

Frequency Domain Method for Real-Time Detection of Oscillations

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Because of the growing interest in real-time stability monitoring, there is a need for online methods that can detect off-nominal behavior in control systems. Therefore, we present a real-time frequency domain method for detecting oscillations in control systems. The method uses recursive Fourier transform for representing the measured time domain signal in frequency domain. Oscillations are then detected by monitoring the norm of the Fourier coefficients over a time window of pre-specified size across a range of frequencies that represent off-nominal behavior for the plant. This method is computationally efficient and requires limited memory, hence it can be used for onboard health monitoring of Unmanned Aerial Systems (UAS). Consequently, we demonstrate the feasibility of using this method for real-time detection of oscillation on the Georgia Tech GT Twinstar (UAS).

I. Introduction

Real-time stability monitoring is becoming an important area of research as autonomous systems continue to take on increasing responsibilities. Technologies are being developed to reduce aircraft accidents resulting from loss of vehicle control as well as failures. The Intelligent Resilient Aircraft Control (IRAC) project is an excellent example of recent efforts to increase safety of aerospace systems in adverse environments¹. The ability to monitor the health of onboard systems and to assess the effectiveness of control action in real-time aids greatly in preventing and recovering from loss of control situations, and in mitigating undesirable control induced scenarios. Undesirable oscillations in the system states are one potential cause for instability. The detection of such oscillations is an important aspect of system health and stability monitoring that cannot be addressed by monitoring only the instantaneous system state or the tracking error.

Oscillations in systems states can be control induced, for example, it is well known that high gains in linear controllers may lead to control system induced oscillations. Furthermore, it is also well known that adaptive systems are susceptible to the “bursting” phenomenon which is characterized by intermittent high frequency oscillations and is a result of large adaptation gain². Such induced oscillations can lead to variety of undesirable phenomena including actuator saturation, excitation of unmodeled dynamics, and loss of system stability. Early detection and mitigation of these effects is extremely important in assuring safe operation and optimal performance. For example, in autopilot systems for modern flight vehicles, fast and efficient detection of oscillations is imperative to prevent deterioration of ride quality and to minimize the possibility of systems failure.

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Several methods have been proposed in the literature for detecting oscillations and their characteristics. Miao and Seborg studied a statistically based off-line approach in which the dependence of time series data was characterized using sample auto-correlation coefficients³. The decay ratio of these coefficients was used as an indicator to determine if the signal was excessively oscillatory. Thornhill et al. studied the use of zero crossing of filtered auto-correlation functions for detection of oscillations at different frequencies in process control loops⁴. Both these methods require an integral of the data over a finite time window and hence do not lend easily to real-time implementation. Korba et al. proposed a Kalman filter based method for real-time estimation of the oscillatory modes in a system using measured data⁵. The dominant frequency and damping of the oscillatory modes was estimated from the measured data by estimating the eigenvalues and eigenvectors of the state matrix of a linearized model. However, if the underlying controller or dynamics do not yield to a linear model, the eigenvalue estimates may not converge.

Cox, Lewis and Suchomel developed a Neural Network based algorithm for detecting and compensating Pilot Induced Oscillations⁶. A Neural Network (NN) based discrimination function was trained using a gradient-descent like method to indicate the presence of oscillations. This method needs an explicit storage of a window of data, furthermore, the reliability of this method is dependent on the convergence of the gradient descent method for NN training which in turn depends on the number of unknown NN weights and the excitation in the system signals². Odgaard and Wickerhauser developed a method wherein the principal components of a signal could be obtained using Karhunen-Loeve analysis⁷. This analysis method was used to detect as well as localize the oscillation to specific measurements/sensors. This is a method suitable primarily for offline analysis and is computationally intensive to implement online.

Harris et al.⁸ compared the variance of the signal to the lowest achievable variance of the control loop. A priori knowledge of the latter, which is hard to obtain, is needed to set a threshold value. Adami, Best and Zhu presented an instability indicator based on Lyapunov's first method⁹. The indicator can be used to indicate if the system is moving away from the equilibrium state. This method however, is concerned more with the detection of instability and the presence of persistent oscillations in the system could give rise to incorrect information. Thompson et al.¹⁰ developed wavelet based methods for detecting loss-of-control in real-time applications. In this method, the time domain signal was decomposed into basis functions that translate in time and whose length and magnitude were scaled as a function of the frequency. Morani et al.¹¹ proposed the use of suitable band-pass filters-bank to capture the sinusoidal components in the signal. The benefit of this method is that it needs only limited knowledge of the process under control and can be used in real-time to detect oscillations, however, the method can be very sensitive to noise.

