# Golay Layer: Limiting Peak-to-Average Power Ratio for OFDM-based Autoencoders

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*Abstract*—In this study, we propose a differentiable layer for OFDM-based autoencoders (OFDM-AEs) to avoid high instantaneous power without regularizing the cost function used during the training. The proposed approach relies on the manipulation of the parameters of a set of functions that yield complementary sequences (CSs) through a deep neural network (DNN). We guarantee the peak-to-average-power ratio (PAPR) of each OFDM-AE symbol to be less than or equal to 3 dB. We also show how to normalize the mean power by using the functions in addition to PAPR. The introduced layer admits auxiliary parameters that allow one to control the amplitude and phase deviations in the frequency domain. Numerical results show that DNNs at the transmitter and receiver can achieve reliable communications under this protection layer at the expense of complexity.

## I. INTRODUCTION

Historically, the physical layer (PHY) of a communication system has been designed based on well-engineered signal processing blocks. This design philosophy has recently been disrupted with the success of machine learning (ML) over handcrafted designs in various fields. In [1] and [2], the composite behavior of the transmitter-channel-receiver is represented as an autoencoder (AE) where the signal processing blocks at the transmitter and receiver are replaced with neural networks. It has been shown that the AE can outperform a Hamming-coded binary phase shift keying (BPSK) scheme and achieves end-to-end learning for small-scale problems without prescribing specific signal processing blocks. However, when a neural network is applied to a large scale PHY design, the training complexity, reliability, scalability, and unexplainable nature of trained neural networks become major issues. Therefore, new methods that can facilitate the PHY automation need to be developed based on the constraints and requirements in the communication systems.

To this end, one direction is to blend the tools that we have already employed in the communication systems with the machine learning blocks. For example, in [3], an AE was combined with orthogonal frequency division multiplexing (OFDM), called OFDM-based autoencoder (OFDM-AE), to reduce the complexity of synchronization stages for a single-carrier modulation scheme in [4]. OFDM-AE is appealing because OFDM is one of the most used schemes in today's wireless communication standards such as 3GPP Long-Term Evolution (LTE), 5G New Radio (NR), and IEEE 802.11 Wi-Fi. It can be implemented via relatively low-complexity architectures while accommodating many crucial aspects of communications, e.g., user multiplexing, multiple antennas, and

simple equalization. In addition, it can easily multiplex different type waveforms, including the ones based on OFDM-AEs, in the frequency domain. In the literature, OFDM-AEs have currently been investigated under multiple antennas [5], training under an unknown channel model [6], one-bit quantization [7], and peak-to-average-power ratio (PAPR) mitigation [8], [9]. In this study, we also focus on OFDM-AE and address the issue of designing OFDM-AE with low PAPR.

The methods that overcome the hardware non-linearity for OFDM-AE in the literature may be grouped into two main categories. In the first category, the approaches rely on the fact that AEs are able to compensate for multiple effects jointly. For example, in [3], the AM-AM distortion due to a power amplifier (PA) is modeled as a third-order non-linear function with a normalized input and the model is included as part of the channel. Similarly, a coarse quantization is considered as a distortion in the channel in [7]. The main challenge in this category is either the availability of an accurate differentiable non-linearity model or the price paid for more comprehensive training as in [6]. The methods in the second category aim at AE design to combat the non-linearity. One common approach in the literature is to regularize the cost function used during the training. For example, in [8] and [5], the cost function is a summation of a metric related to PAPR and another metric for bit-error rate (BER), which are similar to the techniques used for traditional OFDM [10]. In [9], both discrete Fourier transform (DFT)-spread OFDM and a regularization term are considered to reduce the PAPR for light communications. However, these methods require more sophisticated training/optimization procedures, which may increase the training complexity in practice. In this study, we address this issue and propose a protection layer, called Golay layer, by exploiting the properties of complementary sequences (CSs) to limit instantaneous power fluctuations without introducing a regularization term to the cost function.

