A Reconfigurable Method for Time-Correlated Mimo Channels with a Decision Feedback Receiver

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Abstract

This work considers the combined design of bit loading, precoding and receives filters for a multiple-input-multipleoutput (MIMO) digital communication system. Both the transmitter and the receiver are assumed to know the channel matrix perfectly. It is well known that, for linear MIMO transceivers, orthogonal transmission (i.e., diagonalization of the channel matrix) is optimal for some criteria such as maximum mutual information. It has been shown that if the receiver uses the linear minimum mean squared error (MMSE) detector, the optimal transmission strategy is to perform bit loading on orthogonal sub-channels. The transmission rate of the channel is adapted by assigning bits dynamically to the subchannels of the MIMO system. A variable-rate MIMO system with a decision feedback receiver is considered. The nested sub-matrices are generated that can be updated as time evolves. Predictive quantization is used for the feedback of bit loading to take advantage of the time correlation inherited from the temporally correlated channel. To derive the optimal predictor of the next bit loading for predictive quantization & obtain the statistics of the prediction error using this method. The quantizer is designed to achieve a smaller quantization error. The process of comparing the decoding method is proposed to enhance the design and its methodology. This provides better outcome related to the MMSE and bit rate while comparing with the conventional methods.

Keywords: Multiple-Input-Multiple-Output, Wireless Communication, OFDM, LPC, Gauss-Markov.

INTRODUCTION

Multiple -input multiple -output have attracted great attention in recent years. Optimal -output (MIMO) systems precoders of different design criteria for MIMO channels have been considered for a fixed transmission rate. To increase the transmission rate over a fading channel, variable-rate transmission systems are proposed. Adapting the transmission rate according to the channel also has the advantage that the error rate can be easily controlled without deep interleaving. In [1]–[9] the channel state information (CSI) is assumed to be available to both the transmitter and receiver.

In general, the transmitter has no complete CSI and there is an only limited amount of feedback. Depending on the transmission scheme, the receiver feeds back the information of the precoder, power loading, bit loading, or channel Gram matrix to the transmitter. Precoder codebooks for a fixed bit loading are designed. The feedback of power loading and antenna rate control is proposed. Bitloading codebooks for a given precoder are designed. In these works, the channel is assumed to be independent of time.

In practical transmission, the channel is usually correlated in time. Time-correlated channels have been considered. In [19] the precoder is fed back to the transmitter using Givens rotations for correlated MIMO channels. In[20] the channel Gram matrix is differentially coded using geodesic curves and a differential codebook is designed for maximizing the signal to noise ratio or mutual information. In[21], a beamforming system with limited feedback is designed by modeling the quantized CSI as a finite state Markov chain.

In [22]–[24], a temporally correlated channel is modeled as a first-order Gauss-Markov process. The channel capacity for such a channel is analyzed. A rotation based differential codebook for the precoding matrix is proposed. In [25], the minimum feedback rate for a differential feedback system is derived in a closed-form. A polar-cap differential codebook is proposed for a beamforming system with limited feedback. In [26],[27], predictive quantization was applied on the feedback of bit loading for a linear receiver, and the quantizer was designed using the bounds of the prediction error variance.

In [27], this paper, we consider variable-rate transmission for a slowly time-varying MIMO channel when decision feedback is used at the receiver. The full channel information is assumed to be available at the receiver. The transmitter does not know the channel information. The information that is available to the transmitter is the feedback from the receiver. For a given error rate constraint, bits are dynamically assigned to each subchannel according to the channel information as in per-antenna rate control (PARC) so that the transmission rate can be adapted to the current channel. Due to the temporal correlation of the channel, the bit loading is also time correlated. We feedback the bit loading vector using predictive quantization assuming a delay-free feedback loop is available. When decision feedback is employed at the receiver with reverse detection order, it is known that the Cholesky decomposition of the channel Gram matrix can be used to determine the subchannel signal-to-noise ratios and hence also the bit loading.

