CONSTELLATION SHAPING FOR BROADCAST CHANNELS IN PRACTICAL SITUATIONS

Zeina Mheich[1], Pierre Duhamel[1], Leszek Szczecinski[2], and Marie-Line Alberi Morel[3]

[1] LSS - Supélec - Univ Paris-Sud
3 rue Joliot-Curie
91192 Gif-sur-Yvette cedex, France
{zeina.mheich, pierre.duhamel}@lss.supelec.fr

[2] INRS-EMT, University of Quebec
Montreal, Canada
leszek@emt.inrs.ca

[3] Alcatel Lucent , Bell Labs France
Centre de villarceaux, Route de Villejust
91620 Nozay, France
marie_line.alberi-morel@alcatel-lucent.com

ABSTRACT

This paper analysis the problem of constellation shaping for degraded broadcast channels using a finite dimension constellation, where a single source communicates simultaneously with two receivers. We are interested in a particular situation where the private message to be sent to each user is limited to unity. This corresponds to a very practical situation, and hierarchical modulation is a special case of this framework. We investigate if higher achievable rates can be obtained by using non uniform constellations following a nonequiprobable distribution. Achievable rates regions are derived for Additive White Gaussian Noise (AWGN) and for finite input Pulse Amplitude Modulation (PAM) constellations. A noticeable shaping gain (compared with the equiprobable case) is observed with 4 PAM symbols, when achievable rates are maximized over the probability distribution of the input signals, the shape of the constellation, and the labeling.

1. INTRODUCTION

Current communication systems usually make use of orthogonal schemes (either in time and/or frequency) in order to address several receivers. However, it is well known since Cover [1] that if a sender wants to send information simultaneously to several receivers, given specific broadcast channel conditions, superimposing high-rate information and low-rate information may achieve higher bandwidth efficiency than the time-sharing strategy. Superposition coding has been shown to reach the theoretical capacity limit for two-users AWGN broadcast channel using an infinite Gaussian input alphabet [1]. These results mainly deal with theoretical limits and do not straightforwardly apply in practical situations. A practical implementation of superposition coding is known as Hierarchical Modulation (or layered modulation), that enables the transmission of two independent streams on a single frequency channel, with different transmission qualities (SNR scalability). It has been included in various standards, such as DVB-T for digital terrestrial television [2] and DVB-H/SH for mobile digital TV transmission [3] in order to broadcast scalable digital media [4].

In fact, practical implementation constraints impose the use of finite input alphabets which are usually fed with equiprobable symbols. These restrictions contribute to increase the gap between the capacity region achieved with infinite Gaussian inputs (for AWGN channel) and the throughput obtained in practical situations. This gap can be reduced using constellation shaping. For AWGN channel, an approach to obtain a shaping gain is to arrange the points in such a way that the channel input distribution is closer to a Gaussian shape, i.e., using more points at the lower power levels and less at higher. Another approach is to transmit finite-size uniformly spaced symbols with low energy more frequently (near origin) than the ones with high energy (far from origin). In fact, most available results dealing with optimal constellation shape or optimal probability consider only unicast transmission, where one transmitter communicates with one receiver. For example, [5] and [6] investigate the design of optimal non-uniform constellation using signals with equal probability but unequal spacing. In [7], it is shown that using nonuniform signal sets in high order modulation schemes could result in gains of about 1 dB over an AWGN channel. In [8] and [9] the authors obtain a shaping gain by arranging points in such a way that the emitted signal is closer to a Gaussian distribution. In [10], the authors investigate the effects of the nonequiprobable distribution of a high bandwidth efficiency M-ary QAM signal constellation on the error performance for nonlinear channels. Simulations for binary turbo coded modulation show a 3 dB improvement over equiprobable symbols. In [11], the authors studied the achievable rates in a two users AWGN broadcast channel when superposition techniques are applied, assuming a uniform pdf over the finite input set. To our knowledge, no work investigated the achievable rates maximization for broadcast systems considering optimization over both the probability density and constellation symbols locations. Since this was shown useful in a unicast situation, this paper intends to evaluate its impact in a broadcast context.

