

Optimal Power Allocation for Two-Way Decode-and-Forward OFDM Relay Networks

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Abstract—This paper investigates the achievable rate of two-way Orthogonal Frequency Division Multiplexing (OFDM) relay network, in which two terminal nodes exchange information via a single relay node. A novel multi-carrier two-way decode-and-forward (DF) scheme is proposed, which introduces cross-subcarrier channel coding to exploit frequency selective fading, and achieves higher data rate than individual channel coding for each subcarrier. We further study the optimal power allocation to maximize the exchange rate defined for symmetric data traffic. The optimal power allocation is obtained by a dual decomposition algorithm, whose complexity goes in the order of $O(N)$, where N is the number of subcarriers. Simulation results suggest that our scheme obtains substantial gains over conventional per-subcarrier DF scheme, and outperforms the amplify-and-forward (AF) scheme in a wider signal-to-noise-ratio (SNR) region.

I. INTRODUCTION

In recent years, relaying has emerged as a powerful technique to improve the coverage and throughput of wireless networks. Compared with the traditional one-way relaying, two-way relaying provides better spectral efficiency, where a relay establishes communication links between two terminals for simultaneous information exchange [1], [2].

Orthogonal Frequency Division Multiplexing (OFDM) is an essential broadband transmission technique to improve the spectral efficiency of wireless networks. A combination of OFDM and relaying techniques has been advocated by many industry standardization groups of next generation wireless networks, such as IEEE 802.16m and 3GPP's LET-Advanced.

In one-way OFDM relay networks, joint subcarrier pairing and power allocation was considered to improve the throughput of decode-and-forward (DF) scheme [3]–[5]. For two-way OFDM relay networks, the amplify-and-forward (AF) schemes were commonly adopted [6]–[9]. However, their performance is quite poor in the low signal-to-noise-ratio (SNR) region. Contrary to the AF schemes' popularity, only a little attention has been paid to the DF scheme for two-way OFDM relaying. To the best of our knowledge, [10] first explored the two-way DF OFDM relaying along with other transmission modes. A

two-way DF OFDM relaying scheme with private information for relay was proposed in [11], and another one using physical-layer network coding was proposed in [12]. Unfortunately, these works only consider individual DF relaying for each subcarrier, i.e., they were essentially simple accumulations of narrow-band two-way DF relay schemes, or *per-subcarrier* DF schemes.

In this paper, we propose a novel *multi-carrier DF* scheme for two-way OFDM relay networks. Channel coding across subcarriers is introduced to take full advantage of frequency selective fading. Our scheme achieves higher data rate than the scheme with individual channel coding for each subcarrier [10]. We further formulate a power allocation problem to maximize the *exchange rate*, which is defined as the performance measurement of this scheme with symmetric data traffic in both ways. An efficient dual decomposition algorithm is proposed for this problem, which has a linear complexity with respect to the number of subcarriers. Simulation results show that our proposed DF scheme obtains a coding gain comparing with the conventional per-subcarrier DF schemes, and outperforms the AF scheme in a wider SNR region.

II. SYSTEM DESCRIPTION

In this paper, we consider a two-way relaying scenario as shown in Fig. 1: a terminal T_1 exchanges information with a terminal T_2 via a relay terminal T_R . Assume that each of terminals has a single antenna and operates in a half-duplex mode, i.e., transmitting and receiving in orthogonal time or frequency, which is more practical than the full-duplex assumption [1], [2]. All terminals employ OFDM air interface with N subcarriers. In the n th subcarrier, h_{1n} and h_{2n} denote the instantaneous channel coefficients from T_1 and T_2 to T_R , respectively, \tilde{h}_{1n} and \tilde{h}_{2n} denote the instantaneous channel coefficients from T_R to T_1 and T_2 , respectively.

We focus on a *two-phase DF* scheme comprising of a multiple-access (MAC) phase and a broadcast (BC) phase without direct transmissions. In the MAC phase as shown in Fig. 1(a), T_1 and T_2 transmit information X_1 and X_2 simultaneously, and the relay T_R performs multi-user detection to full decode \hat{X}_1 and \hat{X}_2 from the received Y_R ; in the BC phase as shown in Fig. 1(b), the relay T_R re-encodes $X_R = f(\hat{X}_1, \hat{X}_2)$ and broadcasts it to T_1 and T_2 . With

