Stochastic Processes and their Applications 41 (1992) 325-363
North-Holland

Twentieth Conference on Stochastic Processes and their Applications

Nahariya, Israel, June 9-14, 1991

Organized under the auspices of the Committee for Conferences on Stochastic Processes and the Bernoulli Society, with sponsorship and support from the following organizations:

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Abstracts [1]

1. Invited lectures

Multiplicative ergodic theory and stochastic Petri networks

François Baccelli, INRIA – Sophia, Valbonne, France

Stochastic Petri networks provide a general formalism for describing the dynamics of *discrete event systems* arising in particular in computer science, and in manufacturing applications. The present paper focuses on a subclass of stochastic Petri nets called stochastic event graphs, which contains several classical models of queueing theory (e.g. single server queues, queues in tandem, closed cyclic networks, synchronized queueing networks, network of queues with communication or manufacturing blocking etc.). The main practical concerns of the paper are the construction and the analysis of the stationary regime(s) of this class of systems.

It is shown that each FIFO stochastic event graph is amenable to a representation as a linear system in the $(\oplus, \otimes)$ semi-ring, where $\oplus = \max$, $\otimes = +$. More precisely, the state variables $X_j(t)$ associated with the tth firing epoch of transition j of the event graph are shown to satisfy a linear evolution equation of the form

$$X(t+1) = A(t) \otimes X(t) \oplus B(t) \otimes U(t),$$

where $A(t)$ and $B(t)$ are random matrices built from the random durations associated with the tth transition firing, and $U(t)$ is the input process.

Under the assumption that the random durations associated with transition firings are jointly stationary and ergodic, we give the stability condition of the network in terms of $(\oplus, \otimes)$-Liapounov exponents associated with the stationary and ergodic matrices $A(t)$ and $B(t)$. In particular, it is shown that the theorem that gives the stability condition of stochastic event graphs can be interpreted as an Oseledec type multiplicative ergodic theorem in this semi-field. We also address the problem which consists in constructing the stationary regime of the increments of the $X(t)$ stochastic process (e.g. the waiting time process in the single server queue). Both the autonomous case (with no input) and the non autonomous case are considered. The non autonomous case is analyzed using a Loynes type construction, and it is shown that this type of systems admit a unique stationary regime which is reached with coupling. The analysis of the autonomous case is more difficult, as uniqueness is not always granted. We give a sufficient condition for coupling and uniqueness based on Borovkov's theory of renovating events. The results on the stability condition and on the construction of the stationary regime can be seen as stochastic extensions of known results on the periodic regimes of deterministic timed Petri nets, by Ramamoorthy and Ho, and by Cohen, Quadrat and Viot; they also receive simple linear algebraic interpretations.

Finally, we address various computational problems including the derivation of computable bounds for the Liapounov exponents arising in the stability condition and several stochastic monotonicity and convexity properties of the state variables. Lower bounds for the Liapounov exponents are obtained using the convex ordering property of firing epochs considered as functions of the firing durations; upper bounds are derived using the property of association and large deviation estimates for multitype branching processes. We also prove the stochastic concavity of the 'throughput' of any stochastic event graph considered as a function of the initial marking, which generalizes results of Shantikumar on specific queueing systems.

Keywords: Discrete event systems, stochastic Petri networks, event graphs, queueing networks, stationary processes, stability, stochastic recursive sequences, subadditive ergodic theory, multiplicative ergodic theory.

Brownian motion on the Sierpinski carpet

Martin T. Barlow, University of Cambridge, UK

Let $F_0 = [0, 1] \times [0, 1]$ be the unit square. Set $F_1 = F_0 - (\frac{1}{3}, \frac{2}{3})^2$; thus F_1 is obtained from F_0 by dividing F_0 into 9 squares and removing the (open) middle one. Repeating this procedure, of removing the middle

[1] An asterisk is attached to the name of the speaker in the case of a joint paper.

square, one obtains a decreasing sequence (F_n) of closed sets: F_n consists of the union of $8n$ squares, each of side 3^{-n}. The *Sierpinski carpet*, F is defined by $F=\bigcap_{n=0}^{\infty} F_n$. The set F is a 'fractal' subset of $\mathbb{R}^2$, with Hausdorff dimension $d_f=\log 8/\log 3$. *Brownian motion on* F is a continuous, strong Markov F-valued process $\{X_t, t\geq 0\}$, which is locally invariant with respect to the local isometries of F. In this talk I will review the construction and properties of this process: this is joint work with R.F. Bass. It turns out that most of its behaviour can be captured by two indices, d_f and d_w. The second index d_w governs the space-time scaling of X_t: thus one has, for example,

$$c_1 t^{2/d_w} \leq E^x |X_t - x|^2 \leq c_2 t^{2/d_w}, \quad t \leq 1.$$

As $d_w > 2$ this means that the process is *subdiffusive*; the cause is the presence of successively larger obstacles to impede the movement of the process.

References

M.T. Barlow, Random walks and diffusion on fractals, to appear in: Proc. Internat. Congr. Math. Kyoto 1990.

M.T. Barlow and R.F. Bass, The construction of Brownian motion on the Sierpinski carpet, Ann. Inst. H. Poincaré 25 (1989) 225-257.

M.T. Barlow and R.F. Bass, Local times for Brownian motion on the Sierpinski carpet, Probab. Theory Rel. Fields 85 (1990) 91-104.

M.T. Barlow and R.F. Bass, On the resistance of the Sierpinski carpet, Proc. Roy. Soc. London A 431 (1990) 345-360.

M.T. Barlow and R.F. Bass, Transition densities for Brownian motion on the Sierpinski carpet, Preprint (1991).

M.T. Barlow, R.F. Bass and J.D. Sherwood, Resistance and spectral dimension of Sierpinski carpets, J. Phys. A 23 (1990) L253-258.

T. Lindstrøm, Brownian motion on nested fractals, Mem. Amer. Math. Soc. 420 (1990).

H. Osada, Isoperimetric dimension and estimates of heat kernels of pre-Sierpinski carpets, Probab. Theory Rel Fields 86 (1990) 469-490.

R. Rammal and G. Toulouse, Random walks on fractal structures and percolation clusters, J. Phys. Lett. 44 (1983) L13-L22.

Stochastic partial differential equations in anticipative control

Mark H. Davis, University of Oslo, Norway

In this talk we describe an approach to stochastic optimal control in which possibly 'anticipative' controls are allowed and the characteristic 'nonanticipativity' condition is introduced as an explicit equality constraint. This achieves two things: (a) stochastic control is reduced to deterministic control (all optimization is carried out 'pathwise' for each realization of the noise process), and (b) the Lagrange multiplier associated with the nonanticipativity constraint gives a 'price' for future information. This multiplier is characterized in terms of a linear backwards stochastic partial differential equation (SPDE) which also gives the pathwise minimum cost. We also consider purely anticipative control with a view to calculating the reduction in cost that could be achieved were the nonanticipativity condition to be waived (or extra information bought). This is a more difficult problem involving nonlinear SPDEs. The main issue is the existence of global solutions, and some conditions for this will be given.

U-statistics of random-size samples and limit theorems for systems of Markovian particles with non-Poisson initial distributions

R. Epstein Feldman and S. Rachev, University of California at Santa Barbara, CA, USA*

We study the limiting distribution of the amount of charge left in some set by an infinite system of signed Markovian particles when the initial particle density goes to infinity. By selecting the initial particle distribution, we determine the limiting distribution of charge, constructing different non-Gaussian generalized random fields, including Laplace, α-stable, and their multiple integrals. Our proofs are based on limit theorems for U-statistics with random numbers of terms. We extend the results of Dynkin and Mandelbaum (Ann. Statist., 1983) and mandelbaum and Taqqu (Ann. Statist., 1984) to the case of non-Poisson random sample size. We construct multiple integrals of non-Gaussian generalized fields to identify the limiting distributions and establish an invariance principle.

Convex duality for constrained portfolio optimization

Ioannis Karatzas, Columbia University, New York, USA

We study the stochastic control problem of maximizing expected utility from terminal wealth and/or consumption, when the portfolio is constrained to take values in a given closed, convex subset of $\mathbb{R}^d$. The setting is that of a continuous-time, Itô process model for the prices of financial assets. General existence results are established for optimal portfolio/consumption strategies, by suitably embedding the constrained problem in an appropriate family of unconstrained ones. Equivalent conditions for optimality are obtained, and explicit solutions leading to feedback formulae are derived for special utility functions and for deterministic coefficients. Previous results on 'incomplete markets' and on 'short-selling constraints' are covered as special cases. The mathematical tools are those of continuous-time martingales, convex analysis, and duality theory. (This is a joint work with Jakša Cvitanić.)

A Poisson law for the number of lattice points in a random strip with finite area

Péter Major, Mathematical Institute of the Hungarian Academy of Sciences, Budapest, Hungary

Let a smooth curve be given by a function $r=f(\phi)$ in the polar coordinate system in the plane, and let R be a uniformly distributed random variable on the interval $[a_1L, a_2L]$ with some $a_2>a_1>0$ and a large $L>0$. Ya. G. Sinai has conjectured that given some real numbers $c_2>c_1$, the number of lattice points in the domain between the curves $(R+c_1/R)f(\phi)$ and $(R+c_2/R)f(\phi)$ is asymptotically Poisson distributed for 'good' functions $f(\cdot)$. We cannot prove this conjecture, but we show that if a probability measure with some nice properties is given on the space of smooth functions, then almost all functions with respect to this measure satisfy Sinai's conjecture. This is an improvement of an earlier result of Sinai, and the proof also contains many of his ideas. The question itself arose from a problem in physics. It is closely related to some more delicate questions about the distribution of the spectrum of quantum systems.

Sample path properties of the local times of symmetric Markov processes via Gaussian processes

Michael B. Marcus, Texas A&M University, College Station, TX, USA

Let X be a standard Markov process with state space S and assume that X has a symmetric transition probability density and finite 1-potential $u(x,y)$. Associate with X the mean zero Gaussian process $G=\{G(x), x\in S\}$ with covariance $u(x,y)$. An isomorphism of Dynkin, which relates G to the local times of X, is used to obtain information about the sample path properties of the local times of X. For example, X has a jointly continuous local time if and only if G has continuous sample paths, and results on the local and uniform moduli of continuity of the local times, in the spatial variable, follow immediately from corresponding results for G. (Explicit conditions for many of these results on Gaussian processes are known and are given in terms of $u(x,y)$.) These methods are also used to give a fairly complete description of the p-variation, in the spatial variable, of symmetric stable processes and to obtain iterated log laws for the rate of growth, in the temporal variable, of the local times of a wide class of symmetric Lévy processes. All the new results discussed in this talk are joint with J. Rosen.

Markov and non-Markov solutions of SDEs and SPDEs with boundary conditions

E. Pardoux, Université de Provence, Marseille, France

We consider several classes of white noise driven difference, differential and partial differential equations with boundary conditions, where essentially the Markov property holds in the linear case, and fails in most non-linear cases. The talk will review several recent results obtained jointly by the author and D. Nualart, and some recent results by C. Donati-Martin. The main tool in the proof is the extended Girsanov theorem due to Kusuoka.

Conditioned Brownian motion

Tom Salisbury, York University, North York, Ontario, Canada

The talk will survey progress in our understanding of conditioned Brownian motion. Classically, this involves taking an h-transform of Brownian motion by a superharmonic function h. Generalizations include space-time conditioning, in which h is a parabolic function. The latter is closely tied to recent work on intrinsically ultracontractive semi-groups.

Approximate distributions for maxima of random fields

David Siegmund, Stanford University, CA, USA

Some statistical problems involving the distribution of the maximum of a random field are described. For the case of Brownian-like fields, the method to approximate the tail of the distribution developed by Woodroofe (1976, 1978) and extended to the multidimensional case by Siegmund (1988) is described and applied to a number of specific problems involving tests to detect a change-point or a cluster in normal observations or a Poisson process. The locally Brownian case is compared and contrasted with the case of smooth Gaussian fields, for which approximations based on the expected number of local maxima and the closely related Weyl tube formula, are appropriate. Some examples, where an approximation in the center of the distribution is more appropriate than in the tail, are also discussed.

Brownian survival among random obstacles

Alain-Sol Sznitman, ETH-Zentrum, Zürich, Switzerland

In this talk we will present results concerning the long time survival probability of Brownian motion moving among Poissonian obstacles, and the behaviour of Brownian motion conditioned on survival. We will describe the method of 'enlargement of obstacles' which enables one to study such questions.

Optimal switching

Robert J. Vanderbei, Princeton University, NY, USA

For $i = 1, \ldots, d$, let $B^i_{s_i}$ be a one-dimensional Brownian motion on the positive half-line with absorption at zero. At each instant in time, we must decide to run one of these d Brownian motions while holding the others fixed at their current state. The resulting process evolves in the positive orthant $\mathbb{R}^d_+$. If, at some instant, we decide to freeze all of the Brownian motions, then a reward is received in accordance with this final position. We consider two types of reward functions.

First, we assume that the reward is minus infinity everywhere in $\mathbb{R}_+^d$ except along the coordinate axes, where it is given by smooth concave functions $\gamma_i(x_i)$ that are zero at zero. The optimal control for this problem has a simple description in terms of the following *index functions*:

$$I_i(x_i) = \int_0^{x_i} u\gamma_i''(u)\,\mathrm{d}u.$$

Namely, an optimal control runs, at each point in time, any Brownian motion except the one currently having the largest index function. The controlled process is stopped at the first time it reaches one of the coordinate axes.

The second class of reward functions are assumed to be minus infinity everywhere except on the facets of $\mathbb{R}_+^d$. On the ith facet (i.e., where $x_i = 0$), the reward function is the product of $\gamma_j(x_j)$ for $j \neq i$. In this case, the optimal control runs, at each point in time, the Brownian motion having the smallest index function and stops at the first time it reaches one of the facets.

In this talk, we will outline the methods used to prove these results. The relevant tools include the theory of multi-parameter processes, multi-parameter time changes, Itô calculus and the principle of smooth fit. We will also investigate the nature of the optimally controlled process. Here, an interesting connection with local time of Brownian motion arises. Finally, we will describe some related models, open problems and connections to other research.

Reflected Brownian motions in polyhedral domains

R.J. Williams, University of California, San Diego, CA, USA

In this talk, the current state of knowledge and recent research on reflected Brownian motions in polyhedral domains (RBM's) will be described. RBM's arise naturally as diffusion approximations to queueing network models under conditions of heavy traffic. They are also of interest as examples of reflected diffusions with non-smooth boundary conditions, for which there is no extant general theory. The corresponding object in analysis is a partial differential equation with oblique derivative boundary conditions in a polyhedral domain, for which again there is no general theory.

