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*Research Article*

# Average SEP of AF Relaying in Nakagami-*m* Fading Environments

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This paper is devoted to an investigation of an exact average symbol error probability (SEP) for amplify and forward (AF) relaying in independent Nakagami-$m$ fading environments with a nonnegative integer plus one-half $m$, which covers many actual scenarios, such as one-side Gaussian distribution ($m = 0.5$). Using moment generating function approach, the closed-form SEP is expressed in the form of Lauricella multivariate hypergeometric function. Four modulation modes are considered: rectangular quadrature amplitude modulation (QAM), $M$-ary phase shift keying (MPSK), $M$-ary differential phase shift keying (MDPSK), and $\pi/4$ differential quaternary phase shift keying (DQPSK). The result is very simple and general for a nonnegative integer plus one-half $m$, which covers the same range as integer $m$. The tightness of theoretical analysis is confirmed by computer simulation results.

## 1. Introduction

When the transceiver is far away, it is not a wise choice to increase transmit power in green communications. Energy efficiency is placed in a very important position in resource optimization configuration. The relay technology is used as one effective means to achieve diversity gain, reduce transmission power, and improve energy efficiency. Therefore, cooperative diversity technology has attracted considerable benefits for enhancing the performance of wireless networks, such as cell network and ad hoc network. SEP is considered to be an important performance measure. Integral-free bit error probability formulas were derived in [1, 2] for decode and forward (DF) cooperative systems. An asymptotic bit error probability formula was provided in [3–5] for all participate and selective AF cooperation. A new approximation to the symbol error rate was derived in [6]. Multiple input multiple output (MIMO) technology was introduced in [7, 8], which considered exact and asymptotic symbol error rate. Selection combining was added in [9] with the same configuration as that in [7]. Mobile to mobile communication scenario was shown in [10], where asymptotic symbol error rate bound was provided. Signal space cooperation was realized in [11] for full rate transmission, where a tight bit error rate was obtained. Error probabilities of AF multihop variable gain relaying systems were analyzed in [12] by generalized hypergeometric functions. While the two-hop scenario was investigated in [13].

Although the error probability of the Nakagami-$m$ relay channel has been studied extensively, they mostly addressed integer fading parameter $m$. The error probability formula for arbitrary $m$ must resort to infinite series expansion or reasonable approximation in high signal to noise (SNR) region. Some articles provided error probability just by simulation. The actual situation often requires quickly grasping error probability of wireless communication, so it is impossible to carry out a large number of simulation experiments. The importance and generalization of Nakagami-$m$ channel have not been fully exploited, which motivates our work from another perspective.

In this paper, we calculate the exact average SEP formulas in an AF relay system over Nakagami-$m$ fading environment when $m$ is a nonnegative integer plus one-half, while it includes as a special case the one-sided Gaussian distribution ($m = 0.5$). Such closed-form expressions are urgent because they allow fast and efficient evaluation of system reliability.

Note that this parameter $m$ covers the same quantity range as the integer $m$ in most literature, according to basic knowledge of number theory. Four prevailing modulations are investigated: MQAM, MPSK, MDPSK, and DQPSK. Using the properties of moment generating function, precise expressions are a combination of a series of special functions. Simulation results confirm the tightness compactness of the theoretical analysis. For ease of reading and searching, see Mathematical Notations.

## 2. System Model

Consider a cooperative system where a source node $S$ communicates with a destination node $D$ via a relay node $R$. There is not direct link between $S$ and $R$ due to obstacles. Assume that all links between transceivers are subject to independent but different Nakagami-$m$ fading. When $m$ is a nonnegative integer plus one-half, the probability density function of the end to end SNR $\gamma$ at $D$ is given by [14, eq. (7)]

$$f_\gamma(z) = \frac{\sqrt{\pi}e^{-(\sqrt{\beta_s}+\sqrt{\beta_r})^2 z}}{\Gamma(m_s)\Gamma(m_r)} \sum_{k_1=0}^{\lfloor m_s\rfloor}\sum_{k_2=0}^{\lfloor m_r\rfloor}\sum_{l=0}^{\lfloor (m_s-k_1+m_r-k_2)/2\rfloor} \frac{(\lfloor m_s\rfloor+k_1)!(\lfloor m_r\rfloor+k_2)!\beta_s^{m_s/2-1/4-k_1/2}\beta_r^{m_r/2-1/4-k_2/2}}{4^{k_1+k_2}k_1!k_2!(\lfloor m_s\rfloor-k_1)!(\lfloor m_r\rfloor-k_2)!2^{2l}} \times \frac{(m_s-k_1+m_r-k_2)!z^{m_s-k_1+m_r-k_2-l-3/2}}{l!(m_s-k_1+m_r-k_2-2l)!} \times (-1)^l\left(\sqrt{\beta_s}+\sqrt{\beta_r}\right)^{m_s-k_1+m_r-k_2-2l}, \quad (1)$$