The CSs were introduced by M. Golay in 1961 [11]. The PAPR of an OFDM symbol generated with a CS is less than or equal to 3 dB [12], [13]. In [14], it has been shown that a specific Reed-Muller (RM) code along with phase-shift keying (PSK) constellation results in CSs. Hence, this particular scheme achieves coding gain while ensuring 3 dB PAPR for OFDM symbols. In the literature, there are numerous studies to construct distinct CSs. The reader is referred to [15] and [16] for iterative methods, [17] and [18] for the offset method, [19] for a method based on Gaussian integers, and [14] and [20] for algebraic approaches for the CS synthesis. A survey related to CSs can also be found in [21].

Our main contribution in this study is the derivation of

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a layer that converts a set of real numbers to another set of real numbers based on the framework given in [20] to guarantee maximum 3 dB PAPR for an OFDM-AE. We show that the proposed layer is compatible with the existing layers in ML literature and the training methods (e.g., backpropagation) as the corresponding gradients can be expressed in closedform, and the amplitude and the phase of the elements of synthesized CS, which can be tuned through neural networks independently without affecting the PAPR, are real numbers. With numerical results, we show that it is possible to achieve power-efficient reliable OFDM-AE with the proposed layer.

The rest of the paper is organized as follows. In Section II, we provide preliminary discussions on sequences. In Section III, we introduce the Golay layer. In Section IV, we present numerical results and compare it with OFDM with polar code. We conclude the paper in Section V.

*Notation:* The sets of complex numbers, real numbers, non-negative real numbers, integers, non-negative integers, positive integers, and integers modulo H are denoted by  $\mathbb{C}$ ,  $\mathbb{R}$ ,  $\mathbb{R}_0^+$ ,  $\mathbb{Z}$ ,  $\mathbb{Z}_0^+$ ,  $\mathbb{Z}^+$ , and  $\mathbb{Z}_H$ , respectively. The set of *m*-dimensional integers where each element is in  $\mathbb{Z}_H$  is denoted by  $\mathbb{Z}_H^m$ . The modulo 2 operation is denoted by  $(\cdot)_2$ . The constant j denotes  $\sqrt{-1}$ .

## II. PRELIMINARIES

An OFDM symbol with the symbol duration  $T_s$  can be expressed in continuous time as a polynomial given by

$$p_{a}(z) \triangleq a_{N-1} z^{N-1} + a_{N-2} z^{N-2} + \dots + a_{0}$$
, (1)

where N is the number of subcarriers, a is a sequence of length N, and  $z \in \{e^{j\frac{2\pi t}{T_s}} | 0 \le t < T_s\}$ . The properties of the sequence a play a central role for the link-level performance of an OFDM-based communication system. This is due the fact that the sequence a can be not only a representation of S information bits over  $\mathbb{C}^N$  from the perspective of modulation and coding but also a key component that determines the waveform characteristics in time and frequency. For example, the instantaneous envelope power of the OFDM symbol can be written as

$$|p_{a}(z)|^{2}\Big|_{z=e^{j\frac{2\pi t}{T_{s}}}} = \sum_{k=-N+1}^{N-1} \rho_{a}(k) e^{j\frac{2\pi t}{T_{s}}k} , \qquad (2)$$

where  $\rho_a(k)$  is the aperiodic autocorrelation (APAC) of the sequence a at the kth lag [20]. To minimize instantaneous power fluctuations, by inferring (2), the sequence a should have good APAC properties, i.e., low  $|\rho_a(k)|$  for  $k \neq 0$ .

## A. Complementary Sequences

The sequence pair (a, b) of length N is called a Golay complementary pair (GCP) if  $\rho_a(k) + \rho_b(k) = 0$  for  $k \neq 0$ [11]. Each sequence in a GCP is called a CS. Equivalently, the GCP (a, b) can be defined as [21]

$$|p_{a}(z)|^{2} + |p_{b}(z)|^{2} \Big|_{z=e^{j\frac{2\pi t}{T_{s}}}} = \underbrace{\rho_{a}(0) + \rho_{b}(0)}_{\text{constant}} .$$
 (3)

In other words, the sum of the instantaneous power of two OFDM symbols generated with  $\boldsymbol{a}$  and  $\boldsymbol{b}$  adds up to a constant if  $\boldsymbol{a}$  and  $\boldsymbol{b}$  form a GCP. Hence,  $|p_a(\mathrm{e}^{\mathrm{j}\frac{2\pi t}{T_{\mathrm{s}}}})|^2$  is always bounded, i.e.,  $\max_t |p_a(\mathrm{e}^{\mathrm{j}\frac{2\pi t}{T_{\mathrm{s}}}})|^2 \leq \rho_a(0) + \rho_b(0)$ . As a result, the PAPR of the OFDM symbol  $p_a(\mathrm{e}^{\mathrm{j}\frac{2\pi t}{T_{\mathrm{s}}}})$  is less than or equal to  $10 \log_{10}(2) \approx 3$  dB if  $\rho_a(0) = \rho_b(0)$  [12], [13].

