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## SYSTEMS & CONTROL | RESEARCH ARTICLE

# Multi-agent system finite-time consensus control in the presence of disturbance and input saturation by using of adaptive terminal sliding mode method

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**Abstract:** The paper develops finite-time consensus control for multi-agent systems by considering disturbances and input saturation. A new adaptive-terminal sliding mode control is suggested to solve consensus control within a finite time. Two cases are solved in the paper. In the first case, it is assumed that disturbances are with known upper. To achieve the consensus purpose within the finite time, in this case, the control inputs are designed based on terminal sliding mode technique by considering the input signal saturation. Also, the control inputs are modified to reduce the high dependency of reaching times to initial speeds. In the second case, the agents are subjected to disturbances with unknown upper bounds. To handle the problem, the control signals are acquired by combining the adaptive and terminal sliding mode methods. By considering saturation boundary and disturbances with unknown upper band, a new adaptive-terminal sliding mode method is designed to control the multi-agent system in reduced settling and reaching times. The proposed techniques efficiency is confirmed by numerical simulations.



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### PUBLIC INTEREST STATEMENT

Multi-agent systems control is one of the real challenges in the industry. In addition to the problem of finite-time consensus control, disturbance and input saturation should also be taken into account. This paper presents new adaptive-terminal sliding mode control for covering the above effects.

**Subjects: Systems & Control Engineering; Technology; Systems & Controls**

**Keywords: consensus control; terminal sliding mode control; saturation limit; disturbance**

## 1. Introduction

In the past three decades, multi-agent systems control has received plenty attention from researchers due to their vast system applicability, e.g. multi-UAVs path following (Zhang, Liu, Mao, Liu, & Shen, 2014), traffic control (Zhu, Aziz, Qian, & Ukkusuri, 2015), multi-oscillator synchronizations (Zhang, Yang, & Zhao, 2013), network sensor communications (Yu, Chen, Wang, & Yang, 2009), and ship formations (Chen & Tian, 2015). In different researches, several main control objectives are defined and studied for multi-agent systems comprising flocking (Zhang, Hao, Yang, & Chen, 2016), formation (Ge, Guan, Yang, Li, & Wang, 2016; Liu & Geng, 2015), rendezvous (Dong, 2016), containment control (Fu & Wang, 2015; Wang, Wang, & Xie, 2017), tracking (Mondal & Su, 2016), and consensus (Ma, Liu, & Chen, 2016; Yang, Zhang, & Yu, 2017). Among these, the consensus has proven to be more applicable and has thus been studied extensively in recent decades (Zhang, Hua, & Guan, 2016; Zhu, Meng, & Hu, 2016). Consensus means that a group of agents achieve a state agreement based on local information exchange. Fulfilling the consensus aim requires each agent to generate its control protocol (control input) by only employing its neighbors' local information. Based on the required time to achieve the above-mentioned agreement, consensus control objectives can be divided into asymptotic and finite time (fixed time) consensuses. For asymptotic consensuses (Cui, Ma, Lewis, Zhang, & Ma, 2016; Wang, Wang, & Ji, 2016) the agreement between agents is fulfilled within the infinite time, whilst for finite-time consensus (Du, Cheng, He, & Jia, 2016; Sun, Hu, & Xie, 2016) the aforementioned agreement is achieved in the specified adjustable finite time. The finite-time consensus possess some remarkable advantages such as faster transient response, high-precision tracking performance and faster convergence rate as compared to the asymptotic consensus (Fu & Wang, 2016; Li, Chen, & Su, 2016).

Three finite-time stabilization methods suitable for reaching nonlinear system finite-time consensus are Lyapunov-like approach (Huang, Wen, Wang, & Song, 2016), geometric homogeneity-based strategy (Lyu, Qin, Gao, & Liu, 2016), and terminal sliding mode control (TSMC) technique (Bayat, Mobayen, & Javadi, 2016; Rahmani, 2018; Rahmani, Ghanbari, & Ettefag, 2016; Rahmani & Rahman, 2019). The Lyapunov-like approach is applied to guarantee the fixed-time consensus aim for multi-agent systems in (Zhang & Jia, 2015; Zuo & Tie, 2013), while the geometric homogeneity-based method is used to provide the mentioned aim (Guan, Sun, Wang, & Li, 2012; Zhao, Duan, & Wen, 2015). The finite-time consensus can be satisfied by employing the TSMC technique (Li, Liao, & Chen, 2013; Zhao & Hua, 2014; Zhou, Xia, Wang, & Fu, 2015), which is based on the conventional SMC method (Chang, 2012; Fu & Wang, 2015) and is robust against disturbances and uncertainties (Pai, 2011; Phan, Van Huynh, & Tsai, 2015).

On the other hand, in the consensus problem, two important practical issues including agent disturbances (and uncertainties) and each agent actuator saturation should be considered. If these two issues are not considered in multi-agent system consensus problems, some serious-undesired problems, e.g. convergence rate and tracking precision decrease and even divergence or instability, will appear. In Hu, Yu, Chen, & Xie (2013) and Zhu et al. (2016), the asymptotic consensus for multi-agent systems in the presence of agents' disturbance and saturation is guaranteed. The finite-time consensus for a typical multi-agent system with disturbance free and actuator saturation agents is investigated in (Lyu et al., 2016; Zhang & Yang, 2013). The finite-time consensus problem of disturbed multi-agent systems with agents without saturation actuators is considered (Li et al., 2013; Zhao & Hua, 2014; Zhou et al., 2015).

Due to the importance of the three reviewed problems, including finite-time consensus, agent disturbances and actuator saturation of each agent, a novel robust approach is proposed and generalized in this paper to guarantee the consensus control goal by.

Here, the finite-time consensus control problem is discussed and studied for a typical multi-agent system possessing double-integrator agents and a fixed speed leader. Each system agent is subjected to control input disturbances (or uncertainty) and saturation. It is assumed that parameters related to agents' control inputs saturations are known. Agent disturbance is assumed to be bounded, while their upper bounds can be known (Case 1) or unknown (Case 2). As Case 1 is considered a developed TSMC method (or a generalized Fast TSMC method) is used to fulfill the finite-time consensus for the described multi-agent system. For Case 2, a novel adaptive TSMC (ATSMC) method is suggested to both estimate these upper bounds in finite time and also to solve the multi-agent system finite time consensus problem. It is worth noting that for the two aforementioned cases two inequalities are solved to determine the two finite times for achieving the finite-time consensus objective. In addition, the global dynamic finite-time stability of tracking errors (between agents and leader dynamics) is proven in several theorems in this paper.

Further, basic definitions and mathematical preliminaries are presented in Section 2. Section 3 is devoted to finite-time consensus tracking for Case 1. Section 4 investigates on the fast finite-time consensus tracking problem. In Section 5 adaptive terminal sliding mode is generalized to solve the Case 2. Finally, numerical examples and conclusions are presented in Sections 6 and 7, respectively.