where $m$ and $\beta$ are fading parameter and scale parameter, respectively. The subscripts $s$ and $r$ represent $S \to R$ link and $R \to D$ link, respectively. Using [15, eq. (3.381.4)], the moment generating function of the SNR $\gamma$ is given by

$$\begin{aligned} M_\gamma(s) &= \int_0^\infty e^{-sz} f_\gamma(z)\,dz \\ &= \frac{\sqrt{\pi}}{\Gamma(m_s)\Gamma(m_r)} \sum_{k_1=0}^{\lfloor m_s\rfloor}\sum_{k_2=0}^{\lfloor m_r\rfloor}\sum_{l=0}^{\lfloor (m_s-k_1+m_r-k_2)/2\rfloor} \frac{(\lfloor m_s\rfloor+k_1)!(\lfloor m_r\rfloor+k_2)!\beta_s^{m_s/2-1/4-k_1/2}}{4^{k_1+k_2}k_1!k_2!(\lfloor m_s\rfloor-k_1)!(\lfloor m_r\rfloor-k_2)!} \\ &\quad \times \frac{\beta_r^{m_r/2-1/4-k_2/2}(m_s-k_1+m_r-k_2)!}{2^{2l}l!(m_s-k_1+m_r-k_2-2l)!} \times (-1)^l\left(\sqrt{\beta_s}+\sqrt{\beta_r}\right)^{m_s-k_1+m_r-k_2-2l} \\ &\quad \times \left(s+\beta_s+\beta_r+2\sqrt{\beta_s\beta_r}\right)^{1/2+k_1+k_2+l-m_s-m_r} \times \Gamma\left(m_s+m_r-k_1-k_2-l-\frac{1}{2}\right). \end{aligned} \quad (2)$$

The above moment generating function is very useful in error probability analysis.

## 3. Average SEP Analysis

The average SEP is an important measure of communication reliability. Next, we prepare to study the SEP performances of four modulations.

*3.1. Rectangular QAM.* The average SEP of coherent rectangular $M_I \times M_Q$ QAM is given by

$$\begin{aligned} P_{e,\mathrm{QAM}} = \int_0^\infty \Bigg[ & 2\left(1-\frac{1}{M_I}\right)Q\left(\sqrt{q_1\gamma}\right) + 2\left(1-\frac{1}{M_Q}\right) \\ & \times Q\left(\sqrt{q_2\gamma}\right) - 4\left(1-\frac{1}{M_I}\right) \\ & \times \left(1-\frac{1}{M_Q}\right)Q\left(\sqrt{q_1\gamma}\right)Q\left(\sqrt{q_2\gamma}\right)\Bigg] f_\gamma(\gamma)\,d\gamma, \end{aligned} \quad (3)$$

where

$$q_1 = \frac{6}{M_I^2 - 1 + \left(M_Q^2-1\right)r^2}, \quad (4)$$

$$q_2 = \frac{6r^2}{M_I^2 - 1 + \left(M_Q^2-1\right)r^2}. \quad (5)$$

$r$ is the decision distance ratio between constellations of in phase and quadrature components. Based on moment generating function approach, the error probability is written as

$$\begin{aligned} P_{e,\mathrm{QAM}} = & \frac{2}{\pi}\left(1-\frac{1}{M_I}\right)\int_0^{\pi/2} M_\gamma\left(\frac{q_1}{2\sin^2\theta}\right)d\theta + \frac{2}{\pi}\left(1 - \frac{1}{M_Q}\right)\int_0^{\pi/2} M_\gamma\left(\frac{q_2}{2\sin^2\theta}\right)d\theta \\ & - \frac{2}{\pi}\left(1-\frac{1}{M_I}\right)\left(1 - \frac{1}{M_Q}\right) \times \left[\int_0^{\pi/2-\arctan\sqrt{b/a}} M_\gamma\left(\frac{q_1}{2\sin^2\theta}\right)d\theta + \int_0^{\arctan\sqrt{b/a}} M_\gamma\left(\frac{q_2}{2\sin^2\theta}\right)d\theta\right], \end{aligned} \quad (6)$$

where the first and second integrals correspond to the error probability involving a single $Q$ function and the third and fourth integrals correspond to the error probability involving the product of two $Q$ functions. We first begin by the integral containing one Gaussian $Q$ function. Taking the first integral in (6) as an example, we encounter a kind of integral given by

$$J_1 = \int_0^{\pi/2} \left( \frac{q_1}{2\sin^2\theta} + \beta_s + \beta_r + 2\sqrt{\beta_s\beta_r} \right)^{1/2+k_1+k_2+l-m_s-m_r} d\theta. \quad (7)$$

By change of the variable $u = \cos^2\theta$, after some manipulations, $J_1$ can be written as

$$J_1 = \int_0^1 \left( \frac{q_1}{2(1-u)} + \beta_s + \beta_r + 2\sqrt{\beta_s\beta_r} \right)^{1/2+k_1+k_2+l-m_s-m_r} \times \frac{1}{2\sqrt{u(1-u)}} du. \quad (8)$$

Applying [15, eq. (9.100)], (8) can be expressed in closed form in terms of Gauss hypergeometric function.

