Optimization of Relay Placement in Wireless Butterfly Networks

Quoc-Tuan Vien

Abstract As a typical model of multicast network, wireless butterfly networks (WBNs) have been studied for modelling the scenario when two source nodes wish to convey data to two destination nodes via an intermediary node namely relay node. In the context of wireless communications, when receiving two data packets from the two source nodes, the relay node can employ either physical-layer network coding or analogue network coding on the combined packet prior to forwarding to the two destination nodes. Evaluating the energy efficiency of these combination approaches, energy-delay trade-off (EDT) is worth to be investigated and the relay placement should be taken into account in the practical network design. This chapter will first investigate the EDT of network coding in the WBNs. Based on the derived EDT, algorithms that optimize the relay position will be developed to either minimize the transmission delay or minimize the energy consumption subject to constraints on power allocation and location of nodes. Furthermore, considering an extended model of the WBN, the relay placement will be studied for a general wireless multicast network with multiple source, relay and destination nodes.

Key words: Wireless butterfly network; wireless multicast network; network coding; energy-delay tradeoff; relay placement.

1 Introduction

As wireless communications is growing with emerging enhanced technologies, data transmission over wireless medium turns out to be more reliable and more secured. The broadcast nature of the wireless media has been exploited to enable a variety of communication mechanisms and algorithms for enhancing the performance of the wireless communications. Apparently, there exist a number of nodes in a network

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and a question that can be raised is why they do not help each other in the data transmission between two end nodes. The energy waste and the unwanted interference in the shared media between the nodes that used to be regarded as drawbacks of the wireless communications can become a potential resource in assisting the communication between them. The attenuation in signal strength caused by severe fading of the source-destination link or even completely corrupted link could be solved with the help of intermediate nodes whose channels are independent of the channel between the source and destination nodes. The probability of successful transmission is therefore improved for a more reliable communication if these issues are satisfactorily addressed.

Cooperative communications, also known as relay communications or user cooperation in the preliminary works of Sendonaris *et al.* in 2003 [1, 2] and Laneman *et al.* in 2004 [3], has attracted an increasing interest in wireless communications aiming at throughput enhancement and quality improvement by exploiting spatial diversity gains. The relays can be used not only to improve service quality and link capacity for local users which are located near the source but also to enhance coverage and throughput for remote users. Inspired by the benefits of the relays, relay-assisted communications has been incorporated in various types of wireless systems; for instance, cellular networks in Loa *et al.* in 2010 [4] and Sheng *et al.* in 2011 [5], ad hoc networks in Sharma *et al.* in 2011 [6], sensor networks in Sun *et al.* in 2009 [7], ultra-wideband body area networks in Chen *et al.* in 2009 [8], and storage networks in Dimakis *et al.* in 2011 [9].

Conventionally, data traverses along relays in a store-and-forward manner, and thus the use of the relays does not immediately increase network throughput. In 2000, Ahlswede *et al.* [10] proposed the idea of network coding (NC) to increase the system throughput in lossless networks. Later in 2003, Koetter and Medard [11] developed an algebraic approach to enable the applicability of the NC. The NC has been then applied at the relays to dramatically improve the throughput of wireless relay networks, such as Zhang *et al.* in 2006 [12], Katti *et al.* in 2007 [13], and Louie *et al.* in 2010 [14]. By employing the NC at the relay nodes to coordinate the transmission among nodes in an efficient way, the optimality of the bandwidth could be achieved. Many NC-based protocols have also been proposed for some particular relay channel topologies such as relay-assisted bidirectional channels in Ju *et al.* in 2010 [15], broadcast channels in Nguyen *et al.* in 2009 [16], multicast channels in Chen *et al.* in 2010 [17], and unicast channels in Liu *et al.* in 2009 [18]. As a specific model of the multicast channels, butterfly networks have been investigated, e.g., Zhan *et al.* in 2010 [19] and Hu *et al.* in 2011 [20], in which the NC is applied at the relay node to help two source nodes simultaneously transmit their information to two destination nodes.

This chapter is devoted to investigating the energy efficiency for reliable communications in wireless butterfly networks (WBNs) employing various NC techniques. In particular, the relay placement (RP) problem for energy-efficient and reliable relaying in the WBNs will be discussed. In the rest of this chapter, Section 2 will first introduce the background of cooperative diversity starting from its foundation including the concepts of diversity and multiple-input multiple-output (MIMO). Basic principles and specific protocols for cooperative communications in wireless relay networks will be presented along with cooperative diversity techniques via distributed space-time-frequency coding and NC for a variety of relay network topologies. Section 3 will describe the system model of a typical WBN employing the NC techniques. As a common approach to improve the reliability of the wireless communications, Section 4 will discuss hybrid automatic repeat request with incremental redundancy (HARQ-IR) protocol and, in particular, this section will provide a detailed analysis for the energy-delay tradeoff of the HARQ-IR protocol with the NC techniques in the WBN. The RP problem in the WBN will be then formulated and optimized in Section 5. An extension of the RP for a general wireless multicast network will be discussed in Section 6. Finally, Section 7 will conclude this chapter with suggestion for future works.

2 Background

In this section, the basic concepts of diversity techniques will be firstly described, including temporal diversity, frequency diversity, and spatial diversity. As an approach to achieve the spatial diversity in terms of antenna diversity, MIMO systems will be presented with some well-known space-time-frequency coding schemes, based on which the motivation of cooperative diversity will be then discussed with an overview of cooperative protocols and techniques. The section will conclude by introducing NC which is regarded as a new technique to improve the throughput of wireless cooperative relay networks.

2.1 Diversity Techniques in Wireless Communications

In a communication system consisting of a sender and a receiver, the reliability of data transmission can be improved by providing more than one path between them. This technique is the main idea behind the term "diversity". In fact, by providing multiple replicas or copies of the transmitted signals over independent channels, the receiver can more reliably decode the transmitted signal by either combining all the received signal, namely a maximal ratio combiner, or selecting the best signal with the highest signal-to-noise ratio (SNR), namely a selection combiner, or choosing the signal with an SNR exceeding a threshold, namely a threshold combiner. In order to define the diversity quantitatively, Zheng and Tse in 2003 [21] formulated the relationship between the error probability, i.e., P_e, and the received SNR, i.e., γ, through a diversity gain as

$$G_d \triangleq - \lim_{\gamma \to \infty} \frac{\log P_e}{\log \gamma}. \tag{1}$$

It can be seen in Eq. (1) that the diversity gain G_d is the slope of the P_e curve in terms of γ in a log-log scale. This means that a large diversity gain is preferred to achieve a reduced P_e at a higher data rate. The problems are how to provide various copies of the transmitted signal to the receiver in an efficient way in terms of power, time, bandwidth, and complexity, and, how to take advantage of these copies at the receiver to achieve the lowest P_e. To cope with these two issues, various diversity methods, as will be shown below, can be implemented.

2.1.1 Temporal Diversity

In temporal diversity, copies of the transmitted signal are sent at different time intervals. The time interval between two transmitted replicas should be longer than the coherence time of the channel to make the fading channels uncorrelated and thus the temporal diversity can be obtained. However, the temporal diversity is bandwidth inefficient due to the delay that may be suffered at the receiver in the case of a slow fading channel, i.e., a large coherence time of the channel.

2.1.2 Frequency Diversity

Instead of using temporal separation between different replicas of the transmitted signal, the transmission of these copies can be carried out over different carrier frequencies to achieve frequency diversity. Similar to temporal diversity, frequency diversity can be achieved when there exists a necessary separation between two carrier frequencies which should be larger than the coherence bandwidth of the channel. The frequency diversity is therefore also bandwidth inefficient and the capability of frequency tuning is required at the receiver.

2.1.3 Spatial Diversity

In spatial diversity, multiple antennas are employed at the sender and/or the receiver to transmit and/or receive different copies of a signal. It is therefore also known as antenna diversity in Winters *et al.* in 1994 [22]. The spatial diversity does not suffer from bandwidth inefficiency which is a major drawback of the temporal and frequency diversity. However, in order to achieve the spatial diversity, a number of antennas are required at either the transmitter side or the receiver side or both sides. Also, the antennas deployed on a device are normally separated by at least half of a wavelength of the transmission frequency to guarantee that the fading channels are independent or at least low-correlated. Obviously, the condition of the antenna separation could be easily satisfied at large base stations, but may not be applicable for small handheld devices. This accordingly motivates the concept of user cooperation with cooperative diversity, which will be discussed in details in Subsection 2.3.

2.2 MIMO Systems & Space-Time-Frequency Coding

In radio communications, the negative effects of fading phenomena on quality and data rate in wireless communications can be combated with diversity in the spatial domain via the employment of multiple antennas. The concept of MIMO systems is defined for systems where multiple antennas are deployed at source and destination nodes to achieve the spatial diversity. The works of Foschini and Gans in 1998 [23] and Telatar in 1999 [24] on various MIMO techniques are regarded as the first studies of the MIMO systems. These two pioneering publications showed that a large capacity gain could be achieved with the MIMO systems compared to the traditional single-input single-output (SISO) systems. These findings have motivated a large number of research works on MIMO systems.

Also in 1998 and 1999, Tarokh *et al.* [25] and Guey *et al.* [26] derived two space-time coding (STC) design criteria based on the upper bound of pairwise error probability. One is rank criterion or diversity criterion in which an STC is said to achieve full diversity if the code difference matrix is of full rank. The other is the product criterion or determinant criterion in which the coding gain of an STC is determined by the product of eigenvalues of the code difference matrix, and thus it should be large to obtain a high coding gain.

One important means of achieving spatial diversity is by deployment of multiple antennas at the transmitter, which is known as transmit diversity. Tarokh *et al.* in 1998 [25] proposed space-time trellis coding (STTC) that can effectively exploit transmit diversity, but its decoding complexity increases exponentially with the transmission rate. Thus different transmit diversity schemes should be proposed to reduce the complexity of the decoding algorithm in STTC. Dealing with this issue, in the same year with the STTC, Alamouti [27] designed a new orthogonal transmit diversity scheme using two transmit antennas. This coding scheme has been widely known as the Alamouti code in honour of its inventor. The Alamouti scheme was later generalized by Tarokh *et al.* in 2001 [28], Ganesan and Stoica in 2001 [29], and Tirkkonen and Hottinen in 2002 [30] for more than two transmit antennas and characterized as orthogonal space-time block coding (OSTBC) for MIMO systems. Other STCs were also designed using some specific matrix structures; for instance, quasi-orthogonal space-time block code (QOSTBC) in Jafarkhani in 2001 [31], rotated QOSTBC in Su and Xia in 2004 [32], cyclic STC in Hughes in 2000 [33], unitary STC in Hochwald *et al.* in 2000 [34], diagonal algebraic STC in Damen *et al.* in 2002 [35], and groupwise STC in Du and Li in 2006 [36].

Considering wideband wireless communications when the systems are required to operate at a high data rate, the communication channels now become frequency-selective fading. The STC schemes for the narrowband communications are shown to be inappropriate and are thus required to be redesigned. Indeed, the frequency-selective or multipath fading channels cause not only severe attenuation in signal strength, but also a large amount of inter-symbol interference (ISI), which makes the signal detection unreliable. However, these multiple paths can offer multipath diversity or frequency diversity. Many studies were then dedicated to extend OSTBCs to frequency-selective fading channels, such as Linkskog and Paulraj in 2000

[37], Al-Dhahir in 2001 [38], and Zhou and Giannakis in 2001 [39]. The newly designed codes can be viewed as a block implementation of the Alamouti code. Another approach to mitigate the frequency selectivity is orthogonal frequency-division multiplexing (OFDM) which uses multiple subcarriers to mitigate the fading effects. For wideband MIMO-OFDM systems, Agrawal *et al.* in 1998 [40] proposed space-frequency coding (SFC) by converting the time domain in the STC to the frequency domain. Different versions of the SFC were then developed and analyzed in Lu and Wang in 2000 [41], Blum *et al.* in 2001 [42], and Su *et al.* in 2005 [43] based on the mapping from different STCs. Adapting the SFCs to several consecutive OFDM blocks, Gong and Letaief in 2001 [44] designed space-time-frequency coding (STFC) for two transmit antennas, which was then extended by Liu *et al.* in 2002 [45] and Molisch *et al.* in 2002 [46] for multiple transmit antennas.

2.3 Cooperative Diversity Protocols and Techniques

MIMO systems are only feasible when devices employ multiple co-located antennas. However, the installation of multiple antennas may be impractical due to the inherent hardware limitation of some small devices. Instead, these devices can collaborate to form a virtual multi-antenna system. The communication between a source node and a destination node can be realized in a cooperative manner with one or multiple cooperating nodes acting as relay node(s). Drawing from user cooperation to achieve some of the benefits of MIMO systems, this form of diversity is well known as cooperative diversity or user cooperation diversity.