## 2. Mathematical preliminaries

### 2.1. Graph theory

A graph defined by  $G = (\mathbb{V}, E, A)$  consists of a vertex set  $\mathbb{V} = \{v_1, v_2, \dots, v_N\}$ , an edge set  $E \subseteq \mathbb{V} \times \mathbb{V}$ , and an adjacency matrix  $A$ . Each edge  $e_k$  is defined by a pair of vertices  $(v_i, v_j)$ . Matrix  $A = [a_{ij}] \in \mathbb{R}^{N \times N}$  shows the connections between vertices, so that  $a_{ij} = 1$  if  $(v_j, v_i) \in E$  and  $a_{ij} = 0$ . Else, if matrix  $A$  is symmetric, the graph  $G$  is known as undirected. A path is a sequence of edges from vertex  $i$  to vertex  $j$ .  $G$  is called connected if there exist at least one path between any two arbitrary separate vertices.

### 2.2. Finite-time stability

In this section, the main finite-time stability definition and two useful lemmas are presented. These are later used throughout the paper.

**Definition 1** (Bhat & Bernstein, 1998). Assume a nonlinear time-invariant system as:

$$\dot{x} = f(x), f(0) = 0, x \in U_0 \subset \mathbb{R}^n \quad (1)$$

where  $f: U_0 \rightarrow \mathbb{R}^n$  is a continuous vector function on an open neighborhood  $U_0$  of the origin  $x = 0$ . The equilibrium point  $x = 0$  of system (1) is called locally finite-time stable if the following conditions hold.

- (i) It should be finite-time convergent in  $\hat{U}_0$ , namely, there is a convergence time  $T(x_0): \hat{U}_0 \setminus \{0\} \rightarrow [0, \infty)$  that satisfies  $\lim_{t \rightarrow T(x_0)} x(t, x_0) = 0$  and  $x(t, x_0) = 0$  for  $\forall t \geq T(x_0)$ .
- (ii) It should be Lyapunov stable in an open neighborhood  $\hat{U}_0$  such that  $\hat{U}_0 \subseteq U_0$ .

**Lemma 1** (Bhat & Bernstein, 1998). Consider the nonlinear system (1). Assume that there exist a  $C^1$  positive function  $V(x): U_0 \rightarrow \mathbb{R}$ , real constants  $c > 0$ , and  $0 < \alpha < 1$  such that  $\dot{V}(x) + cV^\alpha(x) \leq 0, \forall x \in U_0 \setminus \{0\}$  is satisfied. Then, the equilibrium point  $x = 0$  of system (1) is locally finite-time stable. Furthermore, the convergence time  $T(x_0)$  satisfies the following inequality.

$$T(x_0) \leq (c(1 - \alpha))^{-1} V(x_0)^{1-\alpha} \quad (2)$$

Moreover, if  $U_0 = \mathbb{R}^n$ , then  $x = 0$  is globally finite-time stable.

**Lemma 2** (Hong, Huang, & Xu, 2001). Consider the nonlinear system (1). Suppose there exist a  $C^1$  positive function  $V(x) : U_0 \rightarrow \mathbb{R}$  and real numbers  $c_1, c_2 > 0$  and  $0 < \alpha < 1$  such that  $\dot{V}(x) + c_2V(x) + c_1V^\alpha(x) \leq 0, \forall x_0 \in U_0 \setminus \{0\}$  is satisfied. Then, the convergence time  $T(x_0)$  is given by the following inequality.

$$T \leq (c_2(1 - \alpha))^{-1} (\ln(c_2V^{1-\alpha}(x(0)) + c_1)) - \ln c_1 \tag{3}$$

**2.3. Finite-time consensus tracking**

The dynamic models of  $N$  agents are assumed to be:

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{v}_i &= u_i + d_i, \quad i = 1, \dots, N \end{aligned} \tag{4}$$

where  $x_i$  and  $v_i$  are the  $i^{th}$  agent position and velocity, respectively.  $u_i$  and  $d_i$  denote the control input and bounded disturbance satisfying the inequality  $|d_i| < l_i, i = 1, \dots, N$ . It is assumed that  $l_i$  is a known constant and the control input of each agent is subjected to saturation such that  $|u_i| < Y_s$ . It is worth noting that the saturation bound  $Y_s$  is known.

The leader dynamic is defined as:

$$\begin{aligned} \dot{x}_0 &= v_0, \\ \dot{v}_0 &= 0. \end{aligned} \tag{5}$$

Based on finite-time consensus tracking, positions and velocities of all agents should converge to the position and velocity of the leader in a specific adjustable finite time. This goal can be defined mathematically as:

$$\begin{cases} \lim_{t \rightarrow T} |\tilde{x}_i| \rightarrow 0, \tilde{x}_i = 0, \quad \forall t > T \\ \lim_{t \rightarrow T} |\tilde{v}_i| \rightarrow 0, \tilde{v}_i = 0, \quad \forall t > T, \quad i = 1, \dots, N \end{cases} \tag{6}$$

where  $T$  is the required finite time for achieving the defined goal. Tracking errors  $\tilde{x}_i$  and  $\tilde{v}_i$  are defined as,

**Assumption 1.** In the multi-agent system of (4), it is assumed that each agent is connected to the leader independently or through other agents. To clarify this assumption mathematically, matrix  $B$  has defined.  $b_i$  is the  $i^{th}$  element of the matrix  $B = [b_1, b_2, \dots, b_n]$ .  $b_i = 1$  if the  $i^{th}$  agent have access to the leader independently, otherwise  $b_i = 0$ .

**Remark 1.** As the upper disturbance bound is known in sections 3 and 4, the powerful robust finite-time stabilization method TSMC control will be adopted for Case 1. But in section 5, for Case 2 ATSMC will be used where several finite-time adaptation laws are proposed for the unknown upper bound estimation.

$$\begin{cases} \tilde{x}_i = x_i - x_0, \quad i = 1, \dots, N \\ \tilde{v}_i = v_i - v_0 \end{cases} \tag{7}$$

**3. Finite-time consensus with known bounded disturbance and saturation**

To satisfy the described consensus problem, a TSMC is designed. The terminal sliding surfaces  $s_i, i = 1, \dots, N$  are proposed as:

$$s_i = \tilde{v}_i - \int_0^t \phi_i d\tau, \quad i = 1, \dots, N \tag{8}$$

in which  $\phi_i$  is defined as:

$$\phi_i = \sum_{j=1}^N a_{ij} [\tanh(\text{sig}^{\alpha_1}(x_j - x_i)) + \tanh(\text{sig}^{\alpha_2}(v_j - v_i))] - b_i [\tanh(\text{sig}^{\alpha_1}(x_i - x_0)) + \tanh(\text{sig}^{\alpha_2}(v_i - v_0))] \tag{9}$$

$\text{sig}^\alpha(x)$  is defined as  $\text{sig}^\alpha(x) = |x|^{\alpha} \text{sgn}(x)$ . The optional parameter  $\alpha_1$  is chosen as  $\alpha_1 \in (0, 1)$  and the parameter  $\alpha_2$  is determined as  $\alpha_2 = \frac{2\alpha_1}{1+\alpha_1}$ .