Heuristically, an RBM behaves like Brownian motion with a constant drift and covariance matrix in the interior of a convex polyhedron in $\mathbb{R}^d$ and is confined to the polyhedral state space by instantaneous reflection (or deflection) at the boundary, where the (oblique) direction of reflection is constant on each face of the polyhedron. The mathematical problems of interest for these processes fall into two main categories, namely, (i) synthesis: existence, uniqueness and characterization of RBM's with given geometric data; and (ii) analysis: recurrence classification, characterization and computation of stationary distributions. Until recently, the known rigorous results concerning synthesis were for two dimensional RBM's or for those arising as approximations to queueing networks with a single type of customer; whereas many of the important applications, such as in computer communication and manufacturing systems are for higher dimensional RBM's and RBM's associated with queueing networks having multiple customer types. Recently, Taylor and Williams established an existence and uniqueness result for the large class of semimartingale RBM's. The characterization provided by them has subsequently been used by Dai and Kurtz to characterize the stationary distributions of semimartingale RBM's via an integral relation. This relation, which is an integral version of an elliptic partial differential equation with oblique derivative boundary conditions in a polyhedral domain, has been used by Dai and Harrison for the numerical computation of stationary distributions.

The exit problem for the incremental process

Ofer Zeitouni, Technion – Israel Institute of Technology, Haifa, Israel

Let $x_t^\varepsilon = bt + \varepsilon w t$ where w_t is a d-dimensional Brownian motion. Let $D \subset \mathbb{R}^d$ be a closed, nonempty set. Define the two index incremental process as $z_{t,\tau}^\varepsilon = x_t^\varepsilon - x_\tau^\varepsilon$, $\tau \leq t$. We are interested in the first hitting time of D, i.e. in the time $\tau_\varepsilon = \inf\{t: \exists 0 \leq \tau \leq t, z^{\varepsilon}_{t,\tau} \in D\}$, and in the behaviour of the incremental process under the exit conditioning. These quantities are related to geometrical properties of the set D. Extensions to the general diffusion case are discussed. (This talk is based on a joint work with A. Dembo, Stanford University.)

2. Contributed lectures

LIL and extrema of some real- and TVS-valued infinitely divisible processes

J.M.P. Albin, University of Lund, Sweden

We study LIL- and extrema-properties of some infinitely divisible stochastic processes (with real or 'abstract' time). The results are most complete in the (easiest) case when the Lévy measure v satisfies $\int [1 \wedge |x|] \, dv(x) < \infty$ or related conditions (meaning $\alpha < 1$ in the stable case).

Random stationary processes

Kenneth S. Alexander and Steven A. Kalikow, University of Southern California, Los Angeles, CA, USA*

Given a finite alphabet, there is an inductive method for constructing a stationary measure on doubly infinite words from this alphabet. This construction can be randomized; the main focus here is on a particular uniform randomization which intuitively corresponds to the idea of choosing a generic stationary process. It is shown that with probability one, the random stationary process has zero entropy and gives positive probability to every periodic infinite word.

Pathwise behaviour of non-stationary queues

Ravi R. Mazumdar, Université du Québec, Ile des Soeurs, Canada
Fabrice Guillemin, Centre National d'Études des Télécommunications, Lannion, France
Vivek Badrinath, École Nationale Supérieure des Télécommunications, Paris, France*

We study the evolution of the workload process in a queue without making assumptions of stationarity. Our approach uses sample-path techniques and martingale convergence theorems. In this paper, we show that the workload process is 'well-behaved', i.e. it grows at rates of at most $o(t)$ provided certain natural conditions on the arrival process are satisfied. This leads to a pathwise definition of the notion of traffic-intensity allowing for correlation and time-dependence. Moreover, we show that this condition is 'minimal' in that a relaxation of the condition allows for the workload process to grow like t, even though in a heuristic sense the 'average arrival rate' is strictly less than the 'average service rate' at all times. Finally, we discuss some implications of these results in the stationary case, in particular the relationship with Loyne's theorem.

Let W_t be the workload process of the queue, Y_t the service time requested by the last customer to have arrived before t, A_t the compensator of the counting process N_t embedded in the arrival process. Under reasonable assumptions on the arrival process, we derive:

$$\limsup_{t \to \infty} \frac{1}{t} \int_0^t Y_{s-} \, dA_s = \rho < 1 \quad \Rightarrow \quad \forall t, \ \lambda(\{W_s = 0\}; s \geq t) = \infty,$$

where λ is the Lebesgue measure on $\mathbb{R}$, and thus the state $W_t = 0$ is recurrent for the workload process. We then show that the assumption that $\lim\sup(1/t)\int_0^t Y_{s-}\,\mathrm{d}A_s < 1$ is not sufficient to ensure that $W_t = o(t)$ by providing an example. We therefore strengthen the assumption and prove:

$$\frac{1}{t}\int_0^t Y_{s-}\,\mathrm{d}A_s \to \rho < 1 \quad \Rightarrow \quad \frac{W_t}{t} \to 0, \ \frac{1}{t}\int_0^t 1_{\{W_s=0\}}\,\mathrm{d}s \to 1-\rho$$

and conversely,

$$\liminf_{t\to\infty}\frac{1}{t}\int_0^t Y_{s-}\,\mathrm{d}A_s = \rho > 1 \quad \Rightarrow \quad W_t \to \infty, \ \frac{1}{t}\int_0^t 1_{\{W_s=0\}}\,\mathrm{d}s \to 0.$$

However, this condition still allows for the workload to grow to infinity, though slower than t, as we show through an example. We discuss some consequences of this result in some particular non-stationary cases and under the conditions of Loyne's theorem. In the latter case, these time-averages can be linked to the limiting distribution W_∞ of the workload.

Shot noise fields on the hypersphere

Bruno Bassan, Università "La Sapienza", Roma, Italy

In this work we extend to the case of unit hyperspheres $S_{n-1} \subset \mathbb{R}^n$ the model of shot noise random fields on the sphere introduced by Orsingher in Bollettino dell'Unione Matematica Italiana (6) 3-B (1984). An integral representation in terms of Poisson random measures is obtained, and mean, variance, covariance, and characteristic function of the field are derived. Furthermore, a spectral decomposition of the covariance function in terms of Gegenbauer polynomials is obtained. (This is a joint work with Orietta Luzi. The paper presented here appeared in Bollettino dell'Unione Matematica Italiana (7) 5-A (1991).)

Aggregation of stochastic processes

Richard J. Boucherie, Free University, Amsterdam, Netherlands

For a sequence of K stochastic processes with transition rates $q^{(k)}$ for process k, $k = 1, \ldots, K$, the aggregated process, an amalgamation of the processes in the sequence, with transition rates

$$q = \sum_{k=1}^{K} r^{(k)} q^{(k)},$$

is introduced. A sufficient condition is given, called cross-balance, under which the equilibrium distribution, π, of the aggregated process is shown to be

$$\pi = \sum_{k=1}^{K} r^{(k)} \pi^{(k)},$$

where $\pi^{(k)}$, $k = 1, \ldots, K$, denotes the equilibrium distribution of the processes in the sequence. A number of examples are discussed including time reversal, truncation and a construction method for the equilibrium distribution. For a given process, this method constructs a sequence of processes with known equilibrium distribution such that the aggregated process equals the original process and thus gives the equilibrium distribution of the original process.

Poisson bracket conditions for global solutions of nonlinear stochastic PDE's

G. Burstein and M.H.A. Davis, Imperial College, London, UK*

Consider the first-order Stratonovich nonlinear SPDE ($t \in \mathbb{R}_+, x \in \mathbb{R}^d$),

$$\mathrm{d}v(t,x) = F\left(x, \frac{\partial v}{\partial x}(t,x)\right)\mathrm{d}t + \sum_{j=1}^{d} G_j\left(x, \frac{\partial v}{\partial x}(t,x)\right) \circ \mathrm{d}w_t^j, \quad v(0,x) = \theta(x). \tag{1}$$

For $F(\varphi,\chi)$ of class C_b^{m+1} in (φ,χ), $G_j(\varphi,\chi)$ of class C_b^{m+2} in (φ,χ) and $\theta(x)$ of class C_b^{l+1} in x ($m \geq 3, 2 \leq l \leq m$), Kunita showed that (1) has a *local* $C^{l-1,\beta}$*-semimartingale solution* ($\beta < 1$) defined up to a stopping time $t \leq T(x)$ and expressed in terms of the flows of stochastic characteristics. We show that this solution becomes global (defined for all $t \in \mathbb{R}_+$) if the stochastic characteristics system admits an invariant Lagrangian submanifold or more generally, a time-variant conservation law (used by Bismut in stochastic mechanics) ensuring that the flows of characteristics are global flows of diffeomorphisms a.s. by decoupling them.

Theorem 1. *Assume that* $\{F(\varphi,\chi),\ \chi_i - \theta_{x_i}(\varphi)\}|_L = 0$; $\{G_j(\varphi,\chi),\ \chi_i - \theta_{x_i}(\varphi)\}|_L = 0$; $i, j = 1, \ldots, d$; *with* $L = \{(\varphi,\chi) \in \mathbb{R}^{2d} | \chi = \theta_x(\varphi)\}$. *Then* $v(t,x) = \theta(x) + F(0, \theta_x(0))t + \sum_{j=1}^{d} G_j(0, \theta_x(0))w_t$ *is the unique global (separable) solution of* (1).

Here

$$\{h(\varphi,\chi),\ k(\varphi,\chi)\} = \sum_{i=1}^{d}\left(\frac{\partial k}{\partial \chi_i}\frac{\partial h}{\partial \varphi_i} - \frac{\partial k}{\partial \varphi_i}\frac{\partial h}{\partial \chi_i}\right)$$

is the Poisson bracket.

Theorem 2. *Assume there exist* $\beta_1(t,\varphi), \ldots, \beta_d(t,\varphi)$ *of class* C_b^{m+1} *in* φ *and* C^1 *in* t *satisfying*

$$\left.\frac{\partial \beta_i}{\partial t}\right|_{L_t} + \{F(\varphi,\chi), \chi_i - \beta_i(t,\varphi)\}|_{L_t} = 0, \quad \{G_j(\varphi,\chi), \chi_i - \beta_i(t,\varphi)\}|_{L_t} = 0, \quad \beta_i(T,\varphi) = \theta_x(x)$$

$\forall t \in \mathbb{R}_+$; $i, j = 1, \ldots, d$ *and* $L_t = \{(\varphi,\chi) \in \mathbb{R}^{2d} | \chi = \beta(t,\varphi)\}$. *Then* $v(t,x) = \bar{\eta}_t \circ \bar{\varphi}_t^{-1}(x)$ *is the unique global solution of* (1) *where*

$$\mathrm{d}\bar{\varphi}_t(x) = -F_\chi(\bar{\varphi}_t(x), \beta(t, \bar{\varphi}_t(x)))\,\mathrm{d}t - \sum_{j=1}^{d} G_{j,\chi}(\bar{\varphi}_t(x), \beta(t,\bar{\varphi}_t(x))) \circ \mathrm{d}w_t^j, \quad \bar{\varphi}_0(x) = x,$$

$$\begin{aligned}\mathrm{d}\bar{\eta}_t(x) = {} & (F(\bar{\varphi}_t(x), \beta(t,\bar{\varphi}_t(x))) - F_\chi(\bar{\varphi}_t(x), \beta(t,\bar{\varphi}_t(x)))\beta(t,\bar{\varphi}_t(x)))\,\mathrm{d}t \\ & + \sum_{j=1}^{d} (G_j(\bar{\varphi}_t(x), \beta(t,\bar{\varphi}_t(x))) \\ & \qquad - G_{j,\chi}(\bar{\varphi}_t(x), \beta(t,\bar{\varphi}_t(x)))\beta(t,\bar{\varphi}_t(x))) \circ \mathrm{d}w_t^j, \quad \bar{\eta}_0(x) = \theta(x).\end{aligned}$$

The interest in global solutions (1) comes from the fact that a backward version of (1) is satisfied by the value function of the almost sure optimization problems associated with the anticipative optimal control problem approached by us.

Semigroup method in the theory of infinitely divisible distributions on groups and vector spaces

T. Byczkowski, Wrocław Technical University, Poland

We discuss applications of the Hille-Yosida-Feller method in semigroups of probability operators in the study of infinitely divisible distributions on groups and vector spaces and present some new results related to the Levy-Khinchine formula and 0-1 laws.

Transition probabilities between steady states of bistable Markovian system

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M.I. Dykman and V.N. Smelyanski, Ukranian Academy of Sciences, Kiev, Ukraine

We consider the problem of fluctuational transitions in a two-dimensional Markovian system, which, in the absence of fluctuations, relaxes to one of its steady states. Two attractors exist in the phase plane of this dynamical system and there is one unstable state (the saddle point). In the weak noise limit, only rare fluctuational transitions occur. When characteristic times are much greater than relaxation times, but are much less than the inverse probabilities of transitions between steady states, the system possesses a quasistationary probability distribution localized mainly in the vicinity of one of the attractors. These probabilities and distributions are found to logarithmic accuracy. The problem is reduced to deriving the most probable suitable path [1]. The optimal trajectory of the dynamical system corresponding to that path begins near the initially filled attractor and ends at the saddle point. Corresponding equations and boundary conditions are obtained for this trajectory. A concrete analysis was made for the Duffing nonlinear oscillator interacting with its environment in a resonant external field.

Reference

[1] M.I. Dykman and V.N. Smelyanski, Phys. Rev. 41A (1990) 3090–3102.

New examples of finite dimensional filters for diffusions observed in correlated noise

Michel Cohen de Lara, Cergrene, Ecole Nationale des Ponts et Chaussées, Noisy le Grand, France

Finite dimensional filters are shown to exist for a class of partially observed stochastic systems with correlated noises. An example is the one dimensional system

$$\begin{aligned} \mathrm{d}x_t &= f(x_t)\,\mathrm{d}t + \mathrm{d}v_t + x_t\,\mathrm{d}y_t, \mathrm{d}y_t, \quad x_0 \sim p_0(x)\,\mathrm{d}x, \\ \mathrm{d}y_t &= (x_t f(x_t) + \tfrac{1}{2})\,\mathrm{d}t + \mathrm{d}w_t, \quad y_0 = 0, \end{aligned} \tag{1}$$

where the drift f is such that $(1+x^2)(f''(x)+f^2(x))+2xf'(x)=0$ (for instance $f(x)=1/(1+x^2)\times(\arctan x+\mathrm{cte})$). If $(P_t)_{t\geq 0}$ is the semi group of operators generated by the parabolic equation

$$\frac{\partial u}{\partial t} = \frac{1}{2}\frac{\partial^2 u}{\partial x^2} - f\frac{\partial u}{\partial x} - \frac{1}{2}(f' - f^2 + \tfrac{9}{4})u, \tag{2}$$

the conditional density can be written as

$$\exp\left(r_1 - \tfrac{1}{2}r_3 + \int_{e^{-r_3}x}^{x} f(z)\,\mathrm{d}z\right)(P_{r_2} p_0)(x) \tag{3}$$

with

$$\mathrm{d}r_1 = \tfrac{9}{8}(e^{2r_3} - 1)\,\mathrm{d}t, \qquad \mathrm{d}r_2 = e^{2r_3}\,\mathrm{d}t, \qquad \mathrm{d}r_3 = \mathrm{d}y_t. \tag{4}$$

The proof of the existence of such a class of systems relies on the analysis of the symmetry group of the parabolic operator appearing in the Zakai equation associated to the filtering problem. Such an analysis can be performed by geometric means after having expressed this parabolic operator as the Laplacian operator on a Riemannian manifold [2]. When the drift is assumed to be a gradient vector field (for the new Riemannian metric), the potential satisfies an equation generalizing that of Beneš [1] or of Zeitouni [3].

