Author(s): Jabeen, Tayyaba; Ali, Zain; Khan, Wali Ullah; Jameel, Furqan; Khan, Imran; Sidhu, Guftaar Ahmad Sardar; Choi, Bong Jun

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Article

Joint Power Allocation and Link Selection for Multi-Carrier Buffer Aided Relay Network

Tayyaba Jabeen [1], Zain Ali [1], Wali Ullah Khan [2] , Furqan Jameel [3,*], Imran Khan [4] , Guftaar Ahmad Sardar Sidhu [1,*] and Bong Jun Choi [5,*]

[1] Department of Electrical and Computer Engineering, COMSATS University Islamabad, Islamabad 44000, Pakistan; tayyabajabeen37@gmail.com (T.J.); zainalihanan1@gmail.com (Z.A.)
[2] School of Information Science and Engineering, Shandong University, Qingdao 266237, China; waliullahkhan30@gmail.com
[3] Faculty of Information Technology, University of Jyvaskyla, Jyvaskyla 40014, Finland
[4] Department of Electrical Engineering, University of Engineering and Technology at Peshawar, Peshawar 25120, Pakistan; imran-khan@uetpeshawar.edu.pk
[5] School of Computer Science and Engineering, Soongsil University, Seoul 06978, Korea
* Correspondence: furqanjameel01@gmail.com (F.J.); guftaarahmad@comsats.edu.pk (G.A.S.S.); davidchoi@soongsil.ac.kr (B.J.C.)

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Abstract: In this paper, we present a joint power allocation and adaptive link selection protocol for an orthogonal frequency division multiplexing (OFDM)-based network consists of one source node i.e., base station (BS), one destination node i.e., (MU) and a buffer aided decode and forward (DF) relay node. Our objective is to maximize the average throughput of the system via power loading over different subcarriers at source and relay nodes. A separate power budget is assumed at each transmitting node to make the system more practical. In order to form our solution more tractable, a decomposition framework is implemented to solve the mixed integer optimization problem. Further, less complex suboptimal approaches have also been presented and simulation results are provided to endorse the efficiency of our designed algorithms.

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1. Introduction

In the field of wireless communication, relay networks acquired much consideration to provide better coverage and throughput [1,2]. For the efficient utilization of wireless resources to provide good quality of services to the users, resource allocation is considered as an important factor [3,4]. The problem of outage probability and ergodic capacity was studied in energy harvesting relay networks [5]. Under amplify and forward (AF) relaying protocol, the joint power optimization was studied to maximize the throughput of the secondary users [6]. A dual decomposition framework was adopted to solve the problem. In order to increase the spectrum efficiency with an additional multi-user diversity gain, authors [7] proposed a generalized sub-carrier pairing strategy and a low complexity resource allocation scheme for the decode and forward (DF)-based two-way relay network. For both the AF and the DF-based networks, described in [6,7] respectively, authors assumed that the relay is operating under the conventional protocol (data packets are received from the base station (BS) in one time slot and forwarded to the mobile user (MU) in the next consecutive time slot). A range-division user relay selection strategy was provided to improve the coverage and capacity efficiency of non-orthogonal multiple access (NOMA)-based cooperative networks [8].

Buffer aided relaying (BAR) emerged as a new paradigm for the wireless communication systems and has provided freedom to the link selection, i.e., the choice to choose a particular hop for transmission in a given time slot [9]. With this addition, the resource allocation problem becomes more challenging and is coupled with the link selection. The optimization techniques designed for memoryless relaying nodes cannot be applied for BAR transmission. Thus, the problem of power allocation and link selection for the BAR has received significant attention in the research community [10–20]. Considering a full-duplex network, power allocation at the source and relay node was studied in [10]. Authors maximized the source arrival rate under the assumption of imperfect self-interference cancellation and statistical delay constraints. For the underlay cognitive radio network with buffer aided DF relay, an adaptive link selection scheme was presented in [11]. A closed-form expression for the data rate was derived by assuming peak power and interference constraints at the secondary nodes. The authors in [12] considered a system where multiple source nodes are communicating with a single destination through a common BAR. Under the total transmit power constraint at each node, this work presented a link selection and a power allocation strategy.

The problem of cross-layer resource allocation considering asymmetric time duration over two hops was investigated in [13]. The work in [14] proposed different BAR schemes under full-duplex (FD) relay transmission with self-interference cancellation (SIC) capability at the relaying node. The results showed the considerable gains of the proposed scheme over the conventional FD relay transmission. Further, the authors in [15] studied the security and the delay issues in the buffer enhanced dual-hop transmission. The relay selection schemes for links with equal weights have recently been explored in [16]. Depending on the status of the buffer at each relaying node, the authors in [17] proposed a max-link selection analysis framework. With the Markov chain approach, analytical expressions for the outage probability, the average bit error rate, and the steady-state probability vector were derived. More recently, the problem of link selection for satellite–terrestrial relays was considered in [21] for correlated fading.

