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Research Article

Distributed Formation Control for Multiple Quadrotor Based on Multi-Agent Theory and Disturbance Observer

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This paper presents the disturbance observers-based distributed formation control for multiple quadrotor aircrafts with external disturbances and uncertain parameters using multi-agent theory and finite-time control method. Firstly, the finite-time disturbance observers are proposed to handle the external disturbances on the position-loop. Similarly, when there are both the uncertain parameters and external disturbances on the attitude-loop, the finite-time disturbance observers are designed to estimate the total lump disturbances. By skillfully using homogeneous system theory, Lyapunov theory, and multi-agent theory, the distributed formation control algorithms are developed. Finally, through simulations, the efficiency of the proposed method (including the convergence rate and disturbance rejection) is verified.

1. Introduction

With the development and application of the quadrotor aircraft, nowadays, the formation control problem for multiple quadrotor aircrafts has become a hot topic and attracted a great deal of attention. Compared to the fixed-wing aircraft, the quadrotor aircraft has many different features, such as simple structures, easy maintenance, and maneuverability, etc [1]. Due to these advantages, the quadrotor aircraft has been widely used in the civilian field and the military field, such as aerial photography, geological survey, disaster relief, pipeline inspection, environmental assessment, etc. [2, 3]. Meanwhile, in some complex and specific environments, multiple quadrotor aircrafts have more advantages than a single aircraft. For example, multiple aircrafts can significantly improve the efficiency and the robustness of the whole system. In addition, more aircraft collaboration means more different equipment that can be carried on to accomplish more difficult tasks. From the control point of view, it is quite difficult and challenging to realize the formation control of multiple aircrafts because it is necessary to further coordinated control of multi-aircraft on the basis of controlling a single aircraft. Actually, even the design of control system and the analysis of stability for single quadrotor aircraft are also challenging since the model has highly nonlinear and strong coupling [4].

Up to now, a series of achievements have been made in the formation control problem. For the heterogeneous multi-agent systems with nonlinear dynamics, the leader-following consensus problem has been considered in [5]. The work [6] studied a novel reliable consensus for uncertain nonlinear multi-agent systems in the presence of probabilistic time-varying delay. This article considers the leader-following consensus problem of heterogeneous multi-agent systems The time-varying formation tracking control has been realized for quadrotor aircrafts [7, 8]. Based on complex Laplacian, the work [9] studied the distributed formation control problem for multi-agent systems. Considering sampled-data and communication delays, the distributed formation controllers have been designed for mobile robots [10]. The work [11] discussed the problem of vision-based leader-follower formation control for mobile robots.

However, note that most of the existing control algorithms are required to satisfy the Lipschitz continuity condition. In other words, the desired formation will be achieved at best exponential with an infinite time. As we all know, for the quadrotor aircraft, whether it is multiple formation control or single control, it is hoped that the convergence speed will be as fast as possible. To enhance the convergence rate and the ability of disturbance rejection, recently a nonlinear control method (i.e., finite-time control method) [12–14] was introduced. Theoretically, the finite-time control algorithm can guarantee that the system will stabilize in a finite time. Firstly, there have been already some results with regard to finite-time control for a single quadrotor aircraft [15–22]. Moreover, for the single quadrotor aircraft during the flight, the external disturbances are often unavoidable, such as wind disturbances. Some theoretical and experimental analysis were given in [13, 18, 23], to explain why finite-time control can offer better ability of disturbance rejection. Compared to the study of a single quadrotor aircraft, the results of finite-time research on multiple quadrotor aircrafts, especially the results of finite-time formation control, are still limited. Most of the existing finite-time formation control research results are mainly for multi-agents [24–26], mobile robots [27], etc. The high dimension and nonlinear coupling of multiple quadrotor aircraft systems make the controller design and stability analysis of the closed-loop system more difficult.

Based on the advantages of finite-time control, this paper will employ it to design a distributed formation control for multiple quadrotor aircrafts. It is worth noting that the results in most of the previous papers on formation control problems did not consider the effect of external disturbances, such as payload changes (or mass changes), wind disturbance, inaccurate model parameters, and so on. The robustness is an important aspect in the controlling of quadrotors. To improve the system dynamical performances, the finite-time position and attitude controllers are proposed for the nominal systems based on homogeneous systems theory. A quadrotor controller must be robust enough in order to reject the effect of disturbances and cover the change in model parameter uncertainties and external disturbances. To handle the external disturbances for the position-loop subsystem, the finite-time disturbance observers are deigned to estimate the disturbances in a finite time. Similarly, when there are both the parameter uncertainties and external disturbances for the attitude-loop subsystem, the finite-time disturbance observers are deigned to estimate the lump disturbances in a finite time. Compared with the existing results, the main contributions of this paper are as follows: (1) Construct the finite-time observers to achieve accurate estimation and compensation of disturbances. (2) Based on the finite time control technology, the system dynamical performances can be improved (i.e., faster convergence speed and better anti-interference ability). Finally, based on the finite-time disturbance observers, the finite-time distributed formation control strategy is developed for multiple quadrotor aircrafts, whose efficiency (including the convergence rate and disturbance rejection) is verified through simulations.

2. Preliminaries

2.1. Problem Description. This paper mainly studies the formation control problem of a group of four-rotor aircraft. Then define $\Gamma = \{1, 2, \ldots, n\}$. Generally speaking, the motion information to describe the an aircraft is mainly composed of six-degree-of-freedom variables, i.e., the position information and the attitude information. Specifically, the coordinates of the aircraft can be described as

$$\left(p_i, q_i, r_i, \phi_i, \theta_i, \psi_i\right)^T \in R^6, \quad i \in \Gamma, \tag{1}$$

where $\chi_i = (p_i, q_i, r_i)^T \in R^3$ denotes the position information by inertial coordinate system, and $\Phi_i = (\phi_i, \theta_i, \psi_i)^T \in R^3$ are Euler angles to describe the attitude information based on the inertial coordinate system.

2.1.1. Position Dynamical Model. From [28, 29], the position dynamical model for i-*th* quadrotor aircraft can be described as

$$\begin{aligned} \ddot{p}_i &= \frac{T_i}{m_i}\left(\cos\phi_i \sin\theta_i \cos\psi_i + \sin\phi_i \sin\psi_i\right) + \delta_{i,p}, \\ \ddot{q}_i &= \frac{T_i}{m_i}\left(\cos\phi_i \sin\theta_i \sin\psi_i - \sin\phi_i \cos\psi_i\right) + \delta_{i,q}, \\ \ddot{r}_i &= \frac{T_i}{m_i}\cos\phi_i \cos\theta_i - g + \delta_{i,r}, \quad i \in \Gamma, \end{aligned} \tag{2}$$

where m_i denotes the mass of the aircraft, $\delta_i = (\delta_{i,p}, \delta_{i,q}, \delta_{i,r})^T$ represents the unknown external disturbances, T_i is the total thrust produced by the four rotors, and g is a positive constant, i.e., the gravitational acceleration.