References

[1] V.E. Beneš, Exact finite-dimensional filters for certain diffusions with nonlinear drift, Stochastics 5 (1982) 65–92.

[2] M. Cohen de Lara, A note on the symmetry group and perturbation algebra of a parabolic partial differential operator, to appear in: J. Math. Phys.

[3] O. Zeitouni, On some finite dimensional nonlinear filters for certain diffusions observed in correlated noise, Syst. Contr. Lett. 7 (1986) 61-63.

Weak ergodicity and products of random matrices

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A weakly ergodic sequence of (non-stochastic) matrices allows products of non-negative matrices which eventually become strictly positive to be expressed via products of some associated stochastic matrices and ratios of values of a certain function. This formula used in a random setup leads to a representation for the logarithm of a random matrix product. If the sequence of random matrices is in addition stationary then atuomatically almost all sequences are weakly ergodic, and the representation is expressed in terms of a one-dimensional stationary process. This permits properties of products of random matrices to be deduced from the latter. Second moment assumptions guarantee that central limit theorems and laws of the iterated logarithm hold for the random matrix products if and only if they hold for the corresponding stationary process. Finally, a central limit theorem for ϕ-mixing stationary random matrices is derived doing away with the restriction of boundedness of the ratios of column entries assumed by the previous studies. Extensions beyond stationarity are discussed.

Functional differential equations in percolation theory

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We consider a random field ξ on a homogeneous tree, which satisfies an autoregression equation

$$\xi(x')=\alpha\xi(x)+\theta(x'), \quad 0<\alpha<1,$$

where the $\theta(x)$ are identically distributed independent random variables taking values 0 and -1 with probabilities p and $q=1-p$ respectively. Let us denote: $W_h=\{x\colon \xi(x)>h\}$, $v_h=\{$the number of points in the maximal component of W_h that is connected with the root of the tree$\}$, $m(a,h)=E\nu_h(a)=\langle\nu_h(a)\rangle$.

Definition 1. We shall say that $h_0=h_{\text{cr}}$ is a level of percolation iff $m(a,h)<\infty$ when $h>h_0$, and $m(a,h)\equiv\infty$ when $h\leqslant h_0$.

It is easy to show that if $h>h_0$ then $m_h(a)$ is a positive, nondecreasing function bounded on any finite interval, and

$$\begin{aligned} m(a,h)&=\nu\{pm(\alpha a,h)+qm(\alpha a-1,h)\}+1, && a\geqslant h,\\ m(a,h)&\equiv 0, && a<h. \end{aligned}$$

If $h<h_0$, then $m(a,h)\equiv\infty$.

Theorem 1. *If* $0<\alpha<\frac{1}{2}$ *and*

$$1-\frac{(\nu^{n+1}-1)^{1/(n+1)}}{\nu}\leqslant p<1-\frac{(\nu^{n}-1)^{1/n}}{\nu}$$

then

$$-(1+\alpha+\cdots+\alpha^n)\leqslant h_{\text{cr}}\leqslant-(1+\alpha+\cdots+\alpha^{n-1}).$$

Operators on Hilbert spaces represent stationary processes with finite state space

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Let H be a real Hilbert space, $K \in \mathbb{N}$, $K \geq 2$, $A_1, \ldots, A_K : H \to H$ linear continuous operators, $x \in H$ a fixed vector with $\|x\| = 1$. Assume that

$$(A_1 + \cdots + A_K)x = (A_1^* + \cdots + A_K^*)x = x$$

and assume that

$$\langle A_{i_1} \ldots A_{i_N} x; x \rangle \geq 0$$

for all $N \in \mathbb{N}$ and all $i_1, \ldots, i_N \in \{1, \ldots, K\}$.

Theorem 1. *Under these conditions we claim that*

$$P[X_1 = i_1, \ldots, X_N = i_N] := \langle A_{i_1} \ldots A_{i_N} x; x \rangle$$

defines a probability measure on $\{1, \ldots, K\}^{\mathbb{Z}}$. *We call* $(H, x, A_1, \ldots, A_K)$ *the Hilbert Space Representation (HSR) of* $(X_N)_{N \in \mathbb{Z}}$.

Every stationary stochastic process $(X_N)_{N \in \mathbb{Z}}$ with finite state space $S = \{1, \ldots, K\}$ admits an HSR. Properties of the process $(X_N)_{N \in \mathbb{Z}}$ can be translated to properties of the operators $A_1, \ldots, A_K$. Some recent results are:

- A process is r-Markov iff it admits an HSR such that dim $H \leq K^r$ and $\dim(A_{i_1} \ldots, A_{i_r} H) = 1$ for all $i_1, \ldots, i_r \in S$.
- A process is m-dependent (i.e., $(X_n)_{N \leq -1}$ and $(X_N)_{N \geq m}$ are independent) iff it admits an HSR such that $\dim((A_1 + \cdots + A_K)^m H) = 1$.
- A process is mixing iff it admits an HSR such that

$$(A_1 + \cdots + A_K)^m h \xrightarrow{w} \langle h; x \rangle x \quad \text{if } m \to \infty \quad \text{for all } h \in H.$$

- A process is 3-mixing iff it admits and HSR such that

$$(A_1 + \cdots + A_K)^{m_1} A_{i_1} \cdots A_{i_N} (A_1 + \cdots + A_K)^{m_2} \xrightarrow{w} \langle A_{i_1} \cdots A_{i_N} x; x \rangle \cdot \langle h; x \rangle x$$

if $\min(m_1, m_2) \to \infty$ for all $h \in H$ and all $i_1, \ldots, i_N \in S$.
- A mixing process with compact operators $A_1, \ldots, A_K$ is 3-mixing (and even N-mixing for all $N \in \mathbb{N}$).

A necessary and sufficient condition for the Markov property of the local time process of a symmetric Markov process

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Let X be a symmetric Markov process on an interval $E \subset \mathbb{R}$, with a finite potential density $g(x, y)$. Let $(\phi_x ; x \in E)$ be a zero mean Gaussian process with g as the covariance function. Dynkin and Atkinson have proved that ϕ is a Markov process if, and only if, X has continuous paths. Assuming that g is continuous, we have shown that the local time process $(L_\zeta^x : x \in E)$ of X conditioned to start at a fixed point and die at another fixed point is a Markov process if, and only if, $(\phi_x : x \in E)$ is a Markov process. The sufficiency of this condition follows immediately from Atkinson's results and Walsh's study of the local time process of a real valued diffusion. The necessity is proved using arguments based on excursion theory.

Propagation of chaos for a realistic loss network with alternate routing

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We consider a fully-connected communication network. Each link has a capacity for C calls. Calls arrive independently on each link ab according to a Poisson process and occupy a circuit if the capacity is not attained. If the link ab is full, then a third node c is chosen uniformly; the call is routed through links ac and bc if both are not full, and is lost if either is full. Call durations follow independent exponential laws, and alternately routed calls release both circuits simultaneously.

The processes of interest are the loads of the links, forming a strongly interacting system leading to a non-trivial BBGKY hierarchy. The main problem is to prove a chaos hypothesis, showing the decorrelation of particles as the number of nodes goes to infinity, in order to close the hierarchy. The analysis is further complicated because these processes are not exchangeable and do not form a Markov process.

We shall give a complete result for the problem. For natural initial conditions, we show propagation of chaos in variation norm; if the initial conditions are i.i.d. we have speeds of convergence. We also give a convergence result for the alternate calls. We prove the equivalence of this notion of propagation of chaos (for a non-exchangeable system) and of the convergence of what we call local empirical measures, seen around the nodes. This shows the calls are well distributed around the network.

We work from a sample-path point of view. The tools are interaction graphs, which represent the past history of a given collection of links. We define a limit tree, much as for the spatially-homogeneous Boltzmann equation. Through a coupling argument we obtain interaction graphs which differ from the tree on an event of vanishing probability, and thus show propagation of chaos without any compactness argument. To evaluate the probability of the event that the tree is modified to get the graph, we must study in a precise way the strong interaction and the way indirect interactions are obtained from a succession of direct interactions: this gives the notion of chain reactions. Both the tree construction and the notion of chain reactions are recursive.

Time dynamics of interacting particle systems on a lattice

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We study the behaviour of the mean coverage function $M(t)=E|\varphi(t)|$, $t>0$, where $|\varphi(t)|$ denotes the number of occupied sides of a lattice at time t.

1. It is shown that a sufficient condition for concavity of $M(t)$ is identical to the classical sufficient condition for ergodicity of the process $\varphi(t)$.
2. It is proven that the function $M(t)$ characterizes the parameters of the process.
3. The influence of geometry and the size of a lattice on the behaviour of $M(t)$ are found.

First passage times for perturbed random walks

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Let $X_1, X_2, \ldots$ be i.i.d. random variables with positive, finite mean and set $S_n=\sum_{k=1}^{n} X_k$, $n\geqslant 1$. Further, let $\{\xi_n, n\geqslant 1\}$ be a sequence of random variables, such that ξ_n is independent of $\{X_k, k>n\}$ for all n and set

$$Z_n = S_n + \xi_n, \quad n \geqslant 1. \tag{1}$$

Nonlinear renewal theory is concerned with the family of first passage times

$$\nu(t) = \min\{n: Z_n > t\}, \quad t \geqslant 0, \tag{2}$$

under the assumption that $\{\xi_n, n \geq 1\}$ is *slowly changing*; viz. $n^{-1} \max_{1 \leq k \leq n} |\xi_k| \xrightarrow{p} 0$ as $n \to \infty$ and $\{\xi_n, n \geq 1\}$ are uniformly continuous in probability. For $\xi_n = 0$ a.s., $n \geq 1$, we rediscover classical renewal theory (on the line).

The assumptions and conditions involved in this model are rather technical. An important special case is

$$Z_n = n \cdot g(\bar{Y}_n), \quad n \geq 1, \qquad \text{i.e. } \nu(t) = \min\{n: n \cdot g(\bar{Y}_n) > t\}, \quad t \geq 0, \tag{3}$$

where $Y_1, Y_2, \ldots$ are i.i.d. random variables with positive, finite mean, θ, and finite variance and g is positive and twice continuously differentiable (in some neighbourhood of (at) θ). Although this case is less general, it covers many important applications. On the other hand, the conditions involved are fewer and simpler.

A main observation is that, in the general case, one has, roughly speaking, that $\xi_n = o(n)$ as $n \to \infty$ and, by the law of large numbers that $S_n = O(n)$ as $n \to \infty$, i.e.

$$Z_n = O(n) + o(n) \quad \text{as } n \to \infty. \tag{4}$$

We take this (single) property as our point of departure; we call a process, $\{Z_n, n \geq 1\}$, of the form (1), where $\{S_n, n \geq 1\}$ is a random walk, whose increments have positive, finite mean and $\{\xi_n, n \geq 1\}$ is a sequence of random variables, such that $\xi_n / n \xrightarrow{\text{a.s.}} 0$ as $n \to \infty$ a *perturbed random walk*.

Various limit theorems for first passage times of perturbed random walks as well as for the special case (3) are presented. In the latter case, we impose, in general, less restrictive conditions on g than in earlier work in the area. The proofs are based on the method of stopped random walks; see Gut (1988).

Reference

A. Gut, Stopped Random Walks. Limit Theorems and Applications (Springer, New York, 1988).

Diffusion on a random fractal

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A problem of interest to physicists is the study of diffusion in disordered media. A natural model for the medium is provided by fractals, in that they have irregularities over all length scales but a degree of self similarity which enables them to be analyzed. In order to gain an understanding of the problem of diffusion in a disordered medium, we should first understand diffusion on a fractal. The first fractal considered was the Sierpinski gasket, a regular finitely ramified fractal with exact self similarity. Of more physical interest are irregular fractals which have only statistical self similarity. As a first step toward such fractals, we will consider a simple randomization of the Sierpinski gasket and construct a diffusion process on this class of fractals. The properties of this process can then be found, including its spectral dimension and estimates on its sample path behaviour.

A stability equation for sii-processes and α-monotonicity of self-decomposable distributions

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Let $X(\cdot) = (X(t))_{t \geq 0}$ be a nonnegative *sii-process*, i.e. a process with stationary independent increments with $X(0) \equiv 0$. Consider the following *stability equation*:

$$X(a) \stackrel{d}{=} W^{1/\alpha} \odot X(b), \tag{1}$$

where $0 < a < b$, $\alpha > 0$ and W is uniformly distributed on (0, 1) and independent of $X(\cdot)$. The operation $\odot$ is just ordinary multiplication if $X(\cdot)$ is $\mathbb{R}_+$-valued; in case $X(\cdot)$ *is* $\mathbb{Z}_+$-valued, $\odot$ is the standard 'discrete multiplication' as introduced in [3] for independent r.v.'s V in $(0,1)$ and X in $\mathbb{Z}_+$: $V \odot X$ is binomially distributed with parameters X and V. It can be shown that (1) has a solution iff the triple $(a, b; \alpha)$ satisfies $\gamma := \alpha(b/a - 1) \leq 1$, and that in this case $X(\cdot)$ is a solution iff it is of the form $X(\cdot) \stackrel{d}{=} Y(T(\cdot))$, where $Y(\cdot)$ is a *stable process* of exponent γ and $T(\cdot)$ is a *gamma process*, independent of $Y(\cdot)$, with shape parameter $r := 1/(b-a)$. This result can be reformulated so as to characterize for instance the geometric and exponential distribution. Furthermore, the solution in the $\mathbb{R}_+$-case can easily be obtained from its $\mathbb{Z}_+$-counterpart by way of *Poisson mixtures*.

The generating r.v. $X := X(1)$ of a solution $X(\cdot)$ of (1) is easily shown to be *self decomposable*, i.e. there exists r.v.'s $X_t$with $t > 0$, independent of X, such that

$$X \stackrel{d}{=} e^{-1} \odot X + X_t, \quad t > 0. \tag{2}$$

Hence, as is well known, the distribution of X is *unimodal* and, because of (1), also α*-monotone*, i.e., in the $\mathbb{R}_+$-case, X^α has a monotone density; cf. [1], and see [2] for the discrete case. Now, in the $\mathbb{Z}_+$-case a general self-decomposable rv X can be obtained as the solution of the following equation, more general than (1):

$$X \stackrel{d}{=} W^{1/\alpha} \odot \{X + S\}, \tag{3}$$

where $\alpha > 0$ and S is a r.v. independent of X with necessarily $\mathbb{E} \log^+ S > \infty$. This observation, together with its translation to the $\mathbb{R}_+$-case via Poisson mixtures, enables us to precisely describe the set of $\alpha > 0$ for which a given self-decomposable distribution (on $\mathbb{Z}_+$ or on $\mathbb{R}_+$) is α-monotone. As a simple example it follows that the r.v. $X := \frac{1}{2}(I - 1)$, where I is the first-passage time through state 1 in the symmetric Bernoulli walk starting at 0, is α-monotone iff $\alpha \geq \frac{1}{4}$.

References

[1] R.A. Olshen and L.J. Savage, J. Appl. Probab. 7 (1970) 21–34.
[2] F.W. Steutel, Statist. Neerlandica 42 (1988) 137–140.
[3] F.W. Steutel and K. van Harn, Ann. Probab. 7 (1979) 893–899.