This paper investigates if higher achievable rates are attainable with finite-size input alphabets for two users broadcast systems over a Gaussian channel. The achievable rates are derived using the mutual information for two-user AWGN broadcast channel and 4-PAM constellation. A discussion about the labeling schemes and the joint probability distribution schemes highlights the fact that the well-known hierarchical modulation is a special case of superposition coding. The maximization of the achievable rates is done considering all possible labelings when transmit signals are modulated with 4 PAM constellation, in the specific case where the weakest each can decode at most two symbols, which corresponds to a practical constraint for simple receivers. The improvement is measured in terms of SNR savings for target achievable rates.

2. TWO-USER AWGN BROADCAST CHANNEL

In a two user broadcast channel (BC), the transmitted signal X (drawn from the alphabet $\mathscr{X}$) is sent to the user 1 and the user 2, which receive, respectively , the signals Y_1 and Y_2 belonging to the alphabets $\mathscr{Y}_1$ and $\mathscr{Y}_2$. A broadcast chan-

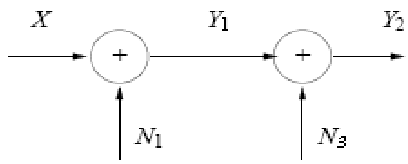

Figure 1: Model of a two-user physical degraded AWGN BC where $N_1 \sim \mathcal{N}(0, \sigma_1^2)$ and $N_3 \sim \mathcal{N}(0, \sigma_2^2 - \sigma_1^2)$.

nel is said to be physically degraded if $X \to Y_1 \to Y_2$ form a Markov chain, i.e., Y_2 is a "degraded" version of the signal Y_1. Consider a two-user AWGN BC with:

$$Y_1 = X + N_1 \tag{1}$$

$$Y_2 = X + N_2 \tag{2}$$

where N_1 and N_2 are zero-mean Gaussian variables whose respective variances satisfy $\sigma_1^2 < \sigma_2^2$. A physical degraded BC can be constructed equivalently to the AWGN BC by introducing a new independent noise $N_3 \sim \mathcal{N}(0, \sigma_2^2 - \sigma_1^2)$ as shown in Fig. 1.

The set of achievable rate pairs (R_1, R_2) satisfies

$$R_1 \leq I(X; Y_1 | U) \tag{3}$$

$$R_2 \leq I(U; Y_2) \tag{4}$$

for some joint distribution $P_{UXY_1Y_2} = P_{UX} \cdot P_{Y_1|X} \cdot P_{Y_2|X}$ on $\{\mathcal{U} \times \mathcal{X} \times \mathcal{Y}_1 \times \mathcal{Y}_2\}$. $I(U;Y_2)$, $I(X;Y_1|U)$ denote respectively the mutual information (MI) between the auxiliary random variable U and the received signal Y_2, and MI between the transmit signal X and the received signal Y_1 after U is observed. P_{UX} is the joint probability distribution of U and X. $P_{Y_1|X}$ and $P_{Y_2|X}$ are the conditional distributions that depend on the nature of the channel. From the inequality (4), $I(U;Y_2)$ is the maximal rate achievable by the user 2. Hence the second receiver can only distinguish U, where $\mathcal{U}$ has cardinality bounded by $|\mathcal{U}| \leq min\{|\mathcal{X}|, |\mathcal{Y}_1|, |\mathcal{Y}_2|\}$.

The theoretical limit of the capacity region of the two users Gaussian broadcast channel is achieved using superposition coding of two Gaussian signals intended for the two users. Superposition coding (SC) is realized here through the addition of both signals $X = X_1 + U$, where X_1 is the "private" message for the user 1 and U is the message for the user 2.

The message U is always decoded first, and in the case of the user 1, it is also subtracted from the received signal Y_1 which eliminates (cancels) the interference so the message X_1 can be then detected.