2.1.2. Attitude Dynamical Model. The description of i-*th* attitude dynamical equation is based on Euler angles [29], which is given as

$$\begin{aligned} J_{i,1}\ddot{\phi}_i &= l_i\tau_{i,1} + d_{i,\phi}(t), \\ J_{i,2}\ddot{\theta}_i &= l_i\tau_{i,2} + d_{i,\theta}(t), \\ J_{i,3}\ddot{\psi}_i &= c_i\tau_{i,3} + d_{i,\psi}(t), \quad i \in \Gamma, \end{aligned} \tag{3}$$

where $J_i = \text{diag}\{J_{i,1}, J_{i,2}, J_{i,3}\}$ is the inertia matrix in the body-fixed frame, $\tau_i = (\tau_{i,1}, \tau_{i,2}, \tau_{i,3})^T$ represents three rotational forces produced by the four rotors, c_i and l_i are the constants, respectively, for the force-to-moment factor and the arm length of the aircraft, and $d_i(t) = (d_{i,\phi}(t), d_{i,\theta}(t), d_{i,\psi}(t))^T$ denotes the unknown time-varying external disturbances.

2.2. Control Objective. The control objective of this paper is to design a a distributed control law for a group of quadrotor aircraft (3)-(4) such that the formation control can be achieved *in a finite time*. Specifically, define the desired formation trajectory $\chi_d = (p_d, q_d, r_d)^T$. At the same time, as we all know, the geometry in 3D space can be described by vector $\Delta_{ij} = (\Delta p_{ij}, \Delta q_{ij}, \Delta r_{ij})^T$, $i, j \in \Gamma$. In other words, the relative positional deviation between aircraft i and aircraft j is expressed as

$$\Delta_{ij} = \Delta_i - \Delta_j = (\Delta p_i, \Delta q_i, \Delta r_i)^T - (\Delta p_j, \Delta q_j, \Delta r_j)^T. \quad (4)$$

Based on mathematical expressions, the control objective of this paper can be described as that there is a finite time T^* such that for any $i, j \in \Gamma$

$$\begin{aligned} &\lim_{t\longrightarrow T^*} \begin{bmatrix} p_i(t) - p_j(t) \\ q_i(t) - q_j(t) \\ r_i(t) - r_j(t) \end{bmatrix} = \Delta_{ij}, \quad t < T^*, \\ &\begin{bmatrix} p_i(t) - p_j(t) \\ q_i(t) - q_j(t) \\ r_i(t) - r_j(t) \end{bmatrix} \equiv \Delta_{ij}, \quad t \geq T^*, \end{aligned} \quad (5)$$

and

$$\begin{aligned} &\lim_{t\longrightarrow T^*} \frac{1}{n}\sum_{i=1}^{n} [p_i(t), q_i(t), r_i(t)]^T = (p_d, q_d, r_d)^T, \\ &\qquad t < T^*, \\ &\frac{1}{n}\sum_{i=1}^{n} [p_i(t), q_i(t), r_i(t)]^T \equiv (p_d, q_d, r_d)^T, \\ &\qquad t \geq T^*. \end{aligned} \quad (6)$$

To achieve the control objective, for each quadrotor aircraft, the following assumptions are imposed.

Assumption 1. Assume that the desired position trajectory is $\chi_d = (p_d, q_d, r_d)^T$, where $\dot{\chi}_d$ and $\ddot{\chi}_d$ are continuous and bounded.

Assumption 2. Assume that the external disturbances and the change rate of the disturbances are bounded; i.e., $d_i(t)$, $\dot{d}_i(t)$, $\delta_i(t)$, $\dot{\delta}_i(t)$ are bounded.

To achieve the finite-time formation control task, we will employ the multi-agent theory to design a distributed formation control algorithm. Next, we first review the knowledge of graph theory.

2.3. Graph Theory. Considering a group of quadrotor aircraft with leader-follower structure, assume that each aircraft is a node and the information exchange among n follower agents is denoted by a directed graph $G(A) = \{V, E, A\}$. $V = \{v_i : i = 1, \ldots, n\}$ is the set of nodes, $E \subseteq V \times V$ is the set of edges, and $A = [a_{ij}] \in R^{n\times n}$ is the weighted adjacency matrix of the graph $G(A)$ with non-negative adjacency elements a_{ij}. If there is an edge from node j to node i, i.e., $(v_j, v_i) \in E$, then $a_{ij} > 0$, which means there exists an available information channel from node j to node i. The set of neighbors of node i is denoted by $N_i = \{j : (v_j, v_i) \in E\}$. The out-degree of node v_i is defined as $\deg_{out}(v_i) = d_i = \sum_{j=1}^{n} a_{ij} = \sum_{j\in N_i} a_{ij}$. Then the degree matrix of digraph G is $D = \operatorname{diag}\{d_1, \ldots, d_n\}$ and the Laplacian matrix of digraph G is $L = D - A$.

A path in graph G from v_{i_1} to v_{i_k} is a sequence of $v_{i_1}, v_{i_2}, \ldots, v_{i_k}$ of finite nodes starting with v_{i_1} and ending with v_{i_k} such that $(v_{i_l}, v_{i_{l+1}}) \in E$ for $l = 1, 2, \ldots, k-1$. The graph G is connected if there is a path between any two distinct vertices.

Assume that the reference state is represented by a leader. The connection weight between the n agent and the leader is denoted by $b_i, i \in \Gamma$. If the i-th agent has access to the information of leader, then $b_i > 0$; otherwise, $b_i = 0$. Let $B = \operatorname{diag}\{b_1, \ldots, b_n\}$.

Assumption 3. For the communication topology, it is assumed that the graph for all follower agents is connected. Meanwhile, at least one agent has access the signal directly from leader.

2.4. Related Definitions and Lemmas. Since the main objective of this paper is to design a finite-time formation control algorithm, we first introduce the concept of finite-time stability.

Definition 4 (see [13]). For a nonlinear system,

$$\dot{x} = f(x), \quad f(0) = 0, \ x \in R^n, \quad (7)$$

where $f(\cdot)$ is a continuous function. If the system is Lyapunov stable and finite-time convergent, then it is called finite-time stable. The finite-time convergence is defined as that there is a finite time $T(x_0)$ such that $\lim_{t\longrightarrow T(x_0)} x(t, x_0) = 0$ and $x(t, x_0) = 0$ for all $t \geq T(x_0)$.

In addition, for brevity, we define the following notation.

Definition 5. Define function

$$\operatorname{sig}^{\alpha}(x) = \operatorname{sign}(x)|x|^{\alpha}, \quad \alpha \geq 0, \ x \in R, \quad (8)$$

where $\operatorname{sign}(\cdot)$ is sign function.

Finally, since the finite-time controller design in this paper is based on the homogeneous systems theory, we give the related definition and lemma about homogeneity.