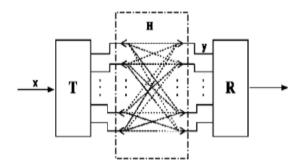


Figure 1: MIMO communication system.

Assuming the time correlated channel is modeled by a slowly varying Gauss-Markov process, we show that the submatrices generated during the process of Cholesky decomposition can be updated with time. The[27], update allows us to obtain the optimal predictor of the next bit loading for predictive quantization in a closed form. Furthermore, we analyze the statistics of the subchannel prediction errors and derive their means and variances. The statistics are then exploited in the design of quantizers for quantizing the prediction errors. By adapting the quantizers according to the statistics of the prediction errors, a smaller quantization error than direct quantization can be achieved. Simulations are given to demonstrate that for slowly varying channels, the proposed predictive quantization of bit loading can achieve a rate very close to the unquantized case with a low feedback rate.

In the first part, we consider the problem of designing the transceiver in order to minimize the probability of error given maximum likelihood (ML) detection. A joint bit loading and linear precoder design is proposed that outperforms the optimal orthogonal transmission. The design uses lattice invariant operations to transform the channel matrix into a lattice generator matrix with large minimum distance separation at a low price in terms of transmit power. An algorithm for this power minimization is presented along with a lower bound on the optimization. Apparently, given the

optimal MLdetector, orthogonal subchannels are (in general) suboptimal.

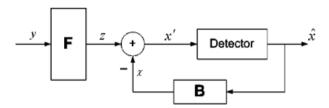


Figure 2: Decision feedback receiver

The ML detector may suffer from high computational complexity, which motivates the use of the suboptimal but less complex MMSE detector. An intermediate detector in terms of complexity and performance is the decision feedback (DF) detector. In the second part of the thesis, we consider the problem of joint bit loading and precoding assuming the DF detector. The main result shows that for a DF MIMO transceiver where the bit loading is jointly optimized with the transceiver filters, orthogonal transmission is optimal. As a consequence, inter-symbol interference is eliminated and the DF part of the receiver is actually not required, only the linear part is needed. The proof is based on a relaxation of the discrete set of available bit rates on the individual subchannels to the set of positive real numbers.

It is shown that the loss due to rounding is small, and an upper bound on the maximum loss is derived. As a by product of the work on decision feedback detectors, we also present some work on the problem of optimizing under a linearly shifted, or skewed, majorization constraint. As applications, two unitary precoder designs for MIMO communication systems that use heterogeneous signal constellations and employ DF detection at the receiver are presented. The Viterbi decoding method is employed to overcome this problem. It is proved to be better than the existing systems.

Mostly, the development of a predictive CSI quantizer for MU-MIMO was pursued. The description and derivation of this quantizer is a central topic. Background information is provided. The main contributions of this dissertation to quantization are:

- i) Efficient predictive quantization of temporally correlated n-dimensional subspaces in them-dimensional Euclidean space. Despite $n \le m$, no restrictions are imposed on the dimensionality of the quantization problem.
- ii) Prediction of the current subspace from previously quantized observations. The prediction is achieved by translating the prediction problem to the tangent space associated with the manifold.
- iii) Derivation of an adaptive quantization codebook to match the temporal evolution of the source. A local codebook is

generated on the manifold that covers a certain volume around the prediction of the current subspace. The size of the volume that needs to be covered is determined by the prediction accuracy, which depends on the strength of the temporal correlation of the source.

iv) Evaluation of the quantization error by means of Monte-Carlo simulations, and comparison to alternative quantizers.

When the number of data streams per user is less than the number of receive antennas, interference-free transmission is ensured by the BD precoder only over dimensional subspaces of the users' channel matrices. An antenna combiner is applied at each user to separate the interference-free signal-space from the interference-contaminated space. To reduce the CSI feedback overhead, a selfish pre-selection of dimensional subspace by the users as CSI feedback is proposed, instead of quantizing the full channel matrix. Within this area, the following contributions are made.