# B. Representation of a Sequence

Let f be a function that maps from  $\mathbb{Z}_2^m = \{(x_1, x_2, \ldots, x_m) | x_j \in \mathbb{Z}_2\}$  to  $\mathbb{S}$  as  $f : \mathbb{Z}_2^m \to \mathbb{S}$ , where  $\mathbb{S}$  is a an arbitrary set. In this study, a sequence f of length  $2^m$  is composed by listing the values of the function  $f(x_1, x_2, \ldots, x_m)$  as  $(x_1, x_2, \ldots, x_m)$  ranges over its  $2^m$ values in lexicographic order. In other words, the (x + 1)th element of the sequence f is equal to  $f(x_1, x_2, \ldots, x_m)$  where  $x = \sum_{j=1}^m x_j 2^{m-j}$  (i.e., the most significant bit is  $x_1$ ). For the sake of simplifying the notation, we denote the sequence  $(x_1, x_2, \ldots, x_m)$  and the function  $f(x_1, x_2, \ldots, x_m)$  as x and f(x), respectively.

The function  $f(\mathbf{x})$  is called a generalized Boolean function for  $\mathbb{S} = \mathbb{Z}_H$  for  $H \in \mathbb{Z}_0^+$ . If H = 2,  $f(\mathbf{x})$  is a Boolean function. The function  $f(\mathbf{x})$  can be uniquely expressed as a linear combination of the monomials over  $\mathbb{S}$  as

$$f(\mathbf{x}) = \sum_{k=0}^{2^m - 1} c_k \prod_{j=1}^m x_j^{k_j} = c_0 1 + \dots + c_{2^m - 1} x_1 x_2 \dots x_m , \quad (4)$$

where  $c_k \in \mathbb{S}$ ,  $k = \sum_{j=1}^m k_j 2^{m-j}$  for  $k_j \in \mathbb{Z}_2$ . If  $f(\mathbf{x})$  is over  $\mathbb{S} = \mathbb{R}$ , each monomial coefficient belongs to  $\mathbb{R}$ , i.e.,  $c_k \in \mathbb{R}$ . The monomials construct a vector space over  $\mathbb{R}$ , where its dimension is  $2^m$ . Therefore, different sets of  $\{c_k | k = 0, ..., 2^m - 1\}$  lead to different sequences. This is also true for  $\mathbb{S} = \mathbb{Z}_H$  as the monomials are linearly independent.

## III. GOLAY LAYER

A direct GCP construction based on four basic functions is given in [20]. By focusing only on one of the sequences in a GCP, we can restate the theorem in [20] as follows:

**Theorem 1** (CS Construction). Let (a, b) be a GCP of length N and  $p = (p_n)_{n=1}^m$  be a sequence defined by a permutation of  $\{1, 2, ..., m\}$ . For any  $\alpha, \beta \in \mathbb{R}_0^+$ ,  $d_n \in \mathbb{Z}_0^+$ ,  $e_n \in \mathbb{R}$ , and  $k_n \in \mathbb{R}_0^+$  for n = 0, 1, ..., m, let

$$f_{\rm r}(\mathbf{x}) = \alpha \left( e_m x_{p_m} + \sum_{n=1}^{m-1} e_n (x_{p_n} + x_{p_{n+1}})_2 + e_0 \right) , \quad (5)$$

$$f_{i}(\mathbf{x}) = \pi \left(\sum_{n=1}^{m-1} x_{p_{n}} x_{p_{n+1}}\right) + \beta \left(\sum_{n=1}^{m} k_{n} x_{p_{n}} + k_{0}\right) , \quad (6)$$

$$f_{\rm s}(\mathbf{x}) = \sum_{n=1} d_n x_{p_n} , \qquad (7)$$

$$p_{\rm o}(\mathbf{x}, z) = p_{\mathbf{a}}(z)(1 - x_{p_1})_2 + p_{\mathbf{b}}(z)x_{p_1} .$$
(8)