The joint link selection and power optimization for the uplink-based BAR network were proposed in [19]. Throughput of the system was maximized under limited power budget at all nodes. To ensure the long lifetime of the system, energy harvesting (EH)-based source and relay nodes were considered in [20]. Authors solved the joint source and relay power optimization along with an adaptive link selection by an iterative spatial branch-and-bound-based method.

1.1. Related Work and Motivation

Orthogonal frequency division multiplexing (OFDM) is considered as a promising solution to provide high data rates due to its robustness against multipath fading, high spectral efficiency, and flexibility in the resource allocation [22]. The resource allocation for traditional relaying transmission without buffer under OFDM modulation has been well studied in the last decade [23,24]. Thus, the combination of OFDM and BAR is a promising candidate for higher throughput in cellular networks. Considering equal power allocation at all nodes, authors in [25] maximized throughput with adaptive link selection and a varying number of antennas. Next, the sub-carrier assignment and transmission mode (direct/relay) selection were presented in [26]. The optimal power loading over different OFDM sub-carriers plays the most significant role in achieving the real benefits of the multi-carrier transmission [27]. However, to the best of our knowledge, the problem of power allocation has not been taken into account for OFDM-based BAR networks.

1.2. Contributions

In this work, our target is to maximize the average throughput in an OFDM-based dual-hop buffer aided relaying transmission. We seek power optimization and the transmission hop selection under the half-duplex one-way relay communication. Specifically, our contributions can be outlined as:

- To maximize the end-to-end rate of the system, an optimization problem is being formulated under individual power constraint at source/relay nodes and the link selection constraint.

- We consider joint power optimization over different sub-carriers at the source node, the optimal power loading over carriers at the relay node and the transmission link selection at a given time slot.
- Under the buffer aided transmission at the relay, we exploit the fact the end-to-end throughput is the sum of the rates received at the second hop while the sum of data transmitted cannot exceed the total received and propose a joint solution over all variables.
- With the DF relaying protocol, we propose an efficient decomposition structure where the optimal power allocation at each node is allocated through the water-filling strategy while the optimal link selection is obtained for the obtain power solution.
- Later, the problem of power allocation over the entire time slot for conventional relay transmission and the corresponding solution is presented. Further, a sub-optimal solution for BAR is also proposed.
- Finally, results are evaluated through extensive simulations.

1.3. Organization

The remaining of this paper is organized as follows. The list of acronyms is in Table 1. The dual hop BAR-based transmission model and the joint optimization problem are presented in Section 2. Section 3 provides the proposed algorithms. Finally, the simulation results and the conclusion are presented in Sections 4 and 5, respectively.

Table 1. List of acronyms.

OFDM	Orthogonal Frequency division multiplexing
DF	Decode and forward
AF	Amplify and forward
BAR	Buffer aided relaying
FD	Full duplex
SOP	Secrecy outage probability
SOC	Secrecy outage capacity
EST	Exact secrecy throughput
EH	Energy harvesting
BS	Base station
MU	Mobile user
SIC	Self interference cancellation
AWGN	Additive white Gaussian noise
SNR	Signal to noise ratio
FIFO	First in first Out
Km	Kilometer
P.L.	Path loss

2. System Model and Problem Formulation

2.1. System Model

We consider an OFDM-based network that consists of one source node i.e., a base station (BS), one destination node i.e., the mobile user (MU) and a buffer-aided DF relay (R_b) node. The BS is communicating with the MU on the downlink, as shown in Figure 1. The direct communication link is missing between source and destination nodes due to the large distance between them. Thus, R_b is mandatory for successful communication. Further, we assume a single antenna on each transmitting node under half-duplex mode. For the total T time slots of equal lengths, in any t-th time, R_b receives data packets from the BS and stores in the buffer. In the next $t + \Delta$ time slot, where Δ is any positive integer, relay forwards the packets to the MU.