**Theorem 1.** Considering the agents, leader, tracking errors, and sliding surfaces described by (4), (5), (6), and (8), respectively, the sliding mode dynamics (sliding motions)  $s_i = \dot{s}_i = 0, i = 1, \dots, N$  are globally finite-time stable. This means that tracking errors  $\tilde{x}_i$  and  $\tilde{v}_i$  on sliding motion  $s_i = \dot{s}_i = 0$  will exactly converge to zero in the finite settling time,  $T_s$ .

**Proof.** Assume that the sliding mode dynamic  $s_i = \dot{s}_i = 0$  has been achieved for the  $i^{\text{th}}$  agent (input control for the  $i^{\text{th}}$  agent will be designed later to guarantee sliding motion existence  $s_i = \dot{s}_i = 0$ ). Based on (7) and (8), sliding mode dynamic  $s_i = \dot{s}_i = 0, i = 1, \dots, N$  can be expressed as

$$\begin{cases} \tilde{x} = \tilde{v} \\ \tilde{v} = \phi_i \end{cases} \tag{10}$$

According to the definition of  $\phi_i$  and by referring to Theorem 1 (Guan et al., 2012), it can be demonstrated that there exist a  $T_s$  such that  $\tilde{x}_i$  and  $\tilde{v}_i$  in (10) become zero for times larger than  $T_s$ . Consequently, sliding motions  $s_i = \dot{s}_i = 0, i = 1, \dots, N$  are globally finite-time stable. This completes the proof.  $\square$

The control inputs are designed to assure the existence of  $s_i = \dot{s}_i = 0, i = 1, \dots, N$  in the finite-reaching time,  $T_r$ , for all agents.

The control law for the  $i^{\text{th}}$  agent is proposed as:

$$u_i = \phi_i - k_i \text{sgn}(s_i) - l_i \text{sgn}(\dot{s}_i), i = 1, \dots, N, \tag{11}$$

where  $k_i, i = 1, \dots, N$  are optional constants satisfying inequalities  $\sum_{j=1}^N 2|a_{ij}| + 2b_i + k_i + l_i \leq \Upsilon_s$ . It is worth noting that  $T_r$  is dependent on these optional constants (demonstrated later). In Theorem 2, it will be shown that (11) can ensure the existence of sliding motions in finite time.

**Theorem 2.** Relation (11) ensure the existence of  $s_i = \dot{s}_i = 0, i = 1, \dots, N$  for all agents at times larger than  $T_r$  described by

$$T_r \leq \frac{\sqrt{\sum_{i=1}^N s_i^2(0)}}{k_m} \tag{12}$$

where  $k_m$  is defined as  $k_m = \min_i(k_i)$ .

**Proof.** Consider the Lyapunov function to be  $V = 0.5 \sum_{i=1}^N s_i^2$  with time derivative  $\dot{V} = \sum_{i=1}^N s_i \dot{s}_i$ . Each sliding surface time derivative  $\dot{s}_i$  is determined as  $\dot{s}_i = \tilde{v}_i - \phi_i$ . By replacing  $\tilde{v}_i$  from (7) and  $u_i$  from (11),  $\dot{s}_i$  becomes

$$\dot{s}_i = -k_i \text{sgn}(s_i) - l_i \text{sgn}(\dot{s}_i) + d_i, i = 1, \dots, N. \tag{13}$$

Replacing (13) in  $\dot{V} = \sum_{i=1}^N s_i \dot{s}_i$  and by considering the definition  $k_m = \min_i(k_i)$ , the following inequality is obtained.

$$\dot{V} \leq -k_m \sum_{i=1}^N |s_i| - \sum_{i=1}^N l_i |s_i| + \sum_{i=1}^N |s_i| |d_i| \tag{14}$$

Since  $|d_i| \leq l_i$ ,  $\sum_{i=1}^N |s_i| (|d_i| - l_i)$  is always less or equal to zero. Thus, (14) could be simplified as  $\dot{V} \leq -k_m \sum_{i=1}^N |s_i|$ . Based on the inequality  $(\sum_{i=1}^N |s_i|)^2 > \sum_{i=1}^N |s_i|^2$ ,  $\dot{V} \leq -k_m \sum_{i=1}^N |s_i|$  can be expressed as

$$\dot{V} \leq -k_m \sqrt{2V}^{\frac{1}{2}} \tag{15}$$

Finally, by setting  $c = \sqrt{2k_m}$  and  $\alpha = 0.5$ , and applying Lemma 1, it is seen that the sliding mode dynamics  $s_i = \dot{s}_i = 0, i = 1, \dots, N$  are always fulfilled for  $t \geq T_r$ , where  $T_r$  can be estimated by (12). This ends the proof.  $\square$

**Remark 1.** The defined consensus tracking object will be fulfilled for  $t \geq T_t$ , where  $T_t = T_s + T_r$ .

**Remark 2.** Since the  $k_i$  parameters,  $i = 1, \dots, N$  are selected to satisfy the inequalities  $\sum_{j=1}^N 2|a_{ij}| + 2b_i + k_i + l_i \leq \Upsilon_s$ , it can be concluded that (11) never reaches the saturation bounds and, consequently, actuator saturation does not occur.

**Remark 3.** Small parameter values  $k_i$  should be chosen,  $i = 1, \dots, N$  to satisfy the mentioned inequalities in Remark 2. On the other hand, based on (12), large  $k_i, i = 1, \dots, N$  should be set to reduce  $T_r$ . Thus,  $k_i, i = 1, \dots, N$  can be defined as a cost function comprising of two weighted terms related to the control effort energy and  $T_r$ , by which proper parameter values can be determined to minimize the cost function.

#### 4. Fast finite-time consensus with known bounded disturbance

The inequality  $T_r \leq (k_m)^{-1} \sqrt{\sum_{i=1}^N s_i^2(0)}$  is strongly dependent on initial conditions. To reduce this high dependency, (16) is defined by modifying (11) by fast terminal sliding mode control method.

$$u_i = \phi_i - k_i \text{sig}^\gamma(s_i) - l_i \text{sgn}(s_i) - \omega_i s_i, i = 1, \dots, N, \tag{16}$$

where  $k_i, \omega_i, i = 1, \dots, N$  and  $0 < \gamma < 1$  are positive arbitrary constants and are tuned to satisfy  $\sum_{j=1}^N 2|a_{ij}| + 2b_i + k_i |s_i(0)|^\gamma + l_i + \omega_i |s_i(0)| \leq \Upsilon_s$ .

The sliding surfaces  $s_i, i = 1, \dots, N$  are identical to (8). The finite-time stability proof for  $s_i = \dot{s}_i = 0, i = 1, \dots, N$  is similar to Theorem 1. Therefore, it can be claimed that there exists a  $T_s$  such that all  $\tilde{x}_i$  and  $\tilde{v}_i$ , described by (10), will converge to zero for  $t \geq T_s$ . In Theorem 3, it is demonstrated that (16) is able to fulfill  $s_i = \dot{s}_i = 0, i = 1, \dots, N$  within a finite time.