Processes governed by signed and complex measures

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Generalized higher-order stochastic processes governed by signed and complex measures are discussed. Even-order signed processes of this type were studied by Hochberg (Ann. Probab., 1978). Here, recent results of Hochberg and Orsingher (Rome) are presented, extending this analysis to odd-order processes and to processes related to heat-type equations with complex coefficients.

On the perturbation problem for occupation densities

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Let X be a semimartingale, perturbed by a process V of bounded variation, but with completely arbitrary measurability properties. We prove that if V is twice continuously differentiable such that its second derivative is Hölder continuous of order $\alpha > \frac{1}{2}$, then the perturbed process $X + V$ possesses occupation densities which are continuous under usual circumstances.

Large deviations for a class of measures on a Banach space

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Given a family of subprobability measures $\{\mu_\lambda\}$ on a Banach space X, let us try to apply a generalized Cramer transform technique [2] to establish a large deviation principle for $\{\mu_\lambda\}$ on X, i.e. for each $x^* \in X$

set

$$H(x)=\lim_{\lambda\to 0}\lambda\ln\int\exp\{1/\lambda\langle x^*,x\rangle\}\mu(\mathrm{d}x)$$

and assume that H is well defined and lower semi-continuous. Then the candidate for the rate function is the Fenchel transform L of H,

$$L(x)=\sup[\langle x^*,x\rangle-H(x^*)].$$

There are two types of reasonable assumptions [1], which imply large deviations of $\{\mu_\lambda\}$ governed by a strictly convex L. They are

T1: X is reflexive, $\{\mu_\lambda\}$ is exponentially bounded and $0\in\operatorname{int}\operatorname{Dom}(H)$.
T2: $\{\mu_\lambda\}$ is exponentially tight and $\operatorname{Dom}(H)=X^*$.

In fact, strict convexity of L can be substituted by the Gateaux differentiability of H which sometimes makes things more verifiable. Furthermore, exponential assumptions on $\{\mu_\lambda\}$ enable one to carry out all the calculations in a space with a weaker topology and even to compute nonconvex rate functions as, for example, in the case of differential equations with rapidly oscillating random noise [3].

References

[1] P. Baldi, Large deviations and stochastic homogenization, Ann. Mat. Pura. App. 151 (1988) 161–177.
[2] J. Gärtner, On large deviations from the invariant measure, Theory Probab. Appl. 22 (1977) 24–39.
[3] D. Ioffe, On some applicable versions of abstract large deviation theorems, to appear in: Ann. Probab.

The asymptotic composition of supercritical, multi-type branching populations

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The life, past and future is described of a typical individual in an old, non-extinct branching population, where individuals may give birth as a point process and have types in an abstract type space. The type, age and birth-rank distributions of the typical individual are explicitly given, as well as the Markov renewal type process that describes her history.

New results on branching random walks

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We consider a supercritical branching process performing a random walk. The Fourier transform (random) of the position of the particles at time n normalized by its expectation is a martingale. We obtain new results on the almost sure uniform convergence of that martingale.

Symmetries on random arrays and set-indexed processes

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This work is part of an extensive general project to explore symmetries of probability measures. The oldest and most basic result in this area is the de Finetti–Ryll-Nardzewski theorem, which asserts that an infinite sequence (X_i) of random variables is *spreadable*, in the sense that all its subsequences have the same distribution, iff the X_i are conditionally i.i.d., given a suitable σ-field. Now consider instead an arbitrary process $X=(X_J)$ indexed by all finite subsets J of N, and say that X is spreadable if

$(X_{pJ} \stackrel{d}{=} (X_J)$ for all subsequences $p = (p_1, p_2, \ldots)$ of N, where $pJ = \{p_j;\ j \in J\}$. Our main result characterizes such processes by a representation formula, similar to those obtained by Aldous and Hoover for separately and jointly exchangeable arrays in two or higher dimensions. Rather surprisingly, it follows that a set-indexed process X as above is spreadable, iff it can be extended to an exchangeable process indexed by all finite N-sequences.

Levy bandits: Multi-armed bandits driven by Levy processes

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Multi-armed bandits are used to model dynamic allocation of a scarce resource among competing projects. It is customary to interpret the resource as 'time', which is dynamically allocated among several independent stochastic processes, each of which represents the evolution of an arm. The goal is then to find an optimal allocation strategy which maximizes, for example, cumulative reward discounted over an infinite horizon.

In this talk we focus on Levy bandits, which are multi-armed bandits whose arms evolve like Levy processes. We assume that the reward depends on the state of the arm being operated and is determined by a continuous non-decreasing function. As might be anticipated from previous research, such bandits are optimally controlled by an index strategy: one can associate with each arm an index function of its state; the index function of an arm is independent of the other arms; it is optimal to allocate time to those arms whose states have the largest index, and the optimal reward can be expressed in terms of all the indices. Somewhat less anticipated, however, is the fact that the index function of an arm driven by a Levy process has a representation in terms of the decreasing ladder sets and the exit systems of its driver. Further, one may use the Wiener Hopf factorization of the Levy exponents of the arms to obtain the characteristic function of the excursion laws through which the indices are defined. We use this representation to explicitly calculate index functions of some interesting Levy arms. We then calculate optimal rewards, rediscovering along the way that local time naturally quantifies continuous-time switching.

A tandem fluid network with Levy input

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We introduce an open network fluid model with stochastic input and deterministic linear internal flows. In particular, we consider several buffers with unlimited capacity in series. The input to the first buffer is a nondecreasing stochastic process with stationary and independent increments. The content flows forward from buffer to buffer through connecting pipes at constant deterministic rates. We obtain simple expressions for the mean content of each buffer and each pipe by exploiting a connection to the classical single-node storage model with nondecreasing Levy input and constant release rate. We obtain the marginal distributions describing the content of each buffer by exploiting a connection to a linear fluid model with random disruptions. We apply martingale theory to derive the joint distribution for the content of the first two buffers, which is not of product form. Finally, we show that the fluid network can be regarded as the limit of a sequence of conventional queueing networks.

Winding numbers for diffusions

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The distribution of winding numbers for regular diffusion processes on the 2-dimensional sphere is Cauchy asymptotically. We discuss this result and an application.

Local times of Brownian motion on curves and surfaces

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Let $\{A_2 : t \geqslant 0\}$ be a continuous additive functional of Brownian motion, indexed by its Revuz measure, μ, as $\{A_t^\mu : t \geqslant 0\}$. We discuss conditions for the joint continuity of $\{A_t^\mu : t \geqslant 0, \mu \in \mathcal{M}\}$, for some family of measures, $\mathcal{M}$. Time permitting, we apply the aforementioned ideas to invariance principles, local times on curves and surfaces – our main motivation – and intersection local times. The techniques here work for a large class of Markov processes other than Brownian motion. (This is a joint work with Rich Bass.)

Schoenberg's problem on positive definite functions

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In [1], I. Schoenberg posed the following problem: For which numbers $\beta > 0$ is the function $\exp(-\|x\|_q^\beta)$ positive definite on $\mathbb{R}^n$, where $\|x\|_q = (|x_1|^q + \cdots + |x_n|^q)^{1/q}$ and $q > 2$. Denote by $B_n(q)$ the set of such numbers β. We prove here that the function $\exp(-\|x\|_q^\beta)$ cannot be positive definite if $n \geqslant 3$ and $q > 2$, i.e. $B_n(q) = \emptyset$ for every $q > 2$ and $n \geqslant 3$. This result gives a complete answer to Schoenberg's question, because it is known that $B_2(q) = (0, 1]$ for every $q > 2$ and the case $q \in (0, 2]$ was settled by Schoenberg, where one has $B_n(q) = (0, q]$. Let us mention some trivial facts:

(a) $B_1(q) = (0, 2]$.
(b) $B_n(q) \supseteq B_{n+1}(q)$ for every $n \in \mathbb{N}$.
(c) If $\beta \in B_n(q)$ then $(0, \beta] \subseteq B_n(q)$.

(b) and (c) imply that it suffices to prove that the set $B_3(q) \cap (0, 2)$ is empty. We treat here, however, a more general situation. Denote by $\Phi_n(q)$ the set of even functions $f : \mathbb{R} \to \mathbb{R}$ for which $f(\|x\|_q)$ is a characteristic function on $\mathbb{R}^n$, i.e. there exists a probability measure μ on $\mathbb{R}^n$ with $\hat{\mu}(x) = f(\|x\|_q)$ for all $x \in \mathbb{R}^n$. It is clear that $\Phi_n(q) \supseteq \Phi_{n+1}(q)$. If $f \in \Phi_n(q)$ then the function $f(\|x\|_q)$ is positive definite, as is f. Thus there exists a probability measure ν on $\mathbb{R}$ with $\hat{\nu} = f$. We prove the following.

Theorem 1. *The following statements are valid*:

(a) *If $n \geqslant 3$, $q > 2$, $f \in \Phi_n(q)$, $f \not\equiv 1$ and $\hat{\nu} = f$ then the measure ν cannot have finite moments of positive orders, i.e. $\int_{\mathbb{R}} |t|^\beta \, d\nu(t) = \infty$ for every $\beta > 0$.*

(b) *If $n \geqslant 4$, $q > 2$, $f \in \Phi_n(q)$ and $\hat{\nu} = f$ then $\int_{\mathbb{R}} |t|^\beta \, d\nu(t) = \infty$ for every $\beta \in (-1, 0)$.*

(c) *If $q > 2$, $f \in \Phi_2(q)$, $f \not\equiv 1$ and $\hat{\nu} = f$ then $\int_{\mathbb{R}} |t|^\beta \, d\nu(t) = \infty$ for every $\beta \in (1, 2)$.*

Reference

[1] V.M. Zolotarev, One-dimensional Stable Distributions (Nauka, Moscow, 1983).

Equilibrium in a simplified dynamic, stochastic economy with heterogeneous agents

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We study a dynamic, stochastic economy with several agents, who may differ in their endowments (of a single commodity) and in their utilities. An equilibrium financial market is constructed, under the condition that all agents have infinite marginal utility at zero. If, in addition, the Arrow-Pratt indices of relative risk aversion for all agents are less than or equal to one, then uniqueness of equilibrium is also proved. When agents consume and invest in this equilibrium market so as to maximize their expected utility of consumption, their aggregate endowment is consumed as it enters the economy and all financial instruments are held in zero net supply. Explicit examples are provided.

An estimation in the presence of nuisance parameters first and second order minimaxity, degenerate diffusion processes, involutive vector fields

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Let $x_1, \ldots, x_n$ be a sample with a density function smoothly depending on s parameters $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_s) \in \Theta \subset \mathbb{R}^s$. Suppose that only p parameters $\boldsymbol{\theta}_1 = (\theta_1, \ldots, \theta_p)$ are estimated. Then the others $\boldsymbol{\theta}_s = (\theta_{p+1}, \ldots, \theta_s)$ are a nuisance. Suppose that $\varepsilon^2 = 1/n$ and let $R_\varepsilon(\boldsymbol{\theta}^1_\varepsilon, \boldsymbol{\theta}) = E_\theta w(\varepsilon^{-1}(\boldsymbol{\theta}^1_\varepsilon - \boldsymbol{\theta}_1))$ be the risk function of the p-dimensional estimator $\boldsymbol{\theta}^1 = (\theta^1_\varepsilon, \ldots, \theta^p_\varepsilon)$ of parameters of interest with nonnegative, symmetric and quasiconvex loss function $w(\boldsymbol{y}_1)$ that depends only on p arguments $\boldsymbol{y}_1 = (y_1, \ldots, y_p)$. From the Hajek-Le Cam-Ibragimov-Hasminskii theory, it follows the first order minimaxity, i.e.

$$r_\varepsilon(\Theta) = \inf_{\boldsymbol{\theta}^1_\varepsilon} \sup_{\boldsymbol{\theta} \in \Theta} (R_\varepsilon(\boldsymbol{\theta}^1_\varepsilon, \boldsymbol{\theta}) - \tilde{R}(\boldsymbol{\theta})) = o(1), \quad \varepsilon \to 0. \tag{1}$$

Here $\tilde{R}(\boldsymbol{\theta}) = E_\theta w(\boldsymbol{\zeta})$, $\boldsymbol{\zeta}$ is Gaussian, $N(0, \tilde{I}^1)$, $\tilde{I}(\boldsymbol{\theta})$ is the $p \times p$ Fisher information matrix for $\boldsymbol{\theta}_1$ in the presence of the nuisance $\boldsymbol{\theta}_2$. In addition to the usual way of defining $\tilde{I}(\boldsymbol{\theta})$, it may be defined using the p vector fields $X_i = \sum_{j=1}^{s} |b_{ij}(\boldsymbol{\theta})\partial/\partial\theta_j$, $i = 1, \ldots, p$, on Θ. These vector fields play a very significant role in the second order minimaxity problem, that is to investigate the o(1) in (1). The degenerate diffusion process Z_t, defined from the stochastic differential equation

$$\mathrm{d}Z^j_t = \sum_{i=1}^{p} b_{ij}(\boldsymbol{Z}_t) \circ \mathrm{d}W^i_t + b_{0j}(\boldsymbol{Z}_t)\,\mathrm{d}t, \quad j = 1, \ldots, x, \qquad \boldsymbol{Z}_0 = \boldsymbol{\theta}, \quad \boldsymbol{\theta} \in \Theta, \tag{2}$$

is connected with this problem. Here $\boldsymbol{W}$ is a Brownian motion. Suppose that the vector fields $X_1, \ldots, X_p$ are involutive, i.e. the Lie brackets

$$[X_i, X_j] = 0, \quad i, j = 1, \ldots, p. \tag{3}$$

Then there exists a p-dimensional Lie transformation group in Θ and a process $\boldsymbol{Z}_t$ that moves only on the orbits of this group. Let L be the Hermander s operator that corresponds to the process $\boldsymbol{Z}_t$.

Theorem 1. *Under condition* (3), *there exists a one-to-one map* $\boldsymbol{\eta} : \Theta \to \Omega \subset \mathbb{R}^s$, *such that* $\eta_i = \theta_i$, $i = 1, \ldots, p$, *and* $\eta_j = \eta_j(\boldsymbol{\theta})$, $j = p+1, \ldots, s$, *are the* $s - p$ *invariants of the same Lie group. Furthermore, the operator* L *may be reduced to an operator* $\tilde{L}$ *that is elliptic with respect to* $\boldsymbol{\eta}_1 = (\eta_1, \ldots, \eta_p)$ *for each fixed* $\boldsymbol{\eta}_2 = (\eta_{p+1}, \ldots, \eta_s)$.

Filtering equations for a partially-observed semi-Markov process

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We consider a semi-Markov process $Y = \{Y(t)\}$ and another jump process $Z = \{Z(t)\}$ that is derived from Y by a certain $\{f^Y_t\}$ a predictable thinning and marking of the marked point process associated with Y. We suppose Z to be observable and ask for the conditional distribution of the last jump of Y before $t \geq 0$ and the corresponding jump value, given the observations up to t. Using the method of the probability of reference, we derive a formula that is explicit up to a solution of a multivariate Markov renewal equation. A corresponding result can also be proved for more general jump processes Y.