In this paper we present a frequency domain method for detection of oscillation. Our method relies on representing the time domain data into frequency domain using a Fourier basis over a frequency range that represents off-nominal behavior for the plant. Once in frequency domain, the dominant frequencies can be analyzed to detect and even to mitigate oscillations. A key benefit of the method presented here arises from the use of recursive Fourier transform¹², which ensures that no explicit storage of measured data is required; rather only a fixed dimension vector of the Fourier coefficients over the desired frequency range needs to be stored. These qualities of the presented method are conducive to real time implementation onboard Unmanned Aerial Systems (UAS) where computational resources may be restricted.

We begin by describing the details of discrete recursive implementation of Fourier transform. The feasibility of this method is then assessed for detection of high-gain control induced oscillation on an exemplary adaptive control problem. Finally, the presented method is used to detect the onset of angle of attack oscillation on the data from

the GT Twinstar UAS recorded during an enforced 50% loss of right wing, and to detect oscillations due to injected time delay.

II. Frequency Domain Detection of Oscillation

In the following, t denotes the time variable, and ω denotes the frequency variable. The Fourier transform of an arbitrary signal $x(t)$ is given by:

$$F[x(t)] \equiv \tilde{x}(\omega) \equiv \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt. \quad (1)$$

The signals of interest in this study are collected via a data sampling system so the discrete version of the Fourier transform is required. The discrete Fourier transform is given by:

$$X(\omega) \equiv \sum_{i=0}^{N-1} x_i(t_i)e^{-j\omega t_i}. \quad (2)$$

Let Δt denote the sampling interval, and N the total number of data points, then the Euler approximation of the Fourier transform in equation 1 is given by:

$$\tilde{x}(\omega) \approx X(\omega)\Delta t. \quad (3)$$

This approximation is suitable if the sampling rate is much higher than any of the frequencies of interest. This is assumed to be the case in this paper. The discrete version of the Fourier transform can be propagated in a recursive manner, for a data point index k , the Fourier transform can be propagated as follows¹²:

$$X_k(\omega) = X_{k-1}(\omega) + x(t_k)e^{-j\omega k\Delta t}. \quad (4)$$

The recursive propagation of the Fourier Transform greatly facilitates real time implementation as the matrix X_k is the only matrix that needs to be stored in the system memory. For a frequency range of interest, $[1 \dots m]$, the matrix representing the Fourier basis of the time domain data is given by:

$$\tilde{X}(\omega) = \begin{bmatrix} \tilde{x}(1) \\ \tilde{x}(2) \\ \vdots \\ \tilde{x}(m) \end{bmatrix}. \quad (5)$$

Using the above equations, the frequency content of the measured data can be approximated by a finite Fourier basis. For detection of oscillation, we simply restrict our attention to a particular frequency range of interest that characterizes the off-nominal behavior of the plant. This is similar to projecting the data over a finite subset of the Fourier basis. The Discrete Fourier Transform of a signal at a particular time instant ($t = T$) contains the frequency information in the signal from time $t = 0$ to $t = T$. This memory-like property can cause past oscillations to mistakenly trigger the detection algorithm. We handle this issue by using a windowed DFT given below.

$$\Delta X(\omega)_{t=T} = X(\omega)_{t=T} - X(\omega)_{t=T-k}. \quad (6)$$

The difference between the DFT at $t = T$ and the DFT at $t = T - k$ is a measure of the frequency content in the signal in the time window between T and $T - k$. In this way, the information about the magnitude and frequency of oscillations over a window of data can be stored in a single variable. To detect oscillations, we monitor the norm of this windowed Fourier transform. If the norm exceeds a pre-specified threshold value, it indicates the presence of excessive activity in that frequency range, which is considered as a strong indication of undesirable oscillation and a flag is raised. Let $x \in \{0,1\}$ denote the output of the flag, with $x = 1$ if the norm exceeds the threshold value, and $x = 0$ otherwise. The flag is passed through a discrete implementation of a first order low pass filter to smoothen the output. Let y denote the filter state, and τ denote the filter time constant, then the low pass filter has the form:

$$\dot{y}(t) = \frac{-(y(t) - x(t))}{\tau}. \quad (7)$$

Equation 7 represents a first order low pass filter, with a continuous output y constrained between 0 and 1. The numerical value of the output can be thought of as an indicator of the confidence of the method in detection of oscillations. Furthermore, the time rate of change of y can be thought of as an indicator of whether the oscillations are increasing or diminishing. Therefore, the time constant τ becomes a design parameter for this method. A larger time constant results in a smoother output, and is desirable for detecting the presence of sustained oscillations; whereas, a smaller time constant increases the sensitivity of the algorithm, and is desirable for detecting the presence of transient oscillations. Along with the time constant τ , the design parameters for this method are: the frequency band of interest to be monitored, the threshold value, and the window size k . The frequency band and the threshold value should be physically motivated to characterize the off-nominal behavior of the plant. The window size, along with the time constant, determines the sensitivity of the algorithm. A smaller window size results in the algorithm considering only recent data for oscillation detection, while a larger window size ensures that the effect of past data are also included. In practice, the size of the window should be chosen to be at least as large as the period of the lowest frequency in the frequency band to be monitored. Instead of windowed Fourier transforms it is also possible to use a forgetting factor to discount old data¹².

III. Detection of Oscillation for an Exemplary Adaptive Control Problem

In order to assess the feasibility of the presented approach, we present an exemplary problem of detection of bursting phenomena² in an adaptive controller simulation. Let x denote the state vector, u denote the control input, w_d denote the disturbance in the state and consider the problem of controlling the following system:

$$\dot{x}(t) = 3x(t) + 2u(t) + w_d(t), \quad (8)$$

$$y(t) = x(t) + w_s(t). \quad (9)$$

Here y denotes the measurement vector and w_s denotes the sensor noise. A linear reference model is chosen to characterize the response of the system. Let x_r denote the states of the reference model, and $r(t)$ denote a desired reference signal which is treated as an input to the reference model given by:

$$\dot{x}_r(t) = -2x_r(t) + 3r(t). \quad (10)$$

The tracking error can be defined as: $e(t) = x_r(t) - x(t)$.

A stable adaptive controller for this problem can be found by using Lyapunov theory^{2,13,14}, and is given as:

$$u(t) = k_x(t)e(t) + k_r(t)r(t) \quad (11)$$

where $k_x(t), k_r(t)$ are the adaptive weights updated by the following well known learning law^{2,13,14}:

$$\dot{k}_x(t) = -\Gamma_x x(t)e(t)\text{sign}(B), \quad (12)$$

$$\dot{k}_r(t) = -\Gamma_r r(t)e(t)\text{sign}(B), \quad (13)$$

where Γ_x, Γ_r denote the adaptation gains (also known as learning rates). High learning rates ($\Gamma_x = 10, \Gamma_r = 0.5$) are used in order to simulate the bursting phenomena². Bursting is a sudden onset of high frequency and high amplitude oscillation in the system state caused due to high adaptation gains. Early detection of the onset of bursting can be used to prevent damage to the system and to mitigate the oscillations. In order to detect the onset of bursting, we monitor the tracking error $e(t)$ over a frequency range of 2Hz to 6Hz using the windowed Fourier Transform method discussed in the previous section. The time step used for the simulation is 0.01 sec. The threshold value and the window size are determined to be 64 and 0.36 seconds respectively by using the nominal transient response characteristics of the reference model. Particularly, the threshold value was obtained by taking the mean of the recorded norm at each time step of the tracking error windowed Fourier transform for a nominal step input until steady state is reached. Alternatively, the threshold value can also be determined using the known frequency response characteristics of the reference model.

Zero mean Gaussian white noise with a standard deviation of 0.1 was added to the system to simulate the effect of sensor noise (w_s) in the measurements. A significant source of state disturbance (w_d) that many mechanical systems are subject to is vibration caused due to loose mechanical parts and rotating elements. We model these disturbances as low amplitude high frequency sinusoidal oscillations. If such oscillations are considered nominal, then they should not be detected by the oscillation detection algorithm.

Figure 1 shows the performance of the adaptive controller for a nominal linear system. The controller attempts to track a constant reference command of $r(t) = 5$. It can be seen that the controller tracks the command with little error until about 1.3 seconds. Between 1.3 seconds and 5.8 seconds the system state exhibits the bursting phenomena characterized by high frequency, high amplitude oscillation about the reference model state.