Let the gain on the links from BS to R_b, from R_b to MU and the additive white Gaussian noise (AWGN) at BS and R_b in any t-th time slot on i-th subcarrier are denoted by $g_{i,t}$, $h_{i,t}$, $\sigma^2_{BS_{i,t}}$, and $\sigma^2_{R_{bi,t}}$,

respectively. Thus, the signal to noise ratio (SNR) at first hop on the i-th sub-carrier during t-th time can be expressed as;

$$\text{SNR}_{i,t}^1 = \frac{P_{i,t}^{BS}|g_{i,t}|^2}{\sigma_{BS_{i,t}}^2},\tag{1}$$

and, $\text{SNR}_{i,t}^2$ i.e., the SNR at second hop on the i-th sub-carrier during t-th time, is represented as;

$$\text{SNR}_{i,t}^2 = \frac{P_{i,t}^{R_b}|h_{i,t}|^2}{\sigma_{R_{bi,t}}^2},\tag{2}$$

where, $P_{i,t}^{BS}$ and $P_{i,t}^{R_b}$ in the above equations are the transmit powers at BS and R_b over the i-th carrier during t-th time slot, respectively.

Under the half duplex relaying protocol, the transmission is allowed only at a single hop in any given time slot. Thus, to incorporate the link selection in problem formulation, we define a binary decision variable $\alpha_t \in \{0,1\}$ such that;

$$\alpha_t = \begin{cases} \textbf{1,} & \text{if the data flow is on first link at time } t, \\[2mm] \textbf{0,} & \text{if the data flow is on second link at time } t. \end{cases}$$

For $\alpha_t = 1$, data rates on both the hops during t-th time slot over the i-th sub-carrier (i.e., $R_{i,t}^1$ and $R_{i,t}^2$, respectively) can be expressed, mathematically, as;

$$R_{i,t}^1 = \log_2(1 + \text{SNR}_{i,t}^1), \qquad R_{i,t}^2 = 0.$$

Similarly, for $\alpha_t = 0$, we have;

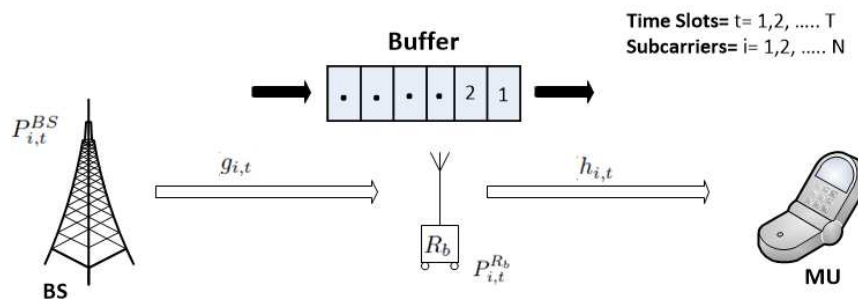$$R_{i,t}^1 = 0, \qquad R_{i,t}^2 = \log_2(1 + \text{SNR}_{i,t}^2).$$

Figure 1. System model.

With this, the end to end average throughput under DF transmission can be written as;

$$R_{avg} = \min\left(\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \log_2(1 + \alpha_t \text{SNR}_{i,t}^1), \right.$$

$$\left. \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \log_2(1 + (1 - \alpha_t)\text{SNR}_{i,t}^2) \right).\tag{3}$$

2.2. Problem Formulation

We assume that the relay's buffer operates under first-in-first-out (FIFO) mode. Thus, the data transmission from relay cannot exceeds the data received and the overall throughput of the system is determined by the rate at the second hop.

Our aim is to maximize average throughput i.e., $\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} (1 - \alpha_t) R_{i,t}^2$ for an OFDM-based BAR network under the constraints that total output at the buffer should not exceeds the total input data and only one link is selected at a given time. The power allocation under a sum power constraint over source and relay may provide analysis and makes the problem simple [28]. However, the solution proposed under global power constraints is not practical, as the relay and source cannot share common energy storage. Thus, we consider separate power limitations at the source and relay nodes to make our solutions more practical. The problem can be stated, mathematically, as;

$$\max_{P_{i,t}^{BS}, P_{i,t}^{R_b}, \alpha_t} \quad \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} (1 - \alpha_t) R_{i,t}^2 \tag{4}$$

$$\text{s.t.} \quad \underbrace{\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} \alpha_t R_{i,t}^1 + \bar{A}}_{\textbf{A}} \geq$$

$$\underbrace{\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} (1 - \alpha_t) R_{i,t}^2}_{\textbf{D}} \tag{5}$$

$$\sum_{i=1}^{N} P_{i,t}^{BS} \leq P_T^{BS}, \quad \forall t, \tag{6}$$

$$\sum_{i=1}^{N} P_{i,t}^{R_b} \leq P_T^{R_b}, \quad \forall t, \tag{7}$$

$$P_{i,t}^{BS} \geq 0, \quad P_{i,t}^{R_b} \geq 0, \quad \forall i, t, \tag{8}$$

$$\alpha_t (1 - \alpha_t) = 0, \tag{9}$$

where $\bar{A}$, P_T^{BS} and $P_T^{R_b}$ are the amount of data in the buffer before the start of current transmission, total available powers at BS and at R_b, respectively. Please note that inclusion of $\bar{A}$ in our model makes the proposed solution more realistic i.e., instead of considering the buffer empty at the start of transmission (as considered in most of literature), this solution is valid starting at any instant of time. According to the law of conservation of flow, state of the buffer is shown by the constraint in (5). The conditions in (6) and (7) represent that sum power at each of the source and relay node can not exceed the total available power at that node. Moreover, (8) and (9) ensure that power at each transmitting node cannot be zero and only one link will be active in a given time slot, respectively.