Next, we cope with the integral containing the product of two Gaussian Q functions. Making change of the variable $u = 1 - b^2/a^2\tan^2\theta$, we obtain one kind of integral given by

$$J_2 = \int_0^1 u^{a-1} (1-u)^{c-a-1} (1-vu)^{-b_1} (1-vu)^{-b_2} du. \quad (9)$$

Using [16, eq. (11)], $J_2$ can be expressed in closed form in terms of Appell hypergeometric function. Finally, combining the results in (8) and (9), the average SEP of rectangular QAM is given by

$$\begin{aligned}
P_{e,\mathrm{QAM}} &= \frac{1}{\Gamma(m_s)\Gamma(m_r)} \sum_{k_1=0}^{\lfloor m_s \rfloor} \sum_{k_2=0}^{\lfloor m_r \rfloor} \sum_{l=0}^{\lfloor (m_s-k_1+m_r-k_2)/2 \rfloor} \frac{(\lfloor m_s \rfloor + k_1)! (\lfloor m_r \rfloor + k_2)! \beta_s^{m_s/2-1/4-k_1/2}}{4^{k_1+k_2} k_1! k_2! (\lfloor m_s \rfloor - k_1)! (\lfloor m_r \rfloor - k_2)!} \\
&\times \frac{\beta_r^{m_r/2-1/4-k_2/2} (m_s - k_1 + m_r - k_2)!}{2^{2l} l! (m_s - k_1 + m_r - k_2 - 2l)!} \times \Gamma\left(m_s + m_r - k_1 - k_2 - l - \frac{1}{2}\right) \times (-1)^l \left(\sqrt{\beta_s} + \sqrt{\beta_r}\right)^{m_s-k_1+m_r-k_2-2l} \times \Bigg\{ \Bigg( 1 \\
&- \frac{1}{M_I} \Bigg) \frac{\Gamma(m_s - k_1 + m_r - k_2 - l)}{\Gamma(m_s - k_1 + m_r - k_2 - l + 1/2)} \times \left( \frac{q_1}{2} + \beta_s + \beta_r + 2\sqrt{\beta_s\beta_r} \right)^{1/2+k_1+k_2+l-m_s-m_r} \times {}_2F_1 \Bigg( \frac{1}{2}, m_s - k_1 + m_r - k_2 \\
&- l - \frac{1}{2}; m_s - k_1 + m_r - k_2 - l + \frac{1}{2}; \frac{2\left(\sqrt{\beta_s} + \sqrt{\beta_r}\right)^2}{2\left(\sqrt{\beta_s} + \sqrt{\beta_r}\right)^2 + q_1} \Bigg) + \left(1 - \frac{1}{M_Q}\right) \frac{\Gamma(m_s - k_1 + m_r - k_2 - l)}{\Gamma(m_s - k_1 + m_r - k_2 - l + 1/2)} \times \Bigg( \frac{q_2}{2} \\
&+ \beta_s + \beta_r + 2\sqrt{\beta_s\beta_r} \Bigg)^{1/2+k_1+k_2+l-m_s-m_r} \times {}_2F_1 \Bigg( \frac{1}{2}, m_s - k_1 + m_r - k_2 - l - \frac{1}{2}; m_s - k_1 + m_r - k_2 - l \\
&+ \frac{1}{2}; \frac{2\left(\sqrt{\beta_s} + \sqrt{\beta_r}\right)^2}{2\left(\sqrt{\beta_s} + \sqrt{\beta_r}\right)^2 + q_2} \Bigg) - \frac{\sqrt{q_1 q_2} (1 - 1/M_I)(1 - 1/M_Q)}{\sqrt{\pi}(q_1 + q_2)(m_s - k_1 + m_r - k_2 - l)} \left( \frac{q_1 + q_2}{2} + \left(\sqrt{\beta_s} + \sqrt{\beta_r}\right)^2 \right)^{1/2+k_1+k_2+l-m_s-m_r} \\
&\cdot \Bigg[ F_1 \Bigg( 1, m_s - k_1 + m_r - k_2 - l - \frac{1}{2}, 1; m_s - k_1 + m_r - k_2 - l + 1; \frac{q_1 + 2\left(\sqrt{\beta_s} + \sqrt{\beta_r}\right)^2}{q_1 + q_2 + 2\left(\sqrt{\beta_s} + \sqrt{\beta_r}\right)^2}, \frac{q_1}{q_1 + q_2} \Bigg) \\
&+ F_1 \Bigg( 1, m_s - k_1 + m_r - k_2 - l - \frac{1}{2}, 1; m_s - k_1 + m_r - k_2 - l + 1; \frac{q_2 + 2\left(\sqrt{\beta_s} + \sqrt{\beta_r}\right)^2}{q_1 + q_2 + 2\left(\sqrt{\beta_s} + \sqrt{\beta_r}\right)^2}, \frac{q_2}{q_1 + q_2} \Bigg) \Bigg] \Bigg\},
\end{aligned} \quad (10)$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ and $F_1(\cdot,\cdot,\cdot;\cdot;\cdot,\cdot)$ stands for the Gauss hypergeometric function defined in [15, eq. (9.100)] and the Appell hypergeometric function [17].

