ZERO-FORCING BLIND EQUALIZATION BASED ON CHANNEL SUBSPACE ESTIMATES FOR MULTIUSER SYSTEMS

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ABSTRACT

The recovery of input signals in a frequency selective fading channel is a problem of great theoretical and practical interest. In this paper, we present several new blind algorithms that utilize second order statistics for multichannel equalization. The algorithms are based on the subspace extraction of a preselected block column of the channel convolution matrix. For multiuser system, user signal separation can be achieved based on partial information of the composite channel response. The equalization algorithms do not rely on the signal and noise subspace separation and therefore tend to be more robust to channel order estimation errors.

1 INTRODUCTION

In digital communication systems, intersymbol interference (ISI) and multiuser interference (MUI) are limiting factors for the performance of the receiver. In order to achieve reliable and high-speed communication, channel equalization is an important technique to overcome the effect of ISI and MUI.

Traditional channel equalization is accomplished by utilizing training sequences. Because of the time variant nature of the channel, training sequences should be transmitted periodically. Blind channel equalization does not require a training sequence. Generally at the receiver, only the output sequence and some *a priori* statistical information on the input sequence are utilized. Such methods offer potential improvements in system capacity by eliminating the training overhead.

A class of second order statistical (SOS) algorithms [1]-[5] have been presented that rely on the Single Input Multiple Output (SIMO) system model. SOS methods require that the channel diversity be available in terms of multiple antennae or over-sampled signals from channels with excess bandwidth. One class of channel equalizations are achieved by first identifying the channel, then equalizing the channel based on the estimated channel. A number of SOS based channel equalizations are done based on some known channel identification algorithms [1]-[5]. However, some existing subspace based blind channel identification algorithms [1][2][4][5] tend to be sensitive to channel order estimation errors. When channel order is unknown, accurate channel order estimation is difficult to achieve in noisy environment and poor channel identification and equalization results are likely.

In this paper, we will develop a family of new direct zero-forcing blind channel equalizers. Our goal is to develop equalization algorithms that are less sensitive to channel order estimation errors. We will present SOS methods that do not need to identify the unknown channel but instead generate equalizer parameters based on the zero-forcing criterion for SIMO and MIMO systems.

For SIMO systems, the zero-forcing blind equalizer can be obtained directly based on second order auto-correlation functions of the channel output signals. For MIMO systems, our zero-forcing equalizer can minimize the ISI and reduce the dynamic MIMO system into an almost memoryless signal mixing system. Source separation can be applied to the equalized MIMO system outputs if additional information (such as the pulse-shaping filter or the CDMA spreading code) is known. The implementations of these algorithms are simple and effective.

2 PROBLEM FORMULATION

Consider a baseband QAM linear system with N users. Assuming that the N user channels are all linear and causal with impulse response $\{h_u(t), u = 1, 2, \cdots, N\}$, the received signal can be written as

$$x(t) = \sum_{u=1}^{N} \sum_{k=-\infty}^{\infty} s_{k,u} h_u(t - kT) + w(t), \quad s_{k,u} \in \mathcal{A}_u, \tag{2.1}$$

where T is the symbol baud period and $\mathcal{A}$ is the input signal constellation of user u. The channel input sequences $\{s_{k,u}\}$ are i.i.d. with unit variance and are independent for different users. The noise $w(t)$ is stationary, white, and independent of channel input sequences $\{s_{k,u}\}$.

Note that $h_u(t)$ is a "composite" channel impulse response that includes transmitter and receiver filters as well as the physical propagation channel response. In a typical multiuser system, multiple channels of observations will be available from multiple sensors. If J subchannels (sensors or antennae) exist, then $x(t)$, $h_u(t)$ and $w(t)$ are all $J \times 1$ vectors. However, for simplicity of presentation, we assume $J = 1$ throughout this paper. The objective of blind channel equalization is to demodulate the input sequence $s_{k,u}$ from the output sequence $x(t)$, given only some *a priori* statistical knowledge of $s_{k,u}$ for unknown channels $\{h_u(t)\}$.