3. SUPERPOSITION CODING WITH FINITE-SIZE CONSTELLATIONS

Since for practical systems, messages are usually transmitted by means of finite signaling constellations, we assume $X \in \mathcal{X} = \{x_0, x_1, x_2, x_3\}$. The received signals Y_1 and Y_2 by user 1 and user 2 respectively are continuous in case of AWGN channel, thus $|\mathcal{Y}_1| = |\mathcal{Y}_2| = \infty$. As a consequence, $|\mathcal{U}|$ is bounded by $min\{|\mathcal{X}|\}$. For the transmission with 4-PAM constellation $|\mathcal{X}| = 4$, thus $|\mathcal{U}| \leq 4$. First consider the case $|\mathcal{U}| = 2$. Thus user 2 can receive at most 1 bit/channel use since it can distinguish only U. The joint distribution is represented in Fig. 3.a where $p_{ij} = \Pr\{U = u_i, X = x_j\}$. In the case of discrete constellations we have to optimize the

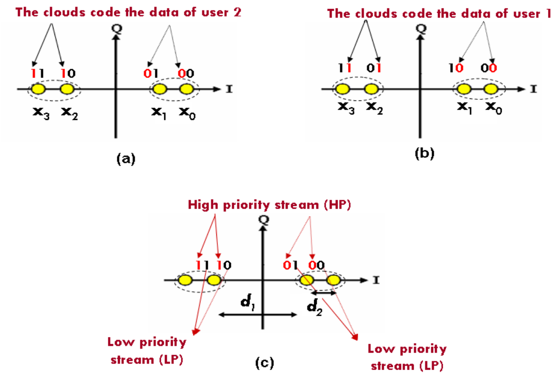

Figure 2: Labeling in 4-PAM when using modulation superposition (MS) and hierarchical modulation (HM) : (a) labeling favoring user 2 - (b) labeling favoring user 1 - (c) HM with $l = d_1/d_2$

distribution $P_{UX}(u,x)$ and, what is important to note, is that there is no guarantee that the optimization results will correspond to the addition of two random variables (as it is the case for continuous variables we mentioned above). Since we want:

- the sum of the joint probabilities p_{ij} be equal to 1,
- the transmitted signal have zero mean : $\mathrm{E}[X] = 0$,
- the power of the transmitter signal be limited : $\mathrm{E}[X^2] = P$,

the constraints on the values of the signals in $\mathcal{X}$ and the distribution $P_{UX}(u,x)$ are given by

$$\sum_{i,j} p_{ij} = 1, \quad \sum_{i,j} p_{ij} \cdot x_j = 0, \quad \sum_{i,j} p_{ij} \cdot x_j^2 = P \tag{5}$$

3.1 Superposition coding by adding the users signals: Modulation superposition

This section derives achievable rates in the case of finite alphabet inputs, in the case where the signals intended for each user are binary, and simply added: $X = X_1 + X_2$. This is indeed very simple as X_1 and $X_2 = U$ can be interpreted as the variables that convey information, respectively, to the user 1 and the user 2. In this paper, this is denoted as modulation superposition (MS) and such an approach is known as hierarchical modulation (HM). Obviously, this is not the general case, but this situation allows to work with simple receivers.

Consider alphabets $\mathcal{X}_1$ and $\mathcal{X}_2$ with average powers P_1 and P_2 corresponding to user 1 and 2 respectively. The encoded bit stream k is modulated using $\mathcal{X}_k$ with $k = \{1, 2\}$ and generates symbols, which is a sequence of 2 signal points $\{x_{kl}\}$ where $x_{kl} \in \mathcal{X}_k$. The messages for both users are encoded separately and added before transmission: the signal on the channel reads $x = x_{1m} + x_{2n} \in \mathcal{X}$, where $l, m, n \in \{0, 1\}$.