- i) Proposal of subspace selection strategies to achieve either a maximal channel gain or a minimal CSI quantization error. The trade-off between these two approaches is investigated.
- ii) Construction of the corresponding antenna combiners to filter out the signal-space. Specifically, the quantization based combining (QBC) method is extended to multiple data-streams.
- iii) Analytic performance investigation of the proposed subspace selection and antenna combining strategies. Upper bounds on the throughput loss compared to perfect CSIT are derived, and the necessary number of feedback bits to achieve a given rate loss is determined.

Finally, the CSI quantizer is extended to frequency-selective multi-carrier OFDM systems by considering two approaches. With feedback pilot interpolation, the frequency-domain correlation of the subspaces on neighboring OFDM subcarriers is exploited by providing CSI feedback only for a subset of subcarriers, so-called CSI pilots, and interpolating the quantized CSI at the base station. This approach has already been investigated for one-dimensional subspaces. It is shown, by means of simulations, that this approach can outperform CSI interpolation if the density of CSI pilots is small compared to the coherence bandwidth of the channel.

SPACE-TIME CODING TECHNIQUES FOR MIMO-OFDM

OFDM is an effective and low-complexity strategy for dealing with frequency-selective channels When the subchannel bandwidth is sufficiently narrow, the frequency response across each subchannel is approximately flat, avoiding the need for complicated time-domain equalization. In this way,

OFDM transforms a frequency-selective channel into a collection of separate flat-fading channels. In the same way, when an OFDM transmitter is used by each of transmit antennas, and an OFDM front-end is used by each of receive antennas, a MIMO frequency-selective channel.

Traditional space—time codes were designed to extract spatial diversity from a flat-fading MIMO channel, and are not generally effective at extracting the additional frequency (or multipath) diversity of a frequency-selective fading channel. Quantitatively, the maximum achievable diversity order is the product of the number of transmit antennas, the number of receiver antennas, and the number of resolvable propagation paths (i.e., the channel impulse response length). To achieve this full diversity requires that the information symbols be carefully spread over the tones as well as over the transmitting antennas. A space-frequency code—ormore generally, space—time-frequency code is a strategy for mapping information symbols to antennas and tones as a means for extracting both spatial and frequency diversity.

Space-frequency codes based directly on space-time codes (with time reinterpreted as frequency) have been proposed but they fail to exploit the frequency diversity of a frequency-selective fading MIMO channel. Guidelines for the design of full-diversity space-frequency codes are given.

A simple method for transforming any full-diversity space—time code into a full-diversity space-frequency code has recently been proposed, at the expense of a reduced rate. An example of a space-frequency code that achieves full spatial and frequency diversity is given. The design of space-frequency and space—time-frequency codes is currently an active area of research.

Multicarrier Delay Diversity Modulation

Delay diversity was the first transmit diversity approach for flat-fading MIMO channels. Multiple transmit antennas send delayed copies of the same signal, and maximum-likelihood sequence estimation or decision-feedback equalization is used at the receiver to estimate the transmitted sequence.MDDM is further investigated with space—time block coding.

Moreover, MDDM provides a very flexible space-time coding approach for any number of transmit antennas, allowing the number of transmit antennas to be changed without changing the codes that are employed, unlike STBC. Fig.3 shows the baseband MDDM transmitter with transmit antennas a length- sequence.

Closed-Loop MIMO-OFDM

The Hermitian transpose, where and have orthogonal columns, and where is a diagonal matrix whose diagonal entries are the non-negative singular values that are ordered

from largest to smallest. A capacity-approaching transmitter will then implement Eigen beamforming by applying the linear filter to symbol vectors before transmission.

A receiver matched to the cascade of this prefilter and the channel will essentially apply the filter to the received vector, which transforms the flat-fading channel into a bank of independent scalar channels.