Figure 2 shows the performance of the detection algorithm. A flag that is passed through a discrete low pass filter with a time constant of 0.05 sec is plotted on the bottom part of the figure. A value of 1 for the filtered flag value indicates high confidence in the presence of oscillations, while a value of 0 indicates that no oscillations over the frequency range of interest are present. The algorithm is able to detect the onset of bursting early on and drives the oscillation detection flag to 1. Furthermore, the algorithm also detects the end of the bursting phenomena and drives the oscillation detection flag back to 0 as the system resumes nominal behavior in presence of noise.

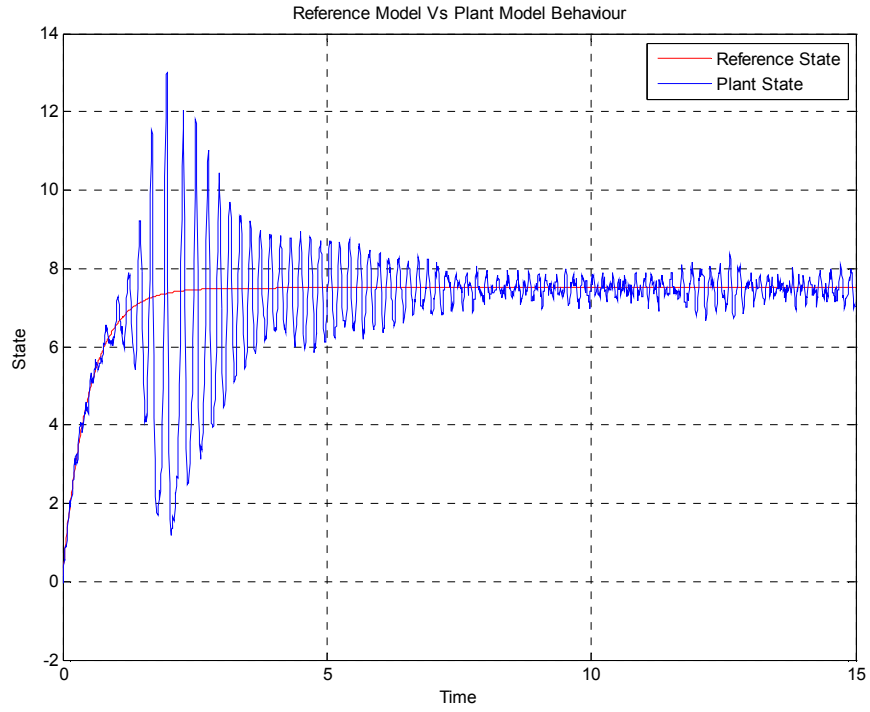


Figure 1 Performance of the Adaptive Controller

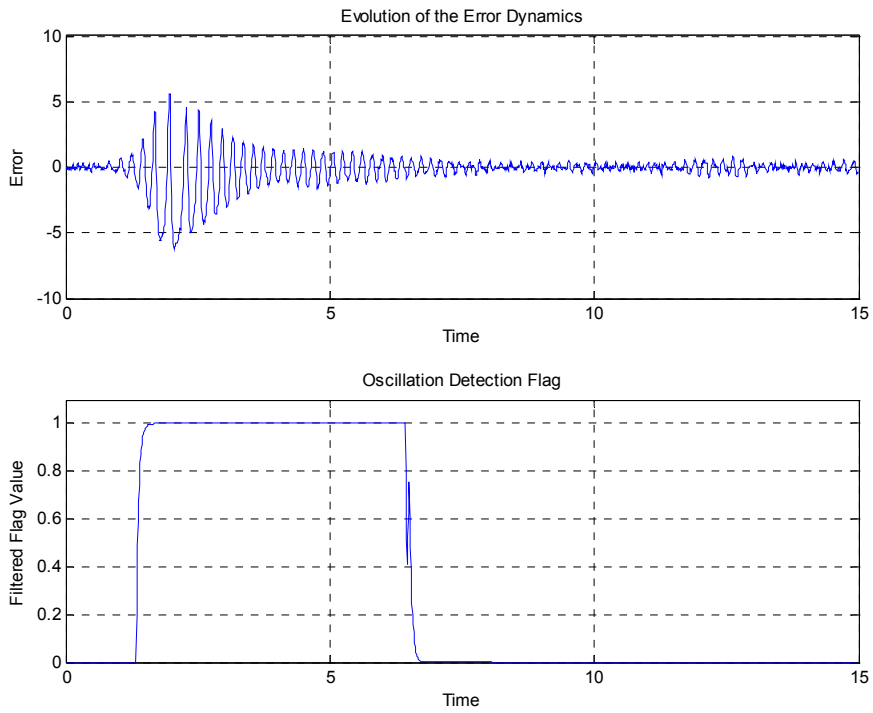


Figure 2 Detection of onset of bursting

IV. Application to Vehicle Health Monitoring for the GT Twinstar UAS

The Georgia Tech (GT) Twinstar, shown in Figure 3 is a fixed Wing UAS. It is a foam built, twin engine aircraft that is equipped with an integrated off-the-shelf autopilot system (Adaptive Flight FCS 20[®]). GT Twinstar has been specifically developed for fault tolerant control system work^{16,17,18}. It comes equipped with an ability to inject actuator time delays and simulate actuator failures, furthermore GT Twinstar is capable of jettisoning 50% right (starboard) wing while in autonomous flight. In this section we use recorded data from GT Twinstar for demonstrating the feasibility of the developed approach for detecting oscillations. This recorded data includes controlled system response to 50% loss of right wing¹⁸, and to injected actuator time delay¹⁹. For both the applications described in the following sections, the threshold value was obtained in the following manner: Using data sets that had been collected previously when the aircraft was performing System-ID maneuvers¹⁷, the windowed Fourier transform of the states was monitored. The norm of the windowed Fourier transform was calculated at each time step and its mean value was used as the threshold for detecting oscillations in the system.

