the proposed method is demonstrated using two different benchmark datasets. The proposed system is simple and can be constructed with a few samples. Furthermore, the proposed scheme does not need to retrain the model whenever a new writer is added to the system.

IV. PROPOSED METHOD

The proposed method includes four main steps: a) pre-processing, b) feature extraction, c) creation of an interval-valued symbolic model for each individual, and d) computation of similarity values and the final decision. An overview of our proposed signature verification method is shown in Fig. 1. Each step of the proposed method is detailed in the following subsections.

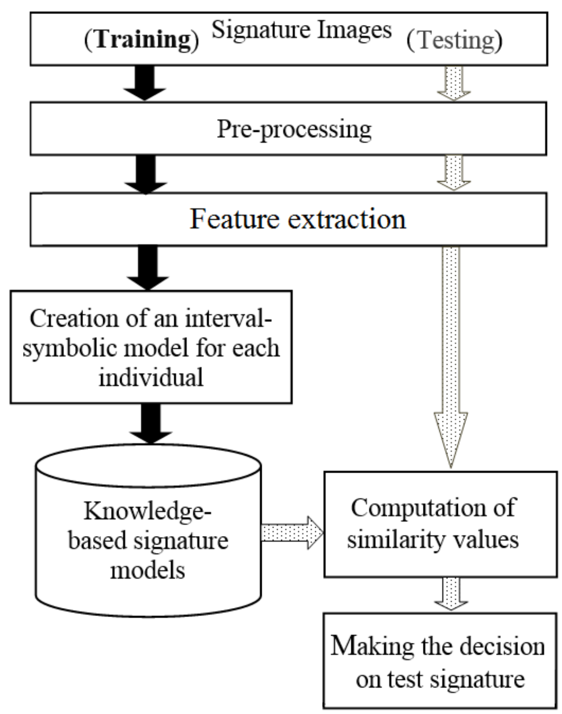


Fig. 1. An overview of the proposed method.

A. *Pre-processing*

Similar to most pattern recognition problems, pre-processing plays an important role in signature verification systems as well. Signature images, even genuine signatures of an individual, include significant variations in terms of size, rotation/slant, pen thickness, etc. Therefore, the pre-processing step, prior to the application of feature extraction methods, is employed on the images to make the images noise-free and have transition invariant features. To do so, first, a histogram-based threshold technique is applied to convert the digitized grey-scale signature images to two-tone images. A mean filter is also employed on the signature images to remove noise. The input images are then cropped to find minimum bounding boxes of the signature images. The cropped signature images of size $\mathrm{M} \times \mathrm{N}$ are then used for feature extraction.

Under-sampled bitmap images have been used in the literature for pattern recognition [50]. In this research work, we have further considered the under-sampled bitmap for feature extraction, since the under-sampled version of an image can be considered as a low resolution version of the image whilst keeping the whole visual appearance of the original one. To compute the under-sampled bitmap, the input image is divided into a number of non-overlapping blocks of similar size, say $\mathrm{b} \times \mathrm{b}$. The number of black pixels in each block is then counted and represents the block intensity. This generates a matrix of size $\mathrm{M}/\mathrm{b} \times \mathrm{N}/\mathrm{b}$ with each element being an integer in the range 0 to the size of the non-overlapping block. Dividing these values by the size of the block and multiplying the results by 255 provides an under-sampled grey image where all pixel values are normalized between 0 and 255. A pictorial representation of the techniques involved in the pre-processing step is shown in Fig. 2(a-h).

B. *Feature extraction*

In the present research work, texture-based features are considered for feature extraction. Texture features, such as the Local Binary Pattern (LBP), the Local Derivative Pattern (LDP), and Grey Level Co-occurrence Matrix (GLCM), have widely been employed in different biometric systems including signature verification and some promising results have also been provided [10, 15, 27]. Notable results obtained in signature verification using the texture features, especially the LBP-based features, are due to the exceptional properties of the LBP-based features, which can provide important information about the personal characteristics of a signer including such elements as the amount of pressure and speed changes, pen-holding, ink distribution, etc. [27]. The LBP features are also computationally efficient and these features have shown their robustness to monotonic illumination change [54]. The LBP features are, however, sensitive to random noise and non-monotonic illumination variation [47-49].

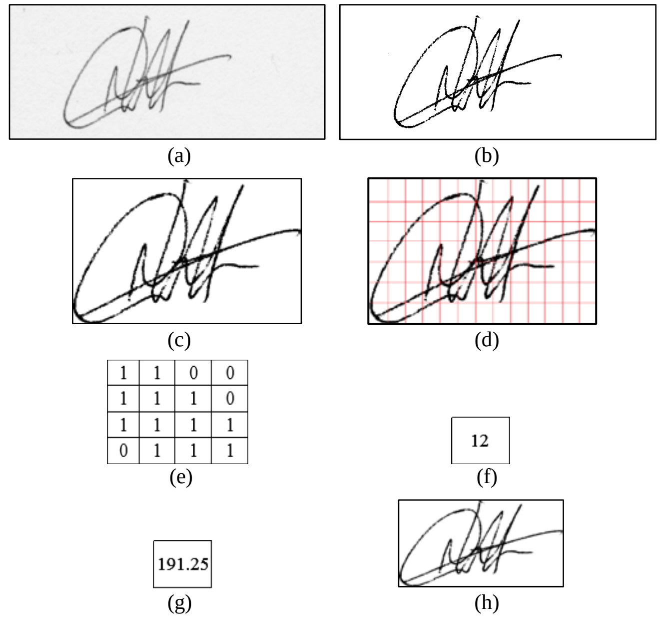


Fig. 2. Examples of different steps of pre-processing used on an input signature: (a) An original grey image, (b) The image after binarization, (c) The image after cropping, (d) Non-overlapping windows of size $\mathrm{b} \times \mathrm{b}$ (here 4×4), e) A block of size 4×4 from (d), f) Obtaining an under-sampled pixel by summing all the pixel values in (e), g) Computing the pixel intensity of the under-sampled pixel using the formula (12/16×255=191.25), and (h) The under-sampled grey image of size $\mathrm{M}/\mathrm{b} \times \mathrm{N}/\mathrm{b}$.