The addition of two binary alphabets results in a 4-PAM constellation with only two possible labelings (see Fig.2). In both configurations user 2 is always mapped to the Most Significant Bit (MSB) and user 1 is always mapped to the Least Significant Bit (LSB). Both schemes differ in the mapping of the codewords '01' and '10' over the symbols x_1 and x_2. They are swapped in the two configurations. In Fig.2.a the codewords {'00','01'} and {'11','10'} with the same MSB

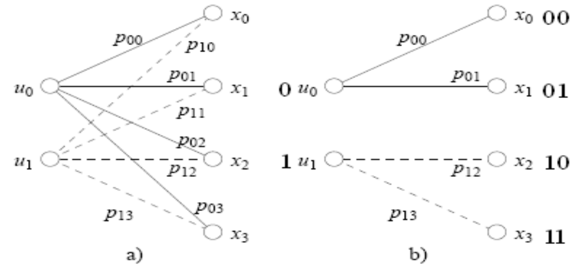

Figure 3: Joint distribution $P_{UX}(u,x)$ of U ($|\mathscr{U}|$=2) and X ($|\mathscr{X}|$=4): a) a general case is characterized by eight values p_{ij}, b) a particular case of MS which corresponds to the hierarchical constellation (if $p_{00} = p_{01} = p_{12} = p_{13} = 0.25$).

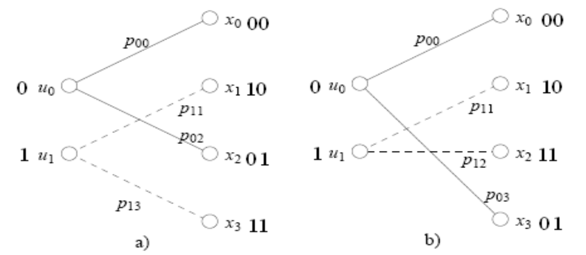

Figure 4: Joint distribution $P_{UX}(u,x)$ of U ($|\mathscr{U}|$=2) and X ($|\mathscr{X}|$=4): a) a particular case which can be obtained from MS, and b) a case that cannot be obtained from the MS.

is mapped over the symbols located in the same half plane. In this configuration the half plane carries information of users 2 while the dots inside the half plane carry data of user 1. Clearly, this labeling of Fig.2.a favors the detection of the information of user 2 since the clouds are more protected again the channel noise. In Fig.2.b the codewords with the same LSB are located in the same region. The labeling of this figure favors the detection of the information of user1.

In the following we describe the composition of the achievable rates when modulation superposition using equiprobable symbols (MS eq.) is used as transmission method. Consider two alphabets $\mathscr{X}_1 = \{x_{10}, x_{11}\}$ and $\mathscr{X}_2 = \{x_{20}, x_{21}\}$, where $x_{k1} = -x_{k0}$. Symbols are taken equiprobably from these two alphabets so their average powers is given by $P_1 = x_{10}^2$ and $P_2 = x_{20}^2$. We now add the symbols from these two binary alphabets which results in a 4-PAM constellation whose symbols $\{x_0, x_1, x_2, x_3\}$ are equiprobable and symmetric ($x_0 = -x_3$ and $x_1 = -x_2$) and verify $x_0 = x_{10} + x_{20}$ and $x_3 = x_{11} + x_{21}$ where $x_0 \geq x_1 \geq x_2 \geq x_3$. The total transmission power P is split between both users so that a power $P_1 = \alpha \cdot P$, with $\alpha \in [0,1]$ is allocated to user 1 and $P_2 = (1-\alpha) \cdot P$ to user 2. Then [12], x_0 is expressed as $\sqrt{\alpha \cdot P} + \sqrt{(1-\alpha) \cdot P}$ and

$$x_1 = \sqrt{\alpha \cdot P} - \sqrt{(1-\alpha) \cdot P} \quad \text{if } \alpha \geq 0.5$$
$$x_1 = -\sqrt{\alpha \cdot P} + \sqrt{(1-\alpha) \cdot P} \quad \text{if } \alpha \leq 0.5.$$