Definition 6 (see [13]). For the system (7), if, for any $\varepsilon > 0$, there exists $(r_1, \ldots, r_n)$ with $r_i > 0$, $i = 1, \ldots, n$, such that

$$f_i(\varepsilon^{r_1}x_1, \ldots, \varepsilon^{r_n}x_n) = \varepsilon^{k+r_i} f_i(x), \quad i = 1, \ldots, n, \quad (9)$$

where $k > -\min\{r_i, i = 1, \ldots, n\}$, then $f(x)$ is said to be homogeneous of degree k with respect to the dilation $(r_1, \ldots, r_n)$.

Lemma 7 (see [30]). *For system (7), if it has a continuous homogeneous vector space and is homogeneous of degree $k < 0$*

with respect to the dilation $(r_1, \ldots, r_n)$, *then it is globally finite-time stable.*

3. Main Results

The controller design method is mainly based on the back-stepping design and disturbance estimation-compensation method. Specifically speaking, the design procedure is divided into two steps.

(i) Step 1: for the position subsystem, the attitude $\Phi_i = (\phi_i, \theta_i, \psi_i)$ is taken as the virtual control input and is designed as the desired attitude such that all aircraft can converge to the desired 3D-pattern and move along the desired leader's trajectory *in a finite time*. For the external disturbance, a finite-time disturbance observer is used to estimate and compensate the disturbance, i.e., disturbance estimation-compensation method.

(ii) Step 2: for the attitude subsystem, by combining disturbance observer and finite-time controller, a control law τ_i is designed such that the each quadrotor desired attitude can be tracked by the real attitude *in a finite time*.

3.1. Finite-Time Position Formation Controller Design. For the sake of statement, denote

$$\begin{aligned} u_{i,p} &= \frac{T_i}{m_i}\left(\cos\phi_i \sin\theta_i \cos\psi_i + \sin\phi_i \sin\psi_i\right), \\ u_{i,q} &= \frac{T_i}{m_i}\left(\cos\phi_i \sin\theta_i \sin\psi_i - \sin\phi_i \cos\psi_i\right), \\ u_{i,r} &= \frac{T_i}{m_i}\cos\phi_i \cos\theta_i - g, \quad i \in \Gamma, \end{aligned} \tag{10}$$

under which the position system's equation (3) can be rewritten as follows:

$$\begin{aligned} \ddot{p}_i &= u_{i,p} + \delta_{i,p}, \\ \ddot{q}_i &= u_{i,q} + \delta_{i,q}, \\ \ddot{r}_i &= u_{i,r} + \delta_{i,r}. \end{aligned} \tag{11}$$

In order to design a finite-time controller for system (11), we first consider the case without external disturbances.

3.1.1. Case 1: No External Disturbance

Lemma 8. *For the position motion model (11) without the external disturbances, i.e.,* $\delta_i(t) = 0$, *if the controller is designed as*

$$\begin{aligned} u_{i,p} = &-\sum_{j\in N_i} a_{ij}\left[k_p \operatorname{sig}^{\alpha_1}\left(p_i - p_j - \Delta p_{ij}\right) + k_d \operatorname{sig}^{\alpha_2}\left(\dot{p}_i - \dot{p}_j\right)\right] - b_i\left[k_p \operatorname{sig}^{\alpha_1}\left(p_i - p_d - \Delta p_i + \frac{1}{n}\sum_{j=1}^{n}\Delta p_i\right) + k_d \operatorname{sig}^{\alpha_2}\left(\dot{p}_i - \dot{p}_d\right)\right] + \ddot{p}_d, \\ u_{i,q} = &-\sum_{j\in N_i} a_{ij}\left[k_p \operatorname{sig}^{\alpha_1}\left(q_i - q_j - \Delta q_{ij}\right) + k_d \operatorname{sig}^{\alpha_2}\left(\dot{q}_i - \dot{q}_j\right)\right] - b_i\left[k_p \operatorname{sig}^{\alpha_1}\left(q_i - q_d - \Delta q_i + \frac{1}{n}\sum_{j=1}^{n}\Delta q_i\right) + k_d \operatorname{sig}^{\alpha_2}\left(\dot{q}_i - \dot{q}_d\right)\right] + \ddot{q}_d, \\ u_{i,r} = &-\sum_{j\in N_i} a_{ij}\left[k_p \operatorname{sig}^{\alpha_1}\left(r_i - r_j - \Delta r_{ij}\right) + k_d \operatorname{sig}^{\alpha_2}\left(\dot{r}_i - \dot{r}_j\right)\right] - b_i\left[k_p \operatorname{sig}^{\alpha_1}\left(r_i - r_d - \Delta r_i + \frac{1}{n}\sum_{j=1}^{n}\Delta r_i\right) + k_d \operatorname{sig}^{\alpha_2}\left(\dot{r}_i - \dot{r}_d\right)\right] + \ddot{r}_d, \end{aligned} \tag{12}$$

where $0 < \alpha_1 < 1$, $\alpha_2 = 2\alpha_1/(1 + \alpha_1)$, $k_p > 0$, $k_d > 0$, *then the desired formation position control can be achieved in a finite time.*

Proof. Since the three axis controllers are similar, without loss of generality, we will only give the proof process about p-axis. Let

$$e_{i,p} = p_i - p_d - \Delta p_i + \frac{1}{n}\sum_{j=1}^{n}\Delta p_i. \tag{13}$$

be the coordinate changes for i-*th* quadrotor aircraft. Under the proposed controller (12), it can be concluded from (11) that the position tracking error equation is

$$\begin{aligned} \dot{e}_{i,p} &= \dot{p}_i - \dot{p}_d, \\ \ddot{e}_{i,p} &= -\sum_{j\in N_i} a_{ij}\left[k_p \operatorname{sig}^{\alpha_1}\left(e_{i,p} - e_{j,p}\right) + k_d \operatorname{sig}^{\alpha_2}\left(\dot{e}_{i,p} - \dot{e}_{j,p}\right)\right] - b_i\left[k_p \operatorname{sig}^{\alpha_1}\left(e_{i,p}\right) + k_d \operatorname{sig}^{\alpha_2}\left(\dot{e}_{i,p}\right)\right], \quad i \in \Gamma. \end{aligned} \tag{14}$$

In the sequel, we will prove that the system (14) is globally finite-time stable and the proof is divided into two steps.