2.3.1 Cooperative Protocols

A very classical relay channel including three terminals was initially introduced by van der Meulen in 1971 [47] where a relay terminal simply listens to the transmitted signal from a source terminal, processes it and then sends it to a destination terminal. For this relay channel model, Cover and Gamal in 1979 [48] was the first work investigating the capacity of the relay channel and also deriving the lower and upper bounds of its capacity. The ergodic capacity of the relay channel with different coding strategies was then analysed by Kramer *et al.* in 2005 [49].

Motivated by the three-terminal channel model, Laneman *et al.* in 2004 [3] proposed low-complexity cooperative protocols for a more general system model taking into account practical aspects. These protocols were developed for time-domain division multiple access (TDMA) systems operating in half-duplex mode. Specifically, two notable cooperative protocols, namely amplify-and-forward (AF) and decode-and-forward (DF), were defined and investigated for two types of relaying techniques including fixed and adaptive relaying. While fixed relaying was shown to be easy in implementation at the cost of low bandwidth efficiency, adaptive relaying via either selective or incremental relaying methods could increase the rate at the expense of high complexity. In fact, the overall rate using the fixed relaying scheme is reduced by half for two transmissions from the source and relay, and thus results in low bandwidth efficiency. Instead of simply processing and forwarding the data to the destination, the relay in the selective relaying scheme has the capability of deciding when to transmit based on a certain threshold of quality of the received signal, i.e., SNR. If the SNR of received signal at the relay is lower than a certain threshold, i.e., the channel from the source to the relay suffers from severe fading, then the relay does not carry out any processing. A relaying scheme known as incremental relaying could improve the performance further if the source knows when to repeat the transmission and the relay knows when the destination needs help. It can be appreciated that the adaptive relaying schemes require a high-complexity processing and a feedback channel from the destination to both the source and relay is specifically required for the incremental relaying.

- *DF Protocol*: In the DF protocol, the relay tries to decode the signal from the source and then transmits the decoded signal to the destination. Since the signal detected at the relay is possibly corrupted, it may cause meaningless cooperation to the eventual decision at the destination. In order to achieve the optimal detection, the destination needs to know the error statistics of the inter-user link. This method was also mentioned by Sendonaris *et al.* in 2003 [1,2] for code division multiple access (CDMA) in cellular networks. Laneman *et al.* showed that the diversity of the DF protocol is limited to one due to the worst link from the source to the relay and from the source to the destination [3].
- *AF Protocol*: In the AF protocol, the relay only amplifies what it receives from the source and then transmits the amplified version to the destination. The destination combines the information sent by the source and the relay, and makes a final decision on the transmitted signal. Although the noises at the relay in the AF protocol are also amplified together with the information, the destination can make a better decision with two independently faded versions of the transmitted signal. Indeed, Laneman *et al.* in 2004 [3] showed that the AF protocol can achieve the full diversity.
- *Other Protocols*: Besides the DF and AF protocols, compress-and-forward (CF) protocol also attracted much attention in Cover and Gamal in 1979 [48] and Kramer *et al.* in 2005 [49]. In the CF protocol, the relay transmits to the destination a quantized and compressed version of the signal received from the source. At the destination, the signal received from the source is used as side information to decode the information from the relay. Another cooperative protocol that was studied by Hunter and Nosratinia in 2006 [50] is coded cooperation where error-control coding is included. In the coded cooperation, the relay transmits incremental redundancy to help the destination recover the original data more reliably by combining the codewords with redundancy from both the source and the relay.

2.3.2 Cooperative Diversity via Distributed Space-Time-Frequency Coding

Relaying protocols are in fact repetition-based cooperative diversity schemes designed to achieve spatial diversity gain as a virtual MIMO system. The benefits of these cooperative protocols are achieved at the price of decreasing bandwidth efficiency with the number of cooperating terminals since each relay requires its own channel or subchannel for repetition. Inspired by the work on STCs for MIMO systems, Laneman and Wornell in 2003 [51] proposed distributed STC (DSTC) to improve the bandwidth efficiency of the cooperative communications. The basic idea of the DSTC is that each single-antenna terminal in the relay network transmits a column of the original OSTBC that was designed for multiple co-located antennas in the MIMO systems. In a distributed fashion, multiple columns of the original OSTBC can be transmitted by the source and the relays to indirectly generate coding matrices which are of the same form as that of the OSTBC, and thus it was named distributed STBC (DSTBC). Various forms of the DSTBC were then devised for flat and frequency-selective fading channels.

With regard to flat fading channels, the first DSTBC was proposed by Laneman and Wornell in 2003 [51] for DF relaying protocol. Nabar *et al.* in 2004 [52] analyzed different DSTBCs for AF relay networks. The original design criteria for the conventional STC in the work of Tarokh *et al.* in 1998 [25] was shown to be able to apply to the DSTBCs. Laneman and Wornell showed that, in order to guarantee full diversity, the number of relay nodes should be less than the number of columns in the conventional OSTBC matrix [51]. The limit on the number of relays was then solved with a new class of the DSTBC for multiple relays designed by Yiu *et al.* in 2006 [53]. In this scheme, the signal transmitted by an active relay node is the product of an information-carrying code matrix and a unique node signature vector to ensure that no active node transmits data using the same coding vector. This method nevertheless operates under the DF protocol and requires high-complexity processing at the relay nodes. For the AF protocol, originated from the idea of linear dispersion STC in Hassibi and Hochwald in 2002 [54], a new DSTBC for the relaying systems was constructed by Jing and Hassibi in 2006 [55] and Jing and Jafarkhani in 2007 [56]. The transmitted signal at each relay is a linear function of its received signals without any decoding but only simple processing. However, these DSTBCs, in general, cannot offer a simple decoding mechanism at the destination. To address this problem, Yi and Kim in 2007 [57] designed a new DSTBC to obtain symbol-wise decodability.

On the subject of dealing with inherent frequency-selective fading phenomena in wideband wireless communications, the DSTBCs for flat fading channels are not directly applicable. In 2005, Scutari and Barbarossa proposed a DSTBC for multi-hop transmission over frequency-selective fading channels with DF protocol [58]. For optimal detection, the error statistics at the relays must be known at the destination. However, this cannot be easily implemented in many current wireless systems. Focused on the uplink communications system with fixed wireless relay stations, Anghel and Kaveh in 2003 [59] introduced the combination of DSTBC and OFDM signaling. Another DSTBC for frequency-selective fading channels was studied in

Mheidat *et al.* in 2007 [60] and Tran *et al.* in 2009 and 2012 [61, 62] where the traditional equalization techniques were extended to the DSTBC with AF protocol. Considering two-relay networks also employing the AF protocol, Vien *et al.* in 2009-2011 proposed other DSTBCs to obtain maximal data rate, maximal diversity gain and decoupling detection of data blocks for a low-complexity receiver structure [63–65]. Based on the QOSTBC designed by Jafarkhani in 2001 [31] for co-located antennas in MIMO systems, distributed QOSTBC (DQOSTBC) was developed for four-relay networks by Vien *et al.* in 2009 in [66]. Inspired by the concept of coded cooperation, Vien *et al.* in 2009 [67] also designed a new DSTBC combined with the hybrid automatic repeat request (HARQ) for turbo-coded relay networks to enhance the performance of the DSTBC achieving both diversity gain and coding gain. The STFC in the MIMO systems was also adapted to the cooperative communications where a distributed space-timefrequency block code (DSTFBC) was proposed by Vien *et al.* in 2013 and 2014 [68, 69] for non-regenerative cognitive wireless relay networks.

2.4 Network Coding Techniques

Network coding (NC) was first proposed in 2000 by Ahswede *et al.* [10] to increase system throughput in lossless networks. This work was regarded as a seminal publication on the NC and has motivated a vast amount of research works. The principle of the NC is that intermediate nodes are allowed to mix signals received from multiple links for subsequent transmissions.

In a typical two-way single-relay network (TWSRN) with no direct link between two end terminals, four transmissions are conventionally required to exchange the data from two terminals through a relay. By applying NC, the number of transmissions could be reduced to three, including two transmissions from two terminals to the relay and one broadcast transmission of the mixed data from the relay to both terminals. Basically, the relay in an NC-based TWSRN mixes the signals received from two terminals, and then forwards the combined signal to both terminals. An end terminal can extract from the combined signal the data sent by another terminal based on its known signal. Since the relay in the NC-based TWSRN has to avoid the collision of the two data packets to detect the data from these two terminals separately, two transmissions are required in the first phase and hence three transmissions in total.

Considering the application of NC at the physical layer, also known as physical-layer NC (PNC), the number of transmissions in a TWSRN could be reduced to two due to the fact that two terminals can transmit simultaneously. The PNC can accordingly improve the network throughput by up to 100% and 50% over the conventional relaying and the NC-based relaying, respectively. Several studies have been dedicated to investigating the application of the PNC in the TWSRN. Specifically, Zhang *et al.* in 2006 [12] and Katti *et al.* in 2007-2008 [13, 70] were the preliminary works that applied the PNC concept for the wireless environment in the TWSRN.

The performance of different PNC-based protocols was summarized by Louie *et al.* in 2010 [14] providing a detailed analysis of the bit error rate and the achievable data rate.

Basically, the processing at the relay with PNC techniques follows DF relaying protocol, while the AF-based PNC was named analog NC (ANC) by Katti *et al.* in 2007 [13]. In the following, these two NC techniques will be presented along with the relevant works on their application in various system models.

2.4.1 Physical-Layer Network Coding

In one-way relay network operated under DF protocol, the relay simply decodes the signal received from the source before forwarding to the destination. However, the relay in TWSRNs receives two data sequences simultaneously from two terminals. The challenging problem is how the relay decodes this mixed signal. Dealing with this problem, Zhang *et al.* proposed the first strategy, namely physical-layer NC (PNC), in 2006 [12]. In the PNC, the relay decodes the sum of two signals instead of decoding each signal individually. The sum of any two signals is characterized by a point in a lattice. Based on this lattice, the relay can decode its received mixed signal and then forward it to both terminals. Using the DF protocol, the PNC technique does not suffer from noise amplification at the relay, and thus a higher data rate is expected. However, the generation of the lattice for mapping would be complicated for a general scenario where the signals transmitted from two terminals use different modulation and coding schemes. Also, this strategy requires a perfect synchronization at the relay in both time and carrier when receiving signals from two terminals.

Another approach to PNC was proposed by Rankov and Wittneben in 2007 [71] where the relay separately decodes two signals from the mixed signal received from two terminals, then combines and forwards them to both terminals. The decoding of these two signals could be implemented using multiuser detection techniques in a well-known textbook of Verdu published in 1998 [72]. Similar to the technique proposed by Zhang *et al.* in 2006 [12], the error amplification at the relay does not have any effects on this strategy. As an advantage of this scheme, the generation of lattice matrices for mapping is not necessary; however, the separate decoding at the relay requires a higher complexity and produces a lower data rate.

2.4.2 Analog Network Coding

With AF protocol, the operation at the relay in TWSRNs is much simpler. The relay only amplifies the mix of two signals received from two terminals and then forwards this amplified version to both terminals. Since the relay performs processing upon the analog signals received from the terminals, this AF-based technique was named analog network coding (ANC) in Katti *et al.* in 2007 [13]. Similar to the AF protocol for one-way relay networks, the ANC has some advantages and disadvantages. The complexity at the relay using the ANC is significantly reduced compared to PNC technique, but the performance and data rate could be nonetheless affected since the noises at the relay are also amplified and forwarded to both terminals. Moreover, in order to extract the interested signal sent by another terminal from the mixed signal, channel information has to be estimated at both terminals to remove its own signal which is regarded as an interference to the interested signal.

2.4.3 Related Works on PNC & ANC

Related works on the application of NC in TWSRNs can be found in Popovski and Yomo in 2007 [73], Zhang *et al.* in 2009 [74], Song *et al.* in 2010 [75], Louie *et al.* in 2010 [14], Ju and Kim in 2010 [15], Wang *et al.* in 2010 [76], and Vien *et al.* in 2010-2014 [77–80], where different PNC and ANC approaches were investigated and evaluated. For instance, the performance analysis of PNC and ANC protocols for TWSRN in Popovski and Yomo in 2007 [73], Louie *et al.* in 2010 [14], and Ju and Kim in 2010 [15], beamforming for ANC in Zhang *et al.* in 2009 [74], differential modulation for ANC in Song *et al.* in 2010 [75], ANC for asynchronous TWSRNs in Wang *et al.* in 2010 [76], and automatic repeat request (ARQ) with PNC in Vien *et al.* in 2010 and 2011 [77,78], channel quality indicator (CQI) reporting with PNC in AF-based TWSRNs in Vien *et al.* in 2012 and 2014 [79, 80].

Many NC-based protocols have also been proposed for a variety of relay channel topologies. A summary of these protocols can be found in the PhD thesis of Vien *et al.* in 2013 [81], such as broadcast channels in Nguyen *et al.* in 2009 [16]; multicast channels in Chen *et al.* in 2010 [17], and Vien *et al.* in 2011-2015 [82–89]; unicast channels in Liu *et al.* in 2009 [18]; and multi-relay channels in Vien *et al.* in 2011-2013 [90–93]. As a specific model of the multicast channels, butterfly networks have been investigated in Zhan *et al.* in 2010 [19], Hu *et al.* in 2011 [20], and Vien *et al.* in 2013 and 2015 [94–96], in which the NC is applied at the relay node to help two source nodes simultaneously transmit their information to two destination nodes.