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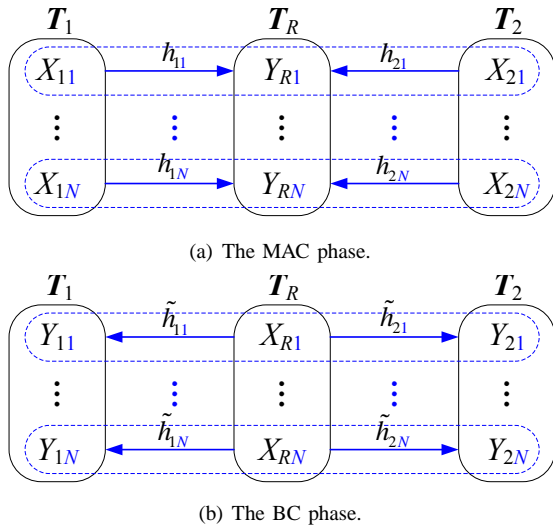


Fig. 1. System model of two-way OFDM relay network.

the help of the stored self-information, T_1 retrieves desired information X_2 from the received Y_1 with X_1 . Similarly, T_2 retrieves desired information X_1 from the received Y_2 with X_2 . When we consider a network-coding-like operation at T_R [13] and symmetric data traffic in both ways, an *exchange rate* $R_X \triangleq \min\{R_{12}, R_{21}\}$ is a relevant throughput measurement, where R_{12} and R_{21} denote the achievable data rates from T_1 to T_2 and from T_2 to T_1 , respectively. We assume that $\mu \in (0, 1)$ denotes the fixed proportion of time or frequency dimension allocated to the MAC phase.

The received signals $Y_{in}s$ in the n th subcarrier at T_i s ($i = 1, 2, R$) are given as

$$Y_{Rn} = \sqrt{P_{1n}}h_{1n}X_{1n} + \sqrt{P_{2n}}h_{2n}X_{2n} + Z_{Rn} \quad (1)$$

$$Y_{1n} = \sqrt{P_{Rn}}\tilde{h}_{1n}X_{Rn} + Z_{1n} \quad (2)$$

$$Y_{2n} = \sqrt{P_{Rn}}\tilde{h}_{2n}X_{Rn} + Z_{2n}, \quad (3)$$

where X_{in} and P_{in} denote the sent signal and its power in the n th subcarrier at T_i , and Z_{in} denotes independent complex *additive white Gaussian noises* with zero mean and unit variance, i.e., $Z_{in} \sim \mathcal{CN}(0, 1)$.

We assume that all terminals are subject to practical separate power constraints $\sum_{n=1}^N P_{in} \leq P_{itot}$ ($i = 1, 2, R$) instead of a total network power constraint in [9], [12], where P_{itot} denotes the maximum available power for T_i . Assume the channel coefficient of each subcarrier is constant over an OFDM frame so that centralized power allocation is feasible. T_R performs power allocation after the channel estimation, then broadcasts the transmission strategy to T_1 and T_2 . In this paper, we assume perfect channel estimation.

III. A NOVEL DF SCHEME FOR TWO-WAY OFDM RELAY NETWORKS

For two-way OFDM relay networks, conventional DF schemes simply applied the narrow-band DF technique in each subcarrier independently, and the overall throughput was the sum rate of all the subcarriers [10], [12]. As we shown later

in this section, these schemes suffer from rate losses due to channel mismatching.

In this section, we propose a novel *multi-carrier* DF scheme, which takes all subcarriers as a whole instead, to exploit the benefit of frequency selective fading and achieve higher throughput. We provide an achievable rate region for this scheme in the following theorem.

Theorem 1. A set of achievable rate pairs (R_{12}, R_{21}) of a DF scheme for two-way OFDM relay networks is given by the closure of the following set of inequalities:

$$R_{12} \leq \mu \sum_{n=1}^N \log_2(1 + |h_{1n}|^2 P_{1n}) \quad (4)$$

$$R_{12} \leq (1 - \mu) \sum_{n=1}^N \log_2(1 + |\tilde{h}_{2n}|^2 P_{Rn}) \quad (5)$$

$$R_{21} \leq \mu \sum_{n=1}^N \log_2(1 + |h_{2n}|^2 P_{2n}) \quad (6)$$

$$R_{21} \leq (1 - \mu) \sum_{n=1}^N \log_2(1 + |\tilde{h}_{1n}|^2 P_{Rn}) \quad (7)$$

$$R_{12} + R_{21} \leq \mu \sum_{n=1}^N \log_2(1 + |h_{1n}|^2 P_{1n} + |h_{2n}|^2 P_{2n}) \quad (8)$$

$$\sum_{n=1}^N P_{in} \leq P_{itot}, \quad i = 1, 2, R \quad (9)$$

$$P_{in} \geq 0, \quad n = 1, \dots, N, \quad i = 1, 2, R. \quad (10)$$

Proof: An achievable rate region of a DF scheme for the discrete-memoryless two-way relay network has been given by the closure of the following set of inequalities [13]:

$$R_{12} \leq \min\{\mu I(X_1; Y_R | X_2), (1 - \mu) I(X_R; Y_2)\} \quad (11)$$

$$R_{21} \leq \min\{\mu I(X_2; Y_R | X_1), (1 - \mu) I(X_R; Y_1)\} \quad (12)$$

$$R_{12} + R_{21} \leq \mu I(X_1, X_2; Y_R). \quad (13)$$

Similar to the idea in the proof of [3, Theorem 1], we set $X_i = (X_{i1}, \dots, X_{iN})$ and $Y_i = (Y_{i1}, \dots, Y_{iN})$, for $i = 1, 2, R$. Therefore, each mutual information item in (11)-(13) corresponds to the achievable rate of a parallel point-to-point channel. We choose the input signals for each subcarrier to be *independent Gaussian* distributed with unit variance, i.e., $X_{in} \sim \mathcal{CN}(0, 1)$. Then, each mutual information item in (11)-(13) is replaced by the sum of N logarithmic rate items decided by (1)-(3) with separate power constraints. The proof is complete. ■

Remark 1: The key idea of this *multi-carrier* two-way DF relay scheme is introducing channel coding across subcarriers to fully exploit frequency selective fading. The information transmitted over one subcarrier in the MAC phase may be forwarded over other subcarriers in the BC phase. By this, the *mismatching* problem of wireless channels in each subcarrier is completely solved. The achievable rate region of **Theorem 1** is no smaller than that achieved by *per-subcarrier* two-way

DF relaying, which is given as [10]

$$\begin{aligned}
R_{12} &\leq \sum_{n=1}^N \min\left\{\mu \log_2(1 + |h_{1n}|^2 P_{1n}), \right. \\
&\quad \left. (1 - \mu) \log_2(1 + |\tilde{h}_{2n}|^2 P_{Rn})\right\} \\
R_{21} &\leq \sum_{n=1}^N \min\left\{\mu \log_2(1 + |h_{2n}|^2 P_{2n}), \right. \\
&\quad \left. (1 - \mu) \log_2(1 + |\tilde{h}_{1n}|^2 P_{Rn})\right\} \\
R_{12} + R_{21} &\leq \mu \sum_{n=1}^N \log_2(1 + |h_{1n}|^2 P_{1n} + |h_{2n}|^2 P_{2n})
\end{aligned}$$

Therefore, multi-carrier two-way relay channel is not a simple combination of multiple narrow-band single-carrier two-way relay subchannels. Similar observations have been found for *one-way* parallel relay networks [3].

IV. OPTIMAL POWER ALLOCATION

In this section, we investigate the largest achievable rate of our proposed DF scheme. This can be approached by an optimal power allocation.

A. Problem Formulation

Our objective is to maximize the *exchange rate* $R_X = \min\{R_{12}, R_{21}\}$, by optimizing the power allocation policy $(\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_R)$, where $\mathbf{P}_i = [P_{i1}, P_{i2}, \dots, P_{iN}]^T$ denotes the power allocation vector at T_i , for $i = 1, 2, R$. This can be expressed as the following *convex* optimization problem:

$$\max_{\mathbf{P}_1, \mathbf{P}_2, R_X} R_X \quad (14a)$$

$$\text{s.t. } R_X \leq \mu \sum_{n=1}^N \log_2(1 + |h_{1n}|^2 P_{1n}) \quad (14b)$$

$$R_X \leq (1 - \mu) \sum_{n=1}^N \log_2(1 + |\tilde{h}_{2n}|^2 P_{Rn}) \quad (14c)$$

$$R_X \leq \mu \sum_{n=1}^N \log_2(1 + |h_{2n}|^2 P_{2n}) \quad (14d)$$

$$R_X \leq (1 - \mu) \sum_{n=1}^N \log_2(1 + |\tilde{h}_{1n}|^2 P_{Rn}) \quad (14e)$$

$$R_X \leq \frac{\mu}{2} \sum_{n=1}^N \log_2(1 + |h_{1n}|^2 P_{1n} + |h_{2n}|^2 P_{2n}) \quad (14f)$$

$$\sum_{n=1}^N P_{in} \leq P_{itot}, \quad i = 1, 2, R \quad (14g)$$

$$P_{in} \geq 0, \quad n = 1, \dots, N, \quad i = 1, 2, R. \quad (14h)$$

In the problem (14), we see that \mathbf{P}_1 and \mathbf{P}_2 are only related to the constraints (14b) (14d) (14f), while \mathbf{P}_R is only related to the constraints (14c) (14e). This observation helps to decompose our original power allocation problem (14) into

