ERGODIC INTERFERENCE ALIGNMENT FOR THE SIMO/MIMO INTERFERENCE CHANNEL

Yohan Lejosne[†*], *Dirk Slock*[*], *Yi Yuan-Wu*[†]

[†]Orange Labs,
38-40 Rue du General Leclerc,
92794 Issy Moulineaux Cedex 9, FRANCE
Email: {yohan.lejosne, yi.yuan}@orange.com

[*]EURECOM, Mobile Communications Dept.
Campus SophiaTech, 450 route des Chappes,
06410 Biot Sophia Antipolis, FRANCE
Email: dirk.slock@eurecom.fr

ABSTRACT

Ergodic interference alignment (IA) is a simple yet powerful tool that not only achieves the optimal $K/2$ degrees of freedom (DoF) of the K-user single-input single-output (SISO) interference channel (IC), but also allows each user to achieve at least half of its interference-free capacity at any SNR. By considering more general message sets, Nazer et al. also covered the MISO case. In this paper, we consider first the SIMO interference channel and extend ergodic IA techniques to this setting with N_r receive antennas. Our scheme achieves $KN_r/(N_r+1)$, which is the DoF yielded by (standard) IA and is also the DoF of the channel when $K > N_r$. Moreover, this technique exhibits spatial scale invariance. By combining the existing MISO and the new SIMO results, we can also cover MIMO with N_t transmit antennas for the cases where either N_t/N_r or N_r/N_t is an integer R, yielding DoF $= \min(N_t, N_r)KR/(R+1)$ which is optimal for $K > R$.

Index Terms— Interference channel, ergodic interference alignment, MIMO

1. INTRODUCTION

The idea of pairing complementary channel realizations, *ergodic interference alignment* (IA), was first proposed by Nazer et al. in [1]. The scheme allows each user of an interference channel (IC) to achieve half of his interference free rate, i.e., half of the rate he would achieve if he had the channel for himself. It thereby reaches the optimal degree of freedom (DoF) K/2 of the K user SISO IC that was first achieved by asymptotic interference alignment [2].

Some improvements have been made to the original ergodic IA scheme, for instance the channel coefficient distribution does not need to be symmetric [3], the sum of channel matrices does not need to be the identity matrix but can be relaxed to an arbitrary diagonal matrix [3], and simple strategies can be deployed to reduce latency [4]. Other efforts were made to generalize the ergodic IA scheme to different networks, for instance for relay networks in [5]. Ergodic IA was also adapted to secrecy scenarios, in which the information leakage is to be minimized in [6]. A variant of ergodic IA for delayed feedback was proposed in [7], it shows that the full sum DoF $K/2$ of the SISO IC can be preserved for feedback delay as long as half the channel coherence time. Another variant, for completely outdated feedback, is developed in [8] and achieves larger DoF than retrospective alignment [9].

However, to the best of our knowledge, the ergodic alignment scheme and its variants do not cover the general symmetric MIMO interference channel. Indeed, both IA and ergodic IA schemes are also directly applicable to the MIMO symmetric square case, by decomposing each multi antenna node in single antenna nodes, but only asymptotic IA was also extended to SIMO and MISO symmetric configuration in [10] whereas only the MISO setting is covered by ergodic IA with the variant for "recovering more messages" proposed in [1].

We extend ergodic IA techniques to the SIMO interference channel and achieve the same DoF as asymptotic IA. Together with the existing MISO result we can also cover MIMO with N_t transmit antennas and N_r receive antennas for the cases where either N_t/N_r or N_r/N_t is an integer R, yielding DoF $= \min(N_t, N_r)KR/(R+1)$ which is optimal for $K > R$ [2].

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Even though different in terms of idea and complexity, asymptotic IA and ergodic IA schemes share certain characteristics. They both create a delay that roughly scales the same way and both have the property of decomposability: the antennas do not need to be collocated neither at the transmitter nor at the receiver. Therefore the SIMO and MISO schemes are also directly applicable to interfering broadcast channels and interfering multiple access channels respectively.

Let us recall that the outer bound for the DoF IC depends on whether the ratio R is more or less than K [10]. Asymptotic IA is only needed for $K > R$ as when $K \leq R$ linear techniques usually yield better multiplexing gains. Ergodic IA is meant as an alternative to asymptotic IA; we will see that it achieves the same DoF as asymptotic IA and is therefore only optimal for $K > R$.