In the basic LBP feature extraction method, an image is processed in such a way that a binary code is generated for each pixel in the image. This code determines whether the intensities of the neighbouring pixels are greater or less than the reference pixel's intensity. For instance, in a 3×3 neighbourhood with the reference pixel being the centre, a binary code of length 8 is generated according to the relative intensities of its neighbours. A histogram of 256 bins is then computed to count the number of occurrences of each binary code, describing the proportion of common textural patterns in the image [47]. By computing the occurrence histogram, structural and statistical information is effectively combined. The LBP map detects microstructures, such as edges, lines, spots and flat areas, whereas their underlying distribution is estimated by the LBP histogram [47]. The basic LBP-based feature extraction technique has further been extended to a generalised rotation invariant feature extraction method in [48]. The generalised LBP feature extraction ($\mathrm{LBP}_{\mathrm{P,R}}^{\mathrm{u2}}$) and rotation invariant ($\mathrm{LBP}_{\mathrm{P,R}}^{\mathrm{riu2}}$) methods have been derived based on a symmetric P members neighbourhood on a circle of radius R. The parameter P controls the quantisation of the angular space and R determines the spatial resolution of the operator. Interested readers can find more about the LBP-based feature extraction method in [47, 48].

Similar to the authors in [27], the $\mathrm{LBP}_{\mathrm{P,R}}^{\mathrm{riu2}}$ with only two variations $(R = 1, P = 8$ and $R = 2, P = 16)$ is initially employed on the pre-processed original image to extract signature features.

Furthermore, an effective feature extraction technique based on under-sampled bitmaps and LBP-based features is proposed in this paper. To do so, first, an under-sampled bitmap image is created and then LBP-based features are extracted from the under-sampled bitmap image. The main reason for the use of the under-sampled image in the proposed feature extraction method is to compute the LBP-based features from the grey level low-resolution version of the input signature image. The significance of LBP-based texture features obtained from the grey images compared to the binary images has been pointed out in [27]. As most of the LBP patterns in an image are generally uniform patterns, and also the uniform LBP ($\mathrm{LBP}_{P,R}^{u2}$) operator can keep the distribution of the LBPs in the image, the $\mathrm{LBP}_{P,R}^{u2}$ with $R = 1$ and $P = 8$ is applied on the resultant under-sampled grey image to obtain a set of 59 LBP-based texture features called $\mathrm{UB}-\mathrm{LBP}_{P,R}^{u2}$. The LBP-based features extracted based on the UB-$\mathrm{LBP}_{8,1}^{u2}$, $\mathrm{LBP}_{8,1}^{riu2}$ and $\mathrm{LBP}_{16,2}^{riu2}$ from the under-sampled and original signature images are concatenated to create a set of 87 (59+10+18) features. It is worth mentioning that both grey-level information as well as binary information are captured in the proposed feature set, ensuring a better interpretation of the signature images.

C. Creation of an interval-valued symbolic model

Patterns/objects are commonly characterized using a set of single real-value data parameters called the feature vector/set. The feature vectors extracted for different objects constitute a data-array, where each cell (i, j) comprises the value of the feature j for the object i. Apart from its simple representation, this kind of modelling cannot take into account the variability and/or uncertainty of the feature values. In the signature verification domain, different fuzzification methods have been employed to take care of the uncertainty in handwritten signatures [43-46]. From a different perspective, interval- and histogram-valued symbolic variables have also been introduced in the domain of symbolic data analysis to represent the variability, uncertainty and distribution of feature values in a specific class object [16]. An interval-variable can be defined using minimum and maximum values of a set of values as the lower or upper bound of the values, respectively. For instance, a set of continuous values X can be defined using finite support $[\underline{x}, \overline{x}]$, where $\underline{x}$ and $\overline{x}$ are the minimum and maximum values of X, respectively.

In this research work, the interval-valued symbolic data is considered to model each feature of every individual's signature extracted during the training phase. To create the interval-valued symbolic data, the minimum and maximum values of the features can be used. However, if the training samples are very similar in shape, the minimum and maximum values of the feature values will be very close to each other and cannot represent an appropriate interval-value of a set of feature values. Therefore the median and standard deviation of each feature, as two main statistics of features along with a tuning parameter, are used to define interval-valued symbolic data for each feature. As a result, an individual's signature is represented by a range of interval-valued data. For easy reading and formulation of the problem, specific mathematical descriptions are further provided in the following.

Let $\mathrm{S_j} = \{\mathrm{s_j^1}, \mathrm{s_j^2}, \dots, \mathrm{s_j^l}\}$ be a set of l samples from a signature class $\mathrm{C_j}$ and let $\mathrm{F_j^i} = \{\mathrm{f_{j1}^i}, \mathrm{f_{j2}^i}, \dots, \mathrm{f_{jn}^i}\}$ be a feature vector of size n extracted from the set $\mathrm{s_j^i}$. For the k^{th} feature $\mathrm{f_{jk}^{\circ}}$ in every class $\mathrm{C_j}$, we compute the statistical median $\mathrm{m_{jk}}$ and standard deviation σ_{jk}. The statistical standard deviation σ_{jk} is computed based on the median $\mathrm{m_{jk}}$. Considering $\mathrm{m_{jk}}$ and σ_{jk}, the lower and upper bound values of the $\mathrm{f_{jk}^{\circ}}$ are computed as follows:

$$m_{jk} = Median(\, f_{jk}^{\circ}) \tag{1}$$

$$\sigma_{jk} = StandardDeviation(\, f_{jk}^{\circ})$$

$$f_{jk}^{-} = m_{jk} - \lambda \times \sigma_{jk}$$

$$f_{jk}^{+} = m_{jk} + \lambda \times \sigma_{jk}$$

where λ is a parameter which should be tuned during the training phase.