Then, the sequence c where its polynomial representation is given by

$$p_{\boldsymbol{c}}(z) = \sum_{x=0}^{2^{m}-1} p_{\mathrm{o}}(\boldsymbol{x}, z) \times \mathrm{e}^{f_{\mathrm{r}}(\boldsymbol{x}) + \mathrm{j}f_{\mathrm{i}}(\boldsymbol{x})} \times z^{f_{\mathrm{s}}(\boldsymbol{x}) + xN}$$
(9)

is a CS of length  $N2^m + \sum_{n=1}^m d_n$ .

The polynomial given in (9) corresponds to an OFDM symbol where the sequence in the frequency domain is a CS c. Therefore, based on the discussions in Section II-A, the instantaneous power for the corresponding OFDM symbol is bounded. In general, the sequence c is a function of an initial GCP  $(\boldsymbol{a}, \boldsymbol{b}), \alpha, \beta, \boldsymbol{p}, d_n, e_n$ , and  $k_n$  for  $n = 0, 1, \dots, m$ , and formed by the functions given in (5)-(8). While the functions related to the real and imaginary part of the exponent, i.e.  $f_{\rm r}(x)$ and  $f_i(\mathbf{x})$ , allow one to adjust the amplitude and phase of the elements of CSs, respectively,  $f_s(\mathbf{x})$  alters the locations of the initial sequences, i.e., a and b, in the frequency domain. The function  $p_{o}(\mathbf{x}, z)$  determines which of the initial sequences is modified through  $f_{\rm r}(\mathbf{x})$ ,  $f_{\rm i}(\mathbf{x})$ , and  $f_{\rm s}(\mathbf{x})$  based on (9). The parameters  $\alpha$  and  $\beta$  are the auxiliary variables introduced in this study to control the amount of the amplitude and phase deviations. For example, if  $\alpha = 0$ , the parameters related to the amplitude, i.e.,  $e_n$  for n = 0, 1, ..., m, do not determine the final sequence. Similarly, if  $\beta = 0$ , the synthesized sequence is a not a function of  $k_n$  for n = 0, 1, ..., m.

In the literature, there are many studies on the enumeration of distinct CSs such that their elements are in the traditional constellations such as M-quadrature amplitude modulation (QAM), particularly after the discovery of the connection between RM codes and CSs in [14]. In their pioneering work, Davis and Jedwab showed that the CSs can occur as the elements of the cosets of the first-order RM code within the second-order RM code where the elements of the CS are in PSK constellation. The corresponding function that defines the RM code can be seen in (6). For instance, by choosing  $k_n \in \mathbb{Z}_{2^q}$  for n = 0, 1, ..., m and  $q \in \mathbb{Z}^+, \beta = 2\pi/2^q, \alpha = 0,$  $p_{0}(\mathbf{x}, z) = 1$ , (9) yields a coded OFDM symbol with the maximum of 3 dB PAPR, where the CS in the frequency domain is a codeword from the coset of the first-order RM code over  $\mathbb{Z}_{2^q}$ with  $2^{q}$ -PSK modulation. In [20], several rules are introduced for CS with M-QAM constellation for Theorem 1. We refer the reader to other approaches based on Gaussian integers and offset methods in [18] and [19], respectively. Nevertheless, CS-based encoding and decoding are still challenging tasks. For example, it is not trivial to map the information bits to the parameters introduced in Theorem 1. In addition, to the best of our knowledge, there is no study that shows that M-QAM is the optimum constellation to achieve good BER performance for an encoder that generates CSs. The decoding at the receiver side can be nontrivial as the modulation and coding are not independent operations in Theorem 1 (e.g., see the discussions in [14]). These design issues motivate us to investigate CSbased encoding and decoding under OFDM-AEs.