On some estimation procedures for the partially observed Markov process

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The quality of filtering and interpolation procedures of a partially observed Markov process is considered. Let X_t, $t \geqslant 0$, be a Markov process with two states $\{0, 1\}$ and transition probability intensities λ and μ. The observed process Y_t, $t \geqslant 0$, is defined by $Y_t = \int_0^t X_s \, ds + \sigma W_t$, where W_t, $t \geqslant 0$, is a standard Wiener process. Let $R(\hat{X}_t)$ be the error probability risk of any estimator $\hat{X}_t$. It is shown that for the optimal, in the sense of the error probability, filter $\hat{X}_t^f$,

$$\lim_{t\to\infty} R(\hat{X}_t^f) = 2\lambda\mu(\lambda+\mu)^{-1}(\sigma^2 \ln(\sigma^{-2}) + 2(1-C)\sigma^2) + o(\sigma^2),$$

where C is the Euler constant and $\sigma \to 0$. The approximate filter $\tilde{X}_t^f$ based on a process $\{Z_t, t \geqslant 0)$ is suggested, where Z_t is defined by $dZ_t = \sigma^{-2}(dY_t - \frac{1}{2}dt)$, $Z_t \in [z-, z+]$, and reflecting barriers at the points $z- = \ln(\sigma^2/\alpha)$, $z+ = \ln(\beta\sigma^{-2})$. It is found that the first term of the limit risk of the approximate filter is equal to that of the optimal one, while the second term is minimal when $\alpha = 2\lambda$, $\beta = 2\mu$. In this case

$$\lim_{t\to\infty} R(\tilde{X}_t^f) = 2\lambda\mu(\lambda+\mu)^{-1}(\sigma^2 \ln(\sigma^{-2}) + \sigma^2) + o(\sigma^2).$$

Let $\hat{X}_t^i$ be the optimal, in the sense of the error probability, interpolator based on the observation of Y_s, $s \geqslant 0$, and $\tilde{X}_t^i$ be the approximate interpolator. It is shown that

$$\lim_{t\to\infty} R(\hat{X}_t^i) = 8 \ln 2\lambda\mu(\lambda+\mu)^{-1}\sigma^2 + o(\sigma^2),$$

$$\lim_{t\to\infty} R(\tilde{X}_t^i) = 6\lambda\mu(\lambda+\mu)^{-1}\sigma^2 + o(\sigma^2).$$

Splitting-up approximation for SDE's and SPDE's with application to nonlinear filtering

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Consider the SDE

$$dX_t = b(X_t)\, dt + \sigma(X_t)\, dW_t + \rho(X_t)\, dV_t, \tag{1}$$

and the stochastic flows of diffeomorphisms associated with the two SDE's

$$dX_t = b(X_2)\, dt + \sigma(X_t)\, dW_t, \tag{2}$$

$$dX_t = \rho(X_t)\, dV_t. \tag{3}$$

A splitting-up approximation of (1) is introduced, based on successive composition of the stochastic flows of diffeomorphisms over a given partition of the time interval. Turning next to the nonlinear filtering problem, where only partial observation is available of the form

$$dY_t = h(X_t)\, dt + dV_t,$$

an equation is derived for the conditional density of the splitting-up approximating process. This equation is interpreted as a splitting-up approximation of the Zakai equation for the conditional density of the original process. (This class of approximation for SPDE's has been considered in P. Florchinger and F. LeGland, Time-discretization of the Zakai equation for diffusion processes observed in correlated noise, to appear in: Stochastics and Stochastics Reports.) Motivated by the application to nonlinear filtering, a further approximation of (1) is introduced, with the property that

- weak convergence holds for the approximation of the component associated with (2),
- convergence in quadratic mean holds for the approximation of the component associated with (3).

Likelihood ratio of Gaussian globally Markovian random fields

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The Maltese cross condition for the Brownian sheet, due to R.C. Dalang and J.B. Walsh, beautifully demonstrates that P. Lévy's sharp Markov property (that is, the conditional independence of the values, taken inside and outside a domain, given the boundary values), badly needs a serious restriction on the domain to hold. Nevertheless, as McKean defined the Markov property, by requiring conditional independence given the germ field (that is, given the values in the infinitesimal environment of the boundary), it holds on *every* domain for fields like the Brownian sheet, the free Markovian field, stationary fields with spectral density one over a polynomial, and Lévy-Brownian motion in odd dimensions. The fields, having this property *on every domain*, we call the globally Markovian ones. As Rozanov proved, Gaussian globally Markovian fields satisfy an equation with a local — in most cases partial differential — operator and a source, which is an independent value field. Our aim is to determine the conditions for the absolute continuity of the measures corresponding to different elliptic operators, and derive a formula for the likelihood ratio. A similar result, supposing *homogeneous boundary values*, can be found in [1], but here we will study the nonhomogeneous case as well. Green's functions will be of help to formulate the result.

Reference

[1] L. Márkus, Likelihood ratio of Gaussian globally Markovian random fields, 2nd World Congress of the Bernoulli Society, Proc. issued by the Soviet Committee, in preparation.

Parameter estimation in differential equations, using random time transformations

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Differential equations with measurements subject to errors are usually handled by Least Squares methods or by Likelihood methods based on diffusion-type stochastic modifications of the differential equation. We study the performance of likelihood methods based on substituting a Gaussian random time transformation as argument in the solution of the original deterministic differential equation. The model is fitted to disease progress curves derived from a real data set consisting of disease assessments of melon plants infected by Zucchini yellow mosaic virus (ZYMV).

Decomposition theorems for set-indexed quasimartingales

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We define the different kind of set-indexed martingales and quasimartingales, and study their relationship. Predictable sigma-fields and admissible functions associated with set indexed processes are defined. Conditions are proposed in order to extend the admissible function associated with a quasimartingale to a measure on the predictable sigma-field. These conditions are related with a uniform integrability property and to the concept of stopping set. We suppose that the processes are indexed by a family of closed subsets of a metric space and that the martingales possesses a regularity property. Therefore we obtain a Doob-Meyer decomposition for quasimartingales, and each quasimartingale is the difference of two submartingales.

Substable and pseudo-isotropic processes

Jolanta K. Misiewicz, Technical University of Wrocław, Poland

We are studying pseudo-isotropic processes: i.e. symmetric stochastic processes with the property that all their one-dimensional projections are the same up to a scale parameter. It turned out that under a weak moment assumption they are mixtures of symmetric stable processes. We show here how the geometry of the level curves for the characteristic function of pseudo-isotropic processes changes after stochastic rescaling each coordinate separately, with the scale variables jointly stable. This gives us an explicit formula for some isometric embeddings between L_p-spaces.

New developments in dynamic programming

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We explore relationships between the long-run-average values of a dynamic programming problem and the limit of the discounted values, when the discount factor goes to one. The lecture is based on three new papers; each of them is written by a pair of authors taken from E. Lehrer, D. Monderer and S. Sorin.

Heavy traffic analysis of fork-join networks: RBM's in non-simple polyhedral domains

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In queueing theory one seeks to predict in quantitative terms the congestion delays that occur when jobs or customers compete for processing resources. At present no satisfactory methods exist for the analysis of systems that allow *simultaneous performance* of tasks associated with a single job or customer. In this paper we study a class of networks called fork-join networks, in which the processing of a customer requires the completion of multiple tasks, subject to precedence constraints that may allow some parallel processing. We present a heavy traffic analysis for the class of fork-join networks. We show that under certain regularity conditions the total job count process converges weakly to a multi-dimensional *reflected Brownian motion* (RBM) whose state space is a *non-simple polyhedral cone* in the nonnegative orthant. (A d-dimensional polyhedral region is said to be *simple* if at most d of its faces intersect.) In addition, the weak limits of workload levels and throughput times are shown to be simple transformations of the RBM. Specifically, the 'steady-state throughput time' (a random variable) is expressed in terms of workload levels and processing times via the 'longest path functional' of classical PERT/CPM analysis.

Generalized sequential procedures for Markov sequences

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Suppose on a probability spacc $(\Omega, \mathscr{F}, \mathscr{P})$ the following are given:

(1) A non-decreasing sequence σ-algebras $\{\mathscr{F}_{m_1,\ldots,m_{i-1},m_i}, m_i > m_{i-1}\}$ of a σ-algebra $\mathscr{F}$ such that $\mathscr{F}_{m_1,\ldots,m_{i-1}} \leqslant \mathscr{F}_{m_1,\ldots,m_i}, m_i > m_{i-1} > \cdots > m_0 = 0$.

(2) A Markov chain $\{X_m, \mathscr{F}_m, \mathscr{P}_x\}$, $m = 1, 2, \ldots, X_1 = x$ in a phase space $(\mathscr{X}, \mathscr{B})$,

(3) For fixed integers $m_1, \ldots, m_{r-1}$, $1 \leqslant m_1 < \cdots < m_{r-1}$, a stochastic process $\{Z_{m_1,\ldots,m_{r-1}}, \mathscr{F}_{m,\ldots,m_{r-1},m_r}, m_r > m_{r-1}\}$, where $Z_{m_1,\ldots,m_r} = g(X_m, \ldots, X_{m_r})$, and $g(x, y, \ldots, v)$ is a real function defined on $\mathscr{X}^r$ and is $\mathscr{B}^r$-measurable.

A generalized sequential procedure (g.s.p.) is a set of random values $\tau = (\tau_1, \ldots, \tau_r)$ with τ_i from $\{1, 2, \ldots, \infty\}$ such that:

(a) $0 < \tau_1 < \cdots < \tau_r < \infty$.

(b$_k$) $\{\omega;\ \tau_1(\omega) = m_1, \ldots, \tau_k(\omega) = m_k\} \in \mathscr{F}_{m_1, \ldots, m_k}$, $k = 1, 2, \ldots, r$.

Let S_m denote the class of g.s.p.'s τ with $\tau_1 \geq m$. Let

$$V_m(x) = \sup_{\tau \in S_m} E_x g(X_{\tau_1, \ldots,} X_{\tau_r}).$$

Let us define $\hat{S}_m \subseteq S_m$ supposing that

$$\tau_1 = \inf\{m_1 > m:\ X_{m_1} \in \mathscr{B}_1), \qquad \tau_k = \inf\{m_k > m_{k-1}:\ X_{m_k} \in \mathscr{B}\}$$

on $\{\omega:\ \tau_1 = m_1, \ldots, \tau_{k-1} = m_{k-1}\}$ where $\mathscr{B}_1, \ldots, \mathscr{B}_k \in \mathscr{B}$, $k = 2, 3, \ldots, r$.

Theorem.

$$V_m(x) = \sup_{\tau \in S} E_x g(X_{\tau_1}, \ldots, X_{\tau_r}).$$

Some closure properties for uniformly convergent classes of functions

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The Vapnik–Cervonenkis theory of uniform convergence extends the classical SLLN from a single real-valued function to an ensemble of such functions. Let $X_1, X_2, \ldots$ be an i.i.d. sequence of random variables X_i that take values in a measurable space $(\mathscr{X}, \mathscr{S})$ and have distribution P. Consider a class $\mathscr{F}$ of measurable functions from $(\mathscr{X}, \mathscr{S})$ into a bounded interval $[-K, K]$ of of the reals; $\mathscr{F}$ is said to be *uniformly convergent* with respect to the sequence $X_1, X_2, \ldots$ if

$$\lim_{n\to\infty} \sup_{\mathscr{F}} \left| \frac{1}{n} \sum_{i=1}^{n} f(X_i) - Ef(X_1) \right| = 0 \quad \text{w.p.1.}$$

In this case we write $\mathscr{F} \in \mathrm{VC}(P)$. We present two closure properties for uniformly convergent classes of functions, and give several applications of these properties. The first closure property concerns the marginal distribution of the sequence $X_1, X_2, \ldots$. Specifically, we show that if $\mathscr{F} \in \mathrm{VC}(\mathscr{P})$ and $Q \ll P$, then $\mathscr{F} \in \mathrm{VC}(Q)$. The second closure property enables us to combine or modify existing uniformly convergent classes to get new ones. In particular, let the function $\phi: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be continuous. Then if the classes $\mathscr{F}_1, \mathscr{F}_2 \in \mathrm{VC}(P)$, the class $\phi(\mathscr{F}_1, \mathscr{F}_2) = \{\phi(f_1, f_2): f_1 \in \mathscr{F}_1, f_2 \in \mathscr{F}_2\}$ is also in $\mathrm{VC}(P)$. As a consequence of this, we can show that any class $\mathscr{F} \in \mathrm{VC}(P)$ is a totally bounded subset of $L_1(P)$. We also obtain natural conditions under which the empirical infinity-norms of functions in a class $\mathscr{F}$ converge uniformly to their true infinity norms under P.

Traces of random variables on Weiner space

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Let F be a square integrable random variable on the Wiener space. The purpose of this paper is to compute the limit as ε tends to zero of the conditional expectation $E(F|\sup_{0\leq t\leq 1}|w(t)| < \varepsilon)$. If the random variable F is a multiple stochastic integral of even order, $F = I_n(f_n)$, one can provide a necessary and sufficient condition on the kernel f_n for the above limit to exist. This condition requires the existence of a multiple iterated trace for f_n, in some particular sense. The case of a general square integrable random variable will be discussed, and some applications to the computation of Onsager–Machlup functionals are given.

The resolvent of a degenerate diffusion on the plane, with application to partially-observed stochastic control

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We compute the resolvent of the degenerate, two-dimensional diffusion process introduced by Beneš, Karatzas and Rishel in the study of a stochastic control problem with partial observations. The process diffuses along lines of slope 1 and −1 when away from the axes, and increases according to the local time of a Brownian motion along the axes. The explicit nature of our computations allows us to show that this diffusion can be constructed uniquely (in the sense of probability law) starting at any point on the plane, including the origin. We are also able to solve for very general cost functions the control problem of Beněs, Karatzes and Rishel in which the controlled process has the form,

$$Y(t)=\int_0^t zu(s)\,\mathrm{d}s+W(t),$$

where z is an unobserved random variable, u is ac ontrol taking values in $[-1,1]$, and u is adapted to a filtration supporting the degenerate, planar diffusion. Our derivation combines probabilistic techniques with the use of the so-called 'principle of smooth fit'.

Estimation of transition kernels using random measures

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We note that some classical estimation problems may be reduced to a general unique estimation problem. Then we define and study a general estimator that includes the classical estimators for each particular case mentioned. This way, we obtain, as a unified result, the classical convergence conditions for each estimator. Finally, we establish conditions for the convergence of the finite dimensional distributions of the associated empirical process.

Some general aspects of Markov dependence in the process of the generation of words

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As we have emphasized formerly, one of the most interesting fields in which Probability Theory can be applied is offered by the *Mathematical Theory of Formal Languages*. Among different classes of formal languages, we consider the languages generated by grammar constituting the so-called 'Chomsky hierachy'.

In our paper "On a Markov dependence case in the process of generation of the words" (to be published), we study some aspects concerning Markov dependence in the process of the generation of words by phrase-structure grammar (the most general type of grammar from the Chomsky hierarchy). In order to generate a word w by such a grammar, it is supposed that a sequence of words $w_0, w_1, \ldots, w_j = w$ exists, each of them being obtained from the previous one by applying some specific rules called 'grammatical rules' or 'productions'. A sequence of words, as the above, is called a *derivation* (of length j) - according to the grammar.