3. Proposed Solution

The optimization problem formulated in previous section can easily be called a mixed integer optimization problem, and it is difficult to solve. For tractability of solution, we chose a decomposition approach to maximize the average throughput at MU in T time slots. At first, we found different link selection strategies for the given power allocation. Let, $P_{i,t}^{BS*}$ and $P_{i,t}^{R_b*}$ represent optimal powers at source and relay nodes for the corresponding maximum rates $R_{i,t}^{1*}$ and $R_{i,t}^{2*}$, respectively. Thus, our problem was reduced to selecting the link such that maximum throughput was obtained, which was:

$$\max_{\alpha_t} \quad \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} (1-\alpha_t) R_{i,t}^{2*} \tag{10}$$

$$\text{s.t.} \quad \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} (\alpha_t) R_{i,t}^{1*} \geq$$

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} (1-\alpha_t) R_{i,t}^{2*}$$

$$\alpha_t(1-\alpha_t) = 0.$$

For $\overline{A}$ to be the amount of data present in the queue of the buffer before the start of communication between *BS* and *MU*, the following two cases can be considered in order to solve the problem.

Case 1: $(\sum_{i=1}^{N} \sum_{t=1}^{T} R_{i,t}^{2*} \leq \overline{A})$:

The solution to the problem (10) becomes;

$$\alpha_t = 0, \qquad \forall t,$$

and from (4) we get,

$$P_{i,t}^{BS*} = 0,$$

$$P_{i,t}^{R_b*} = \arg \max_{P_{i,t}^{R_b}} \sum_{t=1}^{T} \sum_{i=1}^{N} (1-\alpha_t) R_{i,t}^{2} \tag{11}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} P_{i,t}^{R_b} \leq P_T^{R_b}, \quad \forall t.$$

The necessary and sufficient conditions associated with problem (11) are given by:

$$\frac{\partial L(P_{i,t}^{R_b*}, \lambda_t)}{\partial P_{i,t}^{R_b*}} = 0, \tag{12}$$

$$\lambda_t \left(P_T^{R_b} - \sum_{i=1}^{N} P_{i,t}^{R_b*} \right) = 0, \quad \forall t,$$

$$\lambda_t \geq 0, \quad \forall t,$$

when $P_{i,t}^{R_b*}$ is the optimal value that maximizes the objective function in (11), and $L(P_{i,t}^{R_b*}, \lambda_t)$ is the Lagrangian of the problem, written as:

$$L(P_{i,t}^{R_b*}, \lambda_t) = \sum_{t=1}^{T} \sum_{i=1}^{N} (1-\alpha_t) R_{i,t}^{2} + \sum_{t=1}^{T} \lambda_t \left(P_T^{R_b} - \sum_{i=1}^{N} P_{i,t}^{R_b*} \right),$$

here λ_t is called the Lagrangian multiplier of the problem. Applying the necessary and sufficient conditions given in (12) the optimal value of $P_{i,t}^{R_b*}$ can be expressed as:

$$P_{i,t}^{R_b*} = \left[\frac{1}{\lambda_t} - \frac{(\sigma_{i,t}^{1})^2}{|g_{i,t}|} \right]^{+}, \tag{13}$$

the structure of (13) is similar to the water filling solution [29], with, $\frac{1}{\lambda_t}$ as the maximum power stream or water level. In our work, the water level was optimized using the sub-gradient method, where, in each iteration the value of λ_t was updated by:

$$\lambda_t^{itr} = \lambda_t^{itr-1} + \psi \left(P_T^{R_b} - \sum_{i=1}^{N} P_{i,t}^{R_b*} \right),$$

where, *itr* and ψ represent the iteration number and step size, respectively.