**Theorem 3.** Consider the multi-agent system (4) with bounded disturbances. By applying (16),  $s_i = \dot{s}_i = 0, i = 1, \dots, N$  are achieved for  $t \geq T_r$  where  $T_r$  can be estimated by:

$$T_r \leq (\omega_m (1 - \gamma))^{-1} \ln \frac{\omega_m \left( \sqrt{\sum_{i=1}^N s_i^2(0)} \right)^{1-\gamma} + k_m}{k_m} \tag{17}$$

where  $\omega_m$  and  $k_m$  are defined as  $\omega_m = \min_i(\omega_i)$  and  $k_m = \min_i(k_i)$ , respectively.

**Proof.** Consider the Lyapunov function to be  $V = 0.5 \sum_{i=1}^N s_i^2$  with time derivative  $\dot{V} = \sum_{i=1}^N s_i \dot{s}_i$ . The sliding surface time derivative  $\dot{s}_i$  is determined as  $\dot{s}_i = \tilde{\dot{v}}_i - \phi_i$ . By replacing  $\tilde{\dot{v}}_i$  from (7) and  $u_i$  from (16),  $\dot{s}_i$  is expressed as

$$\dot{s}_i = -k_i \text{sig}^\gamma(s_i) - l_i \text{sgn}(s_i) - \omega_i s_i + d_i, i = 1, \dots, N \tag{18}$$

By substituting (18) into  $\dot{V} = \sum_{i=1}^N s_i \dot{s}_i$  and considering the definitions  $k_m = \min_i(k_i)$  and  $\omega_m = \min_i(\omega_i)$ , the following inequality is obtained.

$$\dot{V} \leq -k_m \sum_{i=1}^N |s_i|^{\gamma+1} - \sum_{i=1}^N l_i |s_i| - \omega_m \sum_{i=1}^N s_i^2 + \sum_{i=1}^N |s_i| |d_i| \tag{19}$$

Since  $d_i$ , bounded as  $|d_i| \leq l_i$ ,  $\sum_{i=1}^N |s_i| (|d_i| - l_i)$ , always is none positive, and the inequality  $\sum_i |y_i|^{\gamma+1} \geq \sqrt{(\sum_i |y_i|^2)^{\gamma+1}}$  is correct for all real values  $y_i$  and  $0 < \gamma < 1$ , (19) is simplified to:

$$\dot{V} \leq -k_m \sqrt{(\sum_{i=1}^N |s_i|^2)^{\gamma+1}} - \omega_m \sum_{i=1}^N s_i^2 \tag{20}$$

By considering the definition of  $V$ , (20) can be written as:

$$\dot{V} \leq -k_m \sqrt{2^{\gamma+1}} V^{\frac{\gamma+1}{2}} - 2\omega_m V \tag{21}$$

By setting  $c_1 = k_m \sqrt{2^{\gamma+1}}$ ,  $c_2 = 2\omega_m$ ,  $\alpha = 0.5(\gamma + 1)$ , and applying Lemma 2, it can be proven that  $s_i = \dot{s}_i = 0$ ,  $i = 1, \dots, N$  are always fulfilled for  $t \geq T_r$  where  $T_r$  is calculated by (17). This ends the proof.  $\square$

**Remark 4.** Since  $k_i, \omega_i, i = 1, \dots, N$  and  $0 < \gamma < 1$  are chosen to satisfy inequalities  $2 \sum_{j=1}^N |a_{ij}| + 2b_i + k_i |s_i(0)|^\gamma + l_i + \omega_i |s_i(0)| \leq Y_s$ , it could be proven that the maximum values of (16) are always less than the saturation bounds and, consequently, actuator saturation does not occur.

### 5. Finite-time consensus with unknown-bounded disturbance

Here, it is assumed that the upper disturbance bounds  $l_i, i = 1, \dots, N$  are constant but unknown. By this assumption, (11) and (16) can be expressed as:

$$u_i = \phi_i - k_i \text{sgn}(s_i) - \hat{l}_i \text{sgn}(s_i), i = 1, \dots, N, \tag{22}$$

where  $k_i, i = 1, \dots, N$  are optional positive constants (introduced later).  $\hat{l}_i$  is the unknown upper bound estimations  $l_i$ .

$$\hat{l}_i = \lambda_i |s_i|, \hat{l}_i(0) > 0, i = 1, \dots, N \tag{23}$$

$\lambda_i, i = 1, \dots, N$  are arbitrary parameters that satisfy  $\lambda_i > 1$ . By considering Lemma 1 in (Plestan, Shtessel, Brégeault, & Poznyak, 2010), it can be shown that  $0 \leq \hat{l}_i \leq l_i^*$ , in which the constant  $l_i^*$  is not necessarily equal to the nominal value of  $l_i$ . Therefore,  $l_i^*$  can be assumed to be  $l_i^* = l_i + \eta_i$  in which  $\eta_i > 0$  is an arbitrary number. Notice that optional positive constants  $k_i, i = 1, \dots, N$  should be selected such that  $2 \sum_{j=1}^N |a_{ij}| + 2b_i + k_i + l_i^* \leq Y_s$  is satisfied.

The finite-time stability proof of  $s_i = \dot{s}_i = 0, i = 1, \dots, N$  is similar to that in Theorem 1. In Theorem 4, the existence of  $s_i = \dot{s}_i = 0, i = 1, \dots, N$  for  $t \geq T_r$  will be shown by applying (22) and (23).

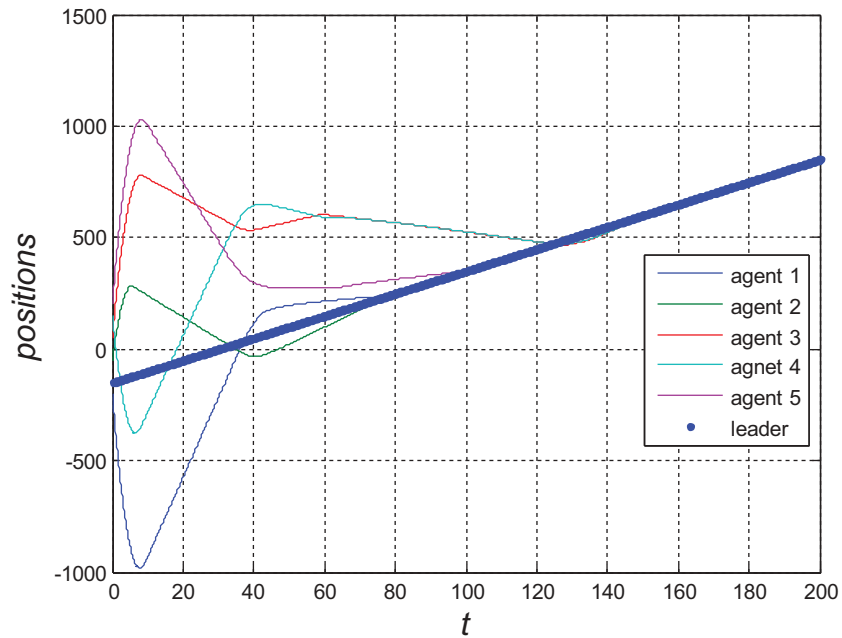
**Theorem 4.** Consider (4) with unknown bounded disturbances. By employing (22) and (23),  $s_i = \dot{s}_i = 0, i = 1, \dots, N$  are achieved for  $t \geq T_r$  where  $T_r$  is determined by

$$T_r \leq \frac{\sqrt{\sum_{i=1}^N s_i^2(0) + \sum_{i=1}^N (\hat{l}_i(0) - l_i^*(0))^2}}{\min(\min_i((1 - \lambda_i) |s_i|), \min_i(k_i))} \tag{24}$$