At the first step, we will show that system (14) is globally asymptotically stable. The Lyapunov function is chosen as

$$V = V_1 + V_2, \tag{15}$$

where

$$\begin{aligned} V_1 &= k_p \sum_{i=1}^{n} \left(\sum_{j=1}^{n} a_{ij} \int_0^{e_{i,p}-e_{j,p}} \operatorname{sig}^{\alpha_1}(\rho)\, d\rho \right. \\ &\quad \left. + 2b_i \int_0^{e_{i,p}} \operatorname{sig}^{\alpha_1}(\rho)\, d\rho \right), \end{aligned} \tag{16}$$

$$V_2 = \sum_{i=1}^{n} \dot{e}_{i,p}^2.$$

Clearly, the Lyapunov function V is positive definite and radially unbounded. On one hand, the derivative of V_1 along system (14) is

$$\begin{aligned} \dot{V}_1 &= k_p \sum_{i=1}^{n} \left(\sum_{j=1}^{n} a_{ij} \operatorname{sig}^{\alpha_1}\left(e_{i,p} - e_{j,p}\right)\left(\dot{e}_{i,p} - \dot{e}_{j,p}\right) \right. \\ &\quad \left. + 2b_i \operatorname{sig}^{\alpha_1}\left(e_{i,p}\right)\dot{e}_{i,p} \right). \end{aligned} \tag{17}$$

On the other hand, based on $a_{ij} = a_{ji}$, $i, j \in \Gamma$ and the function $\operatorname{sig}(\cdot)$ being an odd function, the derivative of V_2 along system (14) is

$$\begin{aligned} \dot{V}_2 &= -2\sum_{i=1}^{n} \dot{e}_{i,p} \left(\sum_{j\in N_i} a_{ij} \left[k_p \operatorname{sig}^{\alpha_1}\left(e_{i,p} - e_{j,p}\right) + k_d \operatorname{sig}^{\alpha_2}\left(\dot{e}_{i,p} - \dot{e}_{j,p}\right)\right] + b_i \left[k_p \operatorname{sig}^{\alpha_1}\left(e_{i,p}\right) + k_d \operatorname{sig}^{\alpha_2}\left(\dot{e}_{i,p}\right)\right] \right) \\ &= -k_p \sum_{i=1}^{n} \left(\sum_{j=1}^{n} a_{ij} \left(\dot{e}_{i,p} - \dot{e}_{j,p}\right) \operatorname{sig}^{\alpha_1}\left(e_{i,p} - e_{j,p}\right) + 2b_i \dot{e}_{i,p} \operatorname{sig}^{\alpha_1}\left(e_{i,p}\right) \right) \\ &\quad - k_d \sum_{i=1}^{n} \left(\sum_{j=1}^{n} a_{ij} \left(\dot{e}_{i,p} - \dot{e}_{j,p}\right) \operatorname{sig}^{\alpha_2}\left(\dot{e}_{i,p} - \dot{e}_{j,p}\right) + 2b_i \dot{e}_{i,p} \operatorname{sig}^{\alpha_2}\left(\dot{e}_{i,p}\right) \right). \end{aligned} \tag{18}$$

In summary, the derivative of V along system (14) is

$$\dot{V} = -k_d \sum_{i=1}^{n} \left(\sum_{j=1}^{n} a_{ij} \left|\dot{e}_{i,p} - \dot{e}_{j,p}\right|^{1+\alpha_2} + 2b_i \left|\dot{e}_{i,p}\right|^{\alpha_2+1} \right). \tag{19}$$

Since $a_{ij} \geq 0$, $b_i \geq 0$, $k_d > 0$, it is obvious that $\dot{V} \leq 0$. Define set $\Psi = \{(e_{i,p}, \dot{e}_{i,p}) \mid \dot{V} \equiv 0\}$. It can be concluded that $\dot{V} \equiv 0$ implies $\dot{e}_{i,p} \equiv 0$ and $\ddot{e}_{i,p} \equiv 0$, which also implies that $e_{i,p} \equiv 0$ from (14), $\forall i \in \Gamma$. Using LaSalle's invariant principle [31] leads to $(e_{i,p}, \dot{e}_{i,p}) \longrightarrow 0$ as $t \longrightarrow 0$; i.e., system (14) is globally asymptotically stable.

Furthermore, according to the definition of homogeneous system (i.e., Definition 6), it can be found that system (14) is homogeneous with degree of $k = (\alpha_1 - 1)/2 < 0$ with respect to the dilation $\Delta = (\underbrace{s_1, \ldots, s_1}_{\text{for } e_{1,p},\ldots,e_{n,p}}, \underbrace{s_2, \ldots, s_2}_{\text{for } \dot{e}_{1,p},\ldots,\dot{e}_{n,p}})$, where $s_1 = 1$, $s_2 = (\alpha_1 + 1)/2$. In summary, based on Lemma 7, it can be concluded that the closed-loop system (14) is globally finite-time stable. □

However, in practice, the external disturbance is usually unavoidable. Next, we will consider this case.

Assumption 9. Assume that there are known positive constants M_1, M_2, such that $|\delta_i| \leq M_1$, $|\dot{\delta}_i| \leq M_2$.

3.1.2. Case 2: The Existence of External Disturbance

Lemma 10. *For system (11) under Assumption 9, the disturbance estimation $(\hat{\delta}_{i,p}, \hat{\delta}_{i,q}, \hat{\delta}_{i,r})$ from the observers*

$$\begin{aligned} \frac{\partial \hat{p}_i}{\partial t} &= u_{i,p} + \hat{\delta}_{i,p} + \lambda_1 \operatorname{sig}^{1/2}\left(\dot{p}_i - \hat{p}_i\right), \\ \dot{\hat{\delta}}_{i,p} &= \lambda_2 \operatorname{sign}\left(\dot{p}_i - \hat{p}_i\right), \\ \frac{\partial \hat{q}_i}{\partial t} &= u_{i,q} + \hat{\delta}_{i,q} + \lambda_1 \operatorname{sig}^{1/2}\left(\dot{q}_i - \hat{q}_i\right), \\ \dot{\hat{\delta}}_{i,q} &= \lambda_2 \operatorname{sign}\left(\dot{q}_i - \hat{q}_i\right), \\ \frac{\partial \hat{r}_i}{\partial t} &= u_{i,r} + \hat{\delta}_{i,r} + \lambda_1 \operatorname{sig}^{1/2}\left(\dot{r}_i - \hat{r}_i\right), \\ \dot{\hat{\delta}}_{i,r} &= \lambda_2 \operatorname{sign}\left(\dot{r}_i - \hat{r}_i\right), \end{aligned} \tag{20}$$

will converge to the real states $(\delta_{i,p}, \delta_{i,q}, \delta_{i,r})$ in a finite time, where

$$\begin{aligned} &\lambda_2 > M_2, \\ &\lambda_1 > \sqrt{\frac{2}{\lambda_2 - M_2}} \cdot \frac{\left(\lambda_2 + M_2\right)\left(1 + h_1\right)}{1 - h_1}, \end{aligned} \tag{21}$$

with $0 < h_1 < 1$.