the following two subproblems:

$$\begin{aligned}
&\max_{\mathbf{P}_1, \mathbf{P}_2, R_{MAC}} R_{MAC} \quad (15) \\
&\text{s.t. } R_{MAC} \leq \mu \sum_{n=1}^N \log_2(1 + |h_{1n}|^2 P_{1n}) \\
&\quad R_{MAC} \leq \mu \sum_{n=1}^N \log_2(1 + |h_{2n}|^2 P_{2n}) \\
&\quad R_{MAC} \leq \frac{\mu}{2} \sum_{n=1}^N \log_2(1 + |h_{1n}|^2 P_{1n} + |h_{2n}|^2 P_{2n}) \\
&\quad \sum_{n=1}^N P_{1n} \leq P_{1tot}, \quad \sum_{n=1}^N P_{2n} \leq P_{2tot} \\
&\quad P_{1n} \geq 0, P_{2n} \geq 0, \quad n = 1, \dots, N,
\end{aligned}$$

$$\begin{aligned}
&\max_{\mathbf{P}_R, R_{BC}} R_{BC} \quad (16) \\
&\text{s.t. } R_{BC} \leq (1 - \mu) \sum_{n=1}^N \log_2(1 + |\tilde{h}_{1n}|^2 P_{Rn}) \\
&\quad R_{BC} \leq (1 - \mu) \sum_{n=1}^N \log_2(1 + |\tilde{h}_{2n}|^2 P_{Rn}) \\
&\quad \sum_{n=1}^N P_{Rn} \leq P_{Rtot} \\
&\quad P_{Rn} \geq 0, \quad n = 1, \dots, N.
\end{aligned}$$

Denote R_{MAC}^* and R_{BC}^* as the optimal values for the MAC subproblem (15) and the BC subproblem (16), respectively. Eventually, the maximal practical exchange rate for our proposed DF scheme is obtained by $R_X^* = \min\{R_{MAC}^*, R_{BC}^*\}$.

B. Proposed Dual Decomposition Algorithm for (15)

The interior-point methods can be used to solve both of the *convex* optimization problems (15) and (16), however, they quickly become computationally intractable as N increases, because they have a $O(N^3)$ complexity at least when solving the search direction in each iteration [14]. Therefore, we present a low-complexity dual decomposition algorithm for the subproblems (15) and (16), to efficiently obtain the optimal solution to (14).

Suppose that problem (15) is strictly feasible. Then, according to the Slater's condition [14], problem (15) is equivalent with the following dual optimization problem:

$$\max_{\lambda, \alpha \geq 0} \left\{ \min_{\mathbf{P}_1, \mathbf{P}_2 \geq 0, R_{MAC}} \mathcal{L}(\mathbf{P}_1, \mathbf{P}_2, R_{MAC}, \lambda, \alpha) \right\}, \quad (17)$$

where

$$\begin{aligned}
\mathcal{L}(\mathbf{P}_1, \mathbf{P}_2, R_{MAC}, \lambda, \alpha) &= -R_{MAC} \\
&+ \lambda_1 \left[R_{MAC} - \mu \sum_{n=1}^N \log_2(1 + |h_{1n}|^2 P_{1n}) \right] \\
&+ \lambda_2 \left[R_{MAC} - \mu \sum_{n=1}^N \log_2(1 + |h_{2n}|^2 P_{2n}) \right] \\
&+ \lambda_3 \left[R_{MAC} - \frac{\mu}{2} \sum_{n=1}^N \log_2(1 + |h_{1n}|^2 P_{1n} + |h_{2n}|^2 P_{2n}) \right]
\end{aligned}$$

$$\begin{aligned}
& + \alpha_1 \left(\sum_{n=1}^N P_{1n} - P_{1tot} \right) + \alpha_2 \left(\sum_{n=1}^N P_{2n} - P_{2tot} \right) \\
& = \sum_{n=1}^N \left[\alpha_1 P_{1n} - \mu \lambda_1 \log_2(1 + |h_{1n}|^2 P_{1n}) \right. \\
& + \alpha_2 P_{2n} - \mu \lambda_2 \log_2(1 + |h_{2n}|^2 P_{2n}) \\
& \left. - \frac{\mu \lambda_3}{2} \log_2(1 + |h_{1n}|^2 P_{1n} + |h_{2n}|^2 P_{2n}) \right] \\
& + (\lambda_1 + \lambda_2 + \lambda_3 - 1) R_{MAC} - \alpha_1 P_{1tot} - \alpha_2 P_{2tot} \quad (18)
\end{aligned}$$

is the Lagrangian of (15), in which $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \lambda_3]^T$, $\boldsymbol{\alpha} = [\alpha_1, \alpha_2]^T$ are nonnegative dual variables associated with three rate constraints and two power constraints, respectively.