Case 2: $(\sum_{i=1}^{N} \sum_{t=1}^{T} R_{i,t}^{2*} > \overline{A})$:

The solution required to put some $\alpha_t = 1$ (or $[1 - \alpha_t] = 0$). To keep $R_{i,t}^{2*}$ at maximum possible level, the link between BS and R_b was selected for the time slots t^* such that the difference between the rates of first and second hops at that slot, denoted as $\sum_{i=1}^{N} R_{i,t}^{3}$ is also maximum. After each selection, the sum rate $\sum_{i=1}^{N} \sum_{t=1}^{T} R_{i,t}^{1*}$ will start to increase and in contrast, $\sum_{i=1}^{N} \sum_{t=1}^{T} R_{i,t}^{2*}$ will gradually decrease. This process will continue to repeat until $\sum_{i=1}^{N} \sum_{t=1}^{T} R_{i,t}^{2*} \approx \sum_{i=1}^{N} \sum_{t=1}^{T} R_{i,t}^{1*} + \overline{A}$. Step wise description of our designed algorithm is elaborated in Algorithm 1.

Algorithm 1 Proposed solution 1.

1: Initialize $\overline{A}$.
2: Let $\sum_{i=1}^{N} R_{i,t}^{3} = \sum_{i=1}^{N} R_{i,t}^{1*} - \sum_{i=1}^{N} R_{i,t}^{2*}$ $\quad \forall t$.
3: Select $t^* = \arg \max \sum_{i=1}^{N} R_{i,t}^{3}$.
4: Find (a) $\sum_{i=1}^{N} \sum_{t=1}^{T} R_{i,t}^{2*} = \sum_{i=1}^{N} \sum_{t=1}^{T} R_{i,t}^{2*} - \sum_{i=1}^{N} R_{i,t^*}^{2}$, (b) $\sum_{i=1}^{N} \sum_{t=1}^{T} R_{i,t}^{1*} = \sum_{i=1}^{N} \sum_{t=1}^{T} R_{i,t}^{1*} + \sum_{i=1}^{N} R_{i,t^*}^{1}$.
5: Assign $\sum_{i=1}^{N} R_{i,t^*}^{3} = -\infty$, $\alpha_{t^*} = 1$.
6: Repeat step 3 to step 5 until $\sum_{i=1}^{N} \sum_{t=1}^{T} R_{i,t}^{2*} \leq \overline{A}$.

Now, it is remain to allocate optimal powers at both source and relay nodes. The power optimization at relay can be obtained through (11) while for the source node we have to solve the following optimization problem:

$$P_{i,t}^{BS*} = \arg \quad \max_{P_{i,t}^{BS}} \sum_{t=1}^{T} \sum_{i=1}^{N} (\alpha_t) R_{i,t}^{1} \tag{14}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} P_{i,t}^{BS} \leq P_T^{BS}, \quad \forall t,$$

Solution of (14) is similar to (11), hence, is excluded from the paper for simplicity. The above designed algorithm gives nearly optimal solution. However, it may be possible that $\sum_{i=1}^{N} R_{i,t}^{3}$ maximizes for more than one value of t^*. Thus, in order to make it more practical, we have also proposed another approach. Step-wise description is given in Algorithm 2.

Please note that the proposed model can be extended to a multi-hop scenario in a straightforward manner. The separate power constraint at each transmitting node provides the opportunity to apply the solution to the multi-hop case without any key difference. Moreover, the link selection, at a given time slot, can also be considered for any hop directly. Thus, to avoid the heavy rotational burden, the solution for the multi-hop case is omitted for simplicity.

Algorithm 2 Proposed Solution 2.

1: Initialize $\overline{A}$.

2: Let $\sum_{i=1}^{N} R_{i,t}^3 = \sum_{i=1}^{N} R_{i,t}^{1*} - \sum_{i=1}^{N} R_{i,t}^{2*}$ $\forall t$.

3: And $x = \max \sum_{i=1}^{N} R_{i,t}^3$.

4: For $t = 1$ to T, find t^* such that $\sum_{i=1}^{N} R_{i,t^*}^3 = x$.

5: Then $\sum_{i=1}^{N} R_{i,t^*}^4 = \sum_{i=1}^{N} R_{i,t^*}^3 - \sum_{i=1}^{N} R_{i,t^*}^2$ $\forall t^*$.

6: Select $tt^* = \arg \max \sum_{i=1}^{N} R_{i,t^*}^4$.

7: Obtain (a) $\sum_{t=1}^{T} \sum_{i=1}^{N} R_{i,t}^{2*} = \sum_{t=1}^{T} \sum_{i=1}^{N} R_{i,t}^{2*} - \sum_{i=1}^{N} R_{i,tt^*}^2$, (b) $\sum_{t=1}^{T} \sum_{i=1}^{N} R_{i,t}^{1*} = \sum_{t=1}^{T} \sum_{i=1}^{N} R_{i,t}^{1*} + \sum_{i=1}^{N} R_{i,tt^*}^1$.