The key observation that we exploit in this study is that the variables in Theorem 1 can be tuned with a deep neural network (DNN) based on the information bits without affecting the PAPR. From the perspective of ML-based PHY design, this approach can also be considered as a framework for developing new layers for power-efficient OFDM-AE as Theorem 1 limits the PAPR to be less than 3 dB *without* introducing constraints on the cost function used during the training while being compatible with state-of-the-art ML approaches. For example, the parameters which control the amplitude and phase of the elements of the CS, i.e.,  $e_n$ , and  $k_n$ , are real numbers and inherently determine the complex numbers in polar coordinate. The gradients for the phase and amplitude functions can also be calculated in closed-form expressions as the functions given in (5) and (6) are multivariate polynomials. Therefore, the backpropagation can be used without any restriction.

## A. Basic Golay Layer

Without loss of generality, in this study, we assume that a = (1) and b = (1), the values of  $d_n$  and p are controlled either by a communication network or they are prescribed to adjust the position of the non-zero elements of the encoded CS. We define the basic Golay layer as

$$y = f(\mathbf{x}) = f_{\rm r}(\mathbf{x}) + jf_{\rm i}(\mathbf{x}).$$
(10)

Based on (6), the derivative of  $f_i(\mathbf{x})$  with respect to  $k_n$  and  $k_0$ , and the derivative of  $g_i(\mathbf{x})$  with respect to  $k_n$  can be calculated as

$$\frac{\partial f_{i}(\boldsymbol{x})}{\partial k_{n}} = \beta x_{p_{n}} , \qquad (11)$$

and

$$\frac{\partial f_{\rm i}(\boldsymbol{x})}{\partial k_0} = \beta \quad , \tag{12}$$

respectively.

The mean OFDM symbol power is a function of the amplitude of each element of the CS. However, Theorem 1 without any constraint does not guarantee a fixed mean power for each set of  $e_n$  for n = 0, 1, ..., m. To resolve this issue, we choose  $e_0$  as a normalization parameter since it is a constant term in (5) (i.e, it scales all the elements of the synthesized CS in (9)), and introduce the condition given by

$$\alpha e_0 = \frac{1}{2} \sum_{n=1}^m \ln \frac{1 + e^{2\alpha e_n}}{2}.$$
 (13)

The condition in (13) can be derived as follows: The parameter  $e_n$  scales half of the elements of the CS by  $e^{\alpha e_n}$  due to the monomials in (5). Therefore, the CS power is scaled by  $(1 + e^{2\alpha e_n})/2$ . For all  $e_{n|n=1,2,...,m}$ , the total factor can be calculated as  $\gamma = \prod_{n=1}^{m} (1 + e^{2\alpha e_n})/2$ . Hence, to normalize the power,  $e^{2\alpha e_n} = 1/\gamma$ , which results in (13). Under the condition in (13), the derivative of  $f_r(\mathbf{x})$  with respect to  $e_n$  for n = 1, 2, ..., m can be calculated as

$$\frac{\partial f_{\mathbf{i}}(\boldsymbol{x})}{\partial e_n} = \begin{cases} \alpha (x_{p_n} + x_{p_{n+1}})_2 - \frac{\mathrm{e}^{2\alpha e_n}}{1 + \mathrm{e}^{2\alpha e_n}} & n < m\\ \alpha (x_{p_m})_2 - \frac{\mathrm{e}^{2\alpha e_m}}{1 + \mathrm{e}^{2\alpha e_m}} & n = m \end{cases}$$
(14)

as

$$\frac{\partial e_0}{\partial e_n} = -\frac{\mathrm{e}^{2\alpha e_m}}{1 + \mathrm{e}^{2\alpha e_m}} \ . \tag{15}$$

As a result, the basic Golay layer consists of two differentiable functions with m inputs for amplitude and m + 1 inputs for the phase and returns  $2^m$  complex (or  $2^{m+1}$  real) values as output. Let  $\mathbb{B}$  be a minibatch of size B. The derivative of the loss J (e.g., cross-entropy function) with respect to *i*th  $k_n^{(i)}$ 



(a) Transmitter and receiver block diagrams for OFDM-AE with Golay layer.



(b) An AE representation of the complete link.