Now, if $\mathscr{D}$ is the set of all the derivations according to a given grammar, let D_j be the class of derivations of length j. Evidently, $\mathscr{D}$ splits into equivalence classes, each of them being represented by one of its elements arbitrarily chosen. A word, w, can or cannot be generated within an equivalence class. If it is, then the probability that w could be generated within D_{j+1} is denoted by j; if w is not generated within D_j, the probability that it could be generated within D_{j+1} is denoted by β. Hence, such is the case when the equivalence classes D_j, $j \geqslant 2$, are connected into a simple Markov chain.

In the present paper, we propose to determine the probability $p_n(m)$ that w could be generated by m derivations of n $(m<n)$ under the following conditions: (i) it will be generated within the first and last class, and there is a production generating w; (ii) it will be generated within the first class and not the last, and there is a production generating w; (iii) it will be generated within the last class and not the first, and there is not a production generating w; (iv) it will not be generated in the first class, nor the last, and there is not a production generating w. We call such a procedure for generating w by m derivations of n as being an 'alternating generation' of w.

Comparison change analysis as a probability concept

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Comparison change analysis is our name for an approach to detection and estimation of change based on comparison density functions $d(u;\, G, F)$; we define $d(u;\, G, F)=f(G^{-1}(u))/g(G^{-1}(u))$ if F and G are continuous, $d(u;\, G, F)=p_F(G^{-1}(u))/p_G(G^{-1}(u))$ if F and G are discrete. Tests of homogeneity (equivalently, change) can be formulated as testing (for independence) bivariate data (X, Y). Assume X, Y discrete; Bayes theorem yields a duality $p_{X|Y=1}(x)/p_X(x)=p_{Y|X=x}(1)/p_Y(1)$. Measure values of X in terms of its quantile function $Q_X(U)=F_X^{-1}(u)$; then

$$d(u;\, F_X, F_{X,Y=1})=\text{prob}\{Y=1\,|\,X=Q_X(u)\}/\text{prob}\{Y=1\},\quad 0<u<1.$$

The left-hand side is a comparison density; the right-hand side is a change density (how Y changes as function of X which could represent time). This talk introduces probability concepts at the foundation of our study of dependence and therefore of change, especially the dependence density functions $d(t, u)$, $d([0, t], u)$ and dependence distribution functions $D(t, u)$, $D([0, t], u)$,

$$d(t, u)=d(u;\, F_Y, F_{Y|X=Q_X(t)}),\quad 0<t<1,\quad 0<u<1,$$

$$d([0, t], u)=d(u;\, F_Y, F_{Y|X\le Q_X(t)}),\quad 0<t<1,\quad 0<u<1.$$

Theorem. *$D(t, u)-tu=t(D([0, t], u)-u)$ if $F_XQ_X(t)=t$, $F_YQ_Y(u)=u$.*

Unification of statistical methods comes from analogies between properties, although proofs are required on a case by case basis; an important unifying fact is that the sample version $D^{\sim}(t, u)$ has asymptotic distribution, under the null hypothesis of no dependence (no change), usually of the form $n^{0.5}\,(D^{\sim}(t, u)-tu)$ which converges to a Brownian sheet.

Large deviations for multiple Wiener integral processes

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For $m\geq 1$, let $I_m(h)$ denote the multiple Weiner–Itô integral of order m of a square-integrable symmetric kernal h. In this work, we consider different conditions on a time-dependent kernel h_t which guarantees that the probability measures induced by $\varepsilon^{m/2}I_m(h_t)$ satisfy a large deviation principle in $C([0, 1])$.

Order statistics for jumps of normalised subordinators

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A subordinator is a process with independent, stationary, non-negative increments. On the unit interval we can view this process as the distribution function of a random measure, and, dividing this random measure by its total mass, we get a random distribution. In this paper only subordinators which give rise to discrete random distributions will be considered. Formulae for the joint distribution of the n largest atoms in this discrete distribution will be derived and then used to rederive a few results about the Dirichlet–Poisson process. Subordinators crop up naturally as inverse local times of diffusions and the atoms in the random measure associated with them correspond to the lengths of excursions of the diffusion away from 0. The formulae for the distribution of the n largest atoms can then be used to find the density of the length of the n longest excursions away from 0 up to $\tau_s = \inf\{u: l_u > s\}$ divided by τ_s where (l_u) is the local time at the origin of the diffusion. For Brownian motion, or, more generally, for a Bessel process of dimension $\delta, 0 < \delta < 2$, these results can be used to find densities of the longest excursion up to a fixed time, longest excursion completed by a given time or longest excursion of the Brownian, or Bessel, bridge.

Keywords: Poisson processes, subordinators, Poisson–Dirichlet process, inverse local time, duration of excursions.

Stratonovich–Taylor expansion

Roger Petterson, Lunds Universitet, Sweden

By a special representation of finite dimensional stochastic differential equations, we give a Stratonovich–Taylor expansion of stochastic integrals. The expansion gives a prescription for numerical simulation of stochastic differential equations and a means for checking the local errors of numerical methods.

The asymptotic behavior of certain harmonic measures for Brownian motion conditioned to remain in a strip

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Let $E = \mathbb{R}^d \times A$, where $A \subset \mathbb{R}^d$ is a smooth bounded domain and let $B(t)$ be a Brownian motion in E, killed at ∂E. Denote points $z \in E$ by $z = (r, \phi, y)$, where $r > 0$, $\phi \in S^{d-1}$ and $y \in A$, and denote the Brownian motion in these coordinates by $B(t) = (r(t), \phi(t), y(t))$. Let

$$D = \{z \in E;\ r \leq 1\}.$$

Define the hitting times

$$\tau = \inf\{t \geq 0: B(t) \notin E\} = \inf\{t \geq 0: y(t) \notin A\}$$

and

$$\tau_D = \inf\{t \geq 0: B(t) \in D\} = \inf\{t \geq 0: r(t) \leq 1\}.$$

Define the harmonic measure $\mu_z(\mathrm{d}\phi, \mathrm{d}y)$ for the Brownian motion starting at $z \in E$ and conditioned to hit D before getting killed at ∂E by

$$\mu_z(\mathrm{d}\phi, \mathrm{d}y) = P_z(\phi(\tau_D) \in \mathrm{d}\phi,\ Y(\tau_D) \in \mathrm{d}y \mid \tau_D < \tau).$$

Let

$$\{z_n\}_{n=1}^{\infty} = \{(r_n, \phi_n, y_n)\}_{n=1}^{\infty} \subset E \quad \text{satisfy} \lim_{n \to \infty} |z_n| = \infty.$$

We show that $\lim_{n\to\infty} \mu_{z_n}$ possesses a weak limit if and only if $\lim_{n\to\infty} \phi_n$ exists. We also describe how this information can be used to determine the Martin boundary of $\frac{1}{2}\Delta$ in E.

Limit properties of processes with linear regression

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We denote the covariance, the conditional expectation, the conditional variance and the infinitesimal moments of a zero-mean stochastic process $\mathscr{X} = \{X, t \geqslant 0\}$ by

$$K(t_i, t_j) = E(X_{t_i} X_{t_j}), \qquad m_{in} = E(X_{t_i} | C_{n \setminus i}), \qquad m_{in}^{(2)} = E\{(X_{t_i} - m_{in})^2 | C_{n \setminus i}\},$$

$$a_{in} = \lim_{\Delta \to 0} \frac{1}{\Delta} E\{(X_{t_i + \Delta} - X_{t_i}) | C_{n \setminus i}\},$$

$$a_{in\delta}^{(2)} = \lim_{\Delta \to 0} \frac{1}{\Delta^{1+\delta}} E\{(X_{t_i + \Delta} - E(X_{t_i + \Delta} | C_n]))^2 | C_{n \setminus i}\},$$

where $\delta \geqslant 0$, $x_{n \setminus i} = (x_j, j = 1, \ldots, n, j \neq i)$, $C_n = (X_{t_j} = x_j, j = 1, \ldots, n)$, $C_{n \setminus i} = (X_{t_j} = x_j, j = 1, \ldots, n, j \neq i)$, $t_1 < t_2 < \cdots$. We suppose that all n-dimensional distributions are non-degenerate.

Proposition 1. *If $K \in C^0$, m_{in} are linear functions, $m_{in}^{(2)}$ do not depend on $x_{n \setminus i}$, then $\mathscr{X}$ is a Gaussian process.*

Proposition 2. *If $K \in C^2$, a_{in} are linear functions, $a_{in1}^{(2)}$ do not depend on $x_{n \setminus i}$, then $\mathscr{X}$ is a Gaussian process.*

Similar characterizations are true for $\delta = 0$. Now let the conditions concern only the preceding states.

Proposition 3. *If m_{nn} are linear functions of $x_1, \ldots, x_{n-1}$, $m_{nn}^{(2)}$ do not depend on $x_1, \ldots, x_{n-1}$, $K(t_1, t_2) = v^2(t_1)$ is an increasing, continuous function, $E(X_{t_2} - X_{t_1})^4 = 3\{v^2(t_2) - v^2(t_1)\} + \mathrm{O}\{v^2(t_2) - v^2(t_1)\}$, then*

$$\mathscr{L}[X_t / v(t)] \to \mathrm{N}(0, 1) \quad \textit{as } t \to \infty.$$

There are different generalizations of Proposition 3. Examples for Proposition 3: Some birth and death processes with linear intensities.

Optimal stopping of a sequence of maxima over a random barrier

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The following problem is considered. The decision maker observes the realization of only one of two independent sequence of i.i.d. random variables. His aim is to stop the observation at the moment when the observable maximum exceeds the maximum of the unobservable sequence with maximal probability. The number of observation is allowed to be a random variable independent of observations with a known distribution. Since N is unknown the decision maker faces an additional risk. If he rejects any observation, he may then discover it was the last one, in which case he loses. The problem is formulated as a classical optimal stopping problem for some Markov chain (ch. Shiryaev [2]). The form of an optimal stopping rule for some class of distributions of N (which contains DFR distributions) is found and the probability of winning is worked out. The special case when N is geometric is examined in detail. For the problem natural situations are when the so-called monotone case does not occur and the OLA policy cannot be optimal. A related problem has been studied by Szajowski [3]. Some continuous-time version of the problem, when sequences appear according to a Poisson process and decision about stopping must be made before a random moment, has been considered by Porosinski [1].

References

[1] Z. Porosinski, On a continuous time version of some optimal stopping problem, Bull. Pol. Acad. Sci. 34 (1986) 609-616.

[2] A.N. Shiryaev, Statistical Sequential Analysis (Nauka, Moscow, 1976).

[3] K. Szajowski, Optimal stopping of a seqence of maxima over an unobservable sequence of maxima, Zastos. Mat. 18 (1984) 359-374.

Intertemporal arbitrage pricing theory

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The paper derives the result that the APT holds in each infinitesimal period of a continuous trading model under the assumption that dividend payoffs are functionals of factor and idiosyncratic uncertainty. This generalizes the one period model that implies that the APT holds under the assumption that price changes in a given period satisfy a factor structure. Since instantaneous returns in a multiperiod model are endogenously determined, the theory in this paper is derived under assumptions that may be viewed as restricting more primitive characteristics of the economy than the assumptions made in the one period model.

Why is a max-stable process hard to predict?

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We consider prediction of stationary max-stable processes. The usual metric between max-stable variables can be defined in terms of the L_1 distance between spectral functions and in terms of this metric a kind of projection can be defined. It is convenient to project onto *max-stable* spaces; i.e. spaces of extreme value distributed random variables which are closed under scalar multiplication and the taking of finite maxima. Some explicit calculations of max-stable spaces generated by processes of interest are given. The concepts of deterministic and purely nondeterministic stationary max-stable processes are defined and illustrated. Differences between linear and non-linear prediction are highlighted and some characterizations of max-moving averages and max-permutation processes are given.

Keywords: extreme value theory, Poisson processes, max-stable processes, prediction, time series, stationary processes.

On the mean sojourn time in queues with stationary input stream

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Customers' sojourn times are important quantities for the assessment of a queueing discipline. In general, one is interested in small mean or expected sojourn times. This raises the question of their existence and finiteness. The answers depend on the queueing discipline and on the input stream $(B_n, T_n)_{n\geq 0}$ where $B_n, T_n : (\Omega, P) \to]0, \infty[$ are the service times of customer n and the interarrival time between customers n and $n+1$, respectively. A seminal result is due to Kiefer and Wolfowitz (1956) who treated the classical case of an i.i.d. input stream and showed inter alia, that the expected stationary waiting time for FCFS is finite if traffic intensity ρ is less than 1 and the service time has finite variance. Results pertaining to more general queueing disciplines are due to Vasicek (1977), Flipo (1981), Shantikumar and Suminta (1987), and others.

We are interested in the case of a *stationary* and *ergodic* input stream (B_n, T_n). Let $V_n : (\Omega, P) \to]0, \infty[$ be the sojourn time of customer n. Ritter and Wacker (1991) showed that for any ε, $0 < \varepsilon < 1$, there exists a stationary and ergodic input stream with constant service times, strictly positive interarrival times, and traffic intensity ε such that P-a.s. $\lim_{n\to\infty} (1/n) \sum_{k<n} V_k = \infty$ with respect to any *work-conserving* queueing discipline. Let us call a queueing discipline *monotone* if, roughly speaking, the server's effort towards a customer in the systems does not become larger if the work load is larger in a sense made precise. The discipline FCFS, preemptive LCFS, server sharing, e.g., are monotone. In this case, the superadditive ergodic theorem shows that P-a.s. the customer means $(1/n) \sum_{k<n} V_k$ tend to a constant limit as $n \to \infty$. If traffic intensity is less than 1, the input stream is φ-mixing and satisfies certain moment restrictions, and the discipline is work-conserving, then the limit can be shown to be finite. The preemptive LCFS queueing discipline corresponds to the *additive* case; it was already treated by Ritter and Wacker (1991).

Self-organizing schemes: A dependence case

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Consider n items labelled $1, 2, \ldots, n$, such that $P(\text{request for item labelled } i) = p_i, 0 \leq p_i \leq 1, i = 1, 2, \ldots, n$, $\sum_{i=1}^{n} p_i = 1$, where $p_1, \ldots, p_n$ are unknown, are arranged in a shelf or in a pile. Also, consider the heaps ordering scheme, which returns a requested item to the top of the pile or to the left extreme of the shelf. We consider the restriction of not requesting an item two requests in a row. A generalization of that (not requesting an item k requests in a row) is also considered. The equilibrium distributions for the processes got by successive requests for both cases are found and shown to be different from the initial distribution $p_1, \ldots, p_n$. Moreover, the equilibrium distributions for the processes got by the sequences of arrangements when one requests and returns items, for both cases, are also found. The proof for both cases is done by going backwards in time and studying the history of those processes.

Second order laws of the iterated logarithm for local times

Jay Rosen, City University of New York, USA

It is well known that Brownian local time L_t^x satisfies

$$\limsup_{t\to\infty} \frac{L_t^x}{\sqrt{2t \lg\lg t}} = 1 \quad \text{a.s.}$$

Recently, Csaki and Foldes have discovered a 'second order' version of this result:

$$\limsup_{t\to\infty} \frac{L_t^0 - L_t^x}{t^{1/4}(\lg\lg t)^{3/4}} = (\tfrac{8}{3})^{3/4}\sqrt{|x|} \quad \text{a.s.}$$

I will report on work with M. Marcus which describes the analogue of the above results for the local times of Lévy processes with regularly varying exponent.