A symbolic representation of the k^{th} feature of class j ($\mathrm{C_j}$) is further defined using an interval-value and 2 continuous values (median and standard deviation).

$$sf_{jk} = ([f_{jk}^{-}, f_{jk}^{+}], m_{jk}, \sigma_{jk}) \tag{2}$$

The symbolic representation of $\mathrm{C_j}$ called $SymbC_j$ is finally defined considering n symbolic features (sf) corresponding to n features as follows:

$$SymbC_j = \{sf_{j1}, sf_{j2}, \dots, sf_{jn}\} \tag{3}$$

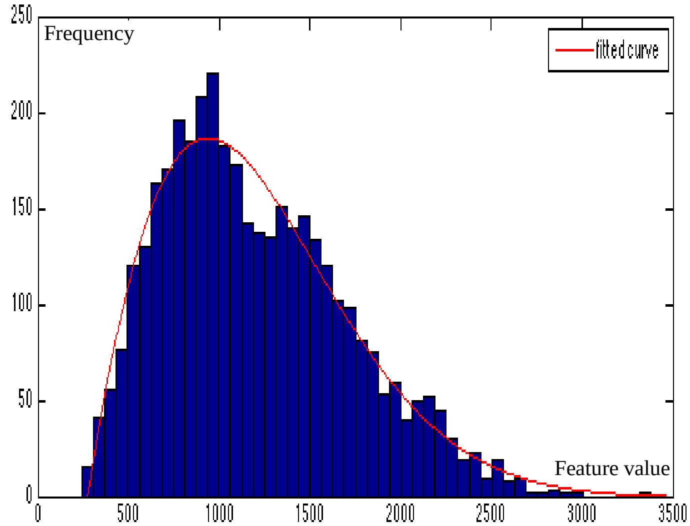
Considering q classes in a particular signature verification problem, the complete interval symbolic representation of the problem contains q signature models composed of n interval values. A complete overview of the symbolic models is shown in Table II.

TABLE II
INTERVAL-SYMBOLIC REPRESENTATION OF A q-CLASS PROBLEM WITH N FEATURES BASED ON THE PROPOSED APPROACH.

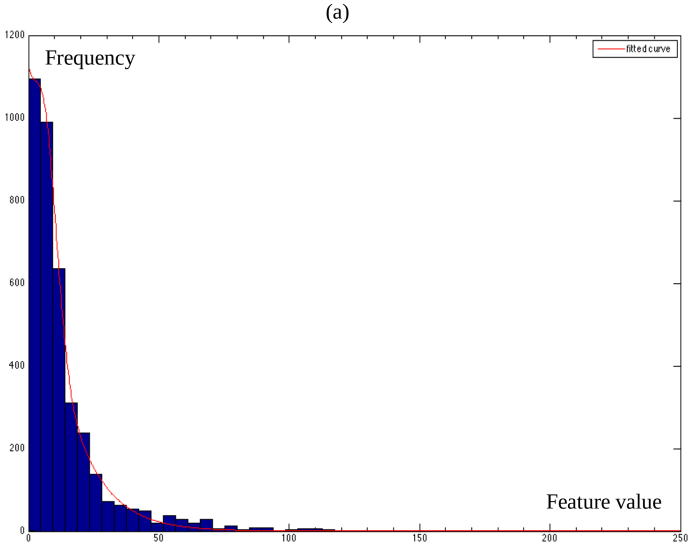
Feature / Class	sf_{j1}	…	sf_{jn}
SymbC₁ → $SymbC_1$	$([f_{11}^{-}, f_{11}^{+}], m_{11}, \sigma_{11})$	…	$([f_{1n}^{-}, f_{1n}^{+}], m_{1n}, \sigma_{1n})$
$SymbC_2$	$([f_{21}^{-}, f_{21}^{+}], m_{21}, \sigma_{21})$	…	$([f_{2n}^{-}, f_{2n}^{+}], m_{2n}, \sigma_{2n})$
⋮	⋮	⋮	⋮
$SymbC_j$	$([f_{j1}^{-}, f_{j1}^{+}], m_{j1}, \sigma_{j1})$	…	$([f_{jn}^{-}, f_{jn}^{+}], m_{jn}, \sigma_{jn})$
⋮	⋮	⋮	⋮
$SymbC_q$	$([f_{q1}^{-}, f_{q1}^{+}], m_{q1}, \sigma_{q1})$	…	$([f_{qn}^{-}, f_{qn}^{+}], m_{qn}, \sigma_{qn})$

D. Computing similarity values and the verification process

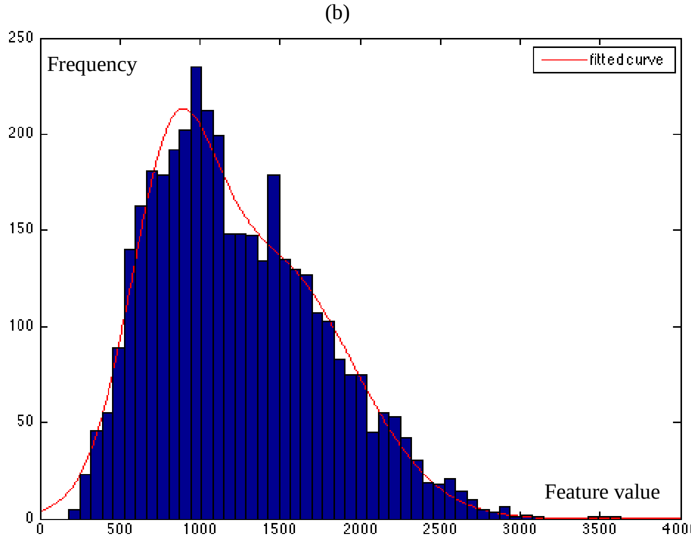
Euclidean, City-block and Mahalanobis distances are some simple, but well-established, distance measures frequently used in the literature for computing similarity/dissimilarity between two feature vectors. These distances cannot be used in our proposed model, since our proposed representation model for each signature class is composed of interval-values and each feature extracted from a test signature is a single numerical real value.