According to (18), the *inner level minimization* problem of (17) can be decomposed as N independent per-subcarrier power allocation problems. Hence, the computational complexity for solving the inner problem is only linear with respect to N . Each per-subcarrier power allocation problem has closed-form solutions for given dual variables $(\boldsymbol{\lambda}, \boldsymbol{\alpha})$, and the optimal (P_{1n}, P_{2n}) must satisfy the following Karush-Kuhn-Tucker (KKT) conditions [14]:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial P_{1n}} & = \alpha_1 - \frac{\mu \lambda_3 |h_{1n}|^2}{2 \ln 2 (1 + |h_{1n}|^2 P_{1n} + |h_{2n}|^2 P_{2n})} \\
& - \frac{\mu \lambda_1 |h_{1n}|^2}{\ln 2 (1 + |h_{1n}|^2 P_{1n})} \begin{cases} \geq 0 & \text{if } P_{1n} = 0 \\ = 0 & \text{if } P_{1n} > 0 \end{cases} \quad (19)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial P_{2n}} & = \alpha_2 - \frac{\mu \lambda_3 |h_{2n}|^2}{2 \ln 2 (1 + |h_{1n}|^2 P_{1n} + |h_{2n}|^2 P_{2n})} \\
& - \frac{\mu \lambda_2 |h_{2n}|^2}{\ln 2 (1 + |h_{2n}|^2 P_{2n})} \begin{cases} \geq 0 & \text{if } P_{2n} = 0 \\ = 0 & \text{if } P_{2n} > 0 \end{cases} \quad (20)
\end{aligned}$$

Thus, the optimal P_{1n} and P_{2n} must belong to one of the following four cases:

Case 1: $P_{1n} > 0, P_{2n} > 0$. Then the formulas (19) and (20) hold with equality. It is hard to solve (19) and (20) directly since they are both *quadratic* equations of two variables P_{1n} and P_{2n} . However, we can utilize an auxiliary variable defined as $x = |h_{1n}|^2 P_{1n} + |h_{2n}|^2 P_{2n}$ to simplify them. More specifically, one can simply obtain from (19) and (20) that

$$|h_{1n}|^2 P_{1n} = \frac{2\mu\lambda_1|h_{1n}|^2}{2\ln 2 \cdot \alpha_1 - \mu\lambda_3|h_{1n}|^2/(1+x)} - 1 \quad (21)$$

$$|h_{2n}|^2 P_{2n} = \frac{2\mu\lambda_2|h_{2n}|^2}{2\ln 2 \cdot \alpha_2 - \mu\lambda_3|h_{2n}|^2/(1+x)} - 1. \quad (22)$$

Taking the sum of these two equations, we obtain a *cubic* equation of x , which has closed solutions given by *Cardano's Formula* [15, Chapter 6]. After deriving the positive root x of this *cubic* equation, we can easily obtain the optimal P_{1n} and P_{2n} from (21) and (22). By this means, the quadratic equations (19) and (20) are solved analytically by converting to an equivalent *cubic* equation. Our closed-form solution is much simpler than ...[?]. Finally, we need to check whether P_{1n} and P_{2n} satisfy the conditions $P_{1n} > 0, P_{2n} > 0$.

Case 2: $P_{1n} > 0, P_{2n} = 0$. Then the solutions to (19) and (20) can be derived as

$$P_{1n} = \frac{\mu(2\lambda_1 + \lambda_3)}{2 \ln 2 \cdot \alpha_1} - \frac{1}{|h_{1n}|^2} \quad (23)$$

$$P_{2n} = 0. \quad (24)$$

Note that this case happens only if $P_{1n} > 0$ and the KKT condition (20) $2 \ln 2 \cdot \alpha_2 \geq 2\mu\lambda_2|h_{2n}|^2 + \frac{\mu\lambda_3|h_{2n}|^2}{1+|h_{1n}|^2P_{1n}}$ is satisfied.

Case 3: $P_{1n} = 0, P_{2n} > 0$. Then the KKT conditions can be reformulated as

$$P_{1n} = 0 \quad (25)$$

$$P_{2n} = \frac{\mu(2\lambda_2 + \lambda_3)}{2 \ln 2 \cdot \alpha_2} - \frac{1}{|h_{2n}|^2}. \quad (26)$$

This case happens only if $P_{2n} > 0$ and the KKT condition (19) $2 \ln 2 \cdot \alpha_1 \geq 2\mu\lambda_1|h_{1n}|^2 + \frac{\mu\lambda_3|h_{1n}|^2}{1+|h_{2n}|^2P_{2n}}$ is satisfied.