Super-extremal processes, the argmax process & dynamic continuous choice

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The theory of classical vector-valued extremal processes is extended to *Super-Extremal Processes*, $\boldsymbol{Y} = \{\boldsymbol{Y}_t, t > 0\}$. At any $t > 0$, $\boldsymbol{Y}_t(\cdot)$ is a random element of $\mathrm{US}(T)$, the space of upper semicontinuous (usc) functions on a compact, metric space T. $\boldsymbol{Y}$ may also be viewed as a *sup-measure* valued random process, where for each t, $\boldsymbol{Y}_t$ is a random sup-measure. General path properties of $\boldsymbol{Y}$ are discussed and it is shown that $\boldsymbol{Y}$ is a $D((0,\infty), \mathrm{US}(T))$-valued Markov process. For each t, $\boldsymbol{Y}_t$ is sup-infinitely divisible, and is associated. Processes which at every time t are continuous functions on T are also examined.

For applications to Choice Theory, we consider the *argmax* process $\boldsymbol{M} = \{M_t, t > 0\}$, where $M_t = \{\tau \in T, Y_t(\tau) = \bigvee_{\zeta \in T} Y_t(\zeta)\}$. $\boldsymbol{M}$ is a closed set-valued random process. We look into the path properties of $\boldsymbol{M}$. When $\boldsymbol{Y}_t$ is max-stable, then $\boldsymbol{Y}$ and $\boldsymbol{M}$ enjoy some independence properties, and $\boldsymbol{M}$ is Markov. This provides a general framework for the analysis of dynamic choice, from a continuum of alternatives T, based on the theory of Random Utility maximization. At any t, $\boldsymbol{Y}_t$ represents the random utility at time t for alternatives in T and M_t is the corresponding set of utility maximizing alternatives.

Optimal service speeds in a competitive environment

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The economic behavior of vendors of service in competition is studied. A simple model with two competing exponential servers and Poisson arrivals is considered. Each server is free to choose his own service rate at a cost (per time unit) that is strictly convex and increasing. There is a fixed reward to a server for each customer that he serves. The model is designed to study one specific aspect of competition. Namely, competition in speed of service as a means for capturing a larger market share in order to maximize long run expected profit per time unit. A two person game is formulated and its solutions are characterized. Depending on the revenue per customer served and on the cost of maintaining service rates, the following three situations may arise; (i) a unique symmetric strategic (Nash) equilibrium in which expected waiting time is infinite; (ii) a unique symmetric strategic equilibrium in which expected waiting time is finite; and (iii) several, non-symmetric strategic equilibria with infinite expected waiting time. An explicit expression for the market share of each server as a function of the service rates of the two servers is also given. (This is a Joint work with Ehud Kalai and Morton I. Kamien of Northwestern University.)

Performance evaluation for the score function method in sensitivity analysis and stochastic optimization

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Estimating systems performance $l(\rho)=\mathbb{E}_{\rho}L$ and the associated sensitivity (the gradient $\nabla l(\rho)$) for several scenarios via simulation generally requires a separate simulation for each scenario. The score function (SF) method handles this problem by using a single simulation run, but little is known about how the estimators perform. Here we discuss the efficiency of the SF estimators in the setting of simple queueing models. In particular, we consider heavy traffic (diffusion) approximations for the sensitivity and the variances of the associated simulation estimators, and discuss how to choose a 'good' reference system (if any) in order to obtain reasonably good SF estimators.

Strong comparison of solutions of one-dimensional stochastic differential equations

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The problem of non-confluence and strong comparison of solutions of one-dimensional Itô stochastic differential equations

$$\mathrm{d}X_t=\sigma(t,X_t)\,\mathrm{d}W_t+b(t,X_t)\,\mathrm{d}t \tag{$*$}$$

is studied. We give sufficient conditions which guarantee these properties in the case of a non-degenerate diffusion coefficient σ. We show, in particular, that if the bounded measurable coefficients σ and b satisfy:

$$|\sigma(t,x)|\geqslant\varepsilon \quad \text{for some } \varepsilon>0, \tag{1}$$

$$(\sigma(t,x)-\sigma(t,y))^2\leqslant(x-y)(\varphi(x)-\varphi(y)) \quad \text{for } x\geqslant y, \tag{2}$$

$$|b(t,x)-b(t,y)|\leqslant|\varphi(x)-\varphi(y)|, \tag{3}$$

for some increasing function φ: $\mathbb{R}\to\mathbb{R}$, then the non-confluent property of solutions of SDE $(*)$ holds. In the case of a possible degenerate diffusion coefficient, the notion of an almost strong comparison is introduced and studied.

References

Y. Ouknine and M. Rutkowski, Strong comparison of solutions of one-dimensional stochastic differential equations, Stochastic Process. Appl. 36 (1990) 217-230.

T. Yamada, On the non-confluent property of solutions of one-dimensional stochastic differential equations, Stochastics 17 (1986) 111-124.

T. Yamada and Y. Ogura, On the strong comparison theorems for solutions of stochastic differential equations, Z. Wahrsch, Verw. Gebiete 56 (1981) 3-19.

Distributions of sublinear functionals of infinitely divisible processes

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Sublinear functionals on the space of sample paths include suprema, integrals of the sample paths, oscillation on sets and many others. By combining ideas from the theory of infinitely divisible distributions on vector spaces with ideas from the theory of subexponential distributions, we characterize the distributions of such functionals for many infinitely divisible processes. In particular, we obtain exact tail behaviour for these distributions, thus improving many recent results in this area (This is a joint work with J. Rosinski.)

The moments of the regenerative estimators for the M/G/1 queue and their interpretation as functionals of an associated branching process

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Of prime interest in the design of queueing simulations is the variance of the estimators as a function of the number of observations. Higher moments and correlations may be useful as well. For regenerative processes, the moments of the one-cycle estimator, to be called regenerative estimators, are of almost equal interest. The analysis of the M/G/1 queue can shed some light on the planning of queueing simulations. In this communication:

(1) We present a functional equation for the joint moment generating function of the regenerative estimators for the mean, variance (and arbitrarily higher moments) of the waiting time in an M/G/1 queue.

(2) We present analytical formulae for the variance of the regenerative estimators for any moment of the waiting time in the M/G/1 queue. The formulas at arbitrary traffic intensity are presented in recursive form, while the heavy traffic approximations are presented in very simple and explicit form.

(3) We present analytical formulas for the variance of the regenerative likelihood ratio estimators for the sensitivity (derivative) of the waiting time moments with respect to either the arrival rate or the service rate, and we compare their performance to that of the finite difference estimators.

(4) We point out that the analysis extends to higher order sensitivities, to the likelihood ratio estimation at one parameter value from a simulation at another parameter value, and (in principle) to the GI/GI/1 queue.

The method of analysis, Shalmon (Probab. Eng. Inf. Sci. 88), relies on a busy period decomposition which has a triple interpretation: in terms of the ladder variable representation of the waiting time process, in terms of a birth-death branching process representation of the virtual time process during a busy period (where the waiting time corresponds to time of birth), and in terms of the LCFS preemptive resume discipline. The estimators can be interpreted as functionals of the birth-death branching process.

A constructive criterion for the ergodic behavior of interacting particle systems

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We consider what is called a Markov process with local interaction. It is a continuous time Markov process, the values of which are configurations of a lattice random field. The time dynamics of the process are given by the following rule: the rate at which each particle changes its state, depends on the current configuration only through its values in some neighbourhood of the particle in question, which is the shift of some fixed neighbourhood of the origin. The specific feature of the infinite state Markov processes is that they can exhibit nonergodic behavior even when all rates are nonzero. The phenomenon of phase transition can be viewed as a special case of this nonergodic behavior. A general sufficient (but not necessary) condition for ergodicity of these systems, known under the name "$M < \varepsilon$", can be found in the book by Liggett. It turns out that there exists a sequence of conditions, such that the validity of any of them implies the ergodicity of the process. The complexity of these conditions increases with their number, but the validity of each of them can be checked with a reasonable amount of labour. Moreover, if the process is ergodic, and the convergence to the limiting distribution is exponential, then some condition on the sequence should hold. The results of the talk, as well as the corresponding results for the ergodicity of discrete time Markov processes with local interaction, were obtained in collaboration with Christian Maes.

A branching random walk with a barrier

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Suppose that a child is likely to be weaker than its parent, and a child who is too weak will not reproduce. What is the condition for a family line to survive? Let b denote the mean number of children a viable parent will have; we suppose that this is independent of strength as long as strength is positive. Let F denote the distribution of the change in strength from parent to child, and define $h = \sup_\theta(-\log(\int e^{\theta t}\, dF(t)))$. We show that the situation is black or white:

(1) If $b < e^h$ then P(family line dies) = 1.

(2) If $b > e^h$ then P(family survives) > 0.

Define $f(x) := E$(number of members in the family |initial strength x). We show that if $b < e^h$, then there exists a positive constant C such that $\lim_{x\to\infty} e^{-\alpha x} f(x) = C$ where α is the smaller of the (at most) two positive roots of $b\int e^{st}\, dF(t) = 1$. We also find an explicit expression for $f(x)$ when the walk is on a lattice and is skip-free to the left. This process arose in an analysis of rollback-based simulation, and these results are the foundation of that analysis.

Keywords: Branching process, random walks, absorbing barrier, survival.

Limit theorems for reaction-diffusion equations with boundary perturbations

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In this paper we consider some limit theorems for reaction-diffusion equations (RDE) with boundary perturbations. We investigate some central limit theorem results and some large deviation results for this class of equations. Our interest here is primarily in the structure of the functional spaces in which to study these equations. The interesting aspect of these functional spaces is in the boundary layer — if the boundary data is white noise, then the boundary behavior is degenerate, and this information must be encoded in the functional space description. Freidlin and Wentzell have carried out this analysis for a one-dimensional RDE, and Sowers, in his thesis, has proposed the functional spaces in which to carry out these investigations when the space-variable is multidimensional. The main effort of this paper is to show some appropriate central limit theorems and large deviations theorems in the correct functional spaces.

Blocking times and the exponential distribution

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In a single-server queue, customers have i.i.d. service times distributed as X. The server is subject to 'fits' separated by i.i.d. exponentially distributed intervals, denoted by Y, with mean $1/\lambda$. When a fit

occurs, the customer being served has to start anew with a new service time. The total time a customer is served, is called the 'blocking time' and is denoted by Z:

$$Z = Z(\lambda) = Y_1 + \cdots + Y_{N-1} + X_n = \sum_{n=1}^{N(\lambda)} \min\{X_n, Y_n(\lambda)\},$$

where $N = N(\lambda) = \min\{n \in \mathbb{N}: X_n < Y_n(\lambda)\}$. Since short service times are favoured, Z is not necessarily (stochastically) larger than X. It is easily verified that $z \overset{d}{=} X$ if X is exponentially distributed. Dimitrov and Khalil have shown that $Z(\lambda) \overset{d}{=} X$ for *all* $\lambda > 0$ (a condition that is hard to interpret) implies that X has an exponential distribution. Here it is shown that X is exponential when $Z(\lambda) \overset{d}{=} X$ for *a single value* of λ.

References

B. Dimitrov and Z. Khalil, On a new characterization of the exponential distribution related to a queueing system with an unreliable server, J. Appl. Probab. 27 (1990) 221-226.

K. Harn and F.W. Steutel, On a characterization of the exponential distribution, to appear in: J. Appl. Probab. (1991).

On multidimensional diffusion processes with time-inhomogeneous singular drift

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We consider stochastic differential equations of the form

$$dX_t = b(t, X_t)\,dt + dW_t, \quad 0 \leqslant t \leqslant T, \; t < \infty,$$

with arbitrary initial (probability) distribution μ on $\mathbb{R}^d$, $d \geqslant 1$. The drift b will be allowed to have singularities. We give several analytic conditions being sufficient for the applicability of the Girsanov (-Maruyama) Theorem. By means of examples, we show that these conditions enable us to construct a wide class of diffusion processes with time-inhomogeneous singular drift. For instance, we present a *nowhere* 'locally a.s. bounded' drift which satisfies the Novikov condition. Furthermore, we give several equivalence conditions of the Novikov condition. Finally, we investigate integrability properties of the Girsanov (-Maruyama) density as well as entropy properties of diffusion processes with (singular) drift b.

Some structural theorems for Markov chains

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The well-known Maisonnevue theorem on exit systems provides background for many results in Markov processes. In the case of simple Markov chains, it may be used for unification of some structural theorems found in applications. Of special interest are (first entrance, last exit) decomposition theorems, association with the Chung equation and the connection with Lévy systems, as well as the investigation of consecutive entrances to a set. Several illustrative examples will be provided.

References

B. Maisonnevue, Exit systems, Ann. Probab. 3 (1975) 399-411.

K.L. Chung, Markov Chains With Stationary Transition Probabilities (Springer, Berlin, 1967).

M. Sharpe, General Theory of Markov Processes (Academic Press, New York, 1988).

H. Kaspi and M. Rubinovitch, Regenerative sets and their applications to Markov storage systems, in: Queueing Theory and its Applications, Liber Amicorum for J.W. Cohen (North-Holland, Amsterdam, 1988) pp. 413-428.

Some remarks on strong laws of large numbers for dependent random variables and Riesz summability of orthogonal series

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We study the convergence behavior of some sequences and series related to a given orthogonal series. Following standard techniques, we define (in terms of fourth mixed moments only) a class of orthonormal functions $\{X_i\}_{i\geqslant 1}$ such that the condition: $\exists k\in\mathbb{N}$ such that $\sum_{i\geqslant 1}\mu_i^2(\ln^{(k)} i)^2<\infty$ implies almost everywhere convergence of the series $\sum_{i\geqslant 1}\mu_i X_i$. Here $\ln^{(1)} i=\ln_2 i$, $\ln^{(j)} i=\ln_2(\max(1,\ln^{(j-1)} i))$, $i=1,2,\ldots, j=1,\ldots,k$. Next, we recall some results concerning strong laws of large numbers for dependent random variables and later apply the above mentioned result concerning orthogonal series to improve the existing strong laws of large numbers.

Stochastic monotonicity and Slepian-type inequalities for stable random vectors

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Let $\boldsymbol{X}=(X_1,\ldots,X_d)$ and $\boldsymbol{Y}=(Y_1,\ldots,Y_d)$ be two symmetric random vectors. The classical Slepian inequality assumes $\boldsymbol{X}$ and $\boldsymbol{Y}$ Gaussian and states conditions on their covariances that ensure $E\max X_i\geqslant E\max Y_i$. Our aim is to extend the Slepian inequality to symmetric stable random vectors whose index falls in the interval $(1,2)$. Covariances, in this case, do not exist. We assume first that $\boldsymbol{X}$ and $\boldsymbol{Y}$ are infinitely divisible and give conditions on their Lévy measures so that $\boldsymbol{X}$ dominates $\boldsymbol{Y}$ stochastically. We then use this result to obtain conditions for the Slepian inequality, that is for $E\max X_i\geqslant E\max Y_i$, when $\boldsymbol{X}$ and $\boldsymbol{Y}$ are symmetric stable random vectors. We also study the relation between stochastic domination of an infinitely divisible random vector $\boldsymbol{X}=(X_1,\ldots,X_d)$ by another infinitely divisible random vector $\boldsymbol{Y}=(Y_1,\ldots,Y_d)$ and their corresponding Lévy measures. The results are used to derive a Slepian-type inequality for random vectors with a symmetric stable distribution. The inequality compares $E\max X_i$ to $E\max Y_i$. (This is a joint work with Gennady Samorodnitsky.)