3 Network Coding in Wireless Butterfly Networks

The basic system model of a WBN is shown in Fig. 1 where data transmitted from two source nodes $\mathscr{S}_1$ and $\mathscr{S}_2$ to two destination nodes $\mathscr{D}_1$ and $\mathscr{D}_2$ is assisted by one relay node $\mathscr{R}$. A half-duplex system is considered where all nodes can either transmit or receive data, but not simultaneously. In the WBN, the NC is applied at $\mathscr{R}$ to help $\mathscr{S}_1$ and $\mathscr{S}_2$ simultaneously transmit their data packets $\mathbf{s}_1$ and $\mathbf{s}_2$, respectively, to $\mathscr{D}_1$ and $\mathscr{D}_2$ in two time slots. In the first time slot, $\mathscr{S}_1$ transmits $\mathbf{s}_1$ to both $\mathscr{R}$ and $\mathscr{D}_1$ while $\mathscr{S}_2$ transmits $\mathbf{s}_2$ to both $\mathscr{R}$ and $\mathscr{D}_2$. Then, $\mathscr{R}$ performs NC on the mixed signals received from $\mathscr{S}_1$ and $\mathscr{S}_2$ and broadcasts the network coded signals to both $\mathscr{D}_1$ and $\mathscr{D}_2$ in the second time slot. Accordingly, $\mathscr{D}_1$ can extract the signal transmitted from $\mathscr{S}_2$, i.e., $\mathbf{s}_2$, and $\mathscr{D}_2$ can extract the signal transmitted from $\mathscr{S}_1$, i.e., $\mathbf{s}_1$. The

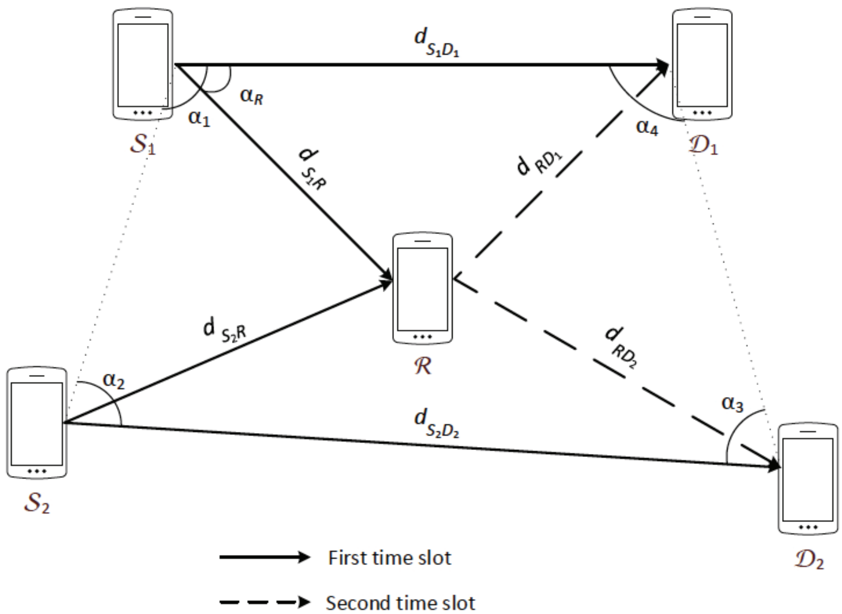


Fig. 1 System model of a wireless butterfly network.

data transmission in the first time slot consists of two direct (DR) transmissions ($\mathscr{S}_1 \to \mathscr{D}_1$ and $\mathscr{S}_2 \to \mathscr{D}_2$) and a multiple access (MA) transmission ($\{\mathscr{S}_1\ \mathscr{S}_2\} \to \mathscr{R}$), while there is only a broadcast (BC) transmission ($\mathscr{R} \to \{\mathscr{D}_1\ \mathscr{D}_2\}$) in the second time slot. Note that the DR and MA transmissions are carried out simultaneously in the first time slot due to the broadcast nature of the wireless medium. This chapter is focused on energy efficiency for a conventional butterfly network when the relay plays a role of coverage extension, facilitating message delivery of indirect links ($\mathscr{S}_1 \to \mathscr{D}_2$ and $\mathscr{S}_2 \to \mathscr{D}_1$), and thus it is assumed that there is no direct link between $\mathscr{S}_1$ and $\mathscr{D}_2$ and between $\mathscr{S}_2$ and $\mathscr{D}_1$.

For convenience, the main notation used in this chapter is listed in Table 1, unless stated otherwise.

4 Hybrid Automatic Repeat Request With Incremental Redundancy Protocol and Energy-Delay Tradeoff

In addition to the merit of NC techniques providing throughput improvement, the reliability and energy efficiency of data transmission should also be taken into consideration within communication systems. This is particularly the case in wireless environments where the communication channels often suffer from deep fading and

Table 1 Main notation in the chapter

Notation	**Meaning**
d_{AB}, $\{A,B\} \in \{S_1,S_2,R,D_1,D_2\}$	distance of link $\mathscr{A}-\mathscr{B}$
α_1, α_2, α_3, α_4, α_R	physical angles $\widehat{\mathscr{D}_1\mathscr{S}_1\mathscr{S}_2}$, $\widehat{\mathscr{S}_1\mathscr{S}_2\mathscr{D}_2}$, $\widehat{\mathscr{S}_2\mathscr{D}_2\mathscr{D}_1}$, $\widehat{\mathscr{D}_2\mathscr{D}_1\mathscr{S}_1}$, $\widehat{\mathscr{D}_1\mathscr{S}_1\mathscr{R}}$, respectively
P_i, $i=1,2$, P_R	transmit powers of $\mathscr{S}_i$,$\mathscr{R}$, respectively
r_i, $i=1,2$, r_R	transmission rate at $\mathscr{S}_i$, $\mathscr{R}$, respectively
h_{ii}, h_{iR}, h_{Ri}, $i=1,2$	channel coefficients of links $\mathscr{S}_i \to \mathscr{D}_i$, $\mathscr{S}_i \to \mathscr{R}$, $\mathscr{R} \to \mathscr{D}_i$, respectively
$\mathbf{n}_{ii}$, $\mathbf{n}_R$, $\mathbf{n}_{Ri}$, $i=1,2$	independent circularly symmetric complex Gaussian (CSCG) noise vectors of links $\mathscr{S}_i \to \mathscr{D}_i$, $\{\mathscr{S}_1,\mathscr{S}_2\} \to \mathscr{R}$, $\mathscr{R} \to \mathscr{D}_i$, respectively, with each entry having zero mean and unit variance
γ_{ii}, γ_{iR}, γ_{Ri}, $i=1,2$	signal-to-noise ratio (SNR) of links $\mathscr{S}_i \to \mathscr{D}_i$, $\mathscr{S}_i \to \mathscr{R}$, $\mathscr{R} \to \mathscr{D}_i$, respectively
ν	pathloss exponent between a pair of transceiver nodes
$\kappa_{(\cdot)}$	number of transmissions required in HARQ-IR protocol to transmit a data packet
$\delta_{(\cdot)}$	effective delay (ED) of HARQ-IR protocol
$\varepsilon_{(\cdot)}$	energy per bit (EB) of HARQ-IR protocol
$[a]_i$	i-th realisation of a random variable a
$\bar{a}$	mean of a random variable a
$\log(\cdot)$	binary logarithm function
$\ln(\cdot)$	natural logarithm function
$E[\cdot]$	statistical expectation function

background noise, and where the energy consumption of various communication and networking devices causes an increasing carbon dioxide emission. To cope with the reliability issue, hybrid automatic repeat request (HARQ) protocols were proposed to reliably deliver information over error-prone channels such as the wireless medium. A detailed study of various error control mechanisms for digital communications is summarized in a textbook of Wicker in 1995 [97]. Specifically, Caire and Tunineti in 2001 [98] showed that the HARQ with incremental redundancy (HARQ-IR) can achieve the ergodic capacity of fading and interference channels. With respect to energy efficiency, an energy-delay tradeoff (EDT) tool was developed by Choi and To in 2012 [99] to evaluate the energy efficiency of HARQ-IR protocols for NC-based two-way relay systems.

4.1 Energy-Delay Tradeoff in Point-to-Point Wireless Links

In order to investigate HARQ-IR protocols with PNC and ANC techniques in WBNs, this subsection will first introduce briefly a simple HARQ-IR protocol for wireless point-to-point (P2P) communications along with EDT evaluation for this system model.

Over a P2P communication channel $\mathscr{S} \rightarrow \mathscr{D}$ employing the HARQ-IR protocol, node $\mathscr{S}$ encodes a data packet $\mathbf{d}$ into a sequence of N coded packets $\{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_N\}$. Then, $\mathscr{S}$ sequentially transmits $\mathbf{c}_k$, $k = 1, 2, \ldots, N$, to $\mathscr{D}$ until a positive acknowledgement (ACK) is received. The signal $\mathbf{y}_k$ received at node $\mathscr{D}$ when transmitted the k-th coded packet $\mathbf{c}_k$ from node $\mathscr{S}$ can be expressed through

$$\mathbf{y}_k = \sqrt{P} h_k \mathbf{x}_k + \mathbf{n}_k, \tag{2}$$

where P is the signal power, h_k is the channel gain of link $\mathscr{S} \rightarrow \mathscr{D}$ for the k-th packet transmission, $\mathbf{x}_k$ is the modulated signal of $\mathbf{c}_k$, and $\mathbf{n}_k$ is an independent circularly symmetric complex Gaussian (CSCG) noise vector with each entry having zero mean and unit variance.

Let κ_{P2P} denote the number of transmissions required in the HARQ-IR protocol to transmit a data packet from $\mathscr{S}$ to $\mathscr{D}$. G. Caire and D. Tuninetti in 2001 [98] expressed κ_{P2P} as

$$\kappa_{\text{P2P}} = \min \left\{ k | \sum_{j=1}^{k} \log(1 + P|[h_k]_j|^2) > r_{\text{P2P}} \right\}, \tag{3}$$

where r_{P2P}, in bits/sec/Hertz (or b/s/Hz), denotes the link spectral efficiency of a capacity-achieving code in P2P communications. By using the same evaluation tool developed by Choi in 2012 [99], the EDT can be characterized by two normalized metrics including energy per bit (EB) in Joules/bit/Hertz (or J/b/Hz] and effective delay (ED) in secs/bit/Hertz (or s/b/Hz). Here, the EB and ED are normalized over the link spectral efficiency r_{P2P}. Let δ_{P2P} and ε_{P2P} denote the ED and EB, respectively, of the HARQ-IR protocol for the P2P communications. These metrics can be written as

$$\delta_{\text{P2P}} = \frac{\bar{\kappa}_{\text{P2P}}}{r_{\text{P2P}}}, \tag{4}$$

$$\varepsilon_{\text{P2P}} = \frac{P\bar{\kappa}_{\text{P2P}}}{r_{\text{P2P}}} = P\delta_{\text{P2P}}, \tag{5}$$

where $\bar{\kappa}_{\text{P2P}}$ denotes the average number of transmissions for reliable P2P communications.

4.2 Energy-Delay Tradeoff in Wireless Butterfly Networks

Basically, the signal processing at relay $\mathscr{R}$ in a WBN (cf. Fig. 1) can be carried out with either PNC or ANC protocols. This subsection will derive the EDTs of the HARQ-IR protocols with PNC and ANC.