Obviously, this is not the general case because the distribution $P_{UX}(u,x)$ is limited to the cases $p_{00} = p_{01} = p_{12} = p_{13} = 0.25$ when $\alpha < 0.5$ (this case is shown in Fig. 3b and is known as HM shown in Fig.2.c) which yields to the labeling scheme shown in Fig.2.a or to $p_{00} = p_{02} = p_{11} = p_{13} = 0.25$ when $\alpha > 0.5$ (this case is shown in Fig. 4a) which yields to the labeling scheme shown in Fig.2.b, and α becomes the only parameter which affects the form of the region of the achievable rates. This is indeed the approach adopted by the industry standard because it is easy to see, for example, that when $\alpha < 0.5$, the positive values of the received signal Y_2 may be directly related to $X_2 = x_{20}$, and the negative ones to $X_2 = x_{21}$; consequently the detection is simplified.

According to P_{UX} schemes and the constellation symbols expressions, the mutual informations $I(X;Y_1|U)$ of user 1 and $I(U;Y_2)$ of user 2 are derived as a function of α. The achievable rates region is obtained by varying α from 0 to 1.

3.2 Maximum private-message achievable rates by a 4-PAM constellation

The main idea in this work is to extend the analysis to a case that is more general than MS eq. and we want to optimize the distribution $P_{UX}(u,x)$. This is a highly nonlinear problem with many optimization variables so we consider only a particular subset of solutions. For full generality, we consider all labeling schemes with 4-PAM constellation when each user can receive 2 symbols by associating two values of X for each value of U. P_{UX} is thus described completely by the three schemes of joint distribution (Fig.3b and Fig.4). The first ones are similar to those obtained for modulation superposition, as shown on Fig.3b and Fig.4a, but the probabilities are unknown. The third possible scheme is given in Fig.4b. It corresponds to the remaining possible labeling of the 4-PAM, where each user can receive a maximum of two symbols. Clearly, this case does not correspond to the addition of two signals X_1 and X_2 carrying information to the corresponding user. This is also practical because, in such a case the information is conveyed to the user 1 using the constellation that does not change the size, which will ease the eventual implementation.

The private-message region of achievable rates for the two-user degraded broadcast channel is composed of all convex combinations $\theta \cdot R_1 + (1-\theta) \cdot R_2$ [13], where $\theta \in [0,1]$. Since right hand side inequalities of (3) and (4) are achievable, the evaluation of $\theta \cdot I(X;Y_1|U) + (1-\theta) \cdot I(U;Y_2)$ for $\theta \in [0,1]$ is sufficient to determine the region of achievable rates.

Now, we determine if higher rates are achievable using constellation shaping technique. Therefore, the maximization has to be performed over the location of the transmit symbols $x_j \in \mathscr{X}$ and over the joint pdf P_{UX} (three possible assignments discussed before are considered) . The maximum achievable rates are obtained by maximizing the multidimensional

$$\max_{P_{UX}, x_0, x_1, x_2, x_3} \theta \cdot I(X;Y_1|U) + (1-\theta) \cdot I(U;Y_2) \qquad (6)$$

subject to $\theta \in [0,1]$ and to the constraints defined in (5).

This is a nonconvex optimization problem, which we solved using simulated annealing which provides a good approximation to the global optimum of a given function in a large search space. Applying this algorithm, good approximation of optimal P_{UX} and the module of $\{x_0, x_1, x_2, x_3\}$ are found for the three labeling schemes considered.

3.3 Simulation results and discussion

This section compares numerically the achievable rates, R_1 for the user 1 (high SNR_1) and R_2 for the user 2 (low SNR_2