A. Detection of oscillations in angle of attack due to asymmetric loss of lifting surface

Figure 4 shows the GT Twinstar in flight with 50% of its right (starboard) wing lost as the aircraft performs an autonomous holding pattern about a given waypoint. Figure 5 shows a slice of recorded accelerometer measurements onboard the GT Twinstar. A sharp spike in the y-axis accelerometer at around 16 seconds indicates the engagement of the jettisoning mechanism. The wing physically separates from the vehicle approximately 4 seconds later, at around 22 seconds into the recorded data. The resulting significant change in the system dynamics is characterized by an oscillation primarily in the angle of attack and roll rate. Figure 6 shows the recorded angular rate measurements onboard GT Twinstar.

Figure 7 shows the performance of the presented oscillation detection algorithm. The algorithm monitors a frequency range between 2Hz to 6Hz based on physical considerations and power spectral density analysis of recorded data in nominal flight conditions. Nominal flight data was used to determine the threshold value used for setting oscillation detection flag to 1. The flag is passed through a low pass filter (equation (7)) with a time constant of 0.67 seconds implemented discretely with an update rate of 50Hz corresponding to the rate at which the data was recorded. The output of the low pass filter is plotted on the bottom part of the figure. In that plot, a value of 1 indicates high confidence in the presence of sustained oscillations, while a value of 0 indicates that no oscillations over the frequency range characterizing off-nominal behavior are present. The algorithm is able to detect the onset of oscillations immediately after the partial loss of wing and drives the output of the low pass filter to 1. The response of the system is characterized by intermittent oscillations after around 50 seconds into the recorded data, this is also reflected by the output of the low pass filter.