Case 4: $P_{1n} = 0, P_{2n} = 0$. This is the default case when the above three cases do not happen.

Then, we optimize the dual variables $(\boldsymbol{\lambda}, \boldsymbol{\alpha})$ for the *outer level maximization* problem of (17). According to the KKT condition for the optimal data rate R_{MAC} , we have

$$\frac{\partial \mathcal{L}}{\partial R_{MAC}} = -1 + \lambda_1 + \lambda_2 + \lambda_3 = 0. \quad (27)$$

Therefore, we can define $\boldsymbol{\nu} = [\lambda_1, \lambda_2, \alpha_1, \alpha_2]^T$, while λ_3 can be calculated by $\lambda_3 = 1 - \lambda_1 - \lambda_2$ from (27). In view of that the objective function may not be differentiable with respect to $(\boldsymbol{\lambda}, \boldsymbol{\alpha})$, we consider to update $\boldsymbol{\nu}$ using the subgradient method [16], [17]. Specifically, in the k th iteration, the subgradient method updates $\boldsymbol{\nu}^k$ by

$$\boldsymbol{\nu}^{k+1} = [\boldsymbol{\nu}^k + s^k \boldsymbol{\eta}(\boldsymbol{\nu}^k)]_+, \quad (28)$$

where $[\boldsymbol{\nu}]_+$ represents the projection of $\boldsymbol{\nu}$ to the dual feasible set $\{\lambda_1 + \lambda_2 \leq 1, \lambda_1, \lambda_2, \alpha_1, \alpha_2 \geq 0\}$ [16], s^k is the step size of the k th iteration, and $\boldsymbol{\eta}(\boldsymbol{\nu}^k)$ is the subgradient of the inner problem of (17) at $\boldsymbol{\nu}^k$, which can be chosen as

$$\boldsymbol{\eta}(\boldsymbol{\nu}^k) = \begin{bmatrix} \frac{\mu}{2} \sum_{n=1}^N \log_2(1 + |h_{1n}|^2 P_{1n}^* + |h_{2n}|^2 P_{2n}^*) \\ - \mu \sum_{n=1}^N \log_2(1 + |h_{1n}|^2 P_{1n}^*) \\ \frac{\mu}{2} \sum_{n=1}^N \log_2(1 + |h_{1n}|^2 P_{1n}^* + |h_{2n}|^2 P_{2n}^*) \\ - \mu \sum_{n=1}^N \log_2(1 + |h_{2n}|^2 P_{2n}^*) \\ \sum_{n=1}^N P_{1n}^* - P_{1tot} \\ \sum_{n=1}^N P_{2n}^* - P_{2tot} \end{bmatrix}, \quad (29)$$

where P_{1n}^* and P_{2n}^* are the optimal solution of the inner minimization problem in the k th iteration. It has been shown that the subgradient updates in (28) can converge to the optimal dual point $\boldsymbol{\nu}^*$ as $k \rightarrow \infty$, provided that the step size s^k is chosen according to a diminishing step size rule [17].

Denote $C_{MAC,i}(\mathbf{P}_1, \mathbf{P}_2)$ ($i = 1, 2, 3$) as the the right side of three rate constraints in (15), respectively, and then we obtain the optimal $R_{MAC}^* = \min\{C_{MAC,i}(\mathbf{P}_1^*, \mathbf{P}_2^*), i = 1, 2, 3\}$.

C. Proposed Dual Decomposition Algorithm for (16)

Similarly, we consider the following dual optimization problem for the BC subproblem (16):

$$\max_{\gamma_1, \gamma_2, \beta \geq 0} \left\{ \min_{P_R \geq 0, R_{BC}} \mathcal{L}'(\mathbf{P}_R, R_{BC}, \gamma_1, \gamma_2, \beta) \right\}, \quad (30)$$

where

$$\begin{aligned} \mathcal{L}'(\mathbf{P}_R, R_{BC}, \gamma_1, \gamma_2, \beta) = & -R_{BC} \\ & + \gamma_1 \left[R_{BC} - (1 - \mu) \sum_{n=1}^N \log_2 \left(1 + |\tilde{h}_{1n}|^2 P_{Rn} \right) \right] \\ & + \gamma_2 \left[R_{BC} - (1 - \mu) \sum_{n=1}^N \log_2 \left(1 + |\tilde{h}_{2n}|^2 P_{Rn} \right) \right] \\ & + \beta \left(\sum_{n=1}^N P_{Rn} - P_{Rtot} \right) \end{aligned} \quad (31)$$

is the Lagrangian of (16), in which γ_1, γ_2 and β are nonnegative dual variables associated with three constraints.