*3.2. MPSK.* Following similar steps in QAM, the average SEP of coherent MPSK is given by

$$\begin{aligned}
P_{e,\mathrm{MPSK}} &= \frac{1}{\pi} \int_0^{\pi-\pi/M} M_\gamma \left[ \frac{\sin^2(\pi/M)}{\sin^2\theta} \right] d\theta = \frac{1}{\pi} \int_0^{\pi/2} M_\gamma \left[ \frac{\sin^2(\pi/M)}{\sin^2\theta} \right] d\theta + \frac{1}{\pi} \int_{\pi/2}^{\pi-\pi/M} M_\gamma \left[ \frac{\sin^2(\pi/M)}{\sin^2\theta} \right] d\theta \\
&= \frac{1}{\Gamma(m_s)\Gamma(m_r)} \sum_{k_1=0}^{\lfloor m_s \rfloor} \sum_{k_2=0}^{\lfloor m_r \rfloor} \sum_{l=0}^{\lfloor (m_s-k_1+m_r-k_2)/2 \rfloor} \frac{(\lfloor m_s \rfloor + k_1)! (\lfloor m_r \rfloor + k_2)! \beta_s^{m_s/2-1/4-k_1/2}}{4^{k_1+k_2} k_1! k_2! (\lfloor m_s \rfloor - k_1)! (\lfloor m_r \rfloor - k_2)!} \times \frac{\beta_r^{m_r/2-1/4-k_2/2} (m_s - k_1 + m_r - k_2)!}{2^{2l} l! (m_s - k_1 + m_r - k_2 - 2l)!}
\end{aligned}$$

$$\times(-1)^l\left(\sqrt{\beta_s}+\sqrt{\beta_r}\right)^{m_s-k_1+m_r-k_2-2l}\times\Gamma\left(m_s+m_r-k_1-k_2-l-\frac{1}{2}\right)$$
$$\times\left(\left(\sqrt{\beta_s}+\sqrt{\beta_r}\right)^2+\sin^2\left(\frac{\pi}{M}\right)^{1/2+k_1+k_2+l-m_s-m_r}\right)\times\left[\frac{\Gamma(m_s-k_1+m_r-k_2-l)}{2\Gamma(m_s-k_1+m_r-k_2-l+1/2)}\right.$$
$$\times{}_2F_1\left(m_s-k_1+m_r-k_2-l-\frac{1}{2},\frac{1}{2};m_s-k_1+m_r-k_2-l+\frac{1}{2};\frac{\left(\sqrt{\beta_s}+\sqrt{\beta_r}\right)^2}{\left(\sqrt{\beta_s}+\sqrt{\beta_r}\right)^2+\sin^2(\pi/M)}\right)+\frac{\cos(\pi/M)}{\sqrt{\pi}}$$
$$\left.\cdot F_1\left(\frac{1}{2},m_s-k_1+m_r-k_2-l-\frac{1}{2},1+k_1+k_2+l-m_s-m_r;\frac{3}{2};\frac{\cos^2(\pi/M)\left(\sqrt{\beta_s}+\sqrt{\beta_r}\right)^2}{\left(\sqrt{\beta_s}+\sqrt{\beta_r}\right)^2+\sin^2(\pi/M)},\cos^2\left(\frac{\pi}{M}\right)\right)\right]. \tag{11}$$