8: Put $\sum_{i=1}^{N} R_{i,tt^*}^4 = -\infty$, $\alpha_{tt^*} = 1$.

9: Repeat step 5 to step 8 until $\sum_{i=1}^{N} \sum_{t=1}^{T} R_{i,t}^{2*} \le \overline{A}$.

10: Update $D = \sum_{i=1}^{N} \sum_{t=1}^{T} R_{i,t}^{2*}$.

11: Repeat steps 3 to 6 of Algorithm 1.

12: Refine $D = \max \left(D, \sum_{i=1}^{N} \sum_{t=1}^{T} R_{i,t}^{2*} \right)$.

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4. Simulation and Results

In this section, we validate the performance of our designed algorithms. We consider a BAR communication network with $N = 1024$ number of subcarriers where each one experiences flat fading. Further, multi-path Rayleigh distributed model-based channels are considered for all links. The throughput of the system is achieved as an average over fifty-time slots of equal length i.e., $T = 50$. Next, we assume that MU is 1 kilometer (Km) apart from the BS. We adopt modified Hata urban propagation model [30] to calculate the path loss (P.L.), such that

$$P.L. = \begin{cases} 38\log(d) + 122, & \text{if } d \ge 0.05 \text{ km,} \\ \\ 38\log(0.05) + 122, & \text{otherwise,} \end{cases} \tag{15}$$

where the distance (i.e., measured in kilometers) between BS and R_b is indicated by d.

The comparison of the following four algorithms is provided in this section:

- BARNS1: this refers to the scheme proposed in Algorithm 1.
- JntS1S2: this corresponds to the solution obtained in Algorithm 2.
- BARNS3: similar to [31], a suboptimal link selection scheme where for the case $\sum_{i=1}^{N} \sum_{t=1}^{T} R_{i,t}^{2*} > \overline{A}$, we put some $\alpha_t = 1$ at the time slots t^* such that $t^* = \arg \min \sum_{i=1}^{N} R_{i,t}^2$. The process will continue until the sum rate of relay to destination link becomes less than or equal to $\overline{A}$.
- NBARS: this shows the non-buffer aided conventional DF relay scheme. The problem can be written as:

$$\max_{P_{i,t}^{BS}, P_{i,t}^{R_b}} \quad \lim_{T \to \infty} \frac{1}{2T} \sum_{t=1}^{T} \sum_{i=1}^{N} R_{i,t}^2$$

$$\text{s.t.} \quad \frac{1}{2} \sum_{i=1}^{N} R_{i,t}^1 \ge \frac{1}{2} \sum_{i=1}^{N} R_{i,t}^2 \ \forall \ t$$

$$\sum_{i=1}^{N} P_{i,t}^{BS} \le P_T^{BS}, \quad \forall t,$$

$$\sum_{i=1}^{N} P_{i,t}^{R_b} \le P_T^{R_b}, \quad \forall t,$$

$$P_{i,t}^{BS} \ge 0, \quad P_{i,t}^{R_b} \ge 0, \quad \forall i, t.$$

Solving this problem using similar steps as in Section 3. We get the optimal values to be:

$$P_{i,t}^{Rb*} = \left(\frac{1}{2T\eta_t} - \frac{\mu_t}{2\eta_t} - \frac{\sigma_{Rbi,t}^2}{|h_{i,t}|^2} \right)^+,$$

$$P_{i,t}^{BS*} = \left(\frac{\mu_t}{2\tau_t} - \frac{\sigma_{BSi,t}^2}{|g_{i,t}|^2} \right)^+,$$

where μ_t, τ_t and η_t are the Lagrangian multipliers associated with the first, second and third constraint of the problem, respectively.

The complexity of BARNS1 is given by $O(3I + C_P)$, where I is the number of iterations required in Algorithm 1 and C_P is the complexity of finding the power allocations. In the case of JntS1S2 the complexity of finding the solution increases to $O(3I + 4J + C_P)$, where the iterations required by step 9 of Algorithm 2 are denoted as J. In the case of BARNS3 and NBARS, the complex part is of finding the power allocations at the source and relay nodes. Thus, the complexity of these algorithms is given by $O(C_P)$. This shows that the JntS1S2 is most complex out of all the schemes. The complexity of BARNS3 and NBARS is the least because of the sub-optimality and system limitation, respectively.