(a)



(b)



(c)

Fig. 3. Examples of different patterns: (a) A bell shape distribution, (b) A half-bell shape distribution, and (c) An approximate bell shape distribution obtained from the values extracted for three different features using genuine signatures considered for training.

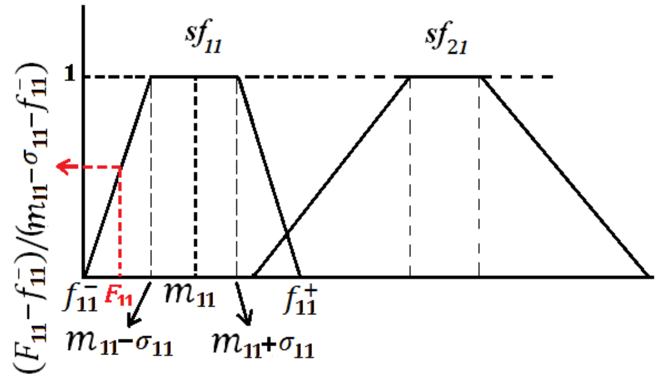


Fig. 4. The proposed interval-valued symbolic representation and a similarity measure obtained for F_{11} based on a fuzzy trapezium-shaped membership function.

Based on a statistical analysis performed on the feature values extracted from the training data for every individual feature, we found three common types of patterns, such as bell shape, half-bell shape and approximate bell shape, as three distributions of feature values repeated in all the features. These three patterns could easily be modelled using different Gaussian distribution models with different means and standard deviations, as shown in Fig. 3. However, the main problem with the mean value is its significant change in the presence of a value, which is highly deviated from other values. This problem results in an erroneous Gaussian model. Furthermore, a Gaussian model generally provides a probability of very close to zero for the sample, which deviates much from the mean in the Gaussian model [43-44]. This provides inaccurate results in the case where a genuine signature has much variation with respect to the other original genuine signatures.

To this end, a fuzzy similarity function is further proposed in this research work to take care of those two issues with a standard Gaussian model. The proposed fuzzy similarity measure is a fuzzy trapezium-shaped membership function, which is relatively similar to the Gaussian distribution as are our feature value distributions. It can further take care of both the mean value and also the low probability of the features, which are far from the mean, by considering the median value and properties of the fuzzy trapezium-shaped membership function. It can also take into account the inter-class variability problem between skilled forgeries and genuine signatures providing small similarity (membership) values for the forged signature features in comparison to the genuine signature features. A pictorial representation of the interval-valued symbolic sf_{11}, and the similarity measure in relation to F_{11} obtained based on a fuzzy trapezium-shaped membership function, is shown in Fig. 4. The sf_{21} shown in Fig. 4 is computed in the same way as the first feature values extracted from l signature samples of the class C_2.

There are many aggregation operators, such as t-norms, t-conorm and averaging operators, proposed in the literature to compute the final fuzzy value for a set of fuzzy values [56]. In this research work, a simple Algebraic Sum from a t-conorm aggregation group is used for the aggregation process. The main reason for choosing the Algebraic Sum aggregation is to equally consider the effect of all fuzzy similarity values computed for all the features in the final fuzzy similarity measure. As a result, the similarity $Sim\,(F_T, SymbC_j)$ between a test sample (T) and a symbolic reference of a particular class j $(SymbC_j)$ is computed as follows:

$$Sim(F_T, SymbC_j) = \frac{1}{n}\sum_{k=1}^{n} \rho_{jk} \tag{4}$$

where ρ_{jk} is a fuzzy trapezium-shaped membership function providing similarity values between every feature value of a test sample feature vector, whereby a signature model $(SymbC_j)$ and $Sim(F_T, SymbC_j)$ is the accumulated similarity value obtained for all the feature values. The proposed fuzzy trapezium-shaped membership function is defined as follows. The $\pm$sigma around the median value provides a support (kernel) of the membership function.

$$\rho_{jk} = \begin{cases} 0 & if \;\; f_{jk}^{-} < F_{Tk} \;\; or \; F_{Tk} > \; f_{jk}^{+} \\ 1 & if \;\; m_{jk} - \sigma_{jk} \le F_{Tk} \le m_{jk} + \sigma_{jk} \\ (F_{Tk} - f_{jk}^{-})/[(m_{jk} - \sigma_{jk}) - f_{jk}^{-}] & if \;\; f_{jk}^{-} \le \; F_{Tk} < m_{jk} - \sigma_{jk} \\ (f_{jk}^{+} - F_{Tk})/[f_{jk}^{+} - (m_{jk} + \sigma_{jk})] & if \;\; m_{jk} + \sigma_{jk} < F_{Tk} \le f_{jk}^{+} \end{cases} \tag{5}$$

In our proposed signature verification method, an adaptive writer-dependent acceptance threshold is defined to decide whether a test signature is genuine or a forgery. Based on the similarity value $(Sim\,(F_{Tr}, SymbC_j))$ computed from the training samples, the acceptance threshold (θ_j) is defined as the confidence value for the class C_j as follows:

$$\theta_j = Mean(Sim(F_{Tr}, SymbC_j) + \alpha \times StandardDeviation(Sim(F_{Tr}, SymbC_j)) \tag{6}$$

where Tr varies from 1 to l training samples from the class C_j and α is a parameter, which is tuned during the training step in order to obtain the optimal results.