*3.3. MDPSK.* With the aid of the common moment generating function, the average SEP for MDPSK is given by

$$P_{e,\text{MDPSK}}=\frac{1}{\pi}\int_0^{\pi-\pi/M}M_\gamma\left[\frac{\sin^2(\pi/M)}{1+\cos(\pi/M)\cos\theta}\right]d\theta=\frac{2\cos(\pi/2M)}{\sqrt{\pi}\Gamma(m_s)\Gamma(m_r)}$$
$$\cdot\sum_{k_1=0}^{\lfloor m_s\rfloor}\sum_{k_2=0}^{\lfloor m_r\rfloor}\sum_{l=0}^{\lfloor(m_s-k_1+m_r-k_2)/2\rfloor}\frac{(\lfloor m_s\rfloor+k_1)!(\lfloor m_r\rfloor+k_2)!\beta_s^{m_s/2-1/4-k_1/2}}{4^{k_1+k_2}k_1!k_2!(\lfloor m_s\rfloor-k_1)!(\lfloor m_r\rfloor-k_2)!}\times\frac{\beta_r^{m_r/2-1/4-k_2/2}(m_s-k_1+m_r-k_2)!}{2^{2l}l!(m_s-k_1+m_r-k_2-2l)!}\times\Gamma\left(m_s+m_r\right.$$
$$\left.-k_1-k_2-l-\frac{1}{2}\right)\times(-1)^l\left(\sqrt{\beta_s}+\sqrt{\beta_r}\right)^{m_s-k_1+m_r-k_2-2l}\times\left(\beta_s+\beta_r+2\sqrt{\beta_s\beta_r}+2\sin^2\left(\frac{\pi}{2M}\right)\right)^{1/2+k_1+k_2+l-m_s-m_r} \tag{12}$$
$$\times F_D^{(3)}\left[\frac{1}{2},m_s+m_r-k_1-k_2-l-\frac{1}{2},\frac{1}{2}+k_1+k_2+l-m_s\right.$$
$$\left.-m_r,\frac{1}{2};\frac{3}{2};\frac{\cos(\pi/M)\left(\sqrt{\beta_s}+\sqrt{\beta_r}\right)^2}{\left(\sqrt{\beta_s}+\sqrt{\beta_r}\right)^2+2\sin^2(\pi/2M)},\cos\left(\frac{\pi}{M}\right),\cos^2\left(\frac{\pi}{2M}\right)\right],$$

where $F_D^{(3)}$ is the Lauricella function [16, eq. (15)].

*3.4. Noncoherent Detection of Equiprobable Correlated Binary Signals and $\pi/4$ DQPSK.* The average SEP of equal energy, equiprobable, and correlated binary signals with noncoherent detection is given by

$$P_e=\frac{1}{2\pi}\int_0^{\pi}M_\gamma\left[\frac{\left(b^2-a^2\right)^2}{2(a^2+b^2)-4ab\cos\theta}\right]d\theta$$
$$=\frac{\sqrt{\pi}}{2\Gamma(m_s)\Gamma(m_r)}\sum_{k_1=0}^{\lfloor m_s\rfloor}\sum_{k_2=0}^{\lfloor m_r\rfloor}\sum_{l=0}^{\lfloor(m_s-k_1+m_r-k_2)/2\rfloor}\frac{(\lfloor m_s\rfloor+k_1)!(\lfloor m_r\rfloor+k_2)!\beta_s^{m_s/2-1/4-k_1/2}}{4^{k_1+k_2}k_1!k_2!(\lfloor m_s\rfloor-k_1)!(\lfloor m_r\rfloor-k_2)!}\times\frac{\beta_r^{m_r/2-1/4-k_2/2}(m_s-k_1+m_r-k_2)!}{2^{2l}l!(m_s-k_1+m_r-k_2-2l)!}$$
$$\times\Gamma\left(m_s+m_r-k_1-k_2-l-\frac{1}{2}\right)\times(-1)^l\left(\sqrt{\beta_s}+\sqrt{\beta_r}\right)^{m_s-k_1+m_r-k_2-2l} \tag{13}$$
$$\times\left(\beta_s+\beta_r+2\sqrt{\beta_s\beta_r}+\frac{(b-a)^2}{2}\right)^{1/2+k_1+k_2+l-m_s-m_r}$$
$$\times F_1\left[\frac{1}{2},m_s+m_r-k_1-k_2-l-\frac{1}{2},\frac{1}{2}+k_1+k_2+l-m_s-m_r;1;\frac{8ab\left(\sqrt{\beta_s}+\sqrt{\beta_r}\right)^2}{(a+b)^2\left(2\left(\sqrt{\beta_s}+\sqrt{\beta_r}\right)^2+(b-a)^2\right)},\frac{4ab}{(a+b)^2}\right],$$