Proof. Similarly, here, only the proof about x-axis is given. Define the disturbance observer error as $\eta_1 = \dot{p}_i - \hat{p}_i$, $\eta_2 =$

$\delta_{i,p} - \widehat{\delta}_{i,p}$, under which the error dynamical equation can be obtained from (11) and (20) that

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 - \lambda_1 \operatorname{sig}^{1/2}(\eta_1), \\ \dot{\eta}_2 &= -\lambda_2 \operatorname{sign}(\eta_1) + \dot{\delta}_{i,p}. \end{aligned} \tag{22}$$

By Assumption 9, we have $-M_2 < \dot{\delta}_{i,p} < M_2$. As a result, it follows from [32] that system (22) is finite-time stable under the gain condition (21), which means that $\widehat{\delta}_{i,p}$ can converge to $\delta_{i,p}$ in a finite time. The proof is completed. □

Based on the precise estimations for the external disturbances, a disturbance-observer-based composite controller is given to achieve finite-time position tracking control.

Theorem 11. *For the i-th position motion model (11) with the disturbance, if the controller is designed as*

$$\begin{aligned} u_{i,p} = &-\sum_{j\in N_i} a_{ij}\left[k_p \operatorname{sig}^{\alpha_1}(p_i - p_j - \Delta p_{ij}) \right.\\ &\left. + k_d \operatorname{sig}^{\alpha_2}(\dot{p}_i - \dot{p}_j)\right] - b_i\left[k_p \operatorname{sig}^{\alpha_1}\left(p_i - p_d - \Delta p_i \right.\right.\\ &\left.\left. + \frac{1}{n}\sum_{j=1}^{n}\Delta p_i\right) + k_d \operatorname{sig}^{\alpha_2}(\dot{p}_i - \dot{p}_d)\right] + \ddot{p}_d - \widehat{\delta}_{i,p}, \\ u_{i,q} = &-\sum_{j\in N_i} a_{ij}\left[k_p \operatorname{sig}^{\alpha_1}(q_i - q_j - \Delta q_{ij}) \right.\\ &\left. + k_d \operatorname{sig}^{\alpha_2}(\dot{q}_i - \dot{q}_j)\right] - b_i\left[k_p \operatorname{sig}^{\alpha_1}\left(q_i - q_d - \Delta q_i \right.\right.\\ &\left.\left. + \frac{1}{n}\sum_{j=1}^{n}\Delta q_i\right) + k_d \operatorname{sig}^{\alpha_2}(\dot{q}_i - \dot{q}_d)\right] + \ddot{q}_d - \widehat{\delta}_{i,q}, \\ u_{i,r} = &-\sum_{j\in N_i} a_{ij}\left[k_p \operatorname{sig}^{\alpha_1}(r_i - r_j - \Delta r_{ij}) \right.\\ &\left. + k_d \operatorname{sig}^{\alpha_2}(\dot{r}_i - \dot{r}_j)\right] - b_i\left[k_p \operatorname{sig}^{\alpha_1}\left(r_i - r_d - \Delta r_i \right.\right.\\ &\left.\left. + \frac{1}{n}\sum_{j=1}^{n}\Delta r_i\right) + k_d \operatorname{sig}^{\alpha_2}(\dot{r}_i - \dot{r}_d)\right] + \ddot{r}_d - \widehat{\delta}_{i,r}, \end{aligned} \tag{23}$$

where $0 < \alpha_1 < 1$, $\alpha_2 = 2\alpha_1/(1+\alpha_1)$, $k_p > 0$, $k_d > 0$, *then the desired formation position control can be achieved in a finite time.*

Proof. The proof is straightforward since the results in Lemmas 8 and 10 are about finite-time convergence. In other word, there is a finite time T^*, for any $i \in \Gamma$,

$$\begin{aligned} &\lim_{t\longrightarrow T^*}\left(e_{i,p}(t), \dot{e}_{i,p}(t)\right) = 0, \quad t < T^*, \\ &\left(e_{i,p}(t), \dot{e}_{i,p}(t)\right) \equiv 0, \quad t \geq T^*. \end{aligned} \tag{24}$$

Therefore, considering the coordination changes (13), for any $i, j \in \Gamma$, it can be concluded that

$$\begin{aligned} &\lim_{t\longrightarrow T^*}\left[p_i(t) - p_j(t)\right] = \Delta p_{ij}, \\ &\lim_{t\longrightarrow T^*}\frac{1}{n}\sum_{i=1}^{n} = p_d, \\ &\qquad t < T^*, \\ &\left[p_i(t) - p_j(t)\right] \equiv \Delta p_{ij}, \\ &\lim_{t\longrightarrow T^*}\frac{1}{n}\sum_{i=1}^{n} \equiv p_d, \\ &\qquad t \geq T^*. \end{aligned} \tag{25}$$

Therefore, the formation control of a group of quadrotor aircraft can be achieved in a finite time. The proof is completed. □

3.2. Attitude Controller Design. In the previous section, the attitude (i.e., Euler angle $\Phi_i = (\phi_i, \theta_i, \psi_i)^T$) is taken as a virtual control input. It is designed such that the desired position trajectory can be tracked in a finite time. Next, in this section, the attitude dynamical system will be considered. According to the relation (10), the desired attitude can be generated from the virtual control input $u_{i,p}, u_{i,q}, u_{i,r}$. Denote $\Phi_{i,d} = (\phi_{i,d}, \theta_{i,d}, \psi_{i,d})^T$ as the desired attitude for i-th quadrotor aircraft; then it follows from (10) that

$$\begin{aligned} T_{i,d} &= m_i\sqrt{u_{i,p}^2 + u_{i,q}^2 + (u_{i,r} + g)^2}, \\ \phi_{i,d} &= \arcsin\left(\frac{m_i}{T_{i,d}}\left(u_{i,p}\sin\psi_{i,d} - u_{i,q}\cos\psi_{i,d}\right)\right), \\ \theta_{i,d} &= \arctan\left(\frac{1}{u_{i,r} + g}\left(u_{i,p}\cos\psi_{i,d} + u_{i,q}\sin\psi_{i,d}\right)\right), \\ &\qquad i \in \Gamma. \end{aligned} \tag{26}$$

Since the variable $\psi_{i,d}$ is a free variable, the desired yaw angle can be set as $\psi_{i,d} = 0$ for the convenience of analysis.

Due to both uncertain parameters and external disturbances, the unknown parameters are broken into two parts, i.e., known nominal parts and unknown uncertainty parts. Specifically, let

$$J_{i,j} = J_{i,j}^o + \triangle J_{i,j}, \quad j = 1, 2, 3, \tag{27}$$

where $J^o_{i,j}$ represents the known part and $\triangle J_{i,j}$ is the uncertainty part. With the help of the above notations, the attitude equation (4) can be rewritten as

$$\begin{aligned} \ddot{\phi}_i &= \frac{l_i}{J^o_{i,1}}\tau_{i,1} + d'_{i,\phi}, \\ \ddot{\theta}_i &= \frac{l_i}{J^o_{i,2}}\tau_{i,2} + d'_{i,\theta}, \\ \ddot{\psi}_i &= \frac{c_i}{J^o_{i,3}}\tau_{i,3} + d'_{i,\psi}, \end{aligned} \tag{28}$$

where

$$\begin{aligned} d'_{i,\phi} &= \left(\frac{1}{J_{i,1}} - \frac{1}{J^o_{i,1}}\right) l_i\tau_{i,1} + \frac{d_{i,\phi}(t)}{J_{i,1}}, \\ d'_{i,\theta} &= \left(\frac{1}{J_{i,2}} - \frac{1}{J^o_{i,2}}\right) l_i\tau_{i,2} + \frac{d_{i,\theta}(t)}{J_{i,2}}, \\ d'_{i,\psi} &= \left(\frac{1}{J_{i,3}} - \frac{1}{J^o_{i,3}}\right) c_i\tau_{i,3} + \frac{d_{i,\psi}(t)}{J_{i,3}}. \end{aligned} \tag{29}$$

Remark 12. Note that the uncertainty functions, i.e., $d'_i = (d'_{i,\phi}, d'_{i,\theta}, d'_{i,\psi})^T$, defined above include not only the external disturbances but also the parameter variations, which are usually called the lumped disturbances in the literature [33].