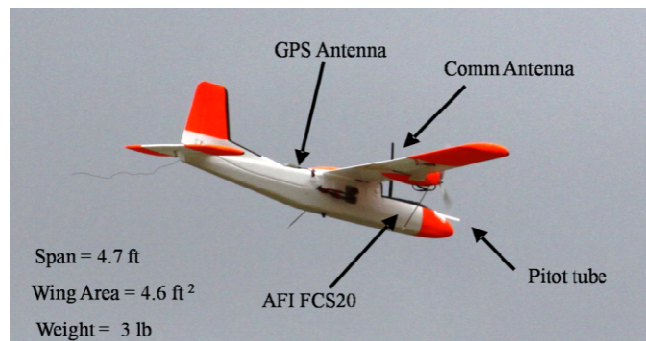


Figure 3 Georgia Tech Twinstar Fixed Wing UAV



Figure 4 GT Twinstar in autonomous flight with 50% right wing failure

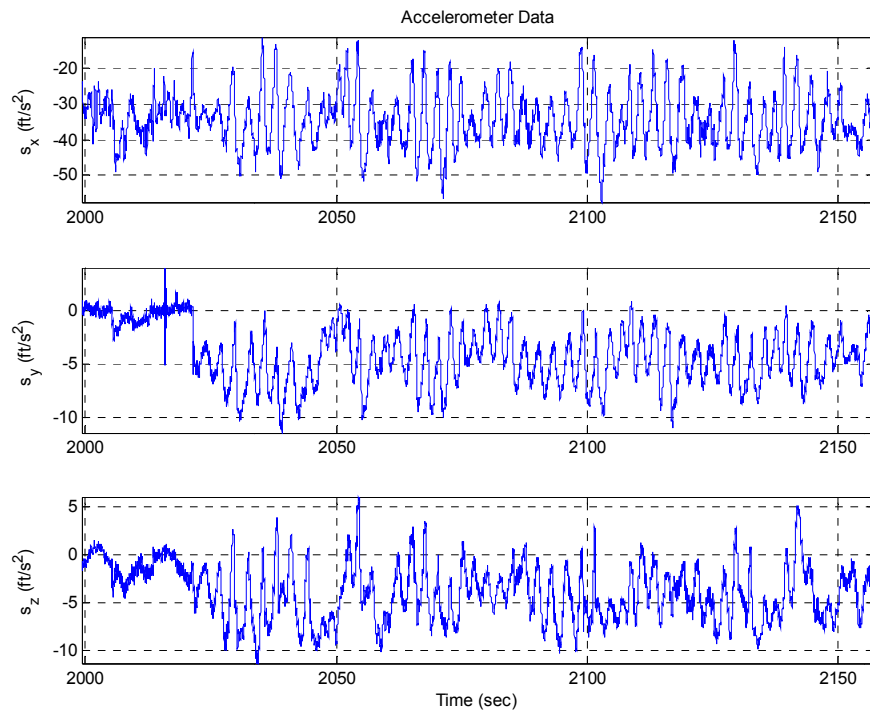


Figure 5 Recorded Accelerometer Measurements

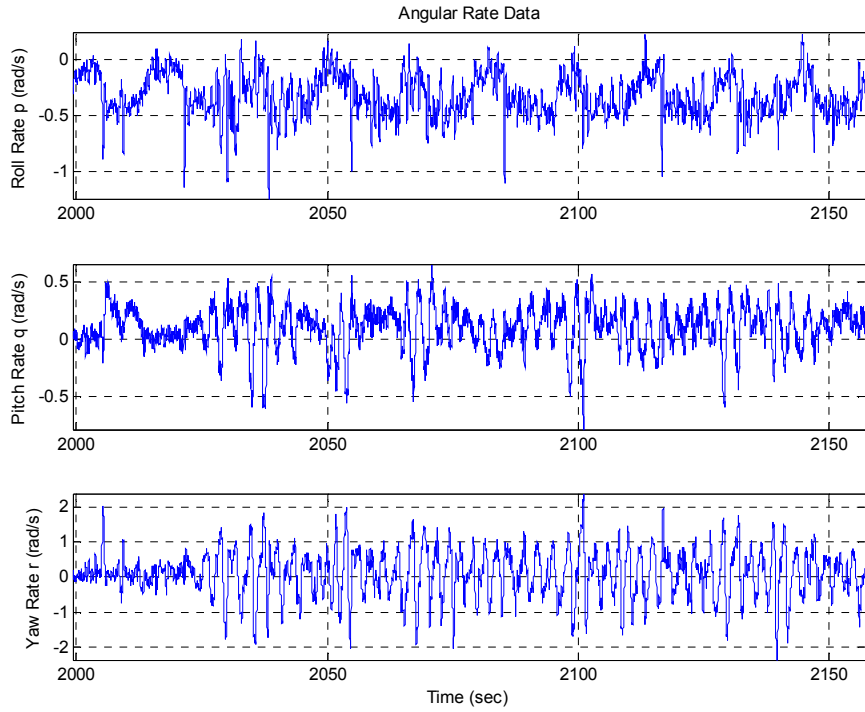


Figure 6 Recorded Angular Rate Measurements

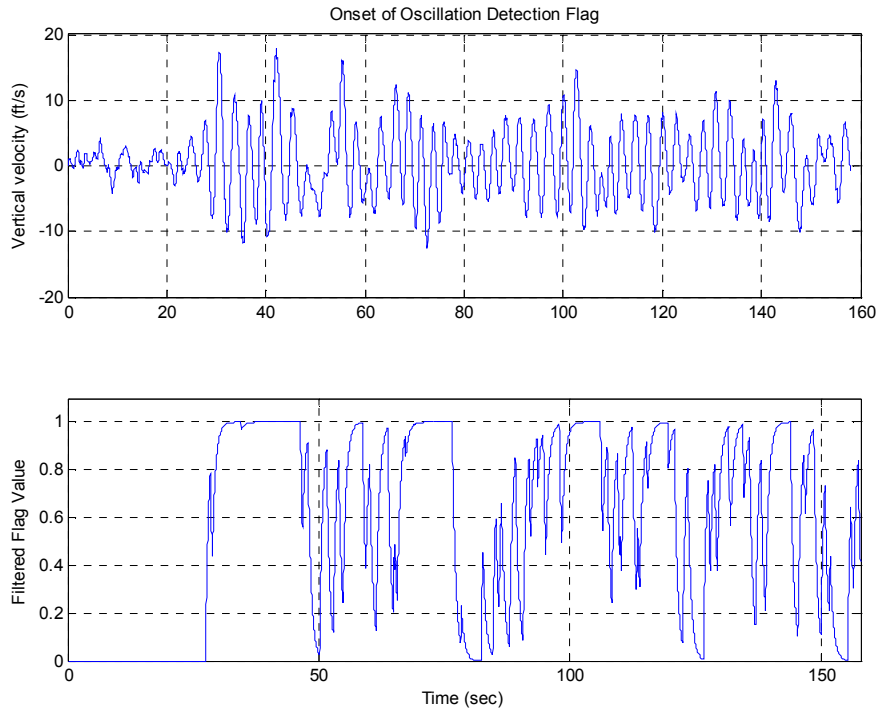


Figure 7 Detection of Oscillation due to asymmetric loss of lifting surface

B. Detection of Oscillations due to Injected Time Delay

The presence of time delay can lead to sustained oscillations and instability in adaptive controllers. To counter this, Calise et al. have developed the Adaptive Loop Recovery¹⁶ (ALR) method capable of recovering the phase margin of the baseline non-adaptive design. In this section we demonstrate the effectiveness of the presented method in detecting oscillations in the control system due to injected time delay on the GT Twinstar UAS. Figure 8 shows the performance presented algorithm in detecting oscillations in the roll rate due to the injected time delay. Nominal flight data was used to determine the threshold value used for setting the oscillation detection flag to 1. The flag data is passed through a low pass filter with a time constant of 0.67 seconds implemented discretely with an update rate of 100Hz corresponding to the rate at which the data was recorded. The algorithm detects the oscillation in the roll rate due to injected time delay at about 27 sec. As the ALR gains are gradually increased starting from around 36 seconds, a gradual reduction in the oscillation is seen; reflected by a downward trend in the filtered flag value. The flag eventually settles down to 0 once the oscillations due to time delay are effectively mitigated by ALR.