The problem has, thus, been reduced to one of communication across a bank of independent parallel scalar subchannels, where the subchannel gains are nonnegative, not increasing singular values of the channel. An MIMO-OFDM system with Eigen beamforming is illustrated for the special case of two transmit and two receive antennas.

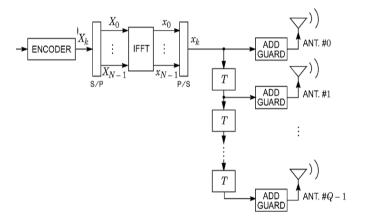


Figure 3: Multicarrier Delay Diversity Modulation

The prefilters and post filters are related to the 2-by-2 matrix of channel gains for the nth tone. Ideally, information bits (constellation size) and symbol energy would be allocated to the subchannels so as to minimize the overall SNR requirement, subject to a target bit rate. (Alternatively, the bits and energy could be allocated so as to maximize the bit rate, subject to a target energy constraint.) Unfortunately, the complexity of an exhaustive search for the bit-allocation is prohibitive when the number of subchannels is large. In a practical MIMO-OFDM application, the number of subchannels can be very large, which motivates a search for low-complexity bit-allocation strategies with near-optimal performance.

Feedback-based communication

This dissertation studies several aspects of feedback-based communication with multiple antennas, such as the estimation of the Channel State Information (CSI), the quantization of the CSI with a finite number of bits to enable its feedback to the transmitter, as well as the effect of errors in the feedback channel on the performance of the communication system. Channel estimation is doubly important in feedback-based communication because inaccurate CSI affects not only the

receiver performance, but also results in suboptimal transmission.

In this context, Multiple Input Multiple Output (MIMO) flat-fading channel estimation when the transmitter employs Maximum Ratio Transmission (MRT) is studied. Hence, the design and analysis of quantizers for Equal Gain Transmission (EGT) systems with finite rate feedback-based communication in flat-fading Multiple Input Single Output (MISO) systems is considered. Two popular approaches for quantizing the phase angles are contrasted: vector quantization (VQ) and scalar quantization (SQ).

Closed-form expressions are derived for the performance of quantized feedback in terms of capacity loss and outage probability in the case of i.i.d. Rayleigh flat-fading channels. In the work described above, the feedback channel is assumed to be free of delay and noise. With the view to understand the effect of errors on quantization, this dissertation considers the more general problem of characterizing the high-rate performance of source coding for noisy discrete symmetric channels with random index assignment

FLOW DIAGRAM- PROPOSED METHOD

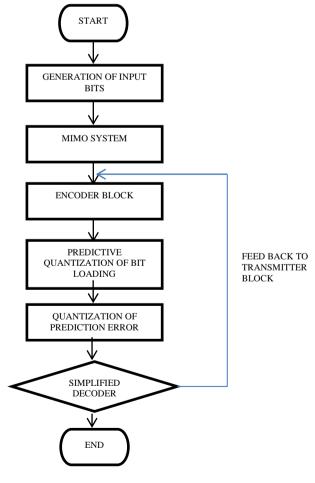


Figure 4: Proposed Approch Flowchart

ERROR CORRECTION CODING FOR MIMO-OFDM

Theoretical expressions for the performance of source coding for noisy channels are derived from a large class of distortion measures. The theoretical expressions are used to derive new results for two specific applications. The first is the quantization of the CSI for MISO systems with beamforming at the transmitter. The second application is in the wideband speech compression problem, i.e., that of quantizing the linear predictive coding parameters in speech coding systems with the log spectral distortion as a performance metric. There are many possible error control strategies in MIMO-OFDM systems. In this section, we highlight some of the methods that have been proposed. This is a very active and rapidly evolving research area. Asin any system there are important performance-complexity tradeoffs. Furthermore, depending on the application, the desired BER may result in the need for only minimal or no error correction; namely, the modulation code might be sufficient to provide the needed BER, for example, 10. However, if either near capacity performance or a very low BER is required, a powerful error control code is needed. From this perspective, much of the recent work has focused on the use of iteratively decodable codes such as turbo codes and low-density parity check (LDPC) codes. As turbo codes are special cases of LDPCs, we focus on those. Also, we start our description with single-input single-output (SISO) channels as many practical error control strategies for MIMO systems will be designed for SISO channels and then mapped to MIMO channels.