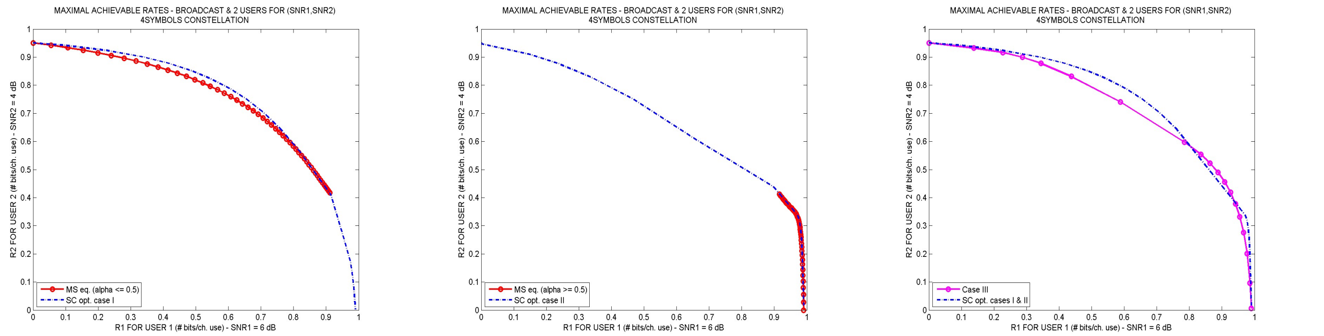

Figure 5: Maximum achievable rates for users 1 and 2 at SNR_1 = 6 dB & SNR_2 = 4 dB : a) for MS eq. $\alpha \in [0, 0.5]$ w.r.t optimal SC-I (SC opt. case I) - b) for MS eq. $\alpha \in [0.5, 1]$ w.r.t optimal SC-II (SC opt. case II) - c) SC opt. case I & case II w.r.t optimal SC-III (SC opt. case III)

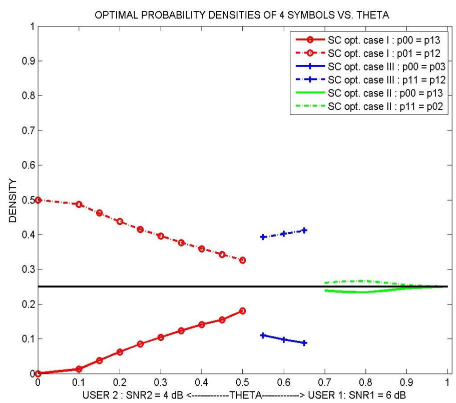

Figure 6: Optimal joint probabilities values versus θ at SNR_1 = 6 dB & SNR_2 = 4 dB obtained by optimizing SC-I, SC-II, and SC-III.

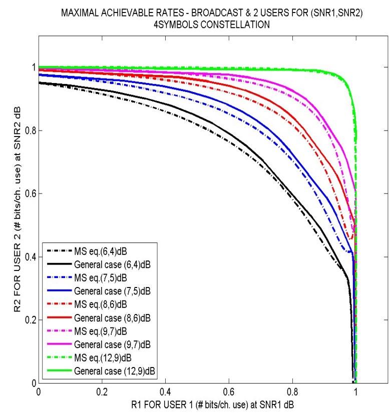

Figure 7: Maximum achievable rates versus SNR_1 & SNR_2 for general case w.r.t MS eq.

for the case of modulation superposition with equiprobable symbols and the optimal broadcast setup with optimized probabilities considering all possible labellings. The improvement will be characterized in terms of attainable rate and in terms on SNR improvement. For our analyze we consider the following cases in which the symbol probabilities have been optimized:

- SC-I: P_{UX} corresponds to Fig. 3b .
- SC-II: P_{UX} corresponds to Fig. 4a.
- SC-III: P_{UX} corresponds to Fig. 4b.

The convex closure of the regions obtained in these three SC cases (denoted as general case) is the final result. The two first cases are compared with the configuration of equiprobable probabilities presenting the same scheme of joint probability distribution. The comparisons are done in the same reception conditions, SNR_1= 6 dB and SNR_2= 4 dB.

In Fig.5 we remark an improvement of the achievable rates region when the constellation symbols and the probabilities are optimized jointly for SC-I when compared to the case with uniform distribution of probability and $\alpha \leq 0.5$ (Fig.5.a). The achievable rate region is not improved with the joint optimization when considering the case SC-II (Fig.5.b). The rates region for the case SC-III (which cannot be obtained via MS), is compared to the convex closure of the regions achievable in the case SC-I and SC-II.