Figure 2 shows how the end-to-end data rate (bps/Hz) changes by changing the total available power at source and relay nodes. We have compared three different scenarios i.e., fixed value of P_T^{BS} for increasing $P_T^{R_b}$, fixed $P_T^{R_b}$ for increasing P_T^{BS} and the simultaneous increase in the values of P_T^{BS} and $P_T^{R_b}$, in a single plot by keeping d = 0.5 Km (i.e., the R_b is at the center of source and destination nodes). For a better understanding, we only display the performance of JntS1S2 and the NBARS in this case. It can be seen that the results are almost the same by either fixing P_T^{BS} for the increasing $P_T^{R_b}$ or vice versa. It can also be noted that JntS1S2 scheme beats in all scenarios and the performance gap between both the JntS1S2 and NBARS remain constant at all data points.
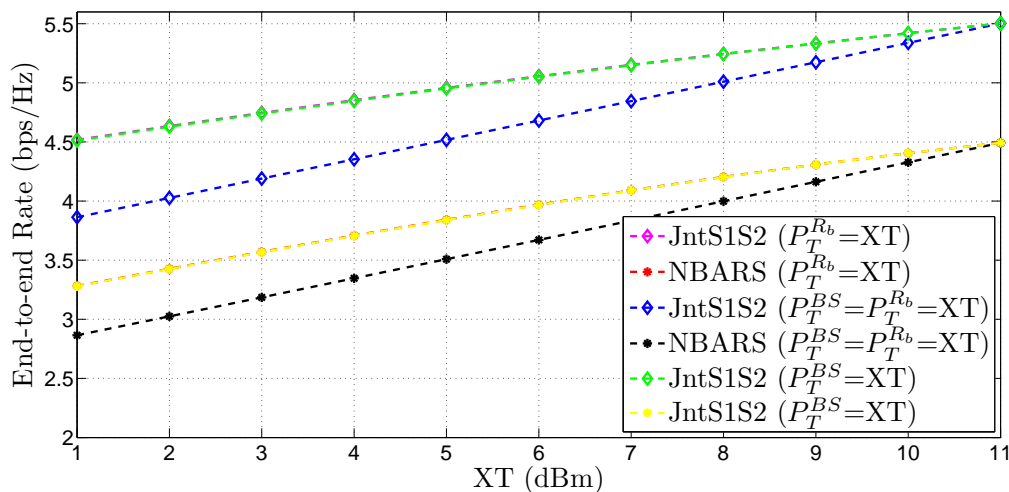

Figure 2. Throughput (bps/Hz) versus total power budget.

In Figure 3, we demonstrate the effect of the total power budget on the system throughput by assuming that the total available power at the relay node is half of the power at BS. It can be noted that we got a higher end-to-end rate by the increase in the total power budget. Further, the graph of JntS1S2 scheme is always at the top at each data point. On the other hand, it was nice to observe that NBARS had no significant benefits over any of our other proposed scheme at any point. It was very interesting to observe the consequences of relay position on the throughput of our designed algorithms, as shown in Figure 4. Similar to [6], it can be noted here as well that our proposed schemes gave their best performance at the point where the relay node was near the center of the source and destination nodes.

Further, the JntS1S2 solution outperforms at all points. The BARNS1 and BARNS3 schemes meet the JntS1S2 solution when the relay is placed near to either end.
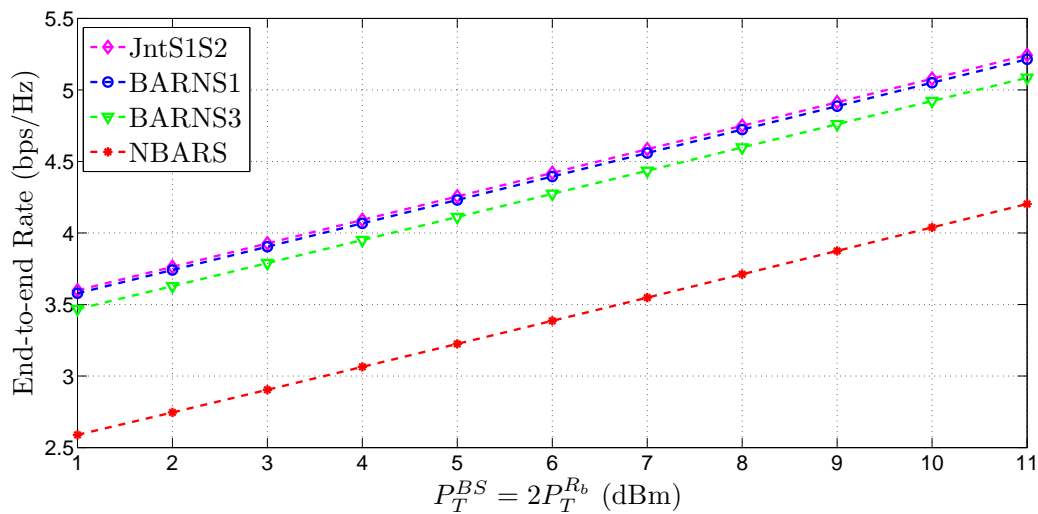