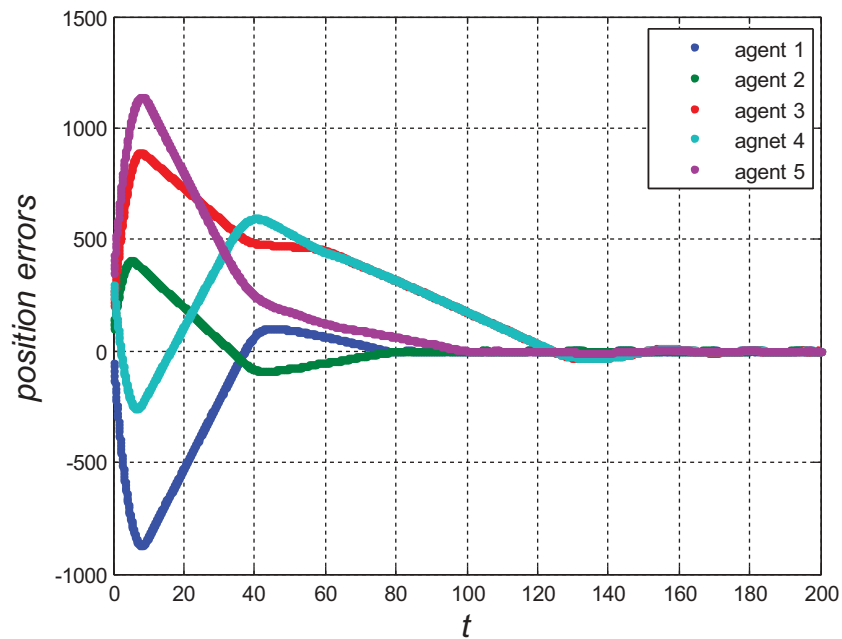
**Proof.** By considering the candidate Lyapunov function  $V = 0.5 \sum_{i=1}^N s_i^2 + 0.5 \sum_{i=1}^N \tilde{l}_i^2$  where  $\tilde{l}_i = \hat{l}_i - l_i^* < 0$ . The sliding surface time derivative is  $\dot{s}_i = \tilde{v}_i - \dot{\phi}_i$ . Now, by replacing  $\tilde{v}_i$  from (7) and  $u_i$  from (22),  $\dot{s}_i$  is obtained as

$$\dot{s}_i = -k_i \text{sgn}(s_i) - \hat{l}_i \text{sgn}(s_i) + d_i, i = 1, \dots, N. \tag{26}$$

**Figure 1. Agent position by applying (11).**

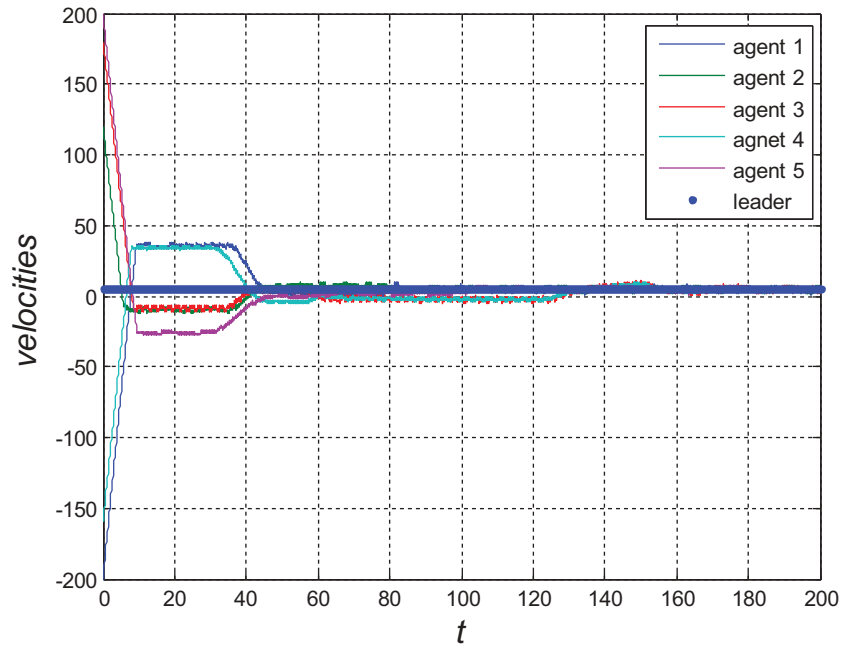


**Figure 2. Agent position error by adopting (11).**

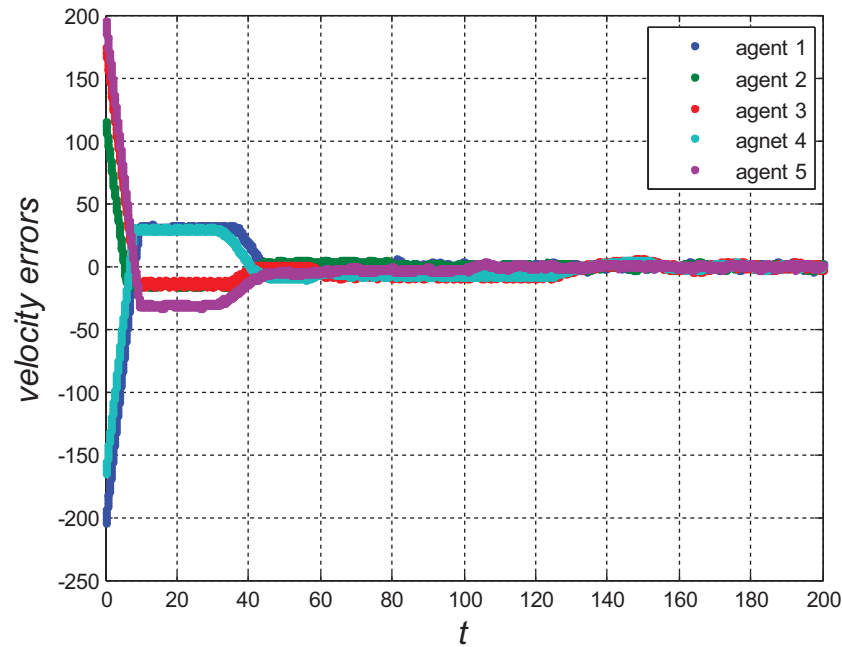




**Figure 3. Agent velocity by adopting (11).**



**Figure 4. Agent velocity error by applying (11).**



By substituting (23) and (25) in  $\dot{V} = \sum_{i=1}^N s_i \dot{s}_i + \sum_{i=1}^N l_i \dot{l}_i$ , the following relation is obtained.