To deal with the lump disturbances in system (29), a disturbance estimation and then compensation method will be employed. As that in [33–35], the following assumption is given on the lump disturbances.

Assumption 13. Assume that there are known positive constants L_1, L_2, such that $|d'_i| \le L_1$, $|\dot{d}'_i| \le L_2$.

Under this assumption, motivated by the work [32], a finite-time disturbance observer is proposed to estimate the lump disturbances in a finite time.

Lemma 14. *For the system (28) under Assumption 13, the states $\hat{d}'_i = (\hat{d}'_{i,\phi}, \hat{d}'_{i,\theta}, \hat{d}'_{i,\psi})$ of the observers*

$$\begin{aligned} \frac{\partial\hat{\dot{\phi}}_i}{\partial t} &= \frac{l_i}{J^o_{i,1}}\tau_{i,1} + \hat{d}'_{i,\phi} + \rho_1 \operatorname{sig}^{1/2}\left(\dot{\phi}_i - \hat{\dot{\phi}}_i\right), \\ \dot{\hat{d}}'_{i,\phi} &= \rho_2 \operatorname{sign}\left(\dot{\phi}_i - \hat{\dot{\phi}}_i\right), \\ \frac{\partial\hat{\dot{\theta}}_i}{\partial t} &= \frac{l_i}{J^o_{i,2}}\tau_{i,2} + \hat{d}'_{i,\theta} + \rho_1 \operatorname{sig}^{1/2}\left(\dot{\theta}_i - \hat{\dot{\theta}}_i\right), \\ \dot{\hat{d}}'_{i,\theta} &= \rho_2 \operatorname{sign}\left(\dot{\theta}_i - \hat{\dot{\theta}}_i\right), \\ \frac{\partial\hat{\dot{\psi}}_i}{\partial t} &= \frac{c_i}{J^o_{i,3}}\tau_{i,3} + \hat{d}'_{i,\psi} + \rho_1 \operatorname{sig}^{1/2}\left(\dot{\psi}_i - \hat{\dot{\psi}}_i\right), \\ \dot{\hat{d}}'_{i,\psi} &= \rho_2 \operatorname{sign}\left(\dot{\psi}_i - \hat{\dot{\psi}}_i\right), \end{aligned} \tag{30}$$

will converge to the states $d'_i = (d'_{i,\phi}, d'_{i,\theta}, d'_{i,\psi})$ in a finite time, where

$$\begin{aligned} &\rho_2 > L_2, \\ &\rho_1 > \sqrt{\frac{2}{\rho_2 - L_2}} \cdot \frac{(\rho_2 + L_2)(1 + h_2)}{1 - h_2}, \end{aligned} \tag{31}$$

with $0 < h_2 < 1$.

Proof. The proof process is the same as that of the position loop disturbance observer, i.e., Lemma 10. The proof is straightforward. □

Based on the precise estimations for the lump disturbances, a disturbance-observer-based composite controller is given to achieve finite-time attitude tracking control.

Theorem 15. *For the attitude subsystem (4) of i-th quadrotor aircraft in the presence of unknown parameters and external disturbances, if the controller is designed as*

$$\begin{aligned} \tau_{i,1} &= \frac{J^o_{i,1}}{l_i}\left(\ddot{\phi}_{i,d} + a_p \operatorname{sig}^{\beta_1}(\phi_{i,d} - \phi_i) + a_d \operatorname{sig}^{\beta_2}\left(\dot{\phi}_{i,d} - \dot{\phi}_i\right) - \hat{d}'_{i,\phi}\right), \\ \tau_{i,2} &= \frac{J^o_{i,2}}{l_i}\left(\ddot{\theta}_{i,d} + a_p \operatorname{sig}^{\beta_1}(\theta_{i,d} - \theta_i) + a_d \operatorname{sig}^{\beta_2}\left(\dot{\theta}_{i,d} - \dot{\theta}_i\right) - \hat{d}'_{i,\theta}\right), \\ \tau_{i,3} &= \frac{J^o_{i,3}}{c_i}\left(\ddot{\psi}_{i,d} + a_p \operatorname{sig}^{\beta_1}(\psi_{i,d} - \psi_i) + a_d \operatorname{sig}^{\beta_2}\left(\dot{\psi}_{i,d} - \dot{\psi}_i\right) - \hat{d}'_{i,\psi}\right), \end{aligned} \tag{32}$$

where $0 < \beta_1 < 1$, $\beta_2 = 2\beta_1/(1 + \beta_1)$, $a_p > 0$, $a_d > 0$, *then the desired attitude can be tracked in a finite time, i.e.,* $(\phi_i, \theta_i, \psi_i)^T \longrightarrow (\phi_{i,d}, \theta_{i,d}, \psi_{i,d})^T$ *in a finite time.*

Proof. Since the disturbance observers (30) are about finite-time convergence, then there is a finite time T_o such that

$$\begin{aligned} \hat{d}'_{i,\phi} &= d'_{i,\phi}, \\ \hat{d}'_{i,\theta} &= d'_{i,\theta}, \\ \hat{d}'_{i,\psi} &= d'_{i,\psi}, \\ &\forall t \ge T_o. \end{aligned} \tag{33}$$

Similarly, only the proof about roll angle ϕ_i is provided. Define the attitude tracking error as $e_{i,\phi} = \phi_{i,d} - \phi_i$, when $t \ge T_o$, whose dynamical equation can be obtained from (28), i.e.,

$$\begin{aligned} \dot{e}_{i,\phi} &= \dot{\phi}_{i,d} - \dot{\phi}_i, \\ \ddot{e}_{i,\phi} &= \ddot{\phi}_{i,d} - \frac{l_i}{J_{i,1}}\tau_{i,1}. \end{aligned} \tag{34}$$

TABLE 1: System's parameters.