A. Convolutional encoding

Start with 'k' memory registers, each holding '1' input bit for convolutionally encoding data. Or otherwise specified, all the memory registers intiated with a value of 0. The encoder has n- modulo-2 adders (a modulo 2 adder can be implemented with a single BooleanXOR gate, where the logic is: 0+0=0, 0+1=1, 1+0=1, 1+1=0), and generator polynomials — one for each adder (see figure below). An input bit m_1 is fed into the leftmost register. Using the generator polynomials and the existing values in the remaining registers, the encoder outputs n symbols. These symbols may be transmitted or punctured depending on the desired code rate. Now bit shift all register values to the right (m_1 moves to m_0 , m_0 moves to m_{-1}) and wait for the next input bit. If there are no remaining input bits, the encoder continues shifting until all registers have returned to the zero state (flush bit termination).

The diagram shown down is a rate 1/3 (m / n) encoder with constraint length (k) of 3. Generator polynomials are $G_1 = (1,1,1)$, $G_2 = (0,1,1)$, and $G_3 = (1,0,1)$. Therefore, result bits are obtained(modulo 2) as follows:

$$\begin{split} &n_1 = m_1 + m_0 + m_{\,\text{-}1} \\ &n_2 = m_0 + m_{\,\text{-}1} \end{split}$$

 $n_3 = m_1 + m_{-1}$.

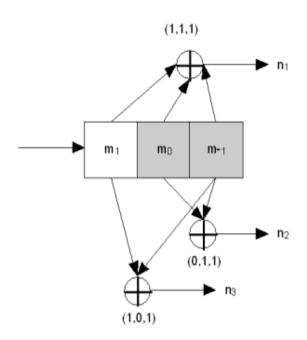


Figure 5: Rate 1/3 non-recursive, non-systematic convolutional encoder with constraint length 3.

The given encoder is systematic, here the input data is used in the output symbols (Output 2). Codes with output symbols that do not include the input data are called anti-systematic.

B. Trellis diagram

Assume that the encoder has '1' in the left memory cell (m_0) , and '0' in the right one $(m_{\text{-}1})$. We will designate such a state as "10". Considering an input bit, the encoder at the next turn can change either to the "01" state or the "11" state.All possible transitions can be shown as below:

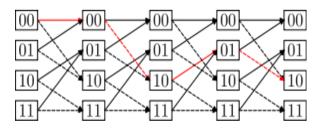


Figure 6: Trellis diagram

A trellis diagram for the encoder on fig.7. A path through the trellis is shown as a red line. The solid lines indicate that the transitions where occurs "0" is input and the dash lines where occurs "1" is input.In this graph,an actual encoded sequence can be noted as path and one valid path is shown in red as an example.

C. Free distance and error distribution

The free distance (d) is the minimal Hamming distance between different encoded sequences.

$$t = \left[\frac{d-1}{2}\right].$$

The multiple groups of 't' errors can be fixed when they are far apart. Free distance can be interpreted as the length of an erroneous burst at the end of a convolutional decoder. The fact that faults appear as burst should be noted for when creating a concatenated code with an inner part of convolutional code.

D. Decoding convolutional codes

Several algorithms exist for decoding convolutional codes. Consider small values of k, the Viterbi algorithm is totally used as it provides maximum likelihood performance and is highly parallelized. The existing implementation of the functionality supports up to 13 bits of quantization, the range can be set up to 13. For reference, 3 bits of quantization is about 2 dB better than hard decision decoding.