For the per-subcarrier *inner level minimization* problem of (30) with given dual variables $(\gamma_1, \gamma_2, \beta)$, the optimal P_{Rn} satisfy the following KKT condition:

$$\begin{aligned} \frac{\partial \mathcal{L}'}{\partial P_{Rn}} = & -\frac{\gamma_1(1-\mu)|\tilde{h}_{1n}|^2}{\ln 2(1+|\tilde{h}_{1n}|^2 P_{Rn})} - \frac{\gamma_2(1-\mu)|\tilde{h}_{2n}|^2}{\ln 2(1+|\tilde{h}_{2n}|^2 P_{Rn})} \\ & + \beta \begin{cases} \geq 0 & \text{if } P_{Rn} = 0 \\ = 0 & \text{if } P_{Rn} > 0 \end{cases} \end{aligned} \quad (32)$$

If $P_{Rn} > 0$, the optimal value of P_{Rn} is given by the positive root x of the quadratic equation (33); otherwise, $P_{Rn} = 0$. Here, the quadratic equation (33) is expressed as

$$\frac{\gamma_1 |\tilde{h}_{1n}|^2}{1 + |\tilde{h}_{1n}|^2 x} + \frac{\gamma_2 |\tilde{h}_{2n}|^2}{1 + |\tilde{h}_{2n}|^2 x} = \frac{\ln 2 \cdot \beta}{1 - \mu}. \quad (33)$$

For the *outer level maximization* problem of (30), considering the KKT condition for the optimal data rate R_{BC}

$$\frac{\partial \mathcal{L}'}{\partial R_{BC}} = -1 + \gamma_1 + \gamma_2 = 0, \quad (34)$$

we define $\mathbf{v} = [\gamma_1, \beta]^T$, and calculate γ_2 by $\gamma_2 = 1 - \gamma_1$ from (34). In the k th iteration, the subgradient method updates \mathbf{v}^k by

$$\mathbf{v}^{k+1} = [\mathbf{v}^k + s^k \boldsymbol{\xi}(\mathbf{v}^k)]_+, \quad (35)$$

where $[\mathbf{v}]_+$ represent the projection of \mathbf{v} to the dual feasible set $\{0 \leq \gamma_1 \leq 1, \beta \geq 0\}$, $\boldsymbol{\xi}(\mathbf{v}^k)$ is the subgradient of the inner problem of (30) at \mathbf{v}^k , which is given by

$$\boldsymbol{\xi}(\mathbf{v}^k) = \begin{bmatrix} (1 - \mu) \sum_{n=1}^N \log_2 \frac{1 + |\tilde{h}_{2n}|^2 P_{Rn}^*}{1 + |\tilde{h}_{1n}|^2 P_{Rn}^*} \\ \sum_{n=1}^N P_{Rn}^* - P_{Rtot} \end{bmatrix}, \quad (36)$$

where P_{Rn}^* is the optimal power allocated for the n th subcarrier of the *inner level minimization* problem of (30) in the k th iteration.

Denote $C_{BC,i}(\mathbf{P}_R)$ ($i = 1, 2$) as the the right side of two rate constraints in (16), respectively, and then we obtain the optimal $R_{BC}^* = \min\{C_{BC,i}(\mathbf{P}_R^*), i = 1, 2\}$.

The proposed dual decomposition algorithms for subproblems (15) and (16) are summarized in **Algorithm 1** and **Algorithm 2**, respectively. Their complexity grow in the order of $O(N)$, which are much lower than the classic convex optimization software package based on interior-point methods. Therefore, our proposed algorithm is more favorable for large value of N , which is quite typical in OFDM systems.

Algorithm 1 Proposed dual decomposition algorithm for (15)

- 1: **Input** the system parameters (N, P_{1tot}, P_{2tot}) , the channel quality $\{h_{1n}, h_{2n}\}_{n=1}^N$ and a solution accuracy ϵ .
 - 2: Set the iteration number $k=1$; initialize dual variables $\boldsymbol{\nu}^1$.
 - 3: Compute the optimal $\{P_{1n}, P_{2n}\}_{n=1}^N$ according to (19) and (20).
 - 4: Update the dual variable $\boldsymbol{\nu}^{k+1}$ according to (28).
 - 5: If $\|\boldsymbol{\nu}^{k+1} - \boldsymbol{\nu}^k\|_2 \leq \epsilon$, go to Step 6; otherwise, set $k = k+1$ and return to Step 3.
 - 6: **Output** the optimal primal solution $\{P_{1n}^*, P_{2n}^*\}_{n=1}^N$ and $R_{MAC}^* = \min\{C_{MAC,i}(\mathbf{P}_1^*, \mathbf{P}_2^*), i = 1, 2, 3\}$.
-