Figure 1. OFDM-AE with Golay layer.

for  $n=0,1,\ldots,m$  and  $e_n^{(i)}$  for  $n=1,2,\ldots,m$  can then be obtained as

$$\frac{\partial J}{\partial k_n^{(i)}} = \sum_{x=0}^{2^m-1} \frac{\partial f_i(\mathbf{x})}{\partial k_n^{(i)}} \frac{\partial J}{\partial f_i(\mathbf{x})} , \qquad (16)$$

$$\frac{\partial J}{\partial e_n^{(i)}} = \sum_{x=0}^{2^m-1} \frac{\partial f_{\mathbf{r}}(\mathbf{x})}{\partial e_n^{(i)}} \frac{\partial J}{\partial f_{\mathbf{r}}(\mathbf{x})} .$$
(17)

where *i* denotes the *i*th sample in the minibatch  $\mathbb{B}$ , and *x* and *x* are defined in Section II-B.

In Figure 1(a), we provide the transmitter and receiver block diagrams for OFDM-AE with the Golay layer. At the transmitter, we consider a DNN with K layers where the kth layer has  $N_k$  output nodes. The DNN at the transmitter maps a sequence of information bits to the aforementioned 2m + 1 parameters and learns the bit mapping. The Golay layer calculates the amplitude and the phase parameters of the corresponding CS based on the output of the preceding layer. After mapping the synthesized CS to the OFDM subcarriers, the inverse DFT of the mapped CS is calculated. A cyclic prefix (CP) can also be prepended to the generated symbol before transmitting the OFDM symbol. At the receiver side, the CP is removed and the DFT of the received signal is calculated. After the subcarrier de-mapping, another DNN with L layers where the lth layer has  $M_l$  output nodes at the receiver side obtain the information bits from the received CS.

#### **B.** Supporting Layers

In this study, we do not introduce any restriction for the DNNs at the transmitter and receiver. However, based on our computer trials, several supporting layers can be helpful for training and manipulating the parameters of the Golay layer in a controlled manner. For example, a clipping layer that limits the range of the parameters for the Golay layer can avoid the growth of exponent in (9). The clipping layer which allows the variable between  $r_{\min}$  and  $r_{\max}$  to pass without any distortion can be defined as

$$y = f(x) = \min\{\max\{x, r_{\min}\}, r_{\max}\}$$
 (18)

where  $r_{\min} \leq r_{\max}$ . The derivative of the loss J with respect to the variable x can be calculated as  $\partial J/\partial y = \partial J/\partial x$  for  $r_{\min} \leq x \leq r_{\max}$ , otherwise it is zero.

Another auxiliary function which may be needed during the training is the polar-to-Cartesian (P2C) layer which can be defined as

$$a = f(x, y) = \Re\{e^{x+jy}\} = e^x \cos(y)$$
, (19)

$$b = f(x, y) = \Im\{e^{x+jy}\} = e^x \sin(y)$$
. (20)

The derivative of the loss with respect to the variable x and y are  $\partial J/\partial x = \partial J/\partial a \times e^x \cos(y) + \partial J/\partial b \times e^x \sin(y)$  and  $\partial J/\partial y = -\partial J/\partial a \times e^x \sin(y) + \partial J/\partial b \times e^x \cos(y)$ , respectively. We also consider an additive white Gaussian noise (AWGN) layer which adds noise to the real and imaginary components with the variance  $\sigma_n^2/2$  after the P2C layer for the sake of training the OFDM-AE offline under certain signal-to-noise ratio (SNR).

A complete AE representation of the link with the supporting layers is shown in Figure 1(b). First, the S information bits are processed by a DNN with K layers. The 2m + 1outputs of the Kth layer at the transmitter are then limited by the two parallel clipping layers. After the calculation of  $f_r(\mathbf{x})$  and  $f_i(\mathbf{x})$ ,  $2^{m+1}$  output of the Golay layer is converted to Cartesian coordinates. The output of P2C layer is perturbed by a noise layer which adds noise with variance  $\sigma_n^2/2$  real and imaginary parts. The transmitted bits are then detected by a DNN with L layers.