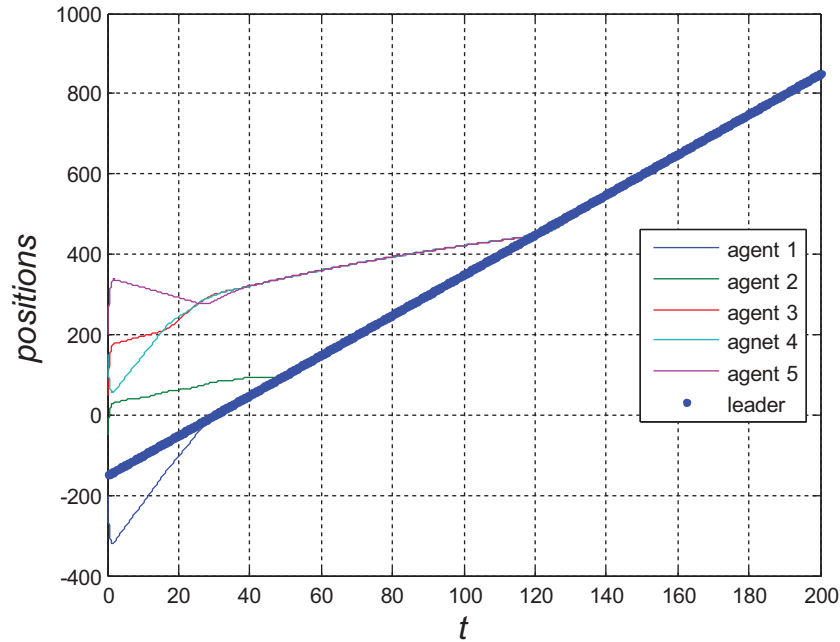
$$\dot{V} = -\sum_{i=1}^N k_i |s_i| - \sum_{i=1}^N \hat{l}_i |s_i| + \sum_{i=1}^N d_i s_i + \sum_{i=1}^N \tilde{l}_i \lambda_i |s_i| \quad (26)$$

By considering  $k_m = \min_i(k_i)$  and  $\sum_{i=1}^N d_i s_i \leq \sum_{i=1}^N l_i^* |s_i|$ ,  $\dot{V}$  becomes:

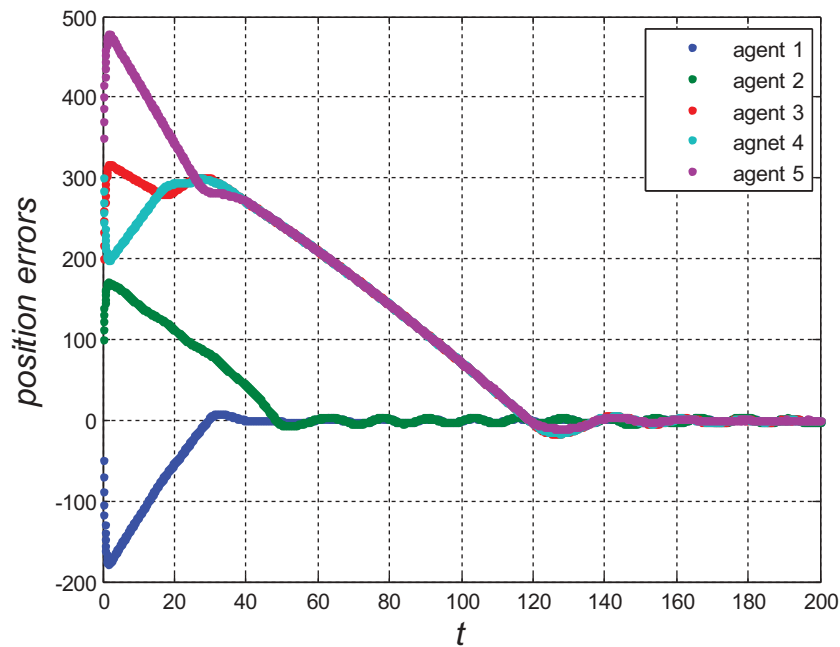
$$\dot{V} \leq -k_m \sum_{i=1}^N |s_i| - \sum_{i=1}^N (\lambda_i - 1) |\tilde{l}_i| |s_i| \quad (27)$$

By defining  $\Omega = \min_i((\lambda_i - 1) |\tilde{l}_i|)$  and  $\theta = \min(\Omega, k_m)$ , (27) is simplified as:

**Figure 5. Agent position by applying (16).**



**Figure 6. Agent position error by adopting (16).**

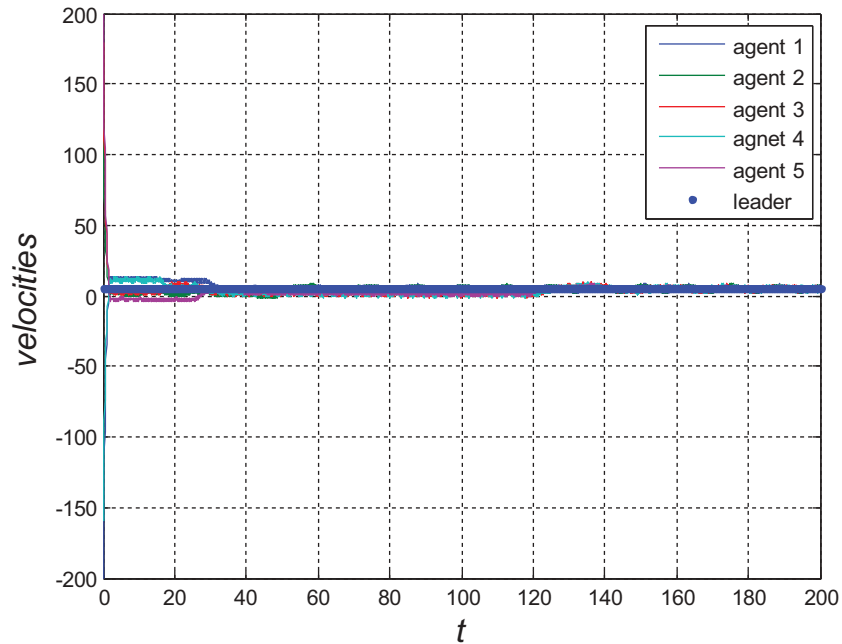


$$\dot{V} \leq -\theta \left( \sum_{i=1}^N |s_i| + \sum_{i=1}^N |\tilde{l}_i| \right) \tag{28}$$

By adopting the well-known inequality  $\sqrt{(\sum_i |y_i|)} < \sum_i \sqrt{|y_i|}$ , (28) is converted to  $\dot{V} \leq -\sqrt{2}\theta V^{\frac{1}{2}}$ . Finally, by setting  $c = \sqrt{2}\theta$ ,  $a = 0.5$ , and applying Lemma 1, it is proven that  $s_i = \dot{s}_i = 0$ ,  $i = 1, \dots, N$  are always fulfilled for  $t \geq T_r$  where  $T_r$  is estimated by (24). This ends the proof.  $\square$