2. SYSTEM MODEL AND BACKGROUND

We consider a K-user SIMO IC, i.e., there are K transmitter-receiver pairs. The transmitters are equipped with $N_t = 1$ antenna and the receivers with N_r antennas. $R = \max(\frac{N_t}{N_r}, \frac{N_r}{N_t})$ is equal to N_r in this case. The channel coefficients are drawn from a continuous distribution, their phases are uniformly distributed and are independent from their magnitude.

The channel realization at time index t is

$$\mathbf{H}(t) = \{\mathbf{h}_{ij}(t)\} \in \mathbb{C}^{K \times KN_r}$$

where $\mathbf{h}_{ij}(t) = \{h_{j_a i}(t)\} \in \mathbb{C}^{1 \times N_r}$ and $h_{j_a i}(t)$ is the channel between transmitter i and receiver's j a^{th} antenna.

The channel output observed at antenna $a \in [1, N_r]$ of receiver $j \in [1, K]$ is a noisy linear combination of the inputs

$$y_{j_a}(t) = \sum_{i=1}^{K} h_{j_a i}(t) x_i(t) + z_{j_a}(t)$$

where $x_i(t)$ is the transmitted symbol of transmitter i, $z_{j_a}(t)$ is the additive white Gaussian noise at antenna a of receiver j.

With

$$\begin{aligned} \mathbf{x}(t) &= [x_1(t), \ldots, x_K(t)], \\ \mathbf{y}(t) &= [y_{1_1}(t), \ldots, y_{1_R}(t), \ldots, y_{K_R}(t)], \\ \mathbf{z}(t) &= [z_{1_1}(t), \ldots, z_{1_R}(t), \ldots, z_{K_R}(t)], \end{aligned}$$

the channel input output relationship becomes

$$\mathbf{y}(t) = \mathbf{x}(t)\mathbf{H}(t) + \mathbf{z}(t).$$

The performance metric is the sum DoF, it is the prelog of the sum rate. Let $R_j(P)$ denote the achievable rate for user j with transmit power P, then the achievable DoF for user j is as follows,

$$d_j = \lim_{P \to \infty} \frac{R_j(P)}{\log_2(P)}$$

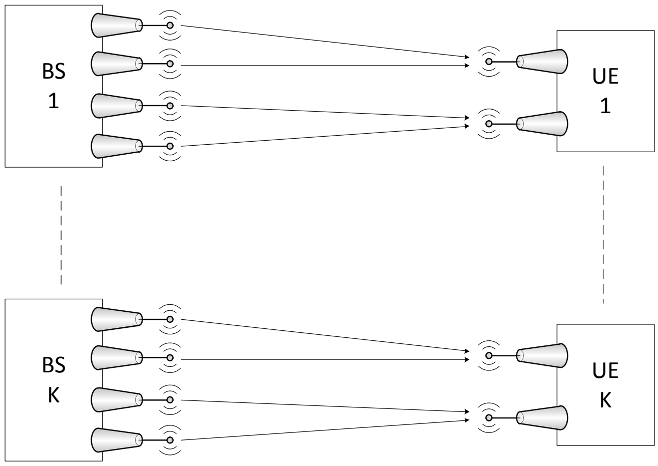

Fig. 1. Decomposition of the K user MIMO IC with $N_t = 4$ and $N_r = 2$, showing only links supporting intended messages.

and the sum DoF of the K-user SIMO IC is $\text{DoF}(K) = \sum_{j=1}^{K} d_j$. Since only the DoF are of interest, Gaussian noise is hereafter omitted for simplicity.

2.1. Ergodic IA [1]

The main idea behind ergodic IA is to transmit the data a first time during channel realization at time t_1, then to wait for the complementary channel realization at times t_2 such that the sum of the two is the $K \times K$ identity matrix. Thereby allowing each receiver to cancel all interference by simply adding the signals received at times t_1 and t_2.

The exact match will never happen when channel coefficients are drawn from a continuous distribution. However, it is still possible to match channel matrices up to an approximation error small enough to allow decoding [1]. This can be done through appropriately precise quantization. The authors of [1] prove that, by considering channel realization sequences that are long enough, it is possible to be sure with a sufficient probability that it will be possible to match up enough channel realizations to achieve a DoF that approaches $\frac{K}{2}$.

3. MAIN RESULTS

Theorem 1. *In the K-user SIMO IC,*

$$K \frac{N_r}{(N_r + 1)} \; DoF$$

are achievable through ergodic IA.

The theorem is proved in section 4.2 by introducing an ergodic IA scheme that assures the transmission of N_r symbols between each transmitter-receiver pair over $N_r + 1$ symbol periods.