To achieve a reliable communications under the fading channels, it is also possible to extend the OFDM-AE shown in Figure 1(b) with several other layers or train it with the

 Table I

 LAYOUT OF THE AUTOENCODER

	Layer	Output Node
TX	Input	9 (binary)
	Dense (Batchnorm+ReLU)	100
	Dense (Batchnorm+ReLU)	100
	Dense (Batchnorm+ReLU)	100
	Dense (Clipping layer)	11
	Golay layer	64
	Polar-to-Cartesian	64
CH	Noise layer	64
RX	Dense (Batchnorm+ReLU)	1000
	Dense (Batchnorm+ReLU)	1000
	Dense (Batchnorm+softmax)	512
	Classification	1 (integer)

existing methods. One approach is to perform the training by including a distortion layer that models the behavior of fading. For example, in [3], it was observed that the AE shifts the center of the constellations to superimpose the pilot information on data symbols. In [22], a different case for an OFDM transmitter is investigated and noted that the receiver complexity can be high and the training can take a longer duration. To overcome these issues, another network between the subcarrier de-mapping and the DNN at the receiver along with a two-stage training is proposed. Another approach is to transmit a fixed OFDM symbol along with the OFDM-AE symbol and let the DNN at the receiver perform joint channel estimation and symbol detection [23]. Given the availability of possible extensions, for the scope of this work, we focus our investigation on the Golay layer for an AWGN channel during the training.

#### **IV. NUMERICAL RESULTS**

Assume that S = 9 bits need to be transmitted. In this case, the transmitter needs to generate  $2^9$  distinct CSs based on the information bits and the receiver should be able distinguish them to receive the information bits. Let m = 5,  $\beta = 2\pi$ , p =(1, 2, 3, 4, 5). Since the length of each CS is  $2^5 = 32$  under these settings, 32 subcarriers are used for the transmission. Therefore, the spectral efficiency (SE) is 9/32 bit/second/Hz.

We consider two OFDM-AE designs with  $\alpha = 1$  and  $\alpha = 0$ . The layer information at the transmitter and receiver are provided in Table I for both OFDM-AEs. At the transmitter, the information bits are first processed by three dense layers with batchnorm and rectified linear units (ReLUs) and one dense layer with two parallel clipping layers as in Figure 1(b). We set the parameters of the clipping layer such that  $-2 \le e_n^{(i)} \le 1$ for n = 1, 2, ..., 5 and  $|k_n^{(i)}| \le 1$  for n = 0, 1, ..., 5. At the Golay layer,  $e_0^{(0)}$  is calculated based on the condition given in (12) for the second se (13) for the normalization, and  $2^5$  outputs for the amplitude function and  $2^5$  outputs for the phase function are calculated for the *i*th sample in the minibatch. The P2C layer converts this information to Cartesian coordinates. During the training, we set the noise variance on the real and imaginary parts to be  $\sigma_{\rm p}^2/2 = 1/2$ . At the receiver, the corresponding subcarriers are processed with two dense layers with batchnorm and ReLUs and one dense layer. The last dense layer of the AE consists of softmax as activation function and the last layer



Figure 2. Temporal characteristics.



Figure 3. PAPR distributions.

is a classification layer which returns the index of the output node with the maximum value. For example, if the information bits are (0,0,0,0,0,0,0,0), the first element of softmax layer should be closer to 1 while other 511 elements are near 0. Then, the classification layer returns 1. During the training, our batch size is B = 5120 and the J is the binary crossentropy function. The learning rate is set to 0.0001 We train the AE by using MATLAB Deep Neural Network Toolbox and utilize NVIDIA GeForce GTX 1060. For the sake of comparison, we consider polar code with length 32 with BPSK modulation. We set the design SNR as 3 dB and consider successive interference cancellation (SIC) at the receiver.

### A. Peak-to-Average Power Ratio

In Figure 2, we provide the temporal characteristics of five randomly generated OFDM symbols (without CP) based on Polar code (with a scrambler to reduce PAPR) and compare it with the OFDM-AE symbols designed for  $\alpha = 1$ . As clearly seen, the instantaneous power for the OFDM-AE symbols never exceeds 2 (i.e., approximately 3 dB) while the OFDM



Figure 4. Constellations on the subcarriers (+: CS for information bit sequence (0,0,0,0,0,0,0,0), o: CS for information bit sequence (0,0,0,0,0,0,0,0,1)).

symbols with traditional encoding can be peaky which may require a large power back-off at the transmitter. The PAPR distributions are compared in Figure 3. The PAPR gain at the 90th percentile is approximately 6 dB as the Golay layers at OFDM-AEs limit the PAPR to be less than or equal to 3 dB.