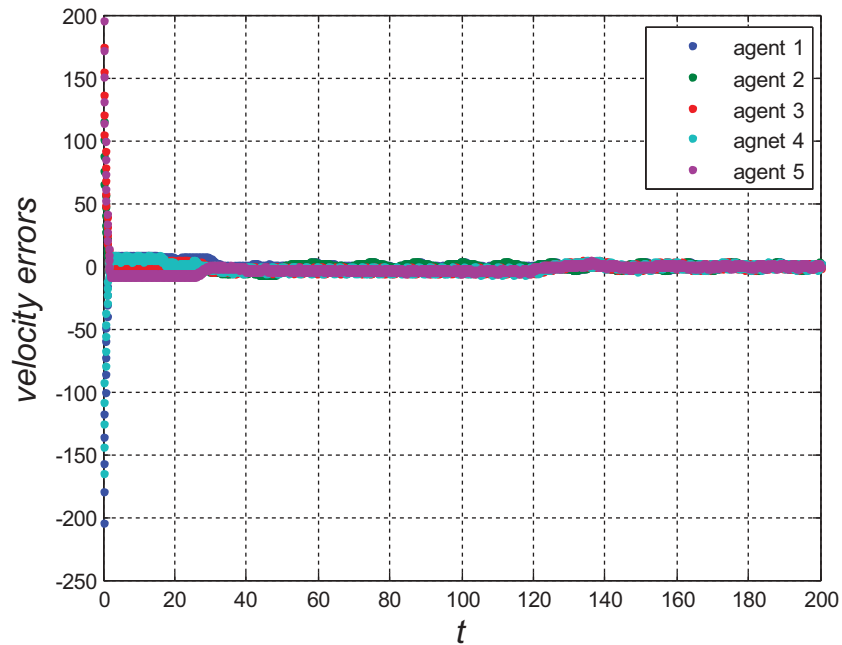
**Remark 5.** As arbitrary constants  $k_i$ ,  $i = 1, \dots, N$  are selected such that  $2 \sum_{j=1}^N |a_{ij}| + 2b_i + k_i + l_i^* \leq \Upsilon_s$  is satisfied, it is concluded that the maximum values of the proposed control inputs (22) are always less than the saturation bounds.

**Figure 7. Agent velocity by adopting (16).**



**Figure 7. Agent velocity by adopting(16).**

**Figure 8. Agent velocity error by applying (16).**

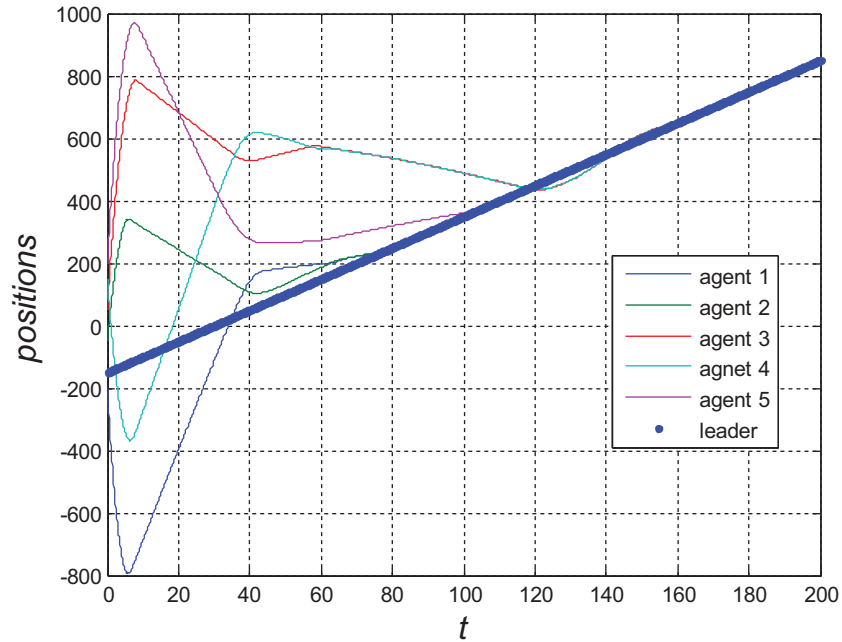


**6. Numerical simulations**

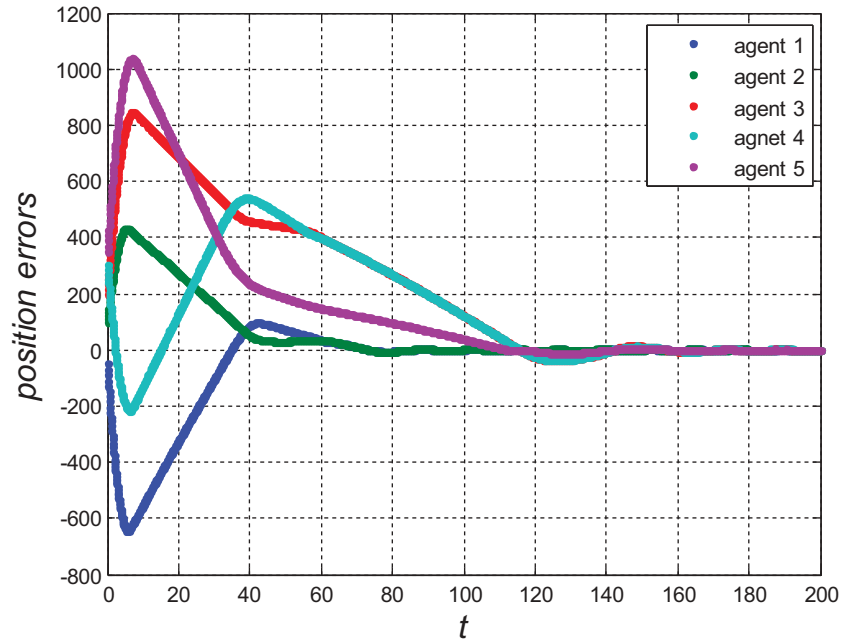
In this section, a multi-agent system consisting of five agents and one leader is simulated and the results are discussed. In all simulations, matrices *A* and *B* are considered to be as presented in (29).

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, B = [ 1 \ 0 \ 0 \ 0 \ 1 ] \tag{29}$$

**Figure 9. Agent position by applying (22).**



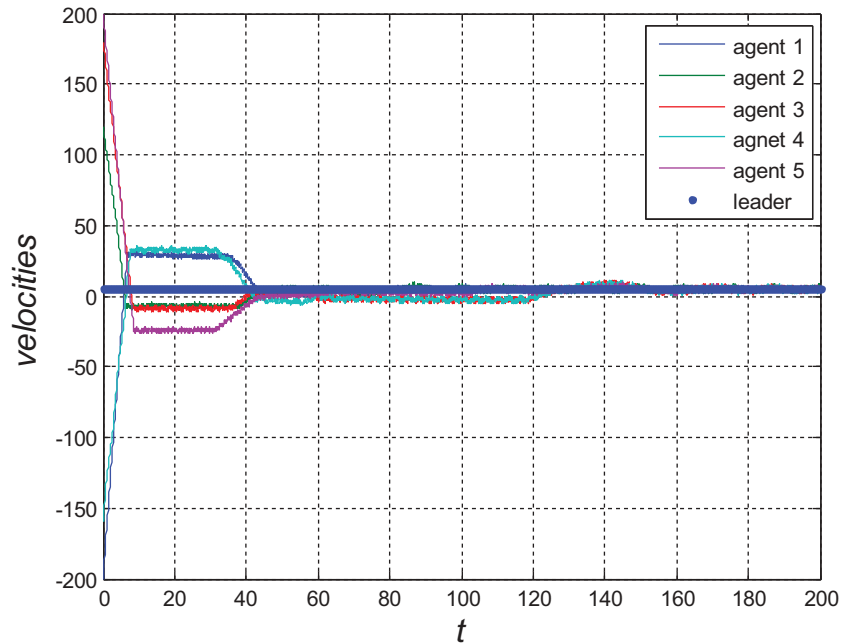
**Figure 10. Agent position error by adopting (22).**