4.2.1 EDT of HARQ-IR protocol with PNC

Using the PNC scheme for HARQ-IR in a WBN, $\mathscr{R}$ performs joint decoding of two signals received from $\mathscr{S}_1$ and $\mathscr{S}_2$ in MA transmission following the approach of Zhang and Liew in 2009 [100]. The number of transmissions in the MA transmission can be determined through the MA channel capacity bound derived in a book on information theory of Cover and Thomas in 2006 [101] as follows:

$$\begin{aligned}\kappa_{\texttt{PNC,MA}} = \min\Bigg\{k\Bigg|\Bigg\{\sum_{j=1}^{k}\log(1+[\gamma_{1R}]_j) > r_1\Bigg\}\\ \cap\Bigg\{\sum_{j=1}^{k}\log(1+[\gamma_{2R}]_j) > r_2\Bigg\}\\ \cap\Bigg\{\sum_{j=1}^{k}\log(1+[\gamma_{1R}]_j+[\gamma_{2R}]_j) > r_1+r_2\Bigg\}\Bigg\},\end{aligned} \tag{6}$$

where γ_{iR} and r_i, $i=1,2$, denote the SNR of the transmission link $\mathscr{S}_i \to \mathscr{R}$ and the transmission rate at $\mathscr{S}_i$, respectively. In parallel with the MA transmission, $\mathscr{D}_i$, $i=1,2$, receives the packet from $\mathscr{S}_i$ in the DR transmission. The received signal at $\mathscr{D}_i$ can be written by

$$\mathbf{y}_{ii} = \sqrt{P_i}h_{ii}\mathbf{s}_i + \mathbf{n}_{ii}, \tag{7}$$

where P_i, h_{ii} and $\mathbf{n}_{ii}$ denote the transmission power, channel coefficient and CSCG noise vector at $\mathscr{D}_i$ of the transmission link $\mathscr{S}_i \to \mathscr{D}_i$, respectively. Similar to the transmission over P2P channels, the number of transmissions required at $\mathscr{S}_i$, $i=1,2$, to transmit $\mathbf{s}_i$ to $\mathscr{D}_i$ in the DR transmission can be computed by

$$\kappa_{\texttt{PNC,DR}_i} = \min\Bigg\{k\Bigg|\sum_{j=1}^{k}\log(1+[\gamma_{ii}]_j) > r_i\Bigg\}, \tag{8}$$

where γ_{ii} denotes the SNR of the transmission link $\mathscr{S}_i \to \mathscr{D}_i$. With HARQ-IR protocol, the data packet is retransmitted by $\mathscr{S}_i$, $i=1,2$, until both $\mathscr{R}$ and $\mathscr{D}_i$ successfully decode. Thus, the number of transmissions at $\mathscr{S}_i$ and the total number of transmissions in the first time slot are given by

$$\kappa_{\texttt{PNC,S}_i} = \max\{\kappa_{\texttt{PNC,MA}}, \kappa_{\texttt{PNC,DR}_i}\}, \tag{9}$$

$$\kappa_{\texttt{PNC},1} = \max\{\kappa_{\texttt{PNC,MA}}, \kappa_{\texttt{PNC,DR}_1}, \kappa_{\texttt{PNC,DR}_2}\}, \tag{10}$$

respectively. Then, $\mathscr{R}$ encodes the superimposed packet, and then broadcasts the encoded packet to both $\mathscr{S}_1$ and $\mathscr{S}_2$ in the second time slot. The number of transmissions required at $\mathscr{R}$ to transmit the mixed packet to $\mathscr{D}_i$, $i=1,2$, in the BC transmission is similarly determined as in P2P communications in Subsection 4.1, i.e.,

$$\kappa_{\mathtt{PNC,BC}_i} = \min\left\{ k \,\middle|\, \sum_{j=1}^{k} \log(1+[\gamma_{Ri}]_j) > r_{i'} \right\}, \tag{11}$$

where $i' = 1$ if $i = 2$ and $i' = 2$ if $i = 1$ (or $i' = i - (-1)^i$). Here, γ_{Ri} denotes the SNR of the transmission link $\mathscr{R} \to \mathscr{D}_i$. In order to help both $\mathscr{D}_1$ and $\mathscr{D}_2$ detect the data packets from $\mathscr{S}_2$ and $\mathscr{S}_1$, respectively, $\mathscr{R}$ retransmits the packet until both $\mathscr{D}_1$ and $\mathscr{D}_2$ successfully detect it. Thus, the number of transmissions in the second time slot is computed by

$$\kappa_{\mathtt{PNC},2} = \max\{\kappa_{\mathtt{PNC,BC}_1}, \kappa_{\mathtt{PNC,BC}_2}\}. \tag{12}$$

Overall, the resulting ED and EB of the HARQ-IR protocol with the PNC are respectively given by

$$\delta_{\mathtt{PNC}} = \frac{\bar{\kappa}_{\mathtt{PNC},1} + \bar{\kappa}_{\mathtt{PNC},2}}{r_1 + r_2}, \tag{13}$$

$$\varepsilon_{\mathtt{PNC}} = \frac{P_1 \bar{\kappa}_{\mathtt{PNC,S}_1} + P_2 \bar{\kappa}_{\mathtt{PNC,S}_2} + P_R \bar{\kappa}_{\mathtt{PNC},2}}{r_1 + r_2}, \tag{14}$$

where P_R denotes the transmission power at $\mathscr{R}$.

4.2.2 EDT of HARQ-IR protocol with ANC

With the ANC protocol, in the MA transmission of the first time slot, $\mathscr{R}$ receives the data packets from both $\mathscr{S}_1$ and $\mathscr{S}_2$, which can be written by

$$\mathbf{r} = \sqrt{P_1} h_{1R} \mathbf{s}_1 + \sqrt{P_2} h_{2R} \mathbf{s}_2 + \mathbf{n}_R, \tag{15}$$

where h_{iR} and $\mathbf{n}_R$ denote the channel coefficient and CSCG noise vector at $\mathscr{R}$ of the transmission link $\mathscr{S}_i \to \mathscr{R}$, respectively. At the same time, $\mathscr{D}_i$, $i = 1,2$, receives the data packet from $\mathscr{S}_i$ in the DR transmission. Similarly, the received signal $\mathbf{y}_{ii}$ at $\mathscr{D}_i$ is given by Eq. (7) and the number of transmissions $\kappa_{\mathtt{ANC,DR}_i}$ is determined as $\kappa_{\mathtt{PNC,DR}_i}$ in Eq. (8).

Prior to broadcasting the received signal to both $\mathscr{D}_1$ and $\mathscr{D}_2$, $\mathscr{R}$ normalises its received signal $\mathbf{r}$ in Eq. (15) by a factor $\lambda = 1/\sqrt{E[|\mathbf{r}|^2]} = 1/\sqrt{\gamma_{1R} + \gamma_{2R} + 1}$ to have unit average energy. Thus, in the BC transmission, the signals received at $\mathscr{D}_i$, $i = 1,2$, can be written as

$$\mathbf{y}_{Ri} = \sqrt{P_R} h_{Ri} \lambda \mathbf{r} + \mathbf{n}_{Ri}, \tag{16}$$

where h_{Ri} and $\mathbf{n}_{Ri}$ denote the channel coefficient and CSCG noise vector at $\mathscr{D}_i$ of the transmission link $\mathscr{R} \to \mathscr{D}_i$, respectively. Then, $\mathscr{D}_i$, $i = 1,2$, detects $\mathbf{s}_{i'}$, $i' = i - (-1)^i$, by canceling $\mathbf{s}_i$ which is detected in the DR transmission. The resulting SNR $\gamma_{i'}$ at $\mathscr{D}_i$ is expressed by

$$\gamma_{i'} = \frac{\gamma_{Ri} \gamma_{i'R}}{\gamma_{Ri} + \gamma_{i'R} + \gamma_{iR} + 1}, \tag{17}$$

where γ_{iR} and γ_{Ri} denote the SNRs of the transmission links $\mathscr{S}_i \to \mathscr{R}$ and $\mathscr{R} \to \mathscr{D}_i$, respectively. In the HARQ-IR protocol with ANC, $\mathscr{D}_1$ and $\mathscr{D}_2$ feedback to $\mathscr{S}_1$ and $\mathscr{S}_2$ over direct links to acknowledge the packets $\mathbf{s}_1$ and $\mathbf{s}_2$, respectively. Since there is no decoding process carried out at $\mathscr{R}$ in the first time slot, $\mathscr{R}$ does not perform any feedback for the links $\mathscr{S}_1 \to \mathscr{R}$ and $\mathscr{S}_2 \to \mathscr{R}$. However, $\mathscr{R}$ can help $\mathscr{D}_1$ and $\mathscr{D}_2$ forward the acknowledgement of the packets $\mathbf{s}_2$ and $\mathbf{s}_1$ to $\mathscr{S}_2$ and $\mathscr{S}_1$, respectively. Therefore, the number of transmissions required at $\mathscr{S}_i$, $i = 1,2$, to transmit $\mathbf{s}_i$ to $\mathscr{D}_{i'}$ is determined by

$$\kappa_{\texttt{ANC}_i} = \min\left\{ k \Big| \sum_{j=1}^{k} \log(1+[\gamma_i]_j) > r_i \right\}. \tag{18}$$

The total number of transmissions at $\mathscr{S}_i$, $i = 1,2$, is accordingly given by

$$\kappa_{\texttt{ANC,S}_i} = \max\{\kappa_{\texttt{ANC}_i}, \kappa_{\texttt{ANC,DR}_i}\}. \tag{19}$$

It is noted that, with the ANC protocol, the retransmission of the lost packets at $\mathscr{D}_1$ and $\mathscr{D}_2$ is carried out by $\mathscr{S}_1$ and $\mathscr{S}_2$. $\mathscr{R}$ only amplifies and forwards to $\mathscr{D}_1$ and $\mathscr{D}_2$ the data received from $\mathscr{S}_1$ and $\mathscr{S}_2$. This means that the number of transmissions at $\mathscr{R}$ to assist $\mathscr{S}_1$ and $\mathscr{S}_2$ is also given by $\kappa_{\texttt{ANC}_1}$ and $\kappa_{\texttt{ANC}_2}$, respectively, and, $\mathscr{R}$ uses half power for each task. Therefore, the resulting ED and EB of the HARQ-IR protocol with the ANC scheme are respectively obtained as

$$\delta_{\texttt{ANC}} = \frac{\max\{\bar{\kappa}_{\texttt{ANC,S}_1}, \bar{\kappa}_{\texttt{ANC,S}_2}\} + \max\{\bar{\kappa}_{\texttt{ANC}_1}, \bar{\kappa}_{\texttt{ANC}_2}\}}{r_1 + r_2}, \tag{20}$$

$$\varepsilon_{\texttt{ANC}} = \frac{P_1\,\bar{\kappa}_{\texttt{ANC,S}_1} + P_2\,\bar{\kappa}_{\texttt{ANC,S}_2} + \frac{P_R}{2}\,\bar{\kappa}_{\texttt{ANC}_1} + \frac{P_R}{2}\,\bar{\kappa}_{\texttt{ANC}_2}}{r_1 + r_2}. \tag{21}$$

4.3 Analysis of EDTs in WBNs

In order to provide insights of the EDT in WBNs, this subsection will derive the approximations of the EDTs for various HARQ-IR protocols in WBNs in high and low power regimes. For comparison, the approximated EDTs of both relay-aided transmission, i.e., PNC and ANC, and non-relay-aided transmission, i.e., DT, are investigated. For fair comparison, both relay-aided transmission and non-relay-aided transmission require the same number of time slots to transmit data packets $\mathbf{s}_1$ and $\mathbf{s}_2$ from $\mathscr{S}_1$ and $\mathscr{S}_2$, respectively, to both $\mathscr{D}_1$ and $\mathscr{D}_2$. Specifically, in the DT scheme, in the i-th, $i = 1,2$, time slot $\mathscr{S}_i$ transmits $\mathbf{s}_i$ to $\mathscr{D}_1$ and $\mathscr{D}_2$ over $\mathscr{S}_i$-$\mathscr{D}_1$ and $\mathscr{S}_i$-$\mathscr{D}_2$ links, respectively. In the PNC and ANC schemes, the data transmission in the first time slot consists of two DR transmissions ($\mathscr{S}_1 \to \mathscr{D}_1$ and $\mathscr{S}_2 \to \mathscr{D}_2$) and a MA transmission ($\{\mathscr{S}_1, \mathscr{S}_2\} \to \mathscr{R}$), and there is a BC transmission ($\mathscr{R} \to \{\mathscr{D}_1, \mathscr{D}_2\}$) in the second time slot. This means that all the PNC, ANC and DT schemes require 2 time slots for the data transmission.

Let P denote the total power constraint of all transmitting nodes, i.e., $P = P_1 + P_2 + P_R$. Also, denote ρ_1, ρ_2 and $(1-\rho_1-\rho_2)$ as the fractions of power allocated to $\mathscr{S}_1$, $\mathscr{S}_2$ and $\mathscr{R}$, respectively. Note that, in the DT scheme, $P_R = 0$ and $\rho_1 + \rho_2 = 1$. Accordingly, $P_1 = \rho_1 P$, $P_2 = \rho_2 P$ and $P_R = (1-\rho_1-\rho_2)P$. All channel links are assumed to suffer from quasi-static Rayleigh block fading with $E[|h_{11}|^2] = 1/d^{\nu}_{S_1D_1}$, $E[|h_{22}|^2] = 1/d^{\nu}_{S_2D_2}$, $E[|h_{iR}|^2] = 1/d^{\nu}_{S_iR}$ and $E[|h_{Rj}|^2] = 1/d^{\nu}_{RD_j}$, $i = 1,2$, $j = 1,2$.