Corollary 1. *In the K-user MIMO IC where*

$$R = \max(\frac{N_t}{N_r}, \frac{N_r}{N_t})$$

is an integer,

$$\min(N_t, N_r)K\frac{R}{(R+1)} \text{ } DoF$$

are achievable through ergodic IA.

Proof. 1. $R = \frac{N_t}{N_r}$: The scheme proposed in [1] for "recovering more messages" can be used. Indeed, by decomposing each node into single antenna nodes, one obtains KN_t transmitters and KN_r receivers. Then, by making each single antenna receiver ask for R different messages from the single antenna transmitters, one falls into the framework of the scheme for "recovering more messages". It achieves $\frac{1}{(R+1)}$ DoF per message which adds up to the $N_r K\frac{R}{(R+1)}$. An example of this kind of decomposition is given in Fig. 1 for the K-user MIMO IC with $N_t = 4$ and $N_r = 2$, showing only links supporting intended messages for clarity.

2. $R = \frac{N_r}{N_t}$: By decomposing each transmitter in single antenna transmitters and each receiver in N_t receivers with R antennas one obtains a KN_t-user SIMO interference IC. According to Theorem 1, in this SIMO IC, $KN_t\frac{R}{(R+1)}$ are achievable through ergodic IA. □

4. SIMO ERGODIC IA

4.1. Example

We start with an example for the SIMO IC with $K = 3$, $R = N_r = 2$. The scheme assures the transmission of 2 symbols between each transmitter-receiver pair in 3 symbol periods, over channel realizations t_1, t_2, t_3. Transmitter $i \in [1, 3]$ has 2 messages for receiver i: $[s_1^i, s_2^i]$ and transmits $x_i(t_n) = s_n^i$ during t_n, $n \in \{1, 2\}$ then $x_i(t_3) = s_1^i + s_2^i$ during t_3. We will see that, by picking channels realizations as below, the interference alignment will be done by simply adding the signals received over the 3 channel realizations at each receiver.

Let the first channel realization be as follows:

$$\mathbf{H}(t_1) = \begin{bmatrix} -h_{11_1} & h_{11_2} & h_{12_1} & h_{12_2} & h_{13_1} & h_{13_2} \\ h_{21_1} & h_{21_2} & -h_{22_1} & h_{22_2} & h_{23_1} & h_{23_2} \\ h_{31_1} & h_{31_2} & h_{32_1} & h_{32_2} & -h_{33_1} & h_{33_2} \end{bmatrix}.$$

Then, wait for t_2 such that

$$\mathbf{H}(t_2) = \begin{bmatrix} h_{11_1} & -h_{11_2} & h_{12_1} & h_{12_2} & h_{13_1} & h_{13_2} \\ h_{21_1} & h_{21_2} & h_{22_1} & -h_{22_2} & h_{23_1} & h_{23_2} \\ h_{31_1} & h_{31_2} & h_{32_1} & h_{32_2} & h_{33_1} & -h_{33_2} \end{bmatrix}$$

and for t_3 such that

$$\mathbf{H}(t_3) = \begin{bmatrix} -h_{11_1} & -h_{11_2} & -h_{12_1} & -h_{12_2} & -h_{13_1} & -h_{13_2} \\ -h_{21_1} & -h_{21_2} & -h_{22_1} & -h_{22_2} & -h_{23_1} & -h_{23_2} \\ -h_{31_1} & -h_{31_2} & -h_{32_1} & -h_{32_2} & -h_{33_1} & -h_{33_2} \end{bmatrix}.$$

First, we can notice that the cross links are chosen to be always the same when the transmitters are sending their symbols one by one, then the opposite when the sum of the symbols are transmitted. This ensures that, by simply adding its received signals, each receiver cancels all inter cell interference. Then, for receiver i to get only its a^{th} message on its a^{th} antenna, the same rule is applied for the direct links, with the exception of the a^{th} coefficient during t_a so that the intended signal is not canceled by the summation. Indeed, we have

$$\begin{aligned} \sum_{t=t_1}^{t_3} \mathbf{y}(t) &= \sum_{t=t_1}^{t_3} \mathbf{x}(t)\mathbf{H}(t) \\ &= -2[h_{11_1}s_1^1, h_{11_2}s_2^1, h_{22_1}s_1^2, \\ &\qquad h_{22_2}s_2^2, h_{33_1}s_1^3, h_{33_2}s_2^3] \end{aligned}$$

and, at each antenna, the intended message can be trivially retrieved. Transmitting 6 messages in 3 channel uses reaches the maximum DoF of $3\frac{2}{3} = 2$ of this SIMO IC.