Suppose that the channel is oversampled by p, then the corresponding discrete MIMO system model is

$$x_n = \sum_{u=1}^{N} \sum_{k=-\infty}^{\infty} s_{k,u} h_u[n - kp] + w_n,$$

where p is the number of output samples derived from oversampling (assuming that there is excess bandwidth). Suppose that $\{h_u(t)\}$ has joint finite support $[0, T_h]$ spanning $m_0 + 1$ integer baud periods. Let superscript $\{.\}'$ represent matrix transpose. By defining notations

$$\begin{aligned} \mathbf{s}_k &\triangleq [s_{k,1} \ \ s_{k,2} \ \ \ldots \ \ s_{k,N}\ ]', \qquad (2.2)\\ \mathbf{s}[k] &\triangleq [\mathbf{s}_k' \ \ \mathbf{s}_{k-1}' \ \ \ldots \ \ \mathbf{s}_{k-m_0-M+1}']', \\ \mathbf{w}_k &\triangleq [w_{kp} \ \ w_{kp+1} \ \ \ldots \ \ w_{kp+p-1}]', \end{aligned}$$

$$\mathbf{w}[k] \triangleq [\mathbf{w}'_k \;\; \mathbf{w}'_{k-1} \;\; \ldots \;\; \mathbf{w}'_{k-M+1}]',$$
$$\mathbf{h}_u[i] \triangleq [h_u[ip] \quad h_u[ip+1] \quad \cdots \quad h_u[ip+p-1]]',$$
$$\mathbf{H}_i \triangleq [\mathbf{h}_1[i] \; \mathbf{h}_2[i] \; \ldots \; \mathbf{h}_N[i]],$$

it is evident that

$$\mathbf{x}_k \triangleq \begin{bmatrix} x_{kp} \\ x_{kp+1} \\ \vdots \\ x_{kp+p-1} \end{bmatrix} = \sum_{i=0}^{m_0} \mathbf{H}_i \mathbf{s}_{k-i} + \mathbf{w}_k. \tag{2.3}$$

Let Mp be the number of sampled channel outputs to be collected in a block, where M is called the *smoothing lag* or the *equalizer memory.* We form an $Mp \times (m_0 + M)N$ block Toeplitz matrix $\mathbf{H}$, which we call *channel convolution matrix* as,

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_0 & \mathbf{H}_1 & \ldots & \mathbf{H}_{m_0} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_0 & \mathbf{H}_1 & \ldots & \mathbf{H}_{m_0} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \ldots & \mathbf{0} & \mathbf{H}_0 & \mathbf{H}_1 & \ldots & \mathbf{H}_{m_0} \end{bmatrix}. \tag{2.4}$$

With these notations, a sampled channel output signal vector with length Mp can be written as

$$\mathbf{x}[k] \triangleq \begin{bmatrix} \mathbf{x}_k \\ \mathbf{x}_{k-1} \\ \vdots \\ \mathbf{x}_{k-M+1} \end{bmatrix} = \mathbf{H}\mathbf{s}[k] + \mathbf{w}[k]. \tag{2.5}$$

This output signal block will be used to determine the zero-forcing blind equalizer parameters.

3 COLUMN SUBSPACE ESTIMATION

Once we have the received signals, the key to getting the zero-forcing equalizer is to estimate the column subspace of a preselected block column in the channel convolution matrix.