SIMULATION RESULT

In this work, the concept of fading is demonstrated clearly and the results obtained from the MATLAB simulations. The Radio frequency signals with standard statistical properties can be simulated readily. The testing can be used to establish the durability of the fading models frequently used in the wireless medium. Simulink is a part of MATLAB used for graphical extension Simulink is linked internally with MATLAB and data can be interchanged between the Programs.

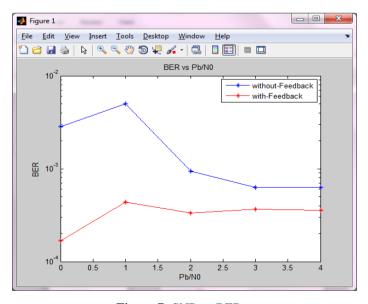


Figure 7: SNR vs BER

BER has been calculated by comparing the uploaded signal with the downloaded signal and compute the error count over the total number of bits. With this graph, we can understand the comparison between conventional work with proposed work its giving better performance than conventional work.

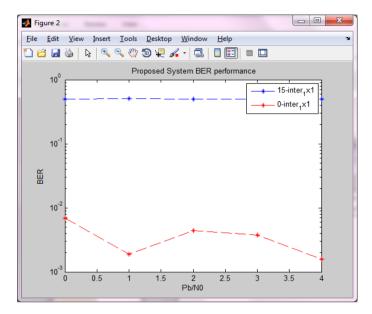


Figure 8: SNR vs BER with various noise interference

The Bit Error Rate is measured by the total number of bits received in error divided by the total number of bits transmitted. BER= (Bits in Error) / (Total bits received). In transmission, the total number of bit errors is the total number of received bits of a data stream over a communication channel that has been altered due to the unwanted signal. The Bit Error Rate is the ratio between the number of bit errors to the total number of transferred bits during a particular duration. The accuracy of the analog modulation process and the effects of the filtering on signal and noise bandwidth also effect quantization errors (see Figure 7.)

In this simulation,(see Figure 8) the BERs are obtained by varying the values of Eb/No in the range of 1 to 10.Although for this case (Eb/No) is increasing linearly but the BER is almost not varying for different values of Eb/No. Similarly, the simulation result for evaluation on BER vs. SNR for AWGN channel for 2 users when the number of data bits are 300 for QPSK modulation shows that when the number of the user is increased average bit error rate is almost same as for the single user.

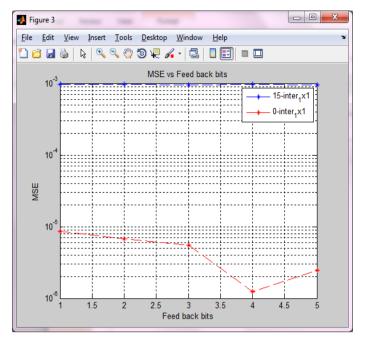


Figure 9: Feedback bits vs MSE

Figure 9 shows the rate of feedback and the corresponding achieved throughput with an average SNR of 20 dB. As seen from Fig. 4.3, the throughputs with this proposed system present a rising tendency along with the increase of feedback rate. When the feedback rate is near 0.33 bits/subcarrier, the throughputs with CS approach the ideal throughputs. Comparatively, the throughputs with proposed system the ideal throughputs when the feedback rate is near 0.56 bits/subcarrier. Consequently, proposed system has better throughput performance than conventional work with the same feedback rate.

CONCLUSION

In this work, we consider a variable-rate MIMO system with a decision feedback receiver. The bit loading is dynamically assigned to the subchannels as per antenna rate control so that the transmission rate is adapted according to the current channel condition. Predictive quantization, which is known to be very useful for coding correlated signals, is used to quantize the bit loading for feedback. Using the update, we derive the optimal predictor of the next bit loading in a closedform. The statistics of the prediction error have also been derived and exploited in the design of the quantizer to achieve a smaller quantization error. The modifications are done in the decoding process to simplify and enhance the low feedback rate than the previous work. Simulations are given to demonstrate that the proposed predictive quantization gives a good approximation of the desired transmission rate to be faster with a low feedback rate.

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