Descriptions	Parameter Values
mass	$m_i = 0.468$ kg
gravitational acceleration	$g = 9.81$ m/s^2
inertia matrix	$J_i = J_i^o + \triangle J_i$, $i = 1, 2, 3$, with $J_i^o = \mathrm{diag}(0.0020, 0.0022, 0.0030)$ $\triangle J_i = \mathrm{diag}(0.0003, 0.0002, -0.0004)$
position-loop disturbances	$\delta_{i,p}(t) = 0.002\sin(t+5)$ $\delta_{i,q}(t) = 0.002\cos(2t+3)$ $i = 1, 2, 3$ $\delta_{i,r}(t) = 0.003\sin(3t+2)$
attitude-loop disturbances	$d_{i,\phi}(t) = 0.8\sin(0.4t+5)$ $d_{i,\theta}(t) = \cos(0.3t+3)$ $i = 1, 2, 3$ $d_{i,\psi}(t) = 0.5\sin(0.3t+2)$

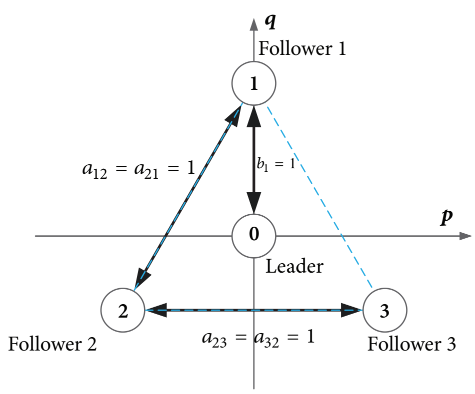

FIGURE 1: The communication topology graph and the desired formation pattern.

Under the proposed controller (32), the closed-loop system is

$$\begin{aligned} \dot{e}_{i,\phi} &= \dot{\phi}_{i,d} - \dot{\phi}_i, \\ \ddot{e}_{i,\phi} &= -a_p \mathrm{sig}^{\beta_1}\left(e_{i,\phi}\right) - a_d \mathrm{sig}^{\beta_2}\left(\dot{e}_{i,\phi}\right). \end{aligned} \tag{35}$$

It can be found that the closed-loop system (35) has the same structure as that of system (14). Hence, based on Lemma 7, it is easy to prove that system (35) is globally finite-time stable, which is omitted here. □

4. Numerical Simulations

To verify the proposed theoretical results in Theorems 11 and 15, some numerical simulations are given. The network system considered in this section consists of one leader and three followers. The undirected topology graph topology is shown in Figure 1. Specifically, the weights of the undirected edges are, respectively, given as $b_1 = 1$, $a_{12} = a_{21} = 1$, $a_{23} = a_{32} = 1$. Meanwhile, the desired formation pattern is to form a regular triangle on in the p-q plane. And from Figure 1, the relative position deviations can been given as

$$\begin{aligned} \Delta p_i &= (0, 1, 0)^T, \\ \Delta q_i &= \left(\cos\left(\frac{5\pi}{6}\right), \sin\left(\frac{-\pi}{6}\right), 0\right)^T, \\ \Delta r_i &= \left(\cos\left(\frac{-\pi}{6}\right), \sin\left(\frac{-\pi}{6}\right), 0\right)^T. \end{aligned} \tag{36}$$

4.1. Model Parameters and Controllers Gains. The desired formation trajectory, i.e., leader's trajectory, is

$$\left(x_d, y_d, z_d\right)^T = \left(5\sin(0.2t), 5\cos(0.2t), 0.5t\right)^T. \tag{37}$$

And the initial conditions for each quadrotor aircraft are as follows:

$$\begin{aligned} &\left(p_1(0), q_1(0), r_1(0), \phi_1(0), \theta_1(0), \psi_1(0)\right) \\ &\quad = (-3, 5, 0, 0, 0, 0), \\ &\left(p_2(0), q_2(0), r_2(0), \phi_2(0), \theta_2(0), \psi_2(0)\right) \\ &\quad = (1, -4, 0, 0, 0, 0), \\ &\left(p_3(0), q_3(0), r_3(0), \phi_3(0), \theta_3(0), \psi_3(0)\right) \\ &\quad = (2, 3, 0, 0, 0, 0). \end{aligned} \tag{38}$$

The model parameter values are given in Table 1. Meanwhile, by Theorems 11 and 15, the controllers' and observers' parameters are selected as $k_p = a_p = 3.5$, $k_d = a_d = 4.5$, $\alpha_1 = \beta_1 = 3/4$, $\lambda_1 = 4$, $\lambda_2 = 1$, $\rho_1 = 5$, $\rho_2 = 2$.

4.2. Numerical Simulation Results. Under the proposed control algorithm, Figure 2 shows the position trajectory of each quadrotor aircraft in 3D space. It can be found that the

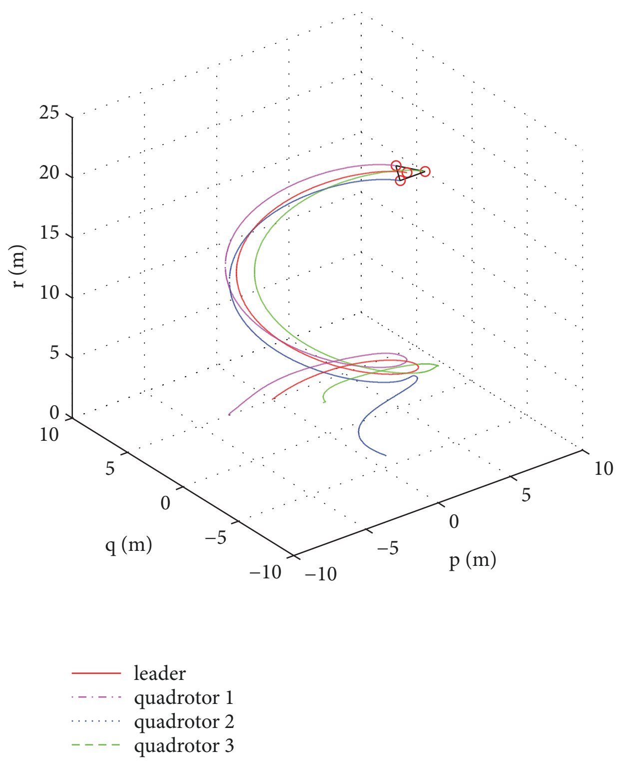

FIGURE 2: The response curves of each aircraft's trajectory in 3D space.