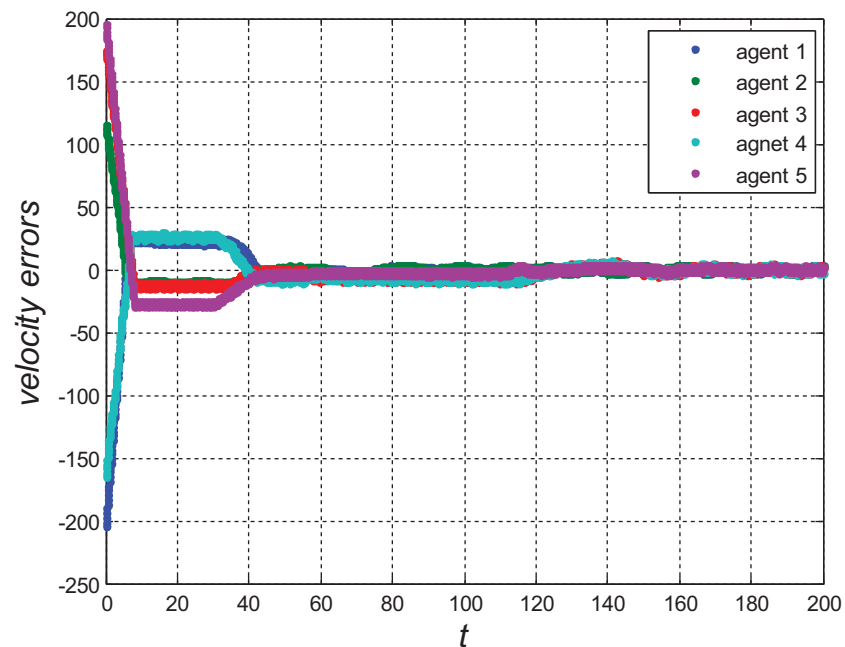
The initial agent positions and velocities are randomly chosen as  $x(0) = [-200 \ 50 \ 50 \ 150 \ 200]^T$  and  $v(0) = [-200 \ 120 \ 180 \ -160 \ 200]^T$ , respectively. The initial leader position and velocity are assumed to be  $x_0(0) = 150$  and  $v_0(0) = 5$ , respectively. Disturbances are selected as  $d_1 = \cos(0.1t)$ ,  $d_2 = 0.5 \sin(0.5t + \pi/4)$ ,  $d_3 = 0.6 \cos(3t)$ ,  $d_4 = 0.8 \sin(2t + \pi/3)$ . The fifth disturbance  $d_5$ , (30), is assumed to be time variant (Yu & Long, 2015).

$$d_5 = \begin{cases} 0.2 \sin(2\pi(\frac{5.9}{60}t + 0.1)t) - 0.2 & t < 30 \\ 0.2 \sin(2\pi(-\frac{5.9}{60}t + 6)t) + 0.2 & t \geq 30 \end{cases} \quad (30)$$

**Figure 11. Agent velocity by adopting (22).**



**Figure 12. Agent velocity error by applying (22).**



Based on the selected disturbances, the upper bound disturbance vectors are obtained as  $l = [1 \ 0.5 \ 0.6 \ 0.8 \ 0.2]^T$ . In all calculations, the optional fractional power  $\alpha_1$ , applied in  $\phi_i$  (9), is chosen as  $\alpha_1 = 0.5$ . Further, the control inputs are assumed  $\pm 27$ . Therefore,  $\Upsilon_s$  is determined as  $\Upsilon_s = 27$ .

Here three scenarios are defined using (11), (16), and (22), respectively. In Scenario 2  $\omega_i = 1$ ,  $i = 1, \dots, N$  and  $\gamma = 0.1$ , and in Scenario 3  $\lambda_i = 1.1$  and  $\hat{l}_i(0) = 0.2$ ,  $i = 1, \dots, N$  are assumed, respectively.

**Scenario 1.** The control inputs are determined based on (11). The tuning parameters  $k_i$  are selected as 20 to satisfy  $\sum_{j=1}^N 2|a_{ij}| + 2b_i + k_i + l_i \leq Y_s, i = 1, \dots, 5$ . Figure 1–4 show agent positions, position error, velocities and velocities error along the leader, respectively. It can be seen that the agent positions and velocities converge to the leader position and velocity in the presence of known bounded disturbances. The numerical results indicate that the control inputs are confined to  $[-25, +25]$ . The maximum control inputs values are adjustable by choosing appropriate  $k_i$ .

**Scenario 2.** Similar to scenario 1, the upper disturbance bounds are assumed to be known, but, (16) are applied to the agents. Here,  $\gamma = 0.1$ ,  $\omega_i = 1$ , and  $k_i = 20$  are selected for  $i = 1, \dots, 5$ . Figures 5–8 depict agent positions, velocities and errors for this scenario. These figures show how all agent positions and velocities reach the leader position and velocity. By comparing Figures 1 and 5 it is noted that the agent convergence rate in Scenario 2 is higher than the same in Scenario 1.

**Scenario 3.** Unlike the two previous scenarios, the considered disturbance upper bounds are assumed to be unknown and should be estimated. In this scenario, the control inputs are based on (22). The tuning parameters are selected as  $k_i = 20$  and  $\lambda_i = 1.01$  for  $i = 1, \dots, 5$ .

The upper bound estimation initial values are chosen as  $\hat{l}_i(0) = 0.2$  for  $i = 1, \dots, 5$ . Agent, positions, velocities and errors in the presence of unknown- bounded disturbances by applying (22) are shown in Figure 9–12.

## 7. Conclusion

In this work, finite-time consensus problem for multi-agent systems with leader in the presence of bounded disturbances and saturation constraints on control inputs have been discussed. To solve the problem, control inputs were designed by considering different assumptions on the upper disturbance bounds. First control laws were proposed, based on a new TSMC method, to tackle the finite-time consensus for disturbed multi-agent systems while the upper disturbance bounds were known.

It has been demonstrated that the proposed control inputs were bounded while their maximum amplitudes could be adjustable by proper tuning parameters selection. Then, by applying a new fast TSMC approach, the control inputs were modified to reduce the high dependency of the finite-reaching time on agent initial conditions. In the second scenario, the same problem was solved for the case where the disturbance upper bounds were unknown. In this case, for fulfilling the finite-time consensus goal, the control inputs and the finite-time estimation laws were designed by applying the adaptive TSMC method. Mathematical analysis of the paper was demonstrated that all suggested control inputs are able to satisfy the finite-time consensus aim within the total adjustable finite-time. Finally, three computer based numerical simulations were illustrated to validate the theoretical results presented in the paper.

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