These results show that the achievable rates region can be split into three parts and the achievable rates at the borders of zones are continuous

1. For $R_1 \in (0, 0.8)$ SC-I should be used $(0 < \theta \leq 0.5)$;
2. For $R_1 \in (0.8, 0.95)$ SC-III should be used $(0.5 < \theta < 0.7)$.
3. For $R_1 \in (0.95, 0.99)$ SC-II should be used $(0.7 \leq \theta \leq 1)$;

The intuition behind the first two conclusions is rather clear. For small rates R_1 we should facilitate detection of the user 2 which is obtained in the case SC-I (positive symbols correspond to $U = u_0$ and negative ones to U_1, so there is an obvious separation of the data as a function of the symbols of U). On the other hand, to guarantee the high rate R_1 the symbols x_j associated with the detected variable U should be well separated, which is obtained applying the SC-II scheme. As shown in Fig.6, the corresponding pdf is approximately uniform when $\theta \geq 0.7$. For these values of θ case II achieves the maximal rates compared to the ones obtained in the cases SC-I or SC-III. We observed also that the rates achieved in the case SC-II are identical to those obtained with MS eq. $(\alpha \geq 0.5)$. Consequently, a uniform and symmetric distribution is optimal in the zone in which the data transmission of user 1 is favored. In the two other zones the data transmission of user 2 is mainly favored. The optimal probabilities of symbols near origin are higher than the ones of symbols far from origin. As an example at $\theta = 0.5$ optimal $p_{00} = p_{13}$

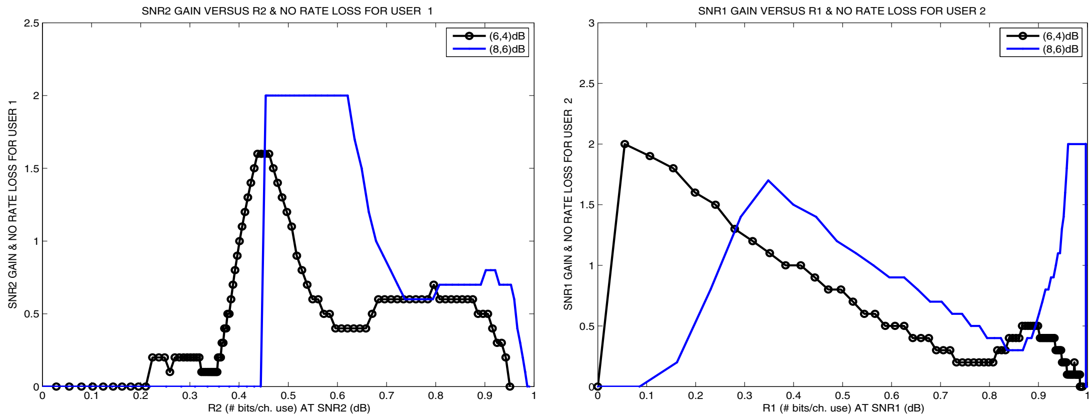

Figure 8: a) SNR_2 gain & no rate loss for user 1 versus R2 -b) SNR_1 gain & no rate loss for user 2 versus R1

is equal to 0.18, optimal $p_{01} = p_{12}$ is equal to 0.32, optimal $x_0 = -x_3$ is equal to 3.4, optimal $x_1 = -x_2$ is equal to 1.1, optimal R_1 is equal to 0.706 and optimal R_2 is equal to 0.703. The numerical results also illustrate that the optimal constellation symbols are still symmetric for each theta considered ($x_0 = -x_3$ and $x_1 = -x_2$).