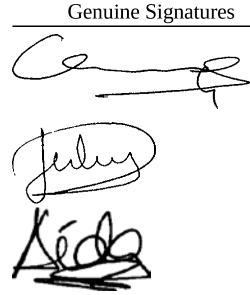
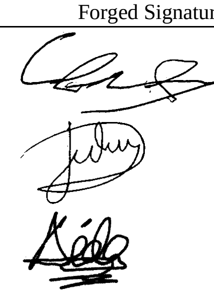
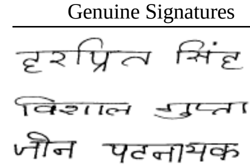
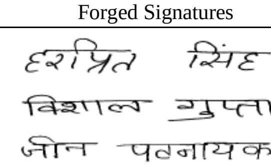
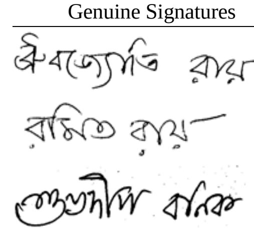
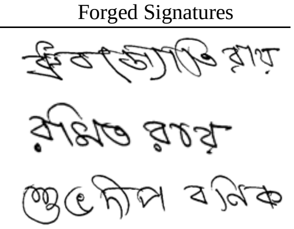
V. Database, Metrics and Experimental Settings

A. Database Details and Evaluation Metrics

To evaluate the proposed verification model we initially used a benchmark off-line signature dataset called the GPDS-300 [52, 51] dataset. We also used a large dataset [53] of Bangla and Hindi off-line signatures (BHSig260) for experimentation. The GPDS-140 was used for tuning the parameters of the proposed signature verification system and GPDS-160 and BHSig260 were considered for testing purposes.

The GPDS-300 signature dataset is composed of 16200 off-line signature images [52]. Three hundred individuals participated to create the dataset. Each signer provided 24 genuine samples in a single day at different writing sessions. For each genuine signature, 30 skilled forged signatures were obtained from 10 different forgers. Forgers could practice as long as they wish to provide the forged signatures. The signatures were binary images and were saved in "bmp" format with a resolution of 300DPI [52]. The BHSig260 dataset [53] consists of 260 sets of handwritten off-line signatures of which 100 sets were written in Bangla and the rest (160 sets) were written in Hindi. The handwritten off-line signatures were collected from 260 different individuals with different educational backgrounds and ages. Individuals used different pens and surfaces for noting down their signatures. Each set consists of 24 genuine signatures and 30 skilled forgeries. Signatures were collected during 2 different sessions. In the first session, the genuine signatures were collected, whereas in the second session, the skilled forgery signatures were collected, showing a genuine signature to an individual to train and mimic the forgeries. A total number of (260×24) 6240 genuine and (260×30) 7800 skilled forgery signatures were collected from all 260 individuals. The data collected was acquired using a flatbed scanner with the resolution of 300DPI in grey scale and stored in TIFF format. A histogram-based thresholding technique was applied for binarization to convert digitized grey-level images to two-tone images. The skilled forgery signatures collected are quite similar to the genuine signatures that make the dataset quite a challenging one [53]. To have an idea about the type of signatures and the complexity of the forged signatures, some binary genuine signature samples of the GPDS-300 and BHSig260 datasets, with their corresponding forgeries, are displayed in Table III.

TABLE III
SAMPLES OF GENUINE AND FORGED SIGNATURE OF THREE DIFFERENT INDIVIDUALS FROM DIFFERENT DATASETS USED IN THIS RESEARCH WORK.

Some samples of the GPDS-300 dataset	
Genuine Signatures	Forged Signatures
	
Hindi Signatures from the BHSig260 dataset	
Genuine Signatures	Forged Signatures
	
Bangla Signatures from the BHSig260 dataset	
Genuine Signatures	Forged Signatures
	

In the literature detailing signature verification methods, two classical types of error have frequently been computed as metrics for evaluation. These are: Type I error or False Rejection Rate (*FRR*) or False Non-Match Rate (*FNMR*), which means a genuine signature is rejected by the system, and a Type II error or False Acceptance Rate (*FAR*) or False Match Rate (*FMR*), which means a forgery is accepted as a genuine signature by the system. As the majority of papers in the literature used the terms *FAR* and *FRR* for evaluation purposes, these terms are used in this paper as well. The mathematical definition of the *FAR* and *FRR* are further provided using the confusion matrix presented in Table IV.

TABLE IV
CONFUSION MATRIX USED TO DEFINE THE EVALUATION METRICS.

		Predicted label	
		G	**F**
True label	**G**	*GG*	*FG*
	F	*GF*	*FF*

$$FRR = \frac{FG}{GG+FG} \tag{7}$$

$$FAR = \frac{GF}{FF+GF} \tag{8}$$

$$AER = \frac{FAR+FRR}{2} \tag{9}$$

In Table IV, F and G represent forged and genuine signatures, respectively. GG is the number of genuine signatures which have been predicted as genuine. FG is the number of genuine signatures which have been predicted as forgeries. GF is the number of forged signatures, which have been predicted as genuine and FF is the number of forged signatures that have been predicted as forgeries. The Equal Error Rate (EER) and the Average Error Rate (AER) have also been used for the evaluation of the signature verification systems. The EER indicates where the FRR and FAR are equal and AER is the average of FAR and FRR. It may be noted that the FAR, FRR, AER and EER metrics were computed knowing that the classes are imbalanced.

B. *Experimental Settings*

In the proposed symbolic representation model in this research work, there are three main parameters, which should be set during the tuning/development stage. To do so, the last 140 classes of the GPDS-300 dataset were considered for tuning the parameters of the proposed model. Only a number of (e.g. 12, 10,…) genuine signatures from the training dataset were used for training the proposed model in the tuning/development step. The rest of the genuine signatures and all the forgeries from the training dataset, were used to tune the parameters of the proposed model and to obtain the optimal acceptance/rejection threshold. In this way, writers have their own symbolic model. However, the parameters are the same for all writers. In Table V a brief description of the dataset and the number of genuine signatures (N_g) and forgeries (N_f) used for each experiment is provided.