Applying HARQ-IR protocol for DT scheme, the ED and EB can be simply derived as

$$\delta_{\mathtt{DT}} = \frac{\bar{\kappa}_{\mathtt{DT},1} + \bar{\kappa}_{\mathtt{DT},2}}{r_1 + r_2}, \tag{22}$$

$$\varepsilon_{\mathtt{DT}} = \frac{P_1 \bar{\kappa}_{\mathtt{DT},1} + P_2 \bar{\kappa}_{\mathtt{DT},2}}{r_1 + r_2}. \tag{23}$$

Here, $\kappa_{\mathtt{DT},i}$, $i = 1,2$, denotes the total number of transmissions required at $\mathscr{S}_i$ to transmit $\mathbf{s}_i$ to both $\mathscr{D}_1$ and $\mathscr{D}_2$, which is given by

$$\begin{aligned}\kappa_{\mathtt{DT},i} = \max\Bigg\{ &\min\left\{ k \Bigg| \sum_{j=1}^{k} \log(1+[\gamma_{ii}]_j) > r_i \right\}, \\ &\min\left\{ k \Bigg| \sum_{j=1}^{k} \log(1+[\gamma_{ii'}]_j) > r_i \right\} \Bigg\},\end{aligned} \tag{24}$$

where $i' = i - (-1)^i$, $i = 1,2$, and $\gamma_{ii'}$ denotes the SNR of the transmission link $\mathscr{S}_i \to \mathscr{D}_{i'}$.

In the high power regime, all HARQ-IR protocols for both relay-aided and non-relay-aided transmissions in a WBN require 2 time slots in total to transmit successfully 2 data packets $\mathbf{s}_1$ and $\mathbf{s}_2$ from $\mathscr{S}_1$ and $\mathscr{S}_2$ to $\mathscr{D}_1$ and $\mathscr{D}_2$. This means that all the PNC, ANC and DT schemes achieve the same EDT performance with $\{\delta_{\mathtt{PNC}}, \delta_{\mathtt{ANC}}, \delta_{\mathtt{DT}}\} \to \frac{2}{r_1+r_2}$ and $\{\varepsilon_{\mathtt{PNC}}, \varepsilon_{\mathtt{ANC}}, \varepsilon_{\mathtt{DT}}\} \to \infty$ as $P \to \infty$, and there is no advantageous scheme in the high power regime.

In the low power regime, the transmission power at all transmitting nodes is assumed to be equally allocated as $P_1 = P_2 = P_R = P/3$ in PNC and ANC schemes and $P_1 = P_2 = P/2$ in the DT scheme. Note that, although the equal power allocation is not optimal in general, it is reasonable to assume the equal power allocation at all transmitting nodes as $P \to 0$. Also, for simplicity, the data transmission from $\mathscr{S}_1$ and $\mathscr{S}_2$ to $\mathscr{D}_1$ and $\mathscr{D}_2$ is assumed to be carried out at the same data rate, i.e., $r_1 = r_2 = R$. The EDTs of the HARQ-IR protocol in the WBN with the DT, PNC and ANC schemes as P approaches to 0 can be derived as in the following theorems.

Theorem 1. *If P approaches 0, then the ED and EB of the HARQ-IR protocol with the DT scheme are approximated by $\delta_{\mathtt{DT},0}$ and $\varepsilon_{\mathtt{DT},0}$, respectively, where*

$$\delta_{\mathtt{DT},0} = \frac{\ln 2}{P}(\max\{d^{\nu}_{S_1D_1}, d^{\nu}_{S_1D_2}\} + \max\{d^{\nu}_{S_2D_1}, d^{\nu}_{S_2D_2}\}), \tag{25}$$

$$\varepsilon_{\mathtt{DT},0} = \frac{\ln 2}{2}(\max\{d^{\nu}_{S_1D_1}, d^{\nu}_{S_1D_2}\} + \max\{d^{\nu}_{S_2D_1}, d^{\nu}_{S_2D_2}\}). \tag{26}$$

Proof. It is noted that when x is sufficiently small,

$$\log(1+ax) \approx \frac{ax}{\ln 2} + O(x^2). \tag{27}$$

Thus, when $P \to 0$,

$$\log(1+[\gamma_{ii}]_j) \approx \frac{|[h_{ii}]_j|^2 P}{2\ln 2}, \quad \log(1+[\gamma_{ii'}]_j) \approx \frac{|[h_{ii'}]_j|^2 P}{2\ln 2},$$

where $i' = i-(-1)^i$, $i = 1,2$. Since $E\{|h_{11}|^2\} = 1/d^{\nu}_{S_1D_1}$, $E\{|h_{22}|^2\} = 1/d^{\nu}_{S_2D_2}$, $E\{|h_{12}|^2\} = 1/d^{\nu}_{S_1D_2}$, $E\{|h_{21}|^2\} = 1/d^{\nu}_{S_2D_1}$ and $r_1 = r_2 = R$, it can be deduced

$$\bar{\kappa}_{\mathtt{DT},1} \approx \frac{2R\ln 2}{P} \max\{d^{\nu}_{S_1D_1}, d^{\nu}_{S_1D_2}\}, \tag{28}$$

$$\bar{\kappa}_{\mathtt{DT},2} \approx \frac{2R\ln 2}{P} \max\{d^{\nu}_{S_2D_2}, d^{\nu}_{S_2D_1}\}. \tag{29}$$

Substituting Eqs. (28) and (29) into Eqs. (22) and (23) with $r_1 = r_2 = R$ and $P_1 = P_2 = P/2$, the theorem is proved.

Theorem 2. *If P approaches 0, then the ED and EB of the HARQ-IR protocol with the PNC scheme are approximated by $\delta_{\mathtt{PNC},0}$ and $\varepsilon_{\mathtt{PNC},0}$, respectively, where*

$$\delta_{\mathtt{PNC},0} = \frac{3\ln 2}{2P}\left(\max\{d^{\nu}_{S_1D_1}, d^{\nu}_{S_2D_2}\} + \max\{d^{\nu}_{RD_1}, d^{\nu}_{RD_2}\}\right), \tag{30}$$

$$\varepsilon_{\mathtt{PNC},0} = \frac{\ln 2}{2}\left(d^{\nu}_{S_1D_1} + d^{\nu}_{S_2D_2} + \max\{d^{\nu}_{RD_1}, d^{\nu}_{RD_2}\}\right). \tag{31}$$

Proof. Consider Eqs. (13) and (14). When $P \to 0$, applying the approximation in Eq. (27) to $\kappa_{\mathtt{PNC,MA}}$, $\kappa_{\mathtt{PNC,DR}_i}$ and $\kappa_{\mathtt{PNC,BC}_i}$, $i = 1,2$, given by Eqs. (6), (8) and (11) with $r_i = R$ and $P_i = P_R = P/3$, i.e.,

$$\bar{\kappa}_{\mathtt{PNC,MA}} \approx \frac{6R\ln 2}{P} \frac{d^{\nu}_{S_1R} d^{\nu}_{S_2R}}{d^{\nu}_{S_1R} + d^{\nu}_{S_2R}}, \tag{32}$$

$$\bar{\kappa}_{\mathtt{PNC,DR}_1} \approx \frac{3R\ln 2}{P} d^{\nu}_{S_1D_1}, \tag{33}$$

$$\bar{\kappa}_{\mathtt{PNC,DR}_2} \approx \frac{3R\ln 2}{P} d^{\nu}_{S_2D_2}, \tag{34}$$

$$\bar{\kappa}_{\mathtt{PNC,BC}_i} \approx \frac{3R\ln 2}{P} d^{\nu}_{Ri}. \tag{35}$$

It is noted that d_{S_1R} and d_{3R} should be both less than $d_{S_1D_1}$ and $d_{S_2D_2}$. Thus,

$$\frac{d^{\nu}_{S_1R} d^{\nu}_{S_2R}}{d^{\nu}_{S_1R} + d^{\nu}_{S_2R}} < \frac{d^{\nu}_{S_1D_1}}{2}, \quad \frac{d^{\nu}_{S_1R} d^{\nu}_{S_2R}}{d^{\nu}_{S_1R} + d^{\nu}_{S_2R}} < \frac{d^{\nu}_{S_2D_2}}{2}.$$

Substitute Eqs. (32), (33), (34) and (35) into Eqs. (9), (10) and (12) as

$$\bar{\kappa}_{\mathtt{PNC},\mathtt{S}_1} \approx \frac{3R\ln 2}{P} d^{\nu}_{S_1D_1}, \tag{36}$$

$$\bar{\kappa}_{\mathtt{PNC},\mathtt{S}_2} \approx \frac{3R\ln 2}{P} d^{\nu}_{S_2D_2}, \tag{37}$$

$$\bar{\kappa}_{\mathtt{PNC},1} \approx \frac{3R\ln 2}{P} \max\{d^{\nu}_{S_1D_1}, d^{\nu}_{S_2D_2}\}, \tag{38}$$

$$\bar{\kappa}_{\mathtt{PNC},2} \approx \frac{3R\ln 2}{P} \max\{d^{\nu}_{RD_1}, d^{\nu}_{RD_2}\}. \tag{39}$$

Then, substituting Eqs. (36), (37), (38) and (39) into Eqs. (13) and (14) with $r_1 = r_2 = R$, the theorem is proved.

Theorem 3. *If P approaches 0, then the ED and EB of the HARQ-IR protocol with the ANC scheme are approximated by $\delta_{\mathtt{ANC},0}$ and $\varepsilon_{\mathtt{ANC},0}$, respectively, where*

$$\delta_{\mathtt{ANC},0} = \frac{9\ln 2}{P^2} \max\{d^{\nu}_{S_1R} d^{\nu}_{RD_2}, d^{\nu}_{S_2R} d^{\nu}_{RD_1}\}, \tag{40}$$

$$\varepsilon_{\mathtt{ANC},0} = \frac{9\ln 2}{4P} (d^{\nu}_{S_1R} d^{\nu}_{RD_2} + d^{\nu}_{S_2R} d^{\nu}_{RD_1}). \tag{41}$$

Proof. Consider Eqs. (20) and (21). When $P \to 0$, applying the approximation in Eq. (27) to $\kappa_{\mathtt{ANC}_i}$, $i = 1,2$, given by Eq. (18) with $r_i = R$ and $P_i = P_R = P/3$, i.e.,

$$\bar{\kappa}_{\mathtt{ANC}_i} \approx \frac{9R\ln 2}{P^2} d^{\nu}_{S_iR} d^{\nu}_{RD'_i}, \tag{42}$$

where $i' = 2$ if $i = 1$ and $i' = 1$ if $i = 2$. Substituting Eqs. (33), (34) and (42) into Eq. (19), it can be deduced

$$\bar{\kappa}_{\mathtt{ANC},\mathtt{S}_1} \approx \max\{\frac{9R\ln 2}{P^2} d^{\nu}_{S_1R} d^{\nu}_{RD_2}, \frac{3R\ln 2}{P} d^{\nu}_{S_1D_1}\}, \tag{43}$$

$$\bar{\kappa}_{\mathtt{ANC},\mathtt{S}_2} \approx \max\{\frac{9R\ln 2}{P^2} d^{\nu}_{S_2R} d^{\nu}_{RD_1}, \frac{3R\ln 2}{P} d^{\nu}_{S_2D_2}\}. \tag{44}$$

Since $P \to 0$, it can be shown that

$$\frac{9R\ln 2}{P^2} d^{\nu}_{S_1R} d^{\nu}_{RD_2} > \frac{3R\ln 2}{P} d^{\nu}_{S_1D_1},$$

$$\frac{9R\ln 2}{P^2} d^{\nu}_{S_2R} d^{\nu}_{RD_1} > \frac{3R\ln 2}{P} d^{\nu}_{S_2D_2}.$$

Thus, Eqs. (43) and (44) can be rewritten as

$$\bar{\kappa}_{\mathtt{ANC},\mathtt{S}_1} \approx \frac{9R\ln 2}{P^2} d^{\nu}_{S_1R} d^{\nu}_{RD_2}, \tag{45}$$

$$\bar{\kappa}_{\mathtt{ANC},S_2} \approx \frac{9R\ln 2}{P^2} d^{\nu}_{S_2R} d^{\nu}_{RD_1}, \tag{46}$$

respectively. Substituting Eqs. (42), (45) and (46) into Eqs. (20) and (21) with $r_1 = r_2 = R$ and $P_1 = P_2 = P_R = P/3$, the theorem is proved.

From the above theorems, the following remarks can be noticed in the low power regime.

Remark 1 (Energy inefficiency with ANC). It can be seen in Eq. (41) that $\varepsilon_{\mathtt{ANC},0}$ increases as P decreases. This means that the ANC scheme is not energy efficient when compared to the DT and PNC schemes for the HARQ-IR protocol in WBN.

Remark 2 (Higher energy efficiency with PNC when relay node is located far from source nodes). When $\mathscr{R}$ is far from $\mathscr{S}_1$ and $\mathscr{S}_2$, $\{d^{\nu}_{RD_1}, d^{\nu}_{RD_2}\} \ll \{d^{\nu}_{S_1D_1}, d^{\nu}_{S_2D_2}\}$. Thus, $d^{\nu}_{S_1D_1} + d^{\nu}_{S_2D_2} + \max\{d^{\nu}_{RD_1}, d^{\nu}_{RD_2}\} \approx d^{\nu}_{S_1D_1} + d^{\nu}_{S_2D_2}$. Accordingly, from Eqs. (26) and (31), it can be shown that $\varepsilon_{\mathtt{PNC},0} < \varepsilon_{\mathtt{DT},0}$, which means the HARQ-IR protocol with the PNC scheme is more energy efficient than the HARQ-IR protocol with the DT scheme.