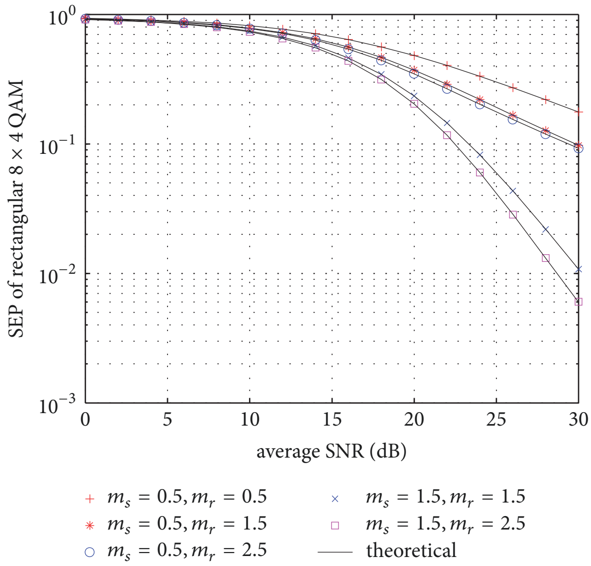


Figure 1: Comparison of SEP for 8 × 4 QAM.

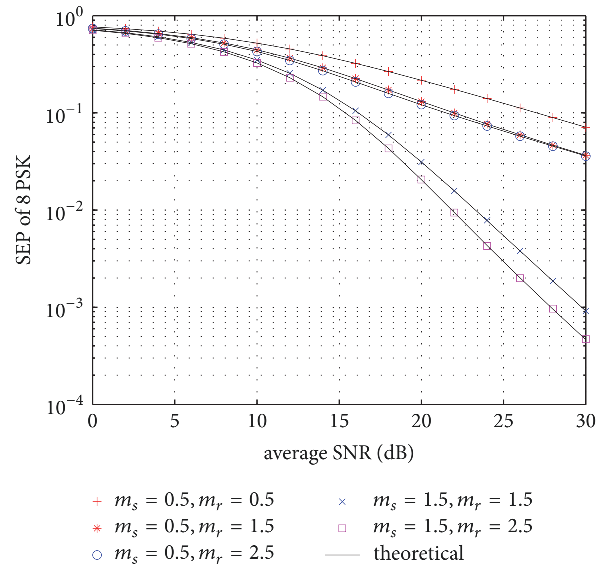


Figure 2: Comparison of SEP for 8 PSK.

where

$$a = \sqrt{\frac{1 - \sqrt{1 - |\rho|^2}}{2}}, \tag{14}$$

$$b = \sqrt{\frac{1 + \sqrt{1 - |\rho|^2}}{2}} \tag{15}$$

and $0 \leq |\rho| \leq 1$ is the magnitude of the cross correlation coefficient between the two signals. When $a = \sqrt{2 - \sqrt{2}}$ and $b = \sqrt{2 + \sqrt{2}}$, (13) corresponds to $\pi/4$ DQPSK with gray coding.

## 4. Simulation Results

In this section, the simulation results of the error probability for QAM, MPSK, MDPSK, and DQPSK are evaluated. The average SNR per symbol is defined as $P_s/N_0 = P_r/N_0$, where $P_s$ and $P_r$ represent transmit power of the source and the relay, respectively. $N_0$ is the noise variances. The channel gain is normalized to unit. The theoretical results highly agree with the simulations for 8 × 4 QAM and integer plus one-half $m$. The same coincidence can be deduced for 8 PSK, 8 DPSK, and DQPSK modulation constellations and fading parameters $m$. This demonstrates the accuracy and validity of the proposed formula.

Figures 1–4 show that the diversity gain is an increasing function of the fading parameter $m$. For example, when the SEP of 8 PSK is 0.1, the diversity gain of the case of $m_s = 0.5$, $m_r = 0.5$ is achieved about 4 dB compared with the case of $m_s = 0.5$, $m_r = 1.5$. Moreover, the diversity gain increased to about 10 dB when $m_s = 1.5$, $m_r = 1.5$. Similar observations can be found in 8 DPSK modulation in Figure 3. The diversity order is dominated by the worse link between $S \rightarrow R$ and $R \rightarrow D$. For example, let us observe two cases of $m_s = 0.5, m_r = 0.5$ and $m_s = 0.5$, $m_r = 1.5$. From Figure 2, when $m_s = m_r = 0.5$, in the high SNR region, the SEP is 0.089645629941062 at average SNR = 28 dB and 0.070958408816342 at average SNR = 30 dB. This implies that the diversity gain is