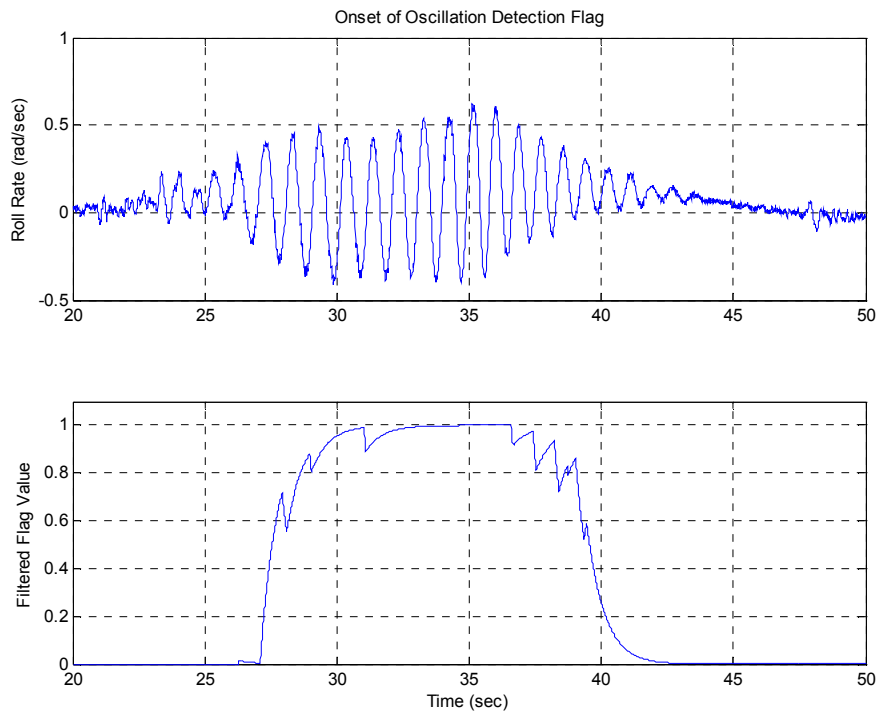


Figure 8 Detection of Oscillation due to the injection of time delay

VI. Conclusion

A frequency domain method of detecting oscillations was presented. The presented method represents time domain data in a frequency domain Fourier basis over a range of frequency that indicates off-nominal behavior for the plant. This method is both computationally efficient and requires limited memory; therefore it was found to be effective for real-time implementation. We presented results showing the feasibility of using the proposed method as a metric for detection of the onset of bursting phenomenon in an exemplary adaptive control problem. The

presented method was also used to detect structural failure and time delay induced oscillations in recorded flight data from the GT Twinstar UAS. A unique benefit of this method is that it enumerates magnitude and frequency information about the oscillations in a single variable. Hence, this method holds great potential for use in mitigation of oscillations in control loops.

VII. References

1. Nguyen N., Krishnakumar K., Frost S., "Proposed Stability and Performance Metrics for Verifiable Adaptive Control", NASA Ames Research Center, IRAC Program guideline proposal.
2. Astrom K. J., Wittenmark B., "Adaptive Control", second edition, Addison – Wesley publishing company Inc., 1995.
3. Tina Miao, Dale E. Seborg, "Automatic Detection of Excessively Oscillatory Feedback Control Loops", Proceedings of the IEEE International Conference on Control Applications, 1999.
4. Thornhill N. F., Huang B., Zhang H., "Detection of multiple oscillations in control loops", Journal of Process control, 2003.
5. Korba P., Larsson M., Rehtanz C., "Detection of Oscillations in Power Systems using Kalman Filtering Techniques", Proceedings of the IEEE Conference on Control Applications, 2003.
6. Chad Cox, Carl Lewis, Charles Suchomel, "A Neural Network Based, Real-Time Algorithm for Detection and Mitigation of Pilot Induced Oscillations", IEEE International Conference on Systems, Man and Cybernetics, 2000.
7. Odgaard P.F., Wickerhauser M.V., "Karhunen-Loeve (PCA) based detection of multiple oscillations in multiple measurement signals from large-scale process plants", IEEE American Control Conference, 2007.
8. Harris T.J., Seppala C.T., Desborough L.D., "A review of performance monitoring and assessment techniques for univariate and multivariate control systems", Journal of Process Control, 1999.
9. Adami T.M., Best E., Zhu J.J., "Stability assessment using Lyapunov's first method", Proceedings of the Southeastern Symposium on System Theory, 2002.
10. Thompson P.M., Klyde D.H., Bachelder E.N., Rosenthal T.J., Brenner M.J., "Development of wavelet-based techniques for detecting loss of control", AIAA Atmospheric Flight Mechanics Conference and Exhibit, 2004.
11. Morani G., Ciniglio U., Corrado F., "An efficient algorithm for on-line automatic detection of flight control instabilities", AIAA Infotech@Aerospace Conference, 2007.
12. Morelli E. A., "Real Time Parameter Estimation in the Frequency Domain", AIAA Journal of Guidance, Control and Dynamics, 2000.
13. Naira Hovakimyan, "Robust Adaptive Control", copyright held by the author, unpublished, 2009.
14. Ioannous P., J. Sun, "Robust Adaptive Control", Prentice Hall USA, 1996.
15. Volyanskyy, K., Calise, A. J., Yang, B. J., and Lavretsky, E., "An Error Minimization Method in Adaptive Control", AIAA Guidance, Navigation and Control Conference, 2006.
16. Blauwe H.D., Johnson E., "Nonlinear modeling and simulation of small Unmanned Aerial Vehicle with substantial damage", Proceedings of the Modeling and Simulation Technologies Conference, 2009.
17. Debusk W., Chowdhary G., Johnson E., "Real-Time system identification of a small multi-engine aircraft", Proceedings of the AIAA Atmospheric Flight Mechanics Conference, 2009.

18. Chowdhary G., Johnson E., "Guidance and Control of an Airplane under Severe Structural Damage", Proceedings of the AIAA@Infotech Conference, 2010.
19. Calise A.J., Yucelen T., Muse J.A., Yang B.J., "A Loop Recovery Method for Adaptive Control", AIAA Guidance, Navigation and Control Conference, 2009.