A possible shaping gain in terms of achievable rates with a 4-PAM constellation is illustrated for various couples of (SNR_1, SNR_2) in Fig.7. The achievable rates region obtained in theses cases is consistently larger than the region obtained with MS eq. Since we known that the constellation shaping we designed brings improvement only for a particular range of rates; when SNR_1= 6 dB and SNR_2= 4 dB, this limit is fixed at $R_1 = 0.95$. To evaluate the contribution of constellation shaping (i.e., SC opt.) we compare it to MS eq. in terms of SNR savings for target achievable rates. For this we fix SNR_i with $i \in \{1,2\}$ and the rate achieved by user i when using MS eq., $R_i^{\text{MS eq.}}$, is kept constant. Then we add δ to SNR_j with $j \neq i$ until $R_j^{\text{MS eq.}}$ of user j coincides with R_j^{opt} achieved by user j at SNR_j in the optimized SC case. The SNR increase yield δ^{opt} is the SNR gain we seek. Fig.8a summarizes the gains on SNR_2 assuming no rate and SNR loss for user 1 for two couples of (SNR_1, SNR_2). Fig. 8b illustrates the gain values on SNR_1 assuming no rate and SNR loss for user 2. In both figures a maximal gain of 2 dB is shown for user 2 and user 1. Using the general case enables to save noticeable SNR for both studied couples although small shaping gain in terms of achievable rates was observed.

4. CONCLUSION

In this paper, we derived the achievable rates for two broadcast transmission methods, modulation superposition with uniformly distributed symbols and for a 4-PAM constellation with fully optimized parameters. It was shown that optimal broadcast transmission including the joint optimization of probability density and constellation symbols position generates slightly higher achievable rates in a 4-PAM setting. However, this small gain is translated into SNR savings up to 2 dB.

REFERENCES

[1] T. M. Cover, "Broadcast Channels" *IEEE Transactions on Information Theory*, vol. IT-18, pp. 2-14, 1972.

[2] ETSI EN 300 744: "Digital Video Broadcasting (DVB); framing structure, channel coding and modulation for digital terrestrial television"

[3] ETSI TS 102 585 : "Digital Video Broadcasting (DVB); System Specifications for Satellite services to Handheld devices (SH) below 3 GHz"

[4] Y. Liu and C. Heneghan, "Optimization of Hierarchical Modulation for Use of Scalable Media", *EURASIP Journal on Advances in Signal Processing*, Volume 2010, 12 pages.

[5] X. Wang, J. Fu, J. Wang, C. Pan, and Z. Yang, "Shaping Gain for AWGN Channel by Non-Uniform Constellation in LDPC-Coded System ", *ICCS*, 2008.

[6] F.R. Kschischang, S. Pasupathy, "Optimal Nonuniform Signaling for Gaussian Channels," *IEEE Trans. Information Theory*, Vol. 39, pp.913-929, May 1993.

[7] D. Sommer, G. P. Fettweis, "Signal Shaping by Non-Uniform QAM for AWGN Channels and Applications Using Turbo Coding", *ITG Conference Source and Channel Coding*, January 2000.

[8] A. R. Calderbank, L. H. Ozarow, "Nonequiprobable Signaling on the Gaussian Channel" , *IEEE Trans. Information Theory*, vol. 36, pp.726-740, July 1990.

[9] Jay N. Livingston, "Shaping Gains Using Non-Equiprobable Signaling", *GLOBECOM*, 1990.

[10] N. H. Ngo, S. A. Barbulescu, S. S. Pietrobon, "Performance of Nonuniform M-ary QAM Constellation on Nonlinear Channels", *Communications Theory Workshop*, 2005.

[11] C. Huppert, M. Bossert,"On Achievable Rates in the Two User AWGN Broadcast Channel with Finite Input Alphabets" , *ISIT 2007*, Nice, France.

[12] V. Singh, "On Superposition Coding for Wireless Broadcast Channels", Master of Science Thesis, Stockholm, Sweden 2005-03-04.

[13] E. Calvo, D. P. Palomar, J. R. Fonollosa and J. Vidal, "The Computation of the Capacity Region of the Discrete Degraded BC is a NonConvex DC Problem" *ISIT 2008*, Toronto, Canada.