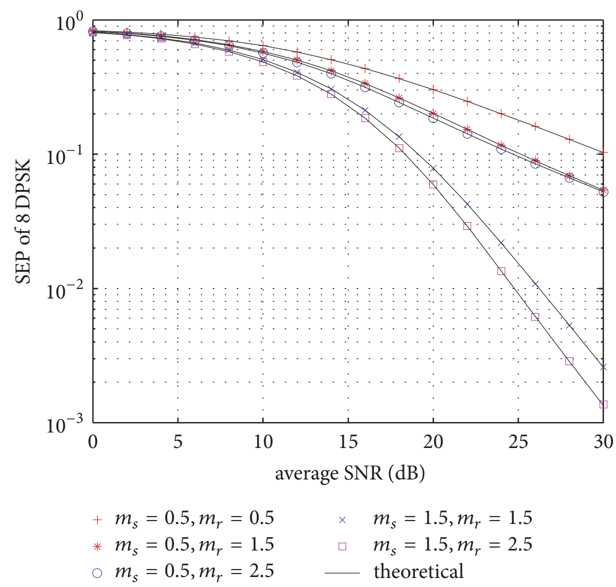


Figure 3: Comparison of SEP for 8 DPSK.

$$\frac{10 \lg (0.089645629941062/0.070958408816342)}{30 - 28} = 0.5076 \approx \min(m_s, m_r). \tag{16}$$

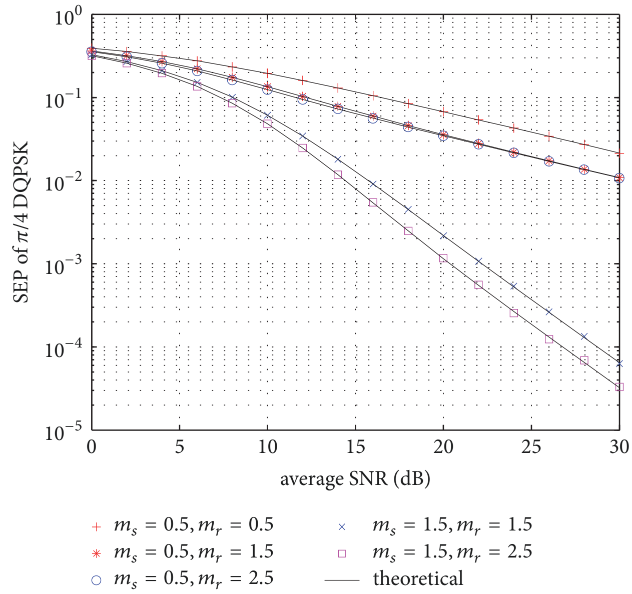


FIGURE 4: Comparison of SEP for $\pi/4$ DQPSK.

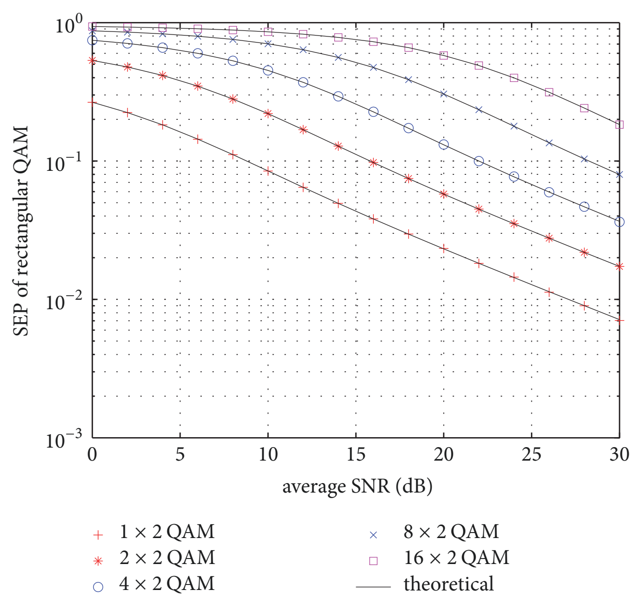


FIGURE 5: Comparison of SEP for MQAM.

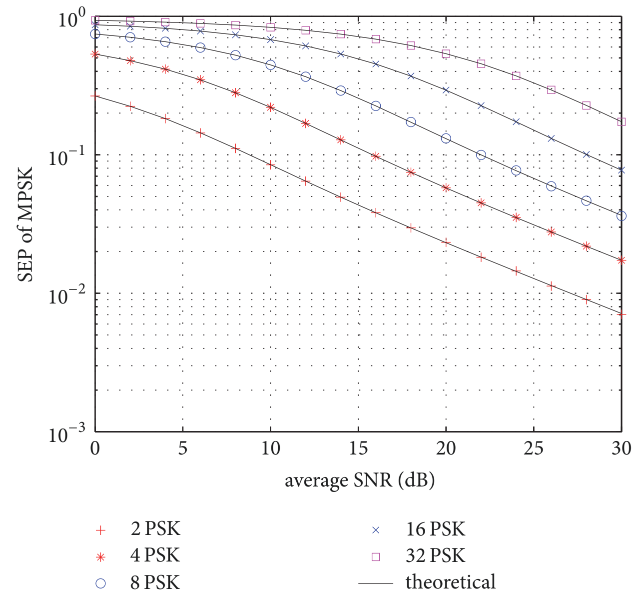


FIGURE 6: Comparison of SEP for MPSK.