For tuning the parameters, initially, 12 genuine signatures of each individual from the training dataset were considered in order to train and build the proposed symbolic representation model. The remaining 12 genuine signatures and all the skilled forgeries (30) of each individual from the training dataset were further used for tuning the parameters. The samples for training and tuning were chosen randomly. The parameters in our proposed models that should be tuned are: λ, which is used to compute the lower and upper bounds of feature values, the size of the block ($b \times b$) for the under-sampling process in our proposed feature extraction method, and the parameter α in the acceptance/rejection threshold (θ_j). We considered different values (2, 3, and 4) for b to analyse the effect of under-sampling in the final signature verification results. To analyse the significance of the parameter λ in the final verification results, λ was set to 2, 4 and 6. When changing each of the parameters λ and b, the other one remains constant. In the tuning process, the parameter α was tuned in such a way that the FAR and FRR became equal as a trade-off between false acceptance rate and false rejection rate to obtain an EER. To obtain reliable and consistent results, the training and tuning procedures were repeated 10 times with different training and tuning subsets of signatures from the GPDS-140 dataset. The results obtained using different values of b and λ, when the average *EER* value was computed from 10 iterations, are presented in Table VI.

From Table VI, it is evident that the results presented in each row are consistent with respect to the changes of parameter λ when the parameter b is the same. However, the best result was obtained when λ was set to 4. Therefore, we fixed the λ to 4 in the rest of our experiments. The proposed method is not very sensitive to parameter λ and can provide stable results with different values of λ, whereas the proposed system is sensitive to the values of b. The best result was obtained when the block size was 2×2. Hence, we set the value of b to 2. The value of α was set to 3.74 based on the average value of α values obtained during 10 repetitions of our experiments. To get an idea about the effect of the parameter α on the signature verification results, the results obtained using different values of α are plotted in Fig. 5, in the form of a Receiver Operating Characteristic (ROC) curve. The results are further plotted in Fig. 6 to demonstrate the relationship between *FAR* and *FRR* metrics. Considering the results, we can see that the value of α was adapted depending on the number of training samples especially to compensate when there is a lack of training data to build a correct support for fuzzy features, integrating well-estimated intra-class variability.

Keeping the same values for the parameters b (=2) and λ (=4), we applied the same process considering a different number of (e.g. 10, 8, 6, 5, 4) genuine signatures of each individual from the GPDS-140 dataset for training the proposed model. Consequently, different values of the parameter α were obtained when 14, 16, 18, 19 and 20 genuine signatures and 30 forged signatures were considered to achieve the *EER* results. A graphical representation of the average *EER* results computed from 10 iterations and their respective parameter α with a different number of signatures for training and tuning are shown in Fig. 7. From Fig. 7, we observed that the value of α has a direct relation to the number of training samples (N_g).

TABLE V
DIFFERENT NUMBER OF SAMPLES USED FOR TUNING AND TESTING IN OUR EVALUATION PROTOCOL

Dataset	Training (Samples per class)		Tuning the parameters (Samples per class)		Training (Samples per class)		Testing (Samples per class)	
	N_g	N_f	N_g	N_f	N_g	N_f	N_g	N_f
GPDS-140	12, 10, 8, 6, 5, 4	0	12, 14, 16, 18, 19, 20	30	-	-	-	-
GPDS-160	-	-	-	-	12, 10, 8, 6, 5, 4	0	12, 14, 16, 18, 19, 20	30
BHSig260	-	-	-	-	12, 10, 8, 6, 5, 4	EÊR	12, 14, 16, 18, 19, 20	30

TABLE VI
RESULTS OBTAINED WITH DIFFERENT BLOCK SIZES AND VARIOUS VALUES OF λ WHEN THE PROPOSED METHOD WAS TRAINED USING 12 SAMPLES OF THE GPDS-140 DATASET.

b × b \ Result λ	EER (%) 2	4	6
2×2	16.76	**16.67**	16.84
3×3	17.47	17.75	17.78
4×4	18.88	18.97	19.11

VI. EXPERIMENTAL RESULTS AND DISCUSSION

To evaluate the proposed signature verification method, several experiments were carried out using different numbers of samples from the GPDS-160, and BHSig260 datasets for testing. The results considering different numbers of training and testing samples are shown in Table VII. Since we have repeated the training and testing/evaluation experiments with different randomly chosen samples, the average of *FAR*, *FRR*, and *AER* were computed as the metrics of evaluation. The standard deviation (σ) of the *AER* results was also provided to demonstrate how the results are stable throughout our experiments with different training and testing samples. From Table VII, it is clear that the standard deviations of the *AER*s, which were obtained employing the proposed method on two datasets, also indicate that the results were quite stable throughout different experiments. In relation to the number of signatures considered for training the proposed model, we noted that the *AER* gradually increases when the number of signatures for training decreases. However, the results suddenly drop when the number of training samples is less than 6 (inclusive). This is because, both the interval-valued symbolic model and the fuzzy similarity measure are based on the statistical median and standard deviation of feature values extracted from the training samples. Sample signatures are not uniformly distributed in GPDS. Therefore, the median and standard deviation computed based on a small number of samples (here less than 5) may not perfectly represent the median and standard deviation of the rest of the data. As a result, our model may not provide high accuracy results when very few samples (< 5) are considered for training.

Furthermore, the results obtained from the BHSig260 are poorer compared to the results achieved from the GPDS dataset. The main reason for this low performance is the complexity of the forgeries collected in the BHSig260, as the majority of the signatures in the BHSig260 dataset are in textual form, the forged signatures obtained from the forgers are substantially similar to those of genuine signatures collected from the genuine signature writers.