4.2. Proof of Theorem 1.

Proof. To achieve the $K\frac{N_r}{(N_r+1)}$ DoF in the K-user SIMO IC, we introduce an alignment scheme that assures the transmission of N_r symbols between each transmitter-receiver pair in $R + 1 = N_r + 1$ symbol periods, over channel realizations $t_1, \ldots, t_{R+1}$. Transmitter $i \in [1, K]$ has R messages for receiver i: $[s_1^i, \ldots, s_R^i]$ and transmits $x_i(t_n) = s_n^i$ during t_n, $n \in [1, R]$ and $x_i(t_{R+1}) = \sum_{n=1}^{R} s_n^i$ during t_{R+1}.

We start with t_{R+1} to simplify the formulas. During the first channel realization, $\mathbf{H}(t_{R+1}) = \{h_{j_a i}(t_{R+1})\}$, the sum of all messages is transmitted. Then channel realizations $\{\mathbf{H}(t_n)\}$, $n \in [1, R]$ are chosen so that

$$\begin{aligned} h_{j_a i}(t_n) &= -h_{j_a i}(t_{R+1}) \text{ for } j \neq i \text{ or } a \neq n \\ h_{i_a i}(t_n) &= h_{j_a i}(t_{R+1}) \text{ for } a \neq n. \end{aligned}$$

By simply summing the signals received over the $N_r + 1$ symbols periods, each receivers gets one intended message at each of his N_r antennas, interference free, thereby achieving the $\frac{N_r}{(N_r+1)}$ DoF per user. □

The certainty that enough pairings can be done to approach the DoF is formally established in [1].

5. DISCUSSIONS

5.1. Decomposability

The ergodic IA scheme for SIMO IC does not require any joint receive antenna processing and can therefore also be used in interfering broadcast channels. With ergodic IA for the MIMO IC, this decomposability property is present at both transmitter and receiver side. This is also true of the asymptotic IA scheme [10] and make the 2 schemes also applicable to interfering multiple access channels.

5.2. Delay

For the SISO case, it was shown in [1] that both ergodic IA and standard asymptotic IA create a delay that roughly scales the same way, exponentially with K^2. This delay, needed to have a capacity that scales with $\frac{1}{2}\log P$, is the length of the symbol extension for asymptotic IA and the expected time before finding a channel realization sufficiently close to the exact complementary for ergodic IA. Going from SISO to SIMO, we have to match $N_r + 1$ channel realizations. However, this does not influence the exponent in the delay which is mainly influenced by the number of possible channel coefficients due to the quantization. Therefore, as a first approximation, the delay of the SIMO variant of ergodic IA should scale exponentially with RK^2. Which again, for large K, is roughly similar to the exponent $\Gamma = RK(K - R - 1)$ of the symbol extension in the SIMO case of asymptotic IA [10].

5.3. Improvements

It was shown that improvements could be made to the original SISO ergodic IA scheme; some of them could also be applied to the MIMO version. If the proposed scheme is DoF optimal, the SNR offset might be improved by finding better pairings as was done in [3]. Different pairing methods could also be considered to reduce the delay, with or without rate loss, as was investigated in [4] and [11].

5.4. Variant for feedback delay

The starting point in [7] is that, in the original ergodic IA scheme, there is no need for the transmitter to know the current channel state information (CSI) during the first transmission. Based on that observation, a different pairing method is proposed to achieve $K/2$ DoF in presence of feedback delay up to half the channel coherence time. In the variant we proposed here for SIMO IC, and in the one for MISO IC in [1], the CSI is not needed for the first transmission either. Therefore, by doing the pairing in a similar fashion, it is possible to achieve the $\min(N_t, N_r)KR/(R+1)$ DoF with feedback delay up to $\frac{1}{R+1}$ of the channel coherence time. Namely, in a block fading environment, one would dedicate the first part of each block, during which the transmitter does not have CSI yet, to first transmissions of possibly multiple instances of the scheme, and the rest of the block for the rest of the transmissions. As the scheme requires $R + 1$ transmissions, this is possible as long as the first part is less than or equal to $\frac{1}{R+1}$ of the block, in other words, for feedback delay up to $\frac{1}{R+1}$ of the channel coherence time.

6. CONCLUSION

In this paper, we extended the ergodic IA technique to the SIMO interference channel. Our scheme achieves $KN_r/(N_r+1)$ DoF, which is the DoF reached by asymptotic IA and is also the DoF of the channel when $K > N_r$. Since no joint antenna processing is needed, this technique trivially exhibits spatial scale invariance. Therefore, these SIMO results, combined with the existing MISO results, allow to cover the symmetric MIMO configurations as well as interfering broadcast channels and interfering multiple access channels.

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