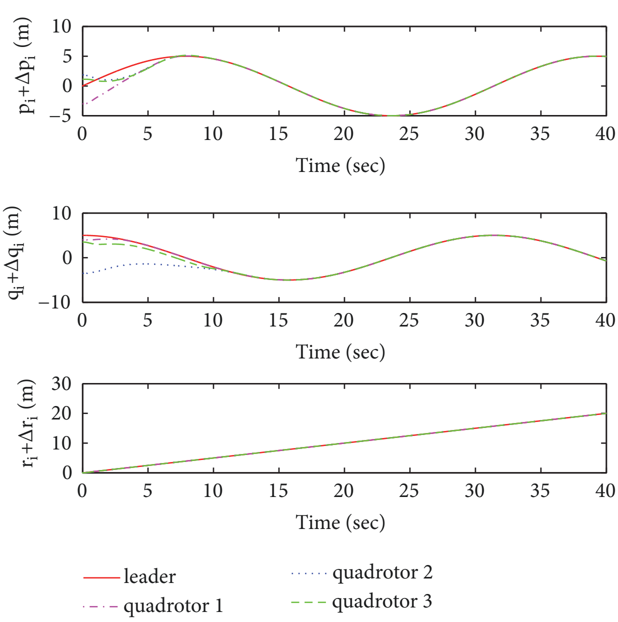

FIGURE 3: The response curves of the position for each aircraft.

desired formation control task can be achieved under the proposed finite-time controller. The tracking curve for each aircraft's position and attitude are given in Figures 3 and 4. And the adjacent distance of all quadrotor aircrafts is shown in Figure 5. The response curves for the estimated position disturbances and the estimated attitude lump disturbances

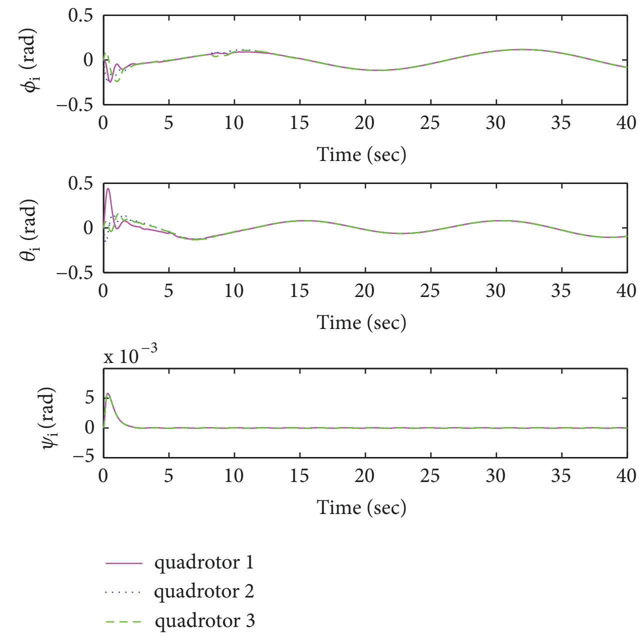

FIGURE 4: The response curves of the attitude for each aircraft.

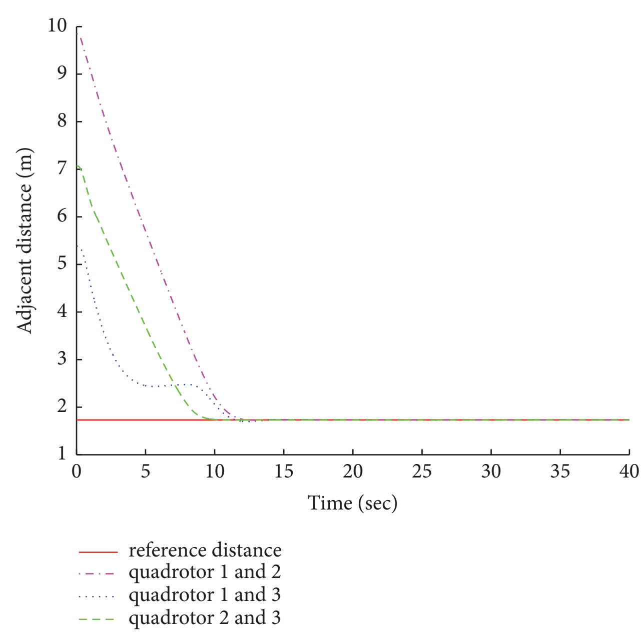

FIGURE 5: The adjacent distance of all quadrotor aircrafts.

are, respectively, shown in Figures 6 and 7. It can be found that the proposed methods of finite-time disturbances estimation are effective.

5. Conclusion

In this paper, observers-based distributed formation control law has been proposed to solve the formation control problem for multiple quadrotor aircrafts in the presence of parameter

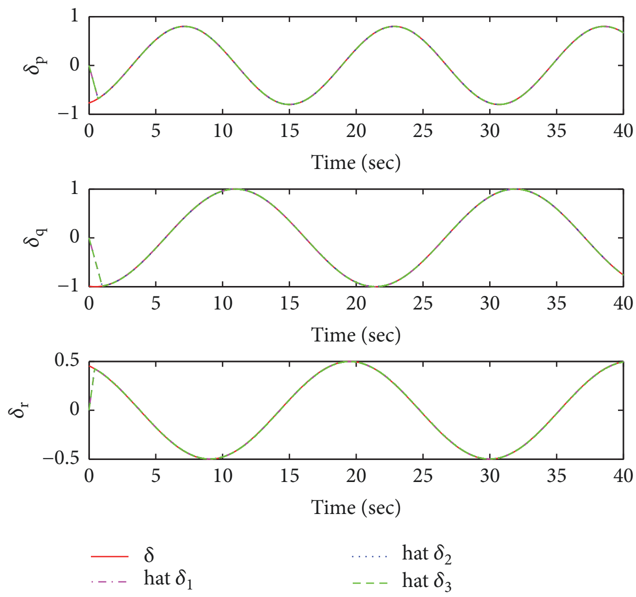

Figure 6: The response curves of the estimated position disturbances for each aircraft.

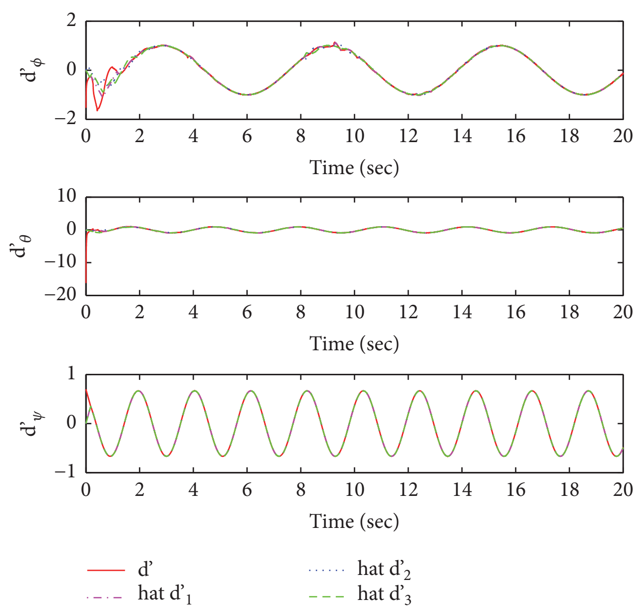

Figure 7: The response curves of the estimated attitude lump disturbances for each aircraft.

uncertainties and external disturbances. Rigorous stability analysis and some simulation results have been presented to show that the formation task can be achieved using the proposed control algorithm.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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