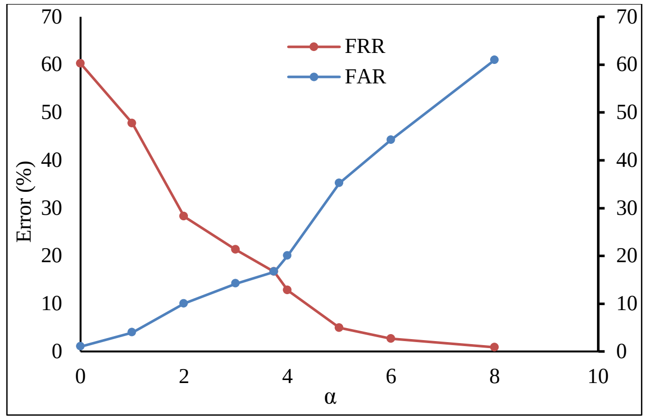


Fig. 5. FAR and FRR curves for different values of the parameter α.

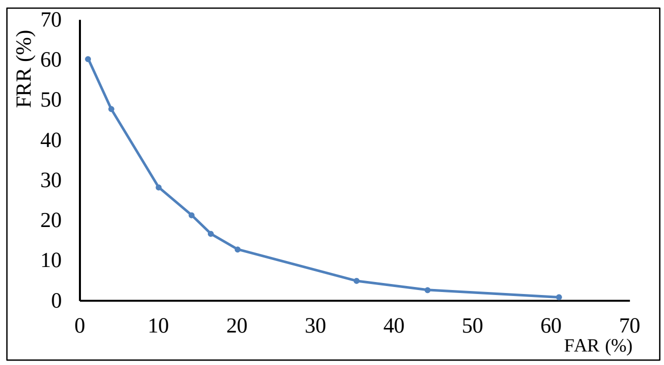


Fig. 6. The ROC curve drawn based on the results obtained from the proposed model using 12 genuine samples for training.

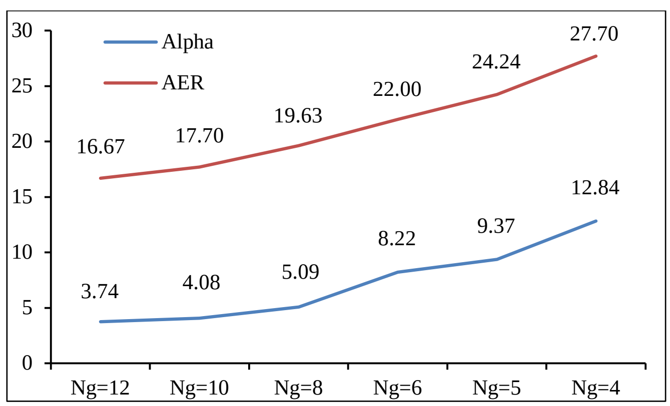


Fig. 7. The EERs and the respective values of α obtained from the proposed model using different genuine samples (N_g) from the GPDS-140 for training.

VII. COMPARATIVE ANALYSIS

To provide a comparative analysis of the results obtained by our proposed method with the results of the state-of-the-art methods, some recent methods evaluated on the GPDS dataset were considered for experimentation. A fair comparison with other methods was difficult due to the use of different numbers of signatures from each writer and also various signature types used in the tuning/designing and evaluation steps for signature verification in different systems. Considering the GPDS dataset for experiments, a comparison of the results obtained from the proposed model and some of the methods in the literature is shown in Table VIII.

TABLE VII
RESULTS OBTAINED FROM THE PROPOSED SYMBOLIC REPRESENTATION MODEL CONSIDERING DIFFERENT DATASETS FOR VALIDATION

Dataset	Training		Testing		FAR	FRR	AER	AER
	N_g	N_f	N_g	N_f	(%)	(%)	(%)	(σ)
GPDS-160	4	0	20	30	20.61	21.84	21.22	1.06
	5		19		17.31	17.22	17.61	0.90
	6		18		16.51	16.53	16.18	0.94
	8		16		10.64	17.43	13.85	1.69
	10		14		9.50	14.49	12.38	1.80
	12		12		7.90	14.76	11.74	1.99
BHSig260	4	0	20	30	32.02	25.73	28.88	0.95
	5		19		26.36	25.85	26.13	0.76
	6		18		24.24	25.31	24.78	0.66
	8		16		19.26	28.23	23.74	0.47
	10		14		17.69	28.61	23.08	0.34
	12		12		16.18	30.12	23.15	0.40

TABLE VIII
AER (%) OBTAINED BY THE PROPOSED MODEL COMPARED TO OTHER SYSTEMS USING THE GPDS-160 DATASET

Method	Feature	AER (%)		
		N_g=4 for training	N_g=8 for training	N_g=12 for training
[5]	Directional features	-	-	17.25
[7]	Surroundedness	Trained with N_g =24 & N_f = 24		13.76
[8]	Local descriptors	Trained with N_g =12 & N_f = 12		15.30
[39]	Boosting feature	-	-	15.24
[38]	Curvelet transform	16.92	15.95	15.07
[41]	Grid segmentation	20.53	17.24	16.84
[51]	Geometric features	**16.10**	14.15	13.35
Proposed method	LBP-based features	21.22	**13.85**	**11.74**

TABLE IX
RESULTS OBTAINED FROM THE PROPOSED MODEL EMPLOYING DIFFERENT NOISES ON THE GPDS-160

Noise type	Noise parameters	AER (FAR, FRR) (%) N_g=12 for training
Salt & Pepper	d = 0.001	11.74 (0, 23.49)
	d = 0.01	11.95 (0, 23.91)
	d = 0.1	12.18 (0, 24.37)
	d = 0.2	12.21 (0, 24.43)
Gaussian White noise	m = 0, v = 0.01	11.88 (0.08, 23.69)
	m = 0, v = 0.1	12.16 (0.0, 24.32)
	m = 0.3, v = 0.01	12.39 (0.0, 24.79)
	m = 0.3, v = 0.1	12.29 (0, 24.58)