Figure 3. Throughput (bps/Hz) versus total power budget (power at the relay is half of the source node's available power.
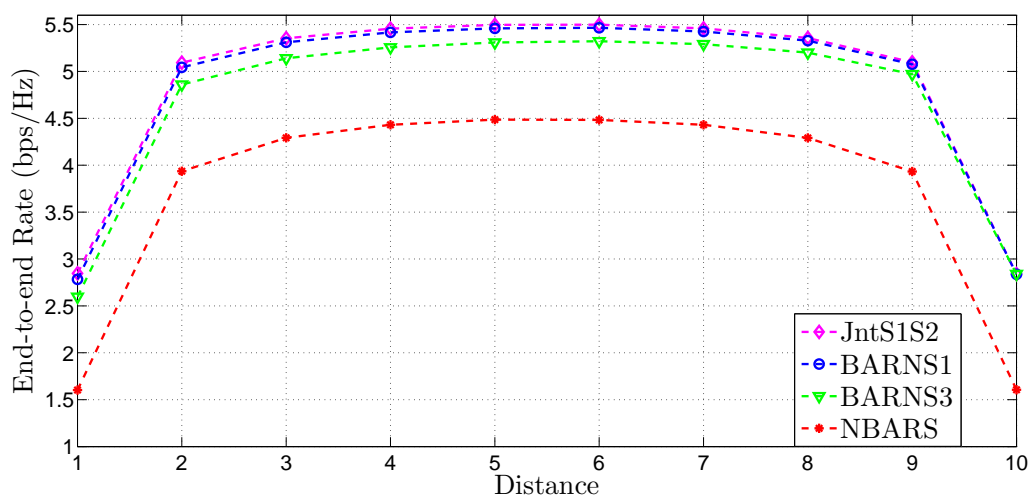

Figure 4. Throughput (bps/Hz) versus relay position (the distance of the relay from base station (BS)).

In Figure 5, we show the performance of our network by varying the state of the buffer (amount of data present in the queue before the start of transmission). It was nice to observe that the throughput of the system improved by increasing the amount of data in the queue. This was because it increased the upper bound on the second link to be selected for more number of time slots, which in turn improves the overall performance. Moreover, it can be clearly seen that there was no effect on the state of the buffer on NBARS.
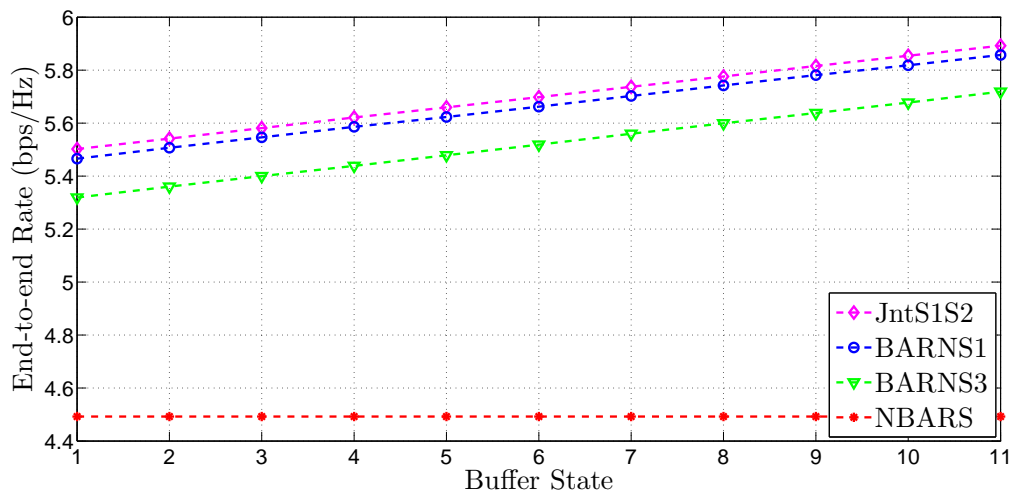
Figure 5. Throughput (bps/Hz) versus the state of the buffer.

5. Conclusions

In this paper, an OFDM-based BAR network was considered. Our objective was to maximize average throughput of the system under separate power constraints at each node and the link selection constraints such that only one link should be selected during one-time slot, and the sum rate at first hop should not exceed the total outward data flow. The results showed that the BAR with joint link selection and power allocation strategy gives significantly better gain as compared to the conventional scheme. Furthermore, suboptimal solutions were also presented and inspected nicely in the simulation section with and without using the buffer at the relay node. It was proved in all the simulation results that the joint optimization solution always outperforms in all scenarios.

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