Approximation theorems of Wong-Zakai type for stochastic differential equations in infinite dimension

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Two generalizations of the Wong-Zakai theorem, including the case of the stochastic delay differential equations, are presented.

Theorem 1. *Consider the equations*

$$X^{n,i}(t,w)=X_0^{n,i}(w)+\int_0^t b^i(X_s^n(\cdot,w))\,\mathrm{d}s+\sum_{p=1}^m\int_0^t \sigma^{ip}(\cdot,w))\dot{B}^{n,p}(s,w)\,\mathrm{d}s,$$

$$X_0^{n,i}(\theta,w)=X_0^{n,i}(w)=Y_0^i(w),$$

$$Y^i(t,w)=Y_0^i(w)+\int_0^t b^i(Y_s(\cdot,w))\,\mathrm{d}s+\sum_{p=1}^m\int_0^t \sigma^{ip}(Y_s(\cdot,w))\,\mathrm{d}w^p(s)$$

$$+\tfrac{1}{2}\sum_{p=1}^m\sum_{j=1}^d\int_0^t D\tilde{\sigma}^{ip}(Y_s(\cdot,w))\sigma^{jp}(Y_s(\cdot,w))\,\mathrm{d}s,$$

where $X_t(\theta, w) = X(t+\theta, w)$ *for* $\theta \in (-\infty, 0] = J$, $b: C_- \to \mathbb{R}^d$, $\sigma: C_- \to L(\mathbb{R}^m, \mathbb{R}^d)$, $\tilde{\sigma}: C_- \to C(J, L(\mathbb{R}^m, \mathbb{R}^d))$, *are nonlinear operators (see [1]),* $B^{n,p}$ *is a piecewise linear approximation of an m-dimensional Wiener process* $B(t, w) = w(t)$, C_- *is a metric space of continuous functions on J. Then, for each* $T > 0$,

$$\lim_{n\to\infty} \sup_{0\le t\le T} E[|X^n(t, w) - Y(t, w)|^2] = 0.$$

Theorem 2. *Consider the equations*

$$u^n(t) = S(t)z_0 + \int_0^t S(t-s)C(u^n(s))\,ds + \int_0^t S(t-s)B(u^n(s))\,dw^n(s), \quad u^n(0) = z_0,$$

$$z(t) = S(t)z_0 + \int_0^t S(t-s)C(z(s))\,ds + \int_0^t S(t-s)B(z(s))\,dw(s)$$

$$+\tfrac{1}{2}\int_0^t S(t-s)\tilde{tr}(QDB(z(s))B(z(s))\,ds, \quad z(0) = z_0,$$

where H, H_1 *are Hilbert spaces* $A: D(A) \subset H_1 \to H_1$ *is an infinitesimal generator of a* C_0*-semigroup* $(S(t))_{t\ge 0}$, *the nonlinear operators* $C: H_1 \to H_1$, $B: H_1 \to L(H, H_1)$, *and the correction term* $\tilde{tr}(QDB(h_1))$ *are introduced in [2], and* $w^n(t)$ *is a piecewise linear approximation of a Wiener process* $w(t) \in H_1$. *Then, for each* $T > 0$,

$$\lim_{n\to\infty} P\left(\sup_{0\le t\le T} \|u^n(t, w) - \hat{z}(t, w)\|_{H_1} \ge \varepsilon\right) = 0.$$

References

[1] K. Twardowska, On the approximation theorem of the Wong–Zakai type for the functional stochastic differential equation, Probab. Math. Statist. 12(1) (1991).

[2] K. Twardowska, An extension of the Wong–Zakai theorem for stochastic evolution equations in Hilbert spaces, submitted to: J. Stoch. Anal. Appl. (1990).

Limit theorems and parameter estimation for the Q-state Curie–Weiss–Potts model

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The q-state Curie–Weiss–Potts model, where $q \ge 3$ is an integer, is a useful statistical mechanical model. It is an exponential family parametrized by the inverse absolute temperature β and the external magnetic field$h$. The first part of the work studies limit theorems for the empirical vector of the model. These limits include the law of large numbers, a central limit theorem when $\beta < \beta_c$ and $h = 0$, and a conditional central limit theorem when $\beta \ge \beta_c$ and $h = 0$, where β_c is the critical inverse temperature. Also a central limit theorem with random centering is proved. The phase transition at β_c is first-order, in contrast to second-order phase transition in the classical Curie–Weiss model. All these limit theorems imply similar limits for the sample mean. Some limit theorems for the classical Curie–Weiss model are also presented. The second part studies the large sample behavior of the maximum likelihood estimators of the external magnetic field and the inverse absolute temperature. The existence, asymptotic normality, and consistency of the estimators are studied in detail.

Queueing network optimization via perturbation analysis

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Consider a queueing network whose performance depends on a variable $x \in \mathbb{R}^d$. Let $Y(x)$ be an average performance measure, and suppose that it is computed (estimated) by Monte Carlo simulation. Thus, a simulation of the network at a fixed value of x gives out a sequence of scalar observables, $y_n(x)$, $n = 1, 2, \ldots$, and we assume that, a.s., as $N \to \infty$, $\sum_{n=1}^{N} y_n(x)/N \to Y(x)$. The problem that we consider is to minimize $Y(x)$. Perturbation Analysis (PA) is a sample path method that gives sensitivity information about $Y(\cdot)$ at a neighborhood of a given point $x \in \mathbb{R}^d$, by simulating the network at x. Two main branches of PA are Infinitesimal Perturbation Analysis (IPA) and Finite Perturbation Analysis (FPA); IPA computes sample derivatives of the form $\sum_{n=1}^{N} \nabla y_n(x)/N$ for a given N, and FPA computes sample differences of the form $\sum_{n=1}^{N} [(y_n(x+\Delta x) - y_n(x))/\Delta x]/N$ for given N and $\Delta x \in \mathbb{R}^d$, both in a single simulation run at x.

In this talk, we will present an algorithm for minimizing $Y(\cdot)$ (to the extent of finding a local minimum) that uses IPA and FPA. When at an iterate $x_i \in \mathbb{R}^d$, it computes the next iterate, x_{i+1}, in two steps: First, it uses IPA to compute an estimate of $\nabla Y(x_i)$, denoted by h_i. Then, it uses FPA to compute an approximate line minimization in the direction $-h_i$, starting from x_i. The result of this line minimization is x_{i+1}. In essence, the algorithm is a stochastic version of the steepest descent method with Armijo stepsizes. The line search requires multiple function evaluations, and these can be quite expensive when simulations are used. However, due to the special structure of FPA, they can be done in a single simulation run, resulting in considerable saving of computing times as compared to multiple simulations. We will discuss some possible tradeoffs between our algorithm and the established stochastic approximation methods, present an asymptotic convergence result, and provide numerical examples of optimization of queueing networks.

A functional limit theorem for waves reflected by a random medium

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We introduce a class of distribution-valued stochastic processes that arise in the study of pulse reflection from random media and we analyze their asymptotic properties when they are scaled in a natural way. We prove the uniqueness of the limit process by characterizing its moments in terms of a random walk and a diffusion process with values in the hyperbolic disc. We also show that this process is a solution of the following stochastic differential equation:

$$\langle W(s), \lambda \rangle = \langle W(\tau), \lambda \rangle + \int_{\tau}^{s} \langle W(u), \mathcal{L}_u^{(1)}\lambda \rangle \, du + \int_{\tau}^{s} \langle W(u), B_u\lambda \rangle \, db_u$$

where $b.$ is a one-dimensional Brownian motion.

A tandem queue with 0–1 dependent service times – The cafeteria process

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An infinite supply of customers are moving through a series of M service stations. Each customer requires one unit time of service in one of the stations, this being station i with probability p_i, independent of the other customers. No service is required at any of the other stations, but there is no overtaking between the customers. The analogy is to a cafeteria queue, in which each customer picks only his own choice of one of the M available items. We study the cases of infinite and of zero buffer between the stations. In the infinite buffer case, we show that the stations are interchangeable. This follows as a special case of a recent interchangeability result of Weber, and can also be inferred by transforming the problem to an equivalent tandem queueing system with exponential servers. In the zero buffer case, we try to obtain the loads $p_1, \ldots, p_M$ of the stations which maximize the throughput. Exact expressions for the throughput of up to 4 stations are used to obtain the optimal loads. A 2^{M-1} state Markov Chain that describes the journeys of successive customers through the system is used to obtain the optimal loads for up to 6 stations numerically, though these may be a local optimum. The optimal load seems to be 'bowl shaped', with heavier loads on the outside stations. The bowl is symmetric and its center is very flat. The throughput of the optimal load is only slightly higher than that of equal loads $p_i = 1/M$. Further, we investigate the throughput of the zero buffer system with equal loads as a function of M. Partial results so far include asymptotic lower and upper bounds of 2 and $\sqrt{(2/\pi)M}$, as M becomes large.

Some results on the extremal index of stationary sequences

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Let $\{X_i\}$ be a stationary sequence of random variables with marginal distribution function F. Let M_n be the largest of $\{X_1, \ldots, X_n\}$. If for large n and x,

$$P\{M_n \leq x\} \approx F^{n\theta}(x)$$

for some $\theta \in (0, 1]$, then θ is said to be the extremal index of the sequence $\{X_i\}$. We discuss several interpretations of the extremal index θ, from which several estimators of θ are derived in a natural way. These estimators are studied and their performance is reported.

On conditional structure of stochastic processes

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Linear conditional expectations and non-random conditional variances with some additional assumptions on the covariance function are a characteristic property of the Gaussian process. It is the main result of the series of papers: Plucinska (1983, Stochastics), Wesolowski (1984, Demonstratio Math.), Bryc (1985, Bull. Polish Acad. Sci., Math.). The conditioning applied in these papers was not only given 'the past' — as it is in the famous martingale charracterizations (the Levy theorem for the Wiener process and the Watanabe theorem for the Poisson process) — but also given the 'future' states of the process. This type of conditioning leads to theorems on finite dimensional distributions of the process with uniform assumptions — concerning moments and conditional moments only. Conditions imposed on trajectories, occurring in each martingale characterization, are omitted.

Since the first results were limited to the Gaussian processes only a natural question of extending this type of characterization to other processes has arisen. At first it was done for the Poisson process by Bryc (1987, Stochastics), then slightly generalized in Wesolowski (1988). Similar characterization for the Gamma process is due to Wesolowski (1989).

The presented paper is devoted to a complete solution of the problem in the case when the conditional variances are quadratic functions of the increment and the conditional expectations are linear functions of increments. In this way we obtain a characterization of the processes with independent increments having distributions (the increments) belonging to the natural exponential family with quadratic variance function introduced by Morris (1982, Ann. Statist.), i.e. the Wiener, Poisson, Gamma, Negative-binomial and Hyperbolic Secant processes.

A coupling technique for point processes and its application to Poisson approximations

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A coupling construction for a Poisson point process — defined as a random measure on a Polish, i.e. complete separable metric space — and a Poisson-Bernoulli process is presented with respect to the total variation distance. The coupling relates the total variation distance of a Poisson point process and a Poisson-Bernoulli process to the total variation distance of the finite dimensional marginal distribution

of an arbitrarily chosen partition of the underlying space and is shown to be optimal whenever the Poisson–Bernoulli process under consideration is the superposition of Bernoulli point processes with mutually disjointed supports (including the case of individual optimal couplings of a Poisson point process to a Bernoulli point process considered by A.F. Karr, 1986). The above coupling is applied to the question of optimal choice of parameters for multivariate Poisson approximations.

On the convergence of random symmetric polynomials

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L. Szeidl has obtained in [1] a description of the class of possible limit distributions of normed random symmetric polynomials under some conditions on the distribution function. The question of what conditions the coefficients of initial polynomials must satisfy so that they converge to some limit distribution, has not been investigated, and is treated in the present report.

References

[1] L. Szeidl, On limit distribution of random symmetric polynomials, Theory Probab. Appl. 33(2) (1988) 266–278.

[2] O. Yanushkevichiene, On limit distributions of nonhomogeneous random symmetric polynomials of third degree, Collect Probl. of Stab. of Stochmod., M, VNIISI.

Integer valued Markov processes and exponential families

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An exponential family of distributions $\mathcal{F}$ being given, a Markov Process is said to be stable if its distribution, non-constant, remains at any instant in the family $\mathcal{F}$. It is the case for instance for the Poisson process and the family of Poisson distributions on $\mathcal{N}$, or for the standard Brownian motion and the family of Gaussian distributions with mean 0. The notion permits the unification of many different situations where the transient behavior of a process is explicitly known. The results we propose here concern the stability of Feller processes with respect to one-parameter natural exponential families on $\mathcal{N}$. We characterize them through explicit relations between the family and the generator of the process. As an application, we obtain a complete description of all stable birth and death processes. Among the processes obtained, some were already known (e.g. linear growth processes), but many more cases appear. The main interest of the results proposed here is to permit explicit constructions of processes, the distributions of which are explicitly known at any time. This will be illustrated by two new examples.

Gaussian measure of translated balls

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Let γ by a symmetric Gaussian measure on a separable Banach space $(E, \| \ \|)$. Denote $B_t = \{x\colon \|x\| \le t\}$. The aim of this note is to summarize the results concerning the behavior of the difference $\gamma(B_t) - \gamma(B_t + r) = f_t(r)$.

The Hilbert space case was investigated in the author's papers [3] and [4]. A new and short proof of the result from [3] has recently been given by Linde and Rosinski [1]. It turned out that $f_t(r) \le c_t(2\lambda_1\lambda_2)^{-1}\|r\|^2$, where λ_1, λ_2 are the greatest eigenvalues of the covariance operator of γ and $c_t \to 0$ if $t \to 0$. The proof in [1] has one more advantage: it gives an unexpected result concerning the direction of the slowest (fastest) decrease of $f_t(\varepsilon r)$ as $\varepsilon \to 0$ and $\|r\|$ is fized. This direction (even in $\mathbb{R}^2$) depends on $\gamma(B_t)$ in a rather complicated way.

In the general case, the behavior of f was examined in [2]. Namely, it was shown that if $(E, \| \ \|)$ is uniformly convex, then $f_t(r) \leq c(\gamma, t)\|r\|^{\alpha}$, where α depends on the smoothness properties of $\| \ \|$. For example, in L_p spaces, $p > 1$, we have $\alpha = \min(2, p)$ and this result is sharp.

References

[1] W. Linde and J. Rosinski, Exact behavior of Gaussian measures of translated balls in Hilbert spaces, preprint (1990).
[2] M. Ryznar and T. Zak, The measure of a translated ball in uniformly convex spaces, J. Theor. Probab. 3 (1990) 547–562.
[3] T. Zak, A formula for the density of the norm of stable random vectors in Hilbert spaces, Probab. Math. Statist. 10 (1989) 257–270.
[4] T. Zak, On the difference of Gaussian measure of two balls in Hilbert spaces, Lecture Notes Math. No. 1391 (1989) 401–405.