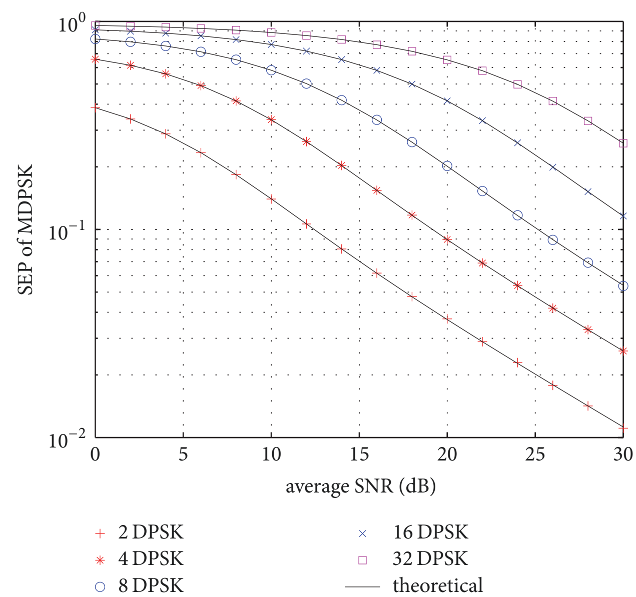


FIGURE 7: Comparison of SEP for MDPSK.

While for $m_s = 0.5$, $m_r = 1.5$, the SEP is 0.046485872872970 at 28 dB and 0.036583504432831 at 30 dB. The diversity gain becomes

$$\frac{10 \lg (0.046485872872970/0.036583504432831)}{30 - 28} = 0.5202 \approx \min (m_s, m_r). \quad (17)$$

Although the parameters $m_r = \{0.5, 1.5\}$ are different, the results are the same. So they achieve the same diversity order.

Figures 5–7 show the average SEP of $M$-ary modulation schemes, where $m_s = 0.5$, $m_r = 1.5$. The average SEP increases with the increase of $M$ because the minimum distance between symbols becomes smaller. But the slopes of the SEP curves are nearly the same, implying the same diversity gain. The influence of cross correlation coefficient on SEP is drawn in Figure 8. When the correlation between two signals is small, the SEP is relatively small.

## 5. Conclusion

In this paper, we study the SEP of QAM, MPSK, MDPSK, and DQPSK modulation in cooperative AF system. Exact closed-form expressions for average SEP are obtained over independent Nakagami-$m$ fading channels with integer plus

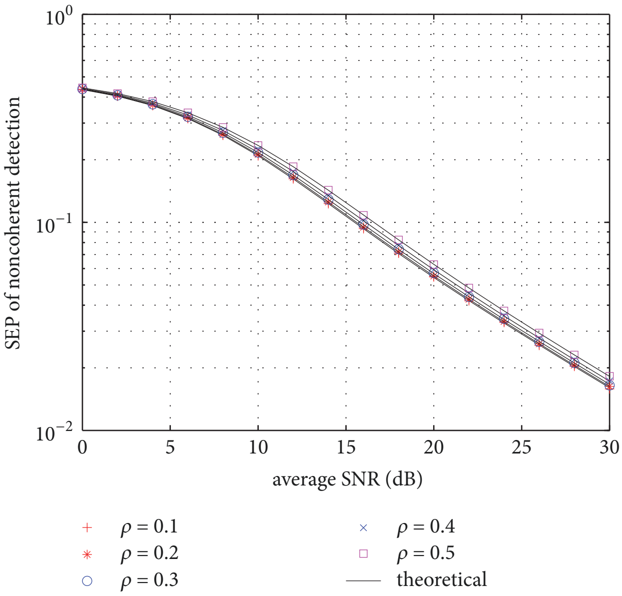


Figure 8: Comparison of SEP for correlated binary signals with noncoherent detection.

one-half $m$. Simulation results agree well with the theoretical analysis.

## Mathematical Notations

$f_\gamma(\cdot)$: Probability density function of the variable $\gamma$
$M_\gamma(\cdot)$: Moment generating function of the variable $\gamma$
$\lfloor\cdot\rfloor$: Floor function
$Q(\cdot)$: Gaussian $Q$ function
${}_2F_1$: Gauss hypergeometric function
$F_1$: Appell hypergeometric function
$F_D^{(n)}$: Lauricella function.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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