Algorithm 2 Proposed dual decomposition algorithm for (16)

- 1: **Input** the system parameters (N, P_{Rtot}) , the channel quality $\{\tilde{h}_{1n}, \tilde{h}_{2n}\}_{n=1}^N$ and a solution accuracy ϵ .
 - 2: Set the iteration number $k=1$; initialize dual variables \mathbf{v}^1 .
 - 3: Compute the optimal $\{P_{Rn}\}_{n=1}^N$ according to (32).
 - 4: Update the dual variable \mathbf{v}^{k+1} according to (35).
 - 5: If $\|\mathbf{v}^{k+1} - \mathbf{v}^k\|_2 \leq \epsilon$, go to Step 6; otherwise, set $k = k+1$ and return to Step 3.
 - 6: **Output** the optimal primal solution $\{P_{Rn}^*\}_{n=1}^N$ and $R_{BC}^* = \min\{C_{BC,i}(\mathbf{P}_R^*), i = 1, 2\}$.
-

V. SIMULATION RESULTS

We consider an OFDM system with $N = 32$ subcarriers. The frequency-domain channels are generated using 8 i.i.d Rayleigh distributed time-domain taps with unit variance [6]. The separate power constraints are set as $P_{1tot} = P_{2tot} = P_{Rtot}$, and $\mu = 0.5$.

Our proposed two-way DF OFDM relaying scheme is denoted as ‘‘Type 1 DF’’ scheme. Two reference schemes are considered in our simulations: The first one is *per-subcarrier* two-way DF OFDM relaying scheme in [10], which is denoted as ‘‘Type 2 DF’’ scheme; the second reference scheme is the two-way AF OFDM relaying scheme with optimized tone permutation in [6].

We divide the sum rate (corresponding to $2R_X$ in Type 1 DF scheme) by N and use this per-subcarrier sum rate to evaluate performance at different average SNRs. Fig. 2 presents the performance of different two-way OFDM relaying schemes. The best performance is given by Type 1 DF scheme with optimal power allocation (PA). At the spectral efficiency of 2 bits/s/Hz, Type 1 DF scheme with optimal PA provides a coding gain of about 2.5 dB compared with Type 2 DF scheme, by allowing channel coding across subcarriers. The PA gain between optimal PA and uniform PA of Type 1 DF scheme is

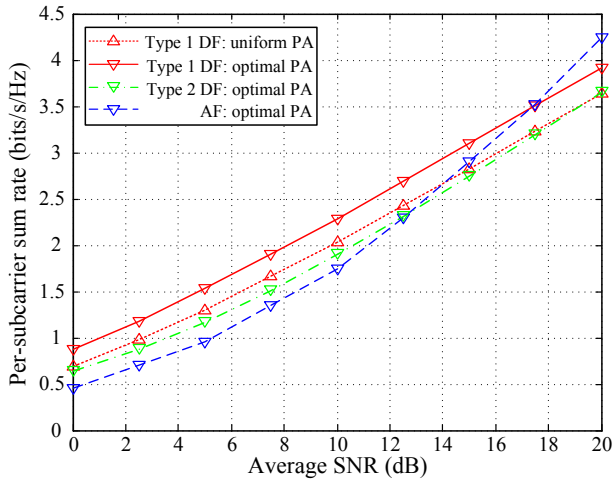


Fig. 2. Per-subcarrier sum rate of different two-way OFDM relaying schemes.

given by 2 dB. It is interesting that Type 1 DF scheme with uniform PA even outperforms Type 2 DF scheme with optimal PA, when the average SNR is in the region [0 dB, 20 dB].

Type 1 DF scheme outperforms the AF scheme in the low and medial SNR region. The intersection of the curves for Type 1 DF scheme and the AF scheme is at about 17.5 dB, which is 5 dB higher than that for Type 2 DF scheme and the AF scheme.

VI. CONCLUSION

We have proposed a novel DF scheme for two-way OFDM relay networks and derived its achievable rate region. The key idea is making use of cross-subcarrier channel coding to fully exploit frequency selective fading. An efficient duality-based power allocation algorithm is proposed to maximize the exchanging data rate. Our simulation results suggest that the proposed DF scheme has better performance than existing DF or AF two-way OFDM relaying schemes in the low SNR region. We believe this two-way DF scheme tends to be optimal, i.e., achieving the capacity region outer bound, in the low SNR region. The optimality of the proposed two-way DF scheme and the effect of channel uncertainty are currently under our investigation.

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