Remark 3 (Higher energy efficiency with DT over PNC when relay node is located nearby source nodes). In this scenario, $\{d^{\nu}_{RD_1}, d^{\nu}_{RD_2}\} \gtrapprox \{d^{\nu}_{S_1D_1}, d^{\nu}_{S_2D_2}\}$. Thus, from Eqs. (26) and (31), it can be shown that $\varepsilon_{\mathtt{PNC},0} > \varepsilon_{\mathtt{DT},0}$. This means that the DT scheme is more energy efficient than the PNC scheme for the HARQ-IR protocol in the WBN. In other words, there is no advantage of employing the relay when the relay is in the neighborhood of the sources.

For illustration, the EDT performance of the HARQ-IR protocols in a WBN is validated in two following examples, i.e., Examples 1 and 2, for different network configurations.

Example 1. A symmetric WBN is considered with $d_{S_1D_1} = d_{S_2D_2}$, $d_{S_1S_2} = d_{D_1D_2}$ and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \pi/2$. The data transmission from $\mathscr{S}_1$ and $\mathscr{S}_2$ to $\mathscr{D}_1$ and $\mathscr{D}_2$ is carried out at the same data rate with spectral efficiency of $r_1 = r_2 = R$ [b/s/Hz]. HARQ-IR protocol is employed with either DT or PNC or ANC schemes. The pathloss exponent between a pair of transceiver nodes is assumed to be $\nu = 3$ and all channels experience quasi-static Rayleigh block fading.

Figure 2 plots the EDT curves of three HARQ-IR protocols with different data rates at $\mathscr{S}_1$ and $\mathscr{S}_2$. The spectral efficiency, i.e., R, is assumed to vary in the ranges $\{1, 4, 16\}$ b/s/Hz. The relay is assumed to be located at the center of the network, i.e., $d_{S_1R} = d_{S_2R} = d_{RD_1} = d_{RD_2}$. The transmission powers at $\mathscr{S}_1$, $\mathscr{S}_2$ and $\mathscr{R}$ are assumed to be equally allocated. It can be seen that the PNC scheme is more energy efficient than both the ANC and DT schemes. In fact, using the HARQ-IR protocol with PNC, $\mathscr{R}$ can help $\mathscr{S}_1$ and $\mathscr{S}_2$ retransmit the corrupted combined packets to both $\mathscr{D}_1$ and $\mathscr{D}_2$. Using the HARQ-IR protocol with the ANC scheme, $\mathscr{S}_1$ and $\mathscr{S}_2$ are required to retransmit the corrupted packets to $\mathscr{R}$, then $\mathscr{R}$ combines the received packets and broadcasts the new combined packets to both $\mathscr{D}_1$ and $\mathscr{D}_2$. Using the DT scheme, there is no relay to assist $\mathscr{S}_1$ and $\mathscr{S}_2$ retransmit the corrupted combined

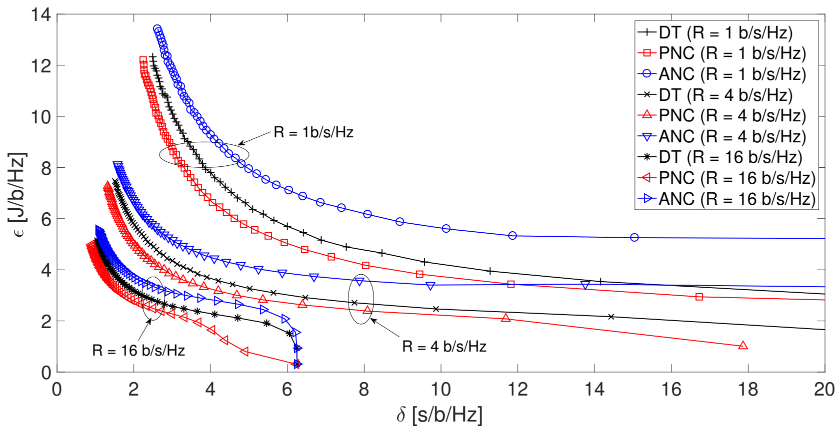


Fig. 2 EDTs of various HARQ-IR protocols in WBN with $d_{S_1D_1} = d_{S_2D_2} = 2$ m, $d_{S_1S_2} = d_{D_1D_2} = 1/2$ m and $d_{S_1R} = d_{S_2R} = d_{RD_1} = d_{RD_2}$.

packets to both $\mathscr{D}_1$ and $\mathscr{D}_2$. Due to the long distances from $\mathscr{S}_1$ to $\mathscr{D}_2$ and from $\mathscr{S}_2$ to $\mathscr{D}_1$, the DT scheme is shown to be less energy efficient than the PNC scheme. However, the EDT of the DT scheme is better than that of the ANC scheme since a re-combination process is required at $\mathscr{R}$ in the ANC scheme, which means more energy consumption at $\mathscr{R}$. This confirms the statements in Remarks 1 and 2 regarding a lower energy efficiency of the ANC scheme and a higher energy efficiency of the PNC scheme over the DT scheme when the relay node is located far from the source nodes. The impact of data rate on the EDT performance can also be observed in Fig. 2 where an improved EDT is achieved for all the HARQ-IR protocols as the data rate increases.

Example 2. Taking into account the practical scenario where the relay is not always located at the center of the network, Fig. 3 plots the EDT curves of various HARQ-IR protocols in the WBN with respect to different relay positions.

Three relay positions are considered, including

(i) $\mathscr{R}$ *near* $\{\mathscr{S}_1, \mathscr{S}_2\}$: $d_{S_1R} = 1/4$ m, $\alpha_R = \pi/4$;
(ii) $\mathscr{R}$ *at the center*: $d_{S_1R} = d_{S_2R} = d_{RD_1} = d_{RD_2}$;
(iii) $\mathscr{R}$ *near* $\{\mathscr{D}_1, \mathscr{D}_2\}$: $d_{S_1R} = 2$ m, $\alpha_R = \pi/6$.

As shown in Fig. 3, the DT scheme is the most energy efficient scheme compared to both the PNC and ANC schemes when the relay is in the neighborhood of the sources. This confirms the statement in Remark 3 in relation to the higher energy efficiency of the DT scheme when the relay is located nearby the sources. In fact, it can be intuitively observed that the relay plays the same role as the sources if the relay is located near the sources, which means the use of the relay in the PNC and ANC schemes is not as energy efficient compared to the DT scheme, though the relay can be used in this case to increase the transmit diversity order. For the scenario

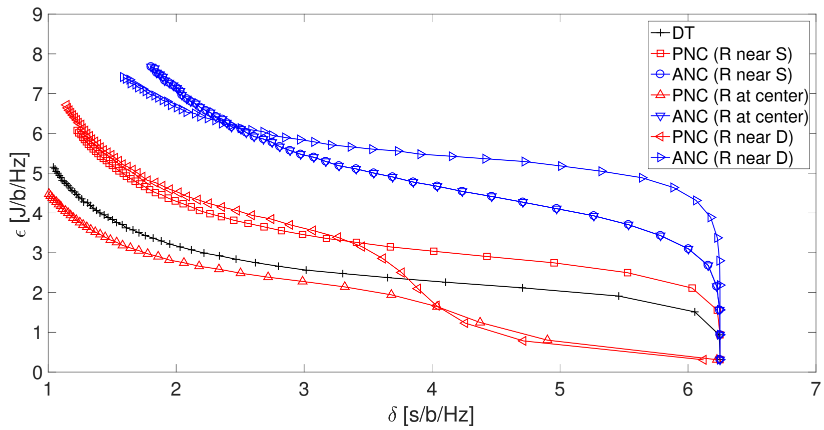


Fig. 3 EDTs of various HARQ-IR protocols in WBN with $R = 16$ b/s/Hz, $d_{S_1D_1} = d_{S_2D_2} = 2$ m, $d_{S_1S_2} = d_{D_1D_2} = 1/2$ m and various relay positions.

when the relay is near the destinations, the relay is shown to be energy efficient with the PNC scheme in the low power regime, while with the ANC scheme it is seen to be always less energy efficient. This scenario is similar to the scenario when the relay is located at the center of the network as observed in Fig. 2 of Example 1.

5 Relay Placement in Wireless Butterfly Networks

Relay placement (RP) problem has been extensively investigated in the literature, e.g., Chen *et al.* in 2012 [102] and Han *et al.* in 2013 [103]. In Chen *et al.* in 2012 [102], the relay position optimization was proposed to improve diversity gain of unbalanced DF relay networks, while the optimal relay placement problem was investigated by Han *et al.* in 2013 [103] for AF relay networks.

In WBNs employing HARQ-IR protocol with either PNC or ANC technique, the location of the relay also has a considerable impact on the energy efficiency of the network. Based on the derived EDT for HARQ-IR protocols with PNC and ANC in Section 3, this section will develop algorithms for solving the RP optimization problem subject to location and power constraints in the WBNs. The best relay location will be determined with respect to different HARQ-IR protocols. This is useful for the system where the mobile users play the role as the relay nodes and thus the user having the best relay location would be selected for the relay communications.

The RP problem relates to how to position the relay node in order to minimize either the total delay or the total energy consumption of all the multicast transmissions from two source nodes to two destination nodes. As shown in Fig. 1, the relay location can be determined through the distance between $\mathscr{S}_1$ and $\mathscr{R}$, i.e., d_{S_1R}, and the angle $\widehat{\mathscr{D}_1\mathscr{S}_1\mathscr{R}}$, i.e., α_R. Based on d_{S_1R} and α_R, the distances from $\mathscr{R}$ to $\mathscr{S}_2$, $\mathscr{D}_1$ and $\mathscr{D}_2$ can be easily obtained as

$$d_{S_2R} = \sqrt{d_{S_1S_2}^2 + d_{S_1R}^2 - 2d_{S_1S_2}d_{S_1R}\cos(\alpha_1 - \alpha_R)}, \tag{47}$$

$$d_{RD_1} = \sqrt{d_{S_1D_1}^2 + d_{S_1R}^2 - 2d_{S_1D_1}d_{S_1R}\cos\alpha_R}, \tag{48}$$

$$d_{RD_2} = \sqrt{d_{S_2D_2}^2 + d_{S_2R}^2 - 2d_{S_2D_2}d_{S_2R}\cos\beta_R}, \tag{49}$$

respectively. Here, β_R denotes the angle $\widehat{\mathscr{D}_2\mathscr{S}_2\mathscr{R}}$, which can be computed by

$$\beta_R = \alpha_2 - \sin^{-1}\left(\frac{d_{S_1R}}{d_{S_2R}}\sin(\alpha_1 - \alpha_R)\right). \tag{50}$$

Let $\{d^*_{S_1R,\delta_{\mathtt{PNC}}},\ \alpha^*_{S_1R,\delta_{\mathtt{PNC}}}\}$, $\{d^*_{S_1R,\delta_{\mathtt{ANC}}},\ \alpha^*_{R,\delta_{\mathtt{ANC}}}\}$, $\{d^*_{S_1R,\varepsilon_{\mathtt{PNC}}},\ \alpha^*_{R,\varepsilon_{\mathtt{PNC}}}\}$ and $\{d^*_{S_1R,\varepsilon_{\mathtt{ANC}}},\ \alpha^*_{R,\varepsilon_{\mathtt{ANC}}}\}$ denote the optimized positioning parameters for the relay location using PNC and ANC protocols subject to minimizing $\delta_{\mathtt{PNC}}$, $\delta_{\mathtt{ANC}}$, $\varepsilon_{\mathtt{PNC}}$ and $\varepsilon_{\mathtt{ANC}}$, respectively. The RP optimization problem can be formulated as

$$\{d^*_{S_1R,\delta_{\mathtt{PNC}}}, \alpha^*_{R,\delta_{\mathtt{PNC}}}\} = \arg\min_{d_{S_1R},\alpha_R} \delta_{\mathtt{PNC}}, \tag{51}$$

$$\{d^*_{S_1R,\delta_{\mathtt{ANC}}}, \alpha^*_{R,\delta_{\mathtt{ANC}}}\} = \arg\min_{d_{S_1R},\alpha_R} \delta_{\mathtt{ANC}}, \tag{52}$$

$$\{d^*_{S_1R,\varepsilon_{\mathtt{PNC}}}, \alpha^*_{R,\varepsilon_{\mathtt{PNC}}}\} = \arg\min_{d_{S_1R},\alpha_R} \varepsilon_{\mathtt{PNC}}, \tag{53}$$

$$\{d^*_{S_1R,\varepsilon_{\mathtt{ANC}}}, \alpha^*_{R,\varepsilon_{\mathtt{ANC}}}\} = \arg\min_{d_{S_1R},\alpha_R} \varepsilon_{\mathtt{ANC}}, \tag{54}$$

where $\delta_{\mathtt{PNC}}$, $\delta_{\mathtt{ANC}}$, $\varepsilon_{\mathtt{PNC}}$ and $\varepsilon_{\mathtt{ANC}}$ are generally given by Eqs. (13), (20), (14) and (21), respectively. Given the fixed location of the source and destination nodes (cf. Fig. 1), d_{S_1R} and α_R are bounded by the following ranges:

$$\begin{aligned} 0 < d_{S_1R} < \max\Big\{ &\sqrt{d_{S_1D_1}^2 + d_{S_1S_2}^2 - 2d_{S_1D_1}d_{S_1S_2}\cos\alpha_1}, \\ &\sqrt{d_{S_1D_1}^2 + d_{D_1D_2}^2 - 2d_{S_1D_1}d_{D_1D_2}\cos\alpha_4}\Big\}, \end{aligned} \tag{55}$$

$$0 < \alpha_R < \alpha_1. \tag{56}$$

The following remarks can be drawn:

Remark 4 (ANC-based relay can be nearly located at the same location for minimizing both the delay and energy). Given a compact set $\mathbb{S}$, $\arg\min_{x_1,x_2\in\mathbb{S}} \max\{f(x_1), f(x_2)\} \approx \arg\min_{x_1,x_2\in\mathbb{S}} (f(x_1) + f(x_2))$. Thus, from Eqs. (20) and (21), it can be approximated that $\arg\min_{d_{S_1R},\alpha_R} \delta_{\mathtt{ANC}} \approx \arg\min_{d_{S_1R},\alpha_R} \varepsilon_{\mathtt{ANC}}$, which means $\{d^*_{S_1R,\delta_{\mathtt{ANC}}}, \alpha^*_{R,\delta_{\mathtt{ANC}}}\} \approx \{d^*_{S_1R,\varepsilon_{\mathtt{ANC}}}, \alpha^*_{R,\varepsilon_{\mathtt{ANC}}}\}$.