## B. Constellation

In Figure 4, we provide the constellation on each subcarrier obtained for OFDM-AE for  $\alpha = 1$  and  $\alpha = 0$ . As opposed to the handcrafted designs for CSs (e.g., [19], [20]), the OFDM-AE does not follow any of the traditional constellations such as *M*-QAM for CSs. This result is expected as the OFDM-AE does not make isolated symbol-level decisions. In Figure 4, we mark the elements of learned CSs for two different information bit sequences with + and o. For  $\alpha = 0$ , we observe that some of the values on the subcarriers are very close to each other (e.g., subcarrier #1, #2, #14, #18) while or there exists points on completely on opposite quadrants (e.g., subcarrier #3, #11, #21, #29). Nevertheless, it is hard to state

that the trained OFDM-AE exploits this property to achieve relatively good error rate performance shown in Figure 5 for AWGN channel. For  $\alpha = 0$ , the constellation on each subcarrier is constrained to be on the unit circle. However, the positions of the elements of the learned CS does not appear to follow the same pattern for the case with  $\alpha = 1$ .

# C. Error Rate

One natural question is if OFDM-AE can still perform well with a Golay layer. Although it is not trivial to provide a definite answer to this question, our numerical results in Figure 5 demonstrate that the OFDM-AE with a Golay layer and OFDM with Polar coding can perform similar (within the range of 1.5 dB) under at least aforementioned simulation settings. For this comparison, we consider the same spectral efficiency for all schemes. As shown in Figure 5, the SNR gains for OFDM-AEs with  $\alpha = 1$  and  $\alpha = 0$  are approximately 1 dB and 0.75 dB, respectively, at 1e-3 BER as compared to the OFDM with Polar coding under AWGN channel. The origin of



Figure 5. BER results.

the gain is the joint design of modulation and coding through (5) and (6). For example, (6) reduces to the first-order RM code within the second-order RM code for PSK constellation [14]. The OFDM-AE with  $\alpha = 1$  performs slightly better than the one with  $\alpha = 0$  since  $\alpha = 1$  allows the backpropagation to exploit the amplitude of the elements of the CSs. For the fading channel, we assume a flat fading channel and consider singletap frequency domain minimum mean square error (MMSE) equalization with ideal channel frequency response. In this case, the performance of the OFDM with polar code is better than that of OFDM-AEs. This result is expected as the SIC exploits the soft bits and the OFDM-AEs are trained under the AWGN channel. Nevertheless, the PAPR gain of OFDM-AEs with the Golay layer is still much larger than the SNR gains in this scenario. Another observation from Figure 5 is that the OFDM-AE with  $\alpha = 0$  is superior to the one with  $\alpha = 1$  under the fading channel. This indicates that the identical energy for the elements of the learned CS can help to achieve better BER under the fading channel.

## V. CONCLUDING REMARKS

In this study, we propose a method that limits the PAPR of an OFDM-AE symbol to be less than or equal to 3 dB without introducing a regularization term to the cost function used in the training. It relies on the modification of the differentiable functions that lead to a complex CS based on Theorem 1. Based on our preliminary results, an OFDM-AE with a Golay layer can provide an acceptable BER performance. Under our simulation assumptions, we obtained 1 dB SNR gain for error rate at 1e-3 for AWGN channel and 6 dB PAPR gain at the 90th percentile as compared to OFDM with a polar code.

The proposed concept needs to be studied further to achieve a more flexible scheme. For example, we limit our investigation in this study to only a small number of information bits (i.e., the SE is only 9/32 bit/second/Hz) because of the highcomplexity of AE. Hence, the methods that can give a larger SE still need to be investigated under transmitter/receiver complexity and power consumption constraints. In this direction, the layers at the DNNs may need to be revised. A multiple parallel Golay layer can also be useful for adjusting the PAPR level while allowing a larger number of information bits. Another extension is to develop advanced Golay layers. For example, Theorem 1 allows the set of  $\{d_n | m = 1, 2, ..., m\}$  to be arbitrary parameters, which can extend the definition of the basic Golay layer. Evaluating the scheme through a practical setup is also another angle that can be explored.

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