The authors in the study [7] used genuine and forged signatures for training the classifier, whilst the methods presented in [38], [41] and [51] used only genuine signatures for training. The results presented in Table VIII show that our proposed method outperforms the state-of-the art methods considered for comparison when 8 or more signatures were used for training. The results demonstrate that a significant improvement was achieved employing our proposed method on the GPDS-160 dataset. From the existing literature we noted that the method presented in [51] reported minimum *AER* was 14.15% (13.35%) considering 8 (12) genuine signatures from each individual for training. Whereas our proposed method shows 13.85% (11.74%) *AER* when 8 (12) genuine signatures were considered for training. Thus, our method provides 1.61% lower *AER* than the existing work when 12 genuine signatures were used for training.

However, from Table VIII we can see that the result obtained based on the method presented in [51] showed better performance when 4 genuine signatures from each individual were used for training. The main reason for the lower performance of our proposed model when trained with a small number of signatures is the use of the statistical measures (mean and standard deviation) in both the symbolic representation process and the fuzzy similarity measure of the proposed model that do not actually reflect the distribution of entire signature samples. Hence, it provided lower performance.

We further noted that for the methods presented in [5, 38, 41], at first, an appropriate kernel needs to be determined and then their parameters should be tuned. Also, these methods have high algorithmic complexity and extensive memory requirements in large-scale tasks. Furthermore, the methods proposed in [7, 38, 41] show poor performance when few signature samples are available for training, and they need reconstruction whenever a new writer is added to the system. The method presented in [8] is very sensitive to irrelevant features as all features equally contribute to the similarity measure and thus to the classification. However, the proposed method does not require to be re-trained when a new class is added to the system. The proposed method is also inexpensive in terms of memory usage and computing time. Moreover, the interval based symbolic model and fuzzy similarity measure proposed in this work can take care of irrelevant features, as every feature does not contribute to the proposed fuzzy similarity measure equally.

To get an idea of the performance of the proposed system on degraded and noisy data, two different types of noise, such as Salt & Pepper, and Gaussian White noise with various noise level were employed on the GPDS-160 dataset to generate noisy signature images. These two types of noise commonly appear in images during the data collection process. Salt & Pepper noise adds black and white noise to the image, where *d* is a parameter that indicates noise density, Gaussian White noise adds Gaussian noise of mean *m* and variance *v* to the image. The results obtained by the proposed signature verification system using noisy data are shown in Table IX when 12 signatures were used for training. From the results presented in Table IX it is clear that the results have slowly decreased when signature images were severely affected by the noise. However, from Tables VIII and XI it can be seen that the results of our method on such noisy data are still better than the results of the state-of-the-art methods when applied on non-noisy data of GPDS-160. This is because the proposed signature verification system can take care the issues of feature variation by using the interval-value representation model and the fuzzy similarity membership function.

To compare the signature verification methods in the literature in relation to their time complexities, a theoretical time complexity analysis is further provided in Table X. As shown in Table X, we compared the time complexity of learning algorithms as most of the methods have used a feature extraction technique of linear complexity. The time complexity of the HMM-based algorithms [41, 51] is of $\mathrm{O}(\mathrm{k}^2 N)$, where N is the length of sequences and k is the number of symbols in the state alphabet of the HMM. The time complexity of the NN-based approach [4, 7] to converge to an optimal solution is of $\mathrm{O}(2^N)$, where N is the dimension of the feature vector. In the case of the SVM-based methods [7, 38, 51], the time complexity is of $\mathrm{O}(\mathrm{N}^3)$. For the GMM-based approach [6] the time complexity is $\mathrm{O}(\mathrm{DKN}^3)$, where D represents the data points, K is the number of Gaussian components and N is the dimension of the feature vector. However, the complexity of the proposed symbolic representation model in this work is of $O(N)$, as all the operations (mean, standard deviation, comparison) used here have a linear complexity. As a result, the proposed signature verification method is computationally less expansive compared to all the existing approaches.

TABLE X
COMPUTATIONAL COMPLEXITY COMPARISON

Method	Time Complexity
HMM-based algorithms [41]	$\mathrm{O}(\mathrm{k}^2 N)$
NN-based approach [7]	$\mathrm{O}(2^N)$
SVM-based methods [38]	$\mathrm{O}(\mathrm{N}^3)$
GMM-based approach [6]	$\mathrm{O}(\mathrm{DKN}^3)$
Proposed method	$O(N)$

VIII. CONCLUSIONS AND FUTURE WORK

In this investigation, the performance of the proposed writer-dependent interval-based symbolic representation model for off-line signature verification is demonstrated, whereby a wide range of experiments was conducted on different datasets. A symbolic representation is used to model the feature vectors, constructing a set of interval values suitable for characterization of intra-class variability of features extracted from different signature samples of an individual. A fuzzy similarity measure applicable to an interval-value symbolic model is proposed to address the inter-class variability of features. A new texture feature based on a low-resolution image obtained employing the under-sampling technique is further introduced. The proposed method provided significantly improved results compared to the state-of-the-art methods considering two different off-line signature datasets. The main advantage of the proposed model is that it allows the design and integration of a model for a new individual using only genuine signatures with the same parameters as before, without any need of re-tuning all the parameters. However, in the case of training with very few samples, the proposed method is not as efficient as when the training is performed with 6 or more signatures.

In future, because of the difficulty of acquiring enough genuine samples, we plan to extend this research work by constructing a representative model, which is composed of different models for feature encoding, using only a very small number of genuine signature samples of each signer for training.

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