Remark 5 (Perspective transformation for a general setting of the node positions in an irregular quadrilateral). Note that the nodes in a quadrilateral can be mapped to the nodes in a rectangle using spatial transformation approach which can be found in a book of Wolberg in 1990 [104] for digital image processing. The optimal relay position in the rectangular region, namely virtual relay positions, can be firstly found for minimizing either delay or energy. Then, the real relay position for the irregular quadrilateral node setting can be determined by an inverse mapping. Specifically, a perspective transformation or projective non-affine mapping with bilinear interpolation can be used to map a quadrilateral to a rectangle as follows: Given four 2-dimensional points A, B, C and D of a quadrilateral located at (x_A, y_A), (x_B, y_B), (x_C, y_C) and (x_D, y_D), and four 2-dimensional points A', B', C' and D' of a rectangle located at $(x_{A'}, y_{A'})$, $(x_{B'}, y_{B'})$, $(x_{C'}, y_{C'})$ and $(x_{D'}, y_{D'})$. $\{A, B, C, D\}$ can be mapped to $\{A', B', C', D'\}$ by finding an 4×4 mapping matrix $\mathbf{M}$ such that

$$\begin{pmatrix} 1 & x_A & y_A & x_A y_A \\ 1 & x_B & y_B & x_B y_B \\ 1 & x_C & y_C & x_C y_C \\ 1 & x_D & y_D & x_D y_D \end{pmatrix} \mathbf{M} = \begin{pmatrix} 1 & x_{A'} & y_{A'} & x_{A'} y_{A'} \\ 1 & x_{B'} & y_{B'} & x_{B'} y_{B'} \\ 1 & x_{C'} & y_{C'} & x_{C'} y_{C'} \\ 1 & x_{D'} & y_{D'} & x_{D'} y_{D'} \end{pmatrix}$$

Wolberg in 1990 [104] and Kim *et al.* in 2002 [105] showed that perspective transformation is planar mapping and thus both forward and inverse mapping are unique. Also, the lines connecting nodes are shown to be preserved in all orientations.

According to Remark 5, for simplicity, a specific scenario can be considered, where $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \pi/2$, $d_{S_1D_1} = d_{S_2D_2}$ and $d_{S_1S_2} = d_{D_1D_2}$. The search range of the relay position given by Eqs. (55) and (56) is thus rewritten as

$$0 < d_{S_1R} < \sqrt{d^2_{S_1D_1} + d^2_{S_1S_2}}, \tag{57}$$

$$0 < \alpha_R < \frac{\pi}{2}. \tag{58}$$

With the total power constraint P and different power allocation at $\mathscr{S}_1$ and $\mathscr{S}_2$, there are three typical cases based on the relationship between P_1 and P_2 which are described as follows:

5.1 Equal Power Allocation at Sources

Due to the equal power allocation at $\mathscr{S}_1$ and $\mathscr{S}_2$, $\mathscr{R}$ is located on the median line between the pair nodes $\{\mathscr{S}_1, \mathscr{D}_1\}$ and $\{\mathscr{S}_2, \mathscr{D}_2\}$. Denote $d_R = \sqrt{d^2_{S_1R} - d^2_{S_1S_2}/4}$. The RP optimization in Eqs. (51), (52), (53) and (54) can be determined through

$$d^*_{R,\delta_{\mathtt{X}}} = \arg \min_{0<d_R<d_{S_1D_1}} \delta_{\mathtt{X}}, \tag{59}$$

$$d^*_{R,\varepsilon_{\mathtt{X}}} = \arg \min_{0<d_R<d_{S_1D_1}} \varepsilon_{\mathtt{X}}, \tag{60}$$

where $\mathtt{X} \in \{\mathtt{PNC}, \mathtt{ANC}\}$. Then, $\{d^*_{S_1R,\delta_{\mathtt{X}}}, \alpha^*_{R,\delta_{\mathtt{X}}}\}$ and $\{d^*_{S_1R,\varepsilon_{\mathtt{X}}}\ \alpha^*_{1R,\varepsilon_{\mathtt{X}}}\}$ can be computed by

$$d^*_{S_1R,\delta_{\mathtt{X}}} = \sqrt{d^{*2}_{R,\delta_{\mathtt{X}}} + \frac{d^2_{S_1S_2}}{4}}, \alpha^*_{R,\delta_{\mathtt{X}}} = \tan^{-1}\left(\frac{d_{S_1S_2}}{2d^*_{R,\delta_{\mathtt{X}}}}\right), \tag{61}$$

$$d^*_{S_1R,\varepsilon_{\mathtt{X}}} = \sqrt{d^{*2}_{R,\varepsilon_{\mathtt{X}}} + \frac{d^2_{S_1S_2}}{4}}, \alpha^*_{R,\varepsilon_{\mathtt{X}}} = \tan^{-1}\left(\frac{d_{S_1S_2}}{2d^*_{R,\varepsilon_{\mathtt{X}}}}\right). \tag{62}$$

It can be observed that the search algorithms using Eqs. (59), (60), (61) and (62) require a lower complexity processing than an exhaustive search of all available relay positions in the whole region encompassing the four source and destination nodes with the constraints of Eqs. (57) and (58).

5.2 Unequal Power Allocation at Sources

Considering unequal power allocation at $\mathscr{S}_1$ and $\mathscr{S}_2$, i.e., $P_1 \neq P_2$, there are two cases including $P_1 > P_2$ and $P_1 < P_2$ as follows:

5.2.1 $P_1 > P_2$

In this scenario, $\mathscr{R}$ should be located in the neighborhood region of the pair node $\{\mathscr{S}_2, \mathscr{D}_2\}$. Thus, the search range for the optimal relay location in Eqs. (57) and (58) can be limited by two regions defined as follows:

$$\text{Region (I):} \begin{cases} \tan^{-1}\left(\frac{d_{S_1S_2}}{2d_{S_1D_1}}\right) < \alpha_R < \tan^{-1}\left(\frac{d_{S_1S_2}}{d_{S_1D_1}}\right), \\ \frac{d_{S_1S_2}}{2\sin\alpha_R} < d_{S_1R} < \frac{d_{S_1D_1}}{\cos\alpha_R}. \end{cases} \tag{63}$$

$$\text{Region (II):} \begin{cases} \tan^{-1}\left(\frac{d_{S_1S_2}}{d_{S_1D_1}}\right) < \alpha_R < \frac{\pi}{2}, \\ \frac{d_{S_1S_2}}{2\sin\alpha_R} < d_{S_1R} < \frac{d_{S_1S_2}}{\sin\alpha_R}. \end{cases} \tag{64}$$

With various relay positions in regions (I) and (II), the optimal relay location $\{d^*_{S_1R,\delta_{\mathtt{X}}}, \alpha^*_{R,\delta_{\mathtt{X}}}\}$ and $\{d^*_{S_1R,\varepsilon_{\mathtt{X}}}, \alpha^*_{R,\varepsilon_{\mathtt{X}}}\}$, $\mathtt{X} \in \{\mathtt{PNC}, \mathtt{ANC}\}$, subject to minimizing either $\delta_{\mathtt{X}}$ or $\varepsilon_{\mathtt{X}}$ can be determined as in Eqs. (51), (52), (53) and (54). Regarding the search range in the context of $P_1 > P_2$, it can be observed that the search regions (I) and

(II) are narrower than the region determined by Eqs. (57) and (58), and thus the complexity of the search for the optimal relay location is reduced.

5.2.2 $P_1 < P_2$

Similarly, in this scenario, $\mathscr{R}$ is located near the two nodes $\mathscr{S}_1$ and $\mathscr{D}_1$. The search range for the optimal relay location in Eqs. (57) and (58) can thus be limited by two regions defined as follows:

$$\text{Region (III):} \begin{cases} 0 < \alpha_R < \tan^{-1}\left(\frac{d_{S_1S_2}}{2d_{S_1D_1}}\right), \\ 0 < d_{S_1R} < \frac{d_{S_1D_1}}{\cos\alpha_R}. \end{cases} \tag{65}$$

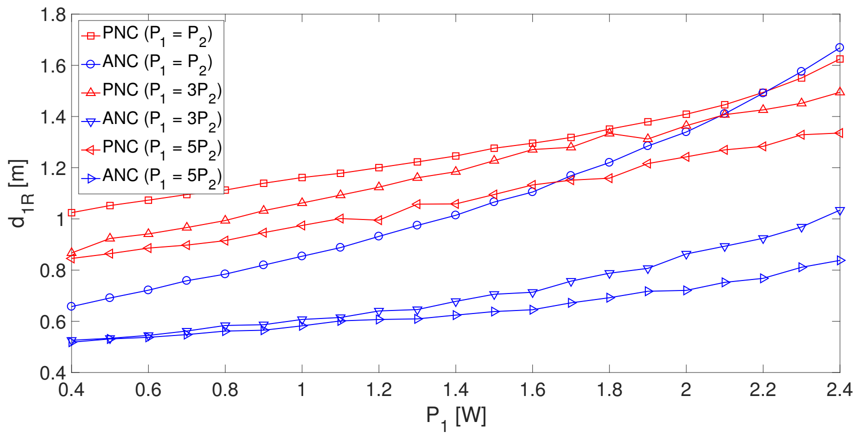
$$\text{Region (IV):} \begin{cases} \tan^{-1}\left(\frac{d_{S_1S_2}}{2d_{S_1D_1}}\right) < \alpha_R < \frac{\pi}{2}, \\ 0 < d_{S_1R} < \frac{d_{S_1S_2}}{2\sin\alpha_R}. \end{cases} \tag{66}$$

Then, the optimal relay location $\{d^*_{S_1R,\delta_{\mathtt{X}}}, \alpha^*_{R,\delta_{\mathtt{X}}}\}$ and $\{d^*_{S_1R,\varepsilon_{\mathtt{X}}}, \alpha^*_{R,\varepsilon_{\mathtt{X}}}\}$, $\mathtt{X} \in \{\mathtt{PNC}, \mathtt{ANC}\}$, can be determined in regions (III) and (IV) so as to minimize either $\delta_{\mathtt{X}}$ or $\varepsilon_{\mathtt{X}}$. Additionally, it can be observed that the search regions (III) and (IV) for the scenario $P_1 < P_2$ are also narrower than the region determined by Eqs. (57) and (58), and again a low-complexity search algorithm is achieved.

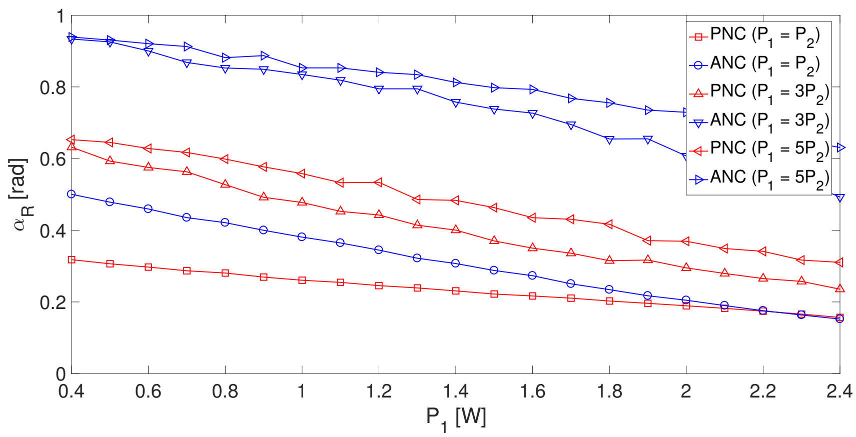
Consider for illustration the following example of the RP optimization problem for minimum ED and EB in a typical WBN.

Example 3. A symmetric WBN as in Example 1 is investigated where $d_{S_1D_1} = d_{S_2D_2} = 2$ m and $d_{S_1S_2} = d_{D_1D_2} = 1/2$ m. Figs. 4 and 5 plot the optimal relay locations for minimizing ED and EB, respectively, as a function of power allocation at source nodes when HARQ-IR protocols are employed with PNC and ANC. The optimal relay locations in Figs. 4 and 5 are determined through d_{S_1R} and α_R. It is assumed that $R = 4$ b/s/Hz and $P = P_1 + P_2 + P_R = 5$ W. Both equal power allocation, i.e., $P_1 = P_2$, and unequal power allocation with $P_1 = 3P_2$ and $P_1 = 5P_2$, are considered. Note that the RP for the scenario $P_1 < P_2$ can be similarly observed to be symmetric with that for the scenario $P_1 > P_2$. The bisection search method is applied to find the optimal relay position in the search region. Investigating the optimal relay location for minimum ED, Fig. 4 shows that for the scenario $P_1 = P_2$, as P_1 and P_2 increase, the optimal location of both the ANC-based and PNC-based $\mathscr{R}$ move from the region near $\mathscr{S}_1$ and $\mathscr{S}_2$ to the region near $\mathscr{D}_1$ and $\mathscr{D}_2$. However, when P_1 and P_2 are small, the ANC-based $\mathscr{R}$ is closer to $\mathscr{S}_1$ and $\mathscr{S}_2$ than the PNC-based $\mathscr{R}$. For the case $P_1 = nP_2$, $n > 1$, as n increases, the optimal location of the ANC-based $\mathscr{R}$ is closer to $\mathscr{S}_2$, while that of the PNC-based $\mathscr{R}$ is farther away from $\mathscr{D}_2$.

For minimum EB, it can be observed in Fig. 5 that for the scenario $P_1 = P_2$, as P_1 and P_2 increase, the optimal location of the ANC-based $\mathscr{R}$ moves from the region near $\mathscr{S}_1$ and $\mathscr{S}_2$ to the region near $\mathscr{D}_1$ and $\mathscr{D}_2$, while that of the PNC-based $\mathscr{R}$ moves in the reverse direction. For the case $P_1 = nP_2$, $n > 1$, similar to Fig. 4,



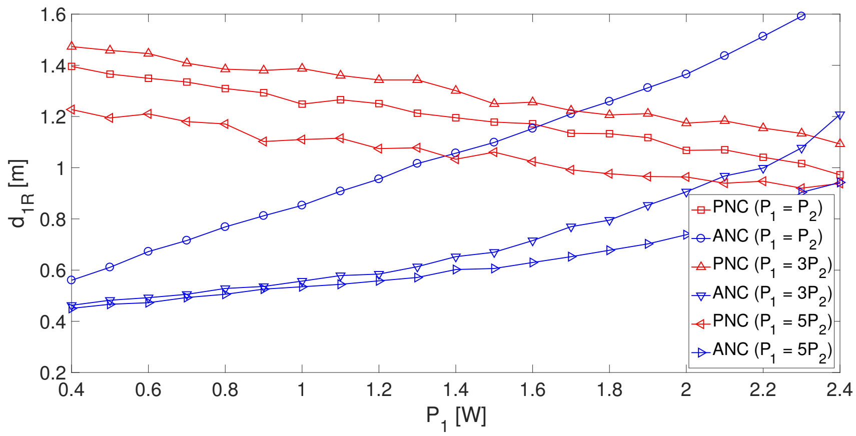
(a)



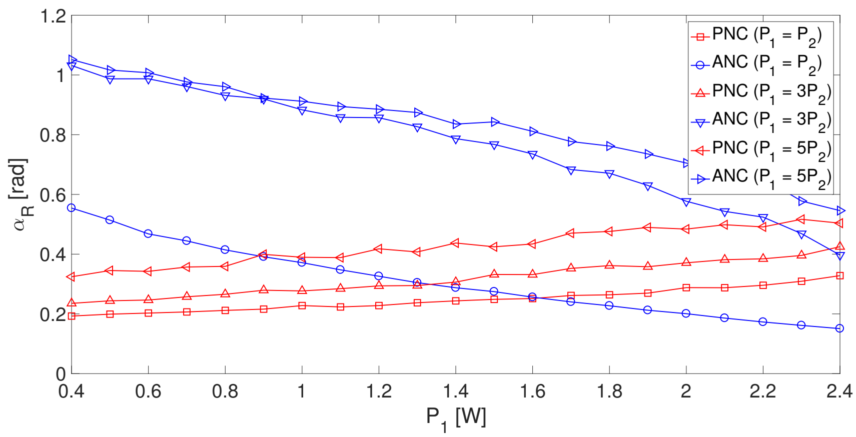
(b)

Fig. 4 Optimization of relay location subject to minimizing ED with $R = 4$ b/s/Hz, $P = 5$ W, $d_{S_1D_1} = d_{S_2D_2} = 2$ m, $d_{S_1S_2} = d_{D_1D_2} = 1/2$ m: (a) d_{S_1R} and (b) α_R.

it is shown that, as n increases, the ANC-based $\mathscr{R}$ should be closer to $\mathscr{S}_2$, while that of the PNC-based $\mathscr{R}$ should be farther away from $\mathscr{D}_2$. Furthermore, the optimal locations for the ANC-based $\mathscr{R}$ are shown to be nearly similar for both objectives of minimum ED and minimum EB, while the optimised locations for the PNC-based $\mathscr{R}$ are different with respect to the objective functions. These nearly similar locations of the ANC-based $\mathscr{R}$ verify the statement in Remark 4.



(a)



(b)

Fig. 5 Optimization of relay location subject to minimizing EB with $R = 4$ b/s/Hz, $P = 5$ W, $d_{S_1D_1} = d_{S_2D_2} = 2$ m, $d_{S_1S_2} = d_{D_1D_2} = 1/2$ m: (a) d_{S_1R} and (b) α_R.

6 Relay Placement in Wireless Multicast Networks

The RP optimization in WBNs can be extended for a general wireless multicast network (WMN) consisting of N_s sources, N_r relays and N_d destinations. The positions of N_s sources and N_d destinations are assumed to be fixed in a two-dimensional plane while the positions of N_r relays vary in a convex set $\mathfrak{S}_T$ having its boundary formed by all the source and destination points. HARQ-IR protocol and NC techniques are also applied at the relays to assist the data transmission between the sources and destinations.

In a WMN, the k-th relay, $k = 1,2,\ldots,N_r$, i.e., $\mathscr{R}_k$, assists the data transmission from a group of $N_{s,k}$ sources, i.e., $\{\mathscr{S}_{k,1},\mathscr{S}_{k,2},\ldots,\mathscr{S}_{k,N_{s,k}}\} \triangleq \mathscr{S}_k^{(N_{s,k})}$, to a group of $N_{d,k}$ destinations, i.e., $\{\mathscr{D}_{k,1},\mathscr{D}_{k,2},\ldots,\mathscr{D}_{k,N_{d,k}}\} \triangleq \mathscr{D}_k^{(N_{d,k})}$. The indices of nodes are determined based upon their vertical axis values in a decreasing order, i.e., the node located higher has a lower index. Denote $\mathfrak{S}_k$ as the convex set generated by points $\{\mathscr{S}_{k,1},\mathscr{S}_{k,2},\ldots,\mathscr{S}_{k,N_{s,k}}\}$ and $\{\mathscr{D}_{k,1},\mathscr{D}_{k,2},\ldots,\mathscr{D}_{k,N_{d,k}}\}$ which are in supporting region of $\mathscr{R}_k$, i.e.,

$$\mathfrak{S}_T \supseteq \bigcup_{k=1}^{N_r} \mathfrak{S}_k \tag{67}$$

The relay-aided transmission is realized in two time slots as follows: In the first time slot, $\mathscr{S}_{k,i_k}$, $i_k = 1,2,\ldots,N_{s,k}$, sends data to $\mathscr{R}_k$ and the corresponding $\mathscr{D}_{k,i'_k}$, $i'_k = 1,2,\ldots,N_{d,k}$ via direct links. Then, $\mathscr{R}_k$ carries out either PNC or ANC on the received signals before broadcasting the combined signal to all $\mathscr{D}_k^{(N_{d,k})}$ in the second time slot. For simplicity, it is assumed that there is no interference caused by non-intended nodes and there is no cooperation between relays, between sources and between destinations in the WMN.

Let (x_A, y_A), $A \in \{\{\mathscr{S}_i\},\{\mathscr{R}_k\},\{\mathscr{D}_j\}\}$, denote the coordinate values of a point $\mathscr{A}$. Exploiting the properties of perspective transformation (cf. Remark 5), the nodes in the irregularly-shaped WMN can be mapped to the nodes in a rectangle. The optimal placement of virtual relays can be found in the rectangular region to minimize either ED or EB. The real optimal positions of the relays can be thus determined by an inverse mapping.

Algorithm 1 Proposed relay placement algorithm

```
for k = 1 to N_r do
    S_k ← {S_{k,1}, S_{k,2}, ..., S_{k,N_{s,k}}, D_{k,1}, D_{k,2}, ..., D_{k,N_{d,k}}}
    Step 1: Map the boundary of S_k to a rectangle S'_k:
    (S'_{k,1} S'_{k,N_{s,k}} D'_{k,1} D'_{k,N_{d,k}}) ← (S_{k,1} S_{k,N_{s,k}} D_{k,1} D_{k,N_{d,k}})
    Find mapping matrix M.
    Step 2: Find virtual positions of remaining nodes in S'_k:
    for i = 2 to N_{s,k} − 1 do
        [1, x_{S'_{k,i}}, y_{S'_{k,i}}, x_{S'_{k,i}} y_{S'_{k,i}}] ← [1, x_{S_{k,i}}, y_{S_{k,i}}, x_{S_{k,i}} y_{S_{k,i}}] M
    end for
    for i = 2 to N_{d,k} − 1 do
        [1, x_{D'_{k,i}}, y_{D'_{k,i}}, x_{D'_{k,i}} y_{D'_{k,i}}] ← [1, x_{D_{k,i}}, y_{D_{k,i}}, x_{D_{k,i}} y_{D_{k,i}}] M
    end for
    Step 3: Find virtual relay placement in S'_k to either minimize ED or minimize EB: (x'_{R_k}, y'_{R_k}).
    Step 4: Find real relay placement in S_k:
        [1, x_{R_k}, y_{R_k}, x_{R_k} y_{R_k}] ← [1, x'_{R_k}, y'_{R_k}, x'_{R_k} y'_{R_k}] M^{-1}
end for
```

For convenience, the entire set $\mathfrak{S}_T$ is divided into N_r subsets with respect to N_r relays (cf. Eq. (67)) and consider a specific subset $\mathfrak{S}_k$, $k = 1, 2, \ldots, N_r$. The RP in the WMN can be carried out as in Algorithm 1, which consists of the following steps:

- *Step 1*: Map the boundary of $\mathfrak{S}_k$ to a rectangle, namely $\mathfrak{S}'_k$, by finding a mapping matrix **M**.
- *Step 2*: Find virtual positions of remaining sources and destinations in $\mathfrak{S}'_k$.
- *Step 3*: Find virtual relay position (x'_{R_k}, y'_{R_k}) in $\mathfrak{S}'_k$ for either minimizing ED or minimizing EB.
- *Step 4*: Find real relay position in $\mathfrak{S}_k$ by inverse mapping.

It can be observed that the RP algorithm only requires the perspective transformation and determination of the optimal relay positions in a particular rectangle.

7 Conclusions

This chapter has provided an overview of cooperative communications with different diversity approaches and cooperative protocols along with NC techniques at the physical layer. In particular, WBN has been investigated as a typical application of the NC techniques. The EDT has been derived for HARQ-IR protocols with PNC and ANC in the WBN by taking into account the effects of both relay location and power allocation. In the high power regime, the use of the relay in both PNC and ANC schemes has been shown to have no advantage over the non-relay-aided DT scheme. In the low power regime, the PNC scheme is more energy efficient than both the ANC and DT schemes when the relay node is located either at the centre of the network or close to the destination nodes, while the DT scheme outperforms both the PNC and ANC schemes when the relay node is in the neighborhood of the source nodes. Furthermore, an RP algorithm for reducing the search region has been developed to find the optimal relay locations for the HARQ-IR protocols with PNC and ANC to minimize either the total delay or the total energy consumption in the WBN. The RP algorithm has also been discussed for a general WMN. For future work, the mobility of nodes as well as network infrastructures in the practical WMNs, such as mobile ad hoc networks, wireless sensor networks, vehicular networks and more generally wireless mesh networks, could be considered in the RP optimization problem subject to constraints on the limited power of nodes and their geographic locations.

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