3.1 Basic Assumptions

We shall assume from here that $\mathbf{H}$ has full column rank and is thus identifiable[1][2]. A more relaxed condition is for $\mathbf{H}$ to have full column rank after removing all zero columns. Assume that both the channel input signal $\mathbf{s}[k]$, and channel noise $\mathbf{w}[k]$ are white with zero mean, and the input signal $\mathbf{s}[k]$ has unit variance. The auto-covariance matrices of $\mathbf{s}[k]$ and $\mathbf{w}[k]$ are

$$\mathbf{R}_s(i) = E\{\mathbf{s}[k+i]\mathbf{s}[k]^H\} = \mathbf{J}^{iN}, \tag{3.1}$$
$$\mathbf{R}_w(i) = E\{\mathbf{w}[k+i]\mathbf{w}[k]^H\} = \sigma_w^2 \mathbf{J}^{ip} \tag{3.2}$$

In the above equations, σ_w^2 is the noise variance. $E\{.\}$ is the expectation operator, $\{.\}^H$ denotes the complex conjugate transpose operator and $\mathbf{I}$ is an $Mp \times Mp$ identity matrix. $\mathbf{J}$ is $Mp \times Mp$ and is called the *Jordan matrix* whose first subdiagonal entries below the main diagonal are unity while all remaining entries are zero.

3.2 Useful Matrix Properties

For notational convenience, we define

$$\mathbf{J}^0 = \mathbf{I}, \quad \mathbf{J}^{-1} = \mathbf{J}'. \tag{3.3}$$

We also define that

$$\mathbf{I}_{iN} \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{(Mp-iN)\times(Mp-iN)} \end{bmatrix}_{Mp\times Mp} \tag{3.4}$$

and

$$\Delta\mathbf{I}_{iN} \triangleq \begin{bmatrix} \mathbf{0}_{iN\times iN} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N\times N} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}_{Mp\times Mp}, \tag{3.5}$$

where $\mathbf{I}_{iN}$ is an identity matrix except for its first iN zero diagonal entries and $\Delta\mathbf{I}_{iN}$ is all zero except for unit entries on its $(iN+1)$ to $(i+1)N$-th diagonal elements. It can be directly shown that

$$\mathbf{I}_{iN} = \mathbf{J}^{iN}\mathbf{J}^{-iN} \quad \text{and} \quad \Delta\mathbf{I}_{iN} = \mathbf{I}_{iN} - \mathbf{I}_{(i+1)N}. \tag{3.6}$$

One important observation is that since $\mathbf{H}$ has full column rank, the following equation is true [9][10]

$$\mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{\#}\mathbf{H} = \mathbf{I}, \tag{3.7}$$

where $\{.\}^{\#}$ denotes the pseudoinverse operator.

3.3 Column Subspace Estimation

In this subsection, we will present two methods to estimate the column subspace matrix that can be used for zero-forcing blind equalizer. We exploit second order statistics of the channel output signal contained in its auto-covariance matrices

$$\mathbf{R}_x(i) \triangleq E\{\mathbf{x}[k+i]\mathbf{x}[k]^H\} = \mathbf{H}\mathbf{J}^{iN}\mathbf{H}^H + \sigma_w^2\mathbf{J}^{ip} \tag{3.8}$$

For simplicity, we first assume that the channel noise is absent such that

$$\mathbf{R}_x(i) = \mathbf{H}\mathbf{J}^{iN}\mathbf{H}^H \quad \text{and} \quad \mathbf{R}_x(0) = \mathbf{H}\mathbf{H}^H. \tag{3.9}$$

The noise parameter σ_w^2 can be estimated [3] and subtracted from the covariance matrices.

3.3.1 Method A

Observe that from the expression of $\mathbf{R}_x(i)$ and (3.7),

$$\mathbf{D}_i \triangleq \mathbf{R}_x(i)\mathbf{R}_x(0)^{\#}\mathbf{R}_x(i)^H$$
$$= \mathbf{H}\mathbf{J}^{iN}\mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{\#}\mathbf{H}(\mathbf{J}^{iN})^H\mathbf{H}^H \tag{3.10}$$
$$= \mathbf{H}\mathbf{I}_{iN}\mathbf{H}^H. \tag{3.11}$$

Similarly,

$$\mathbf{D}_{i+1} \triangleq \mathbf{R}_x(i+1)\mathbf{R}_x(0)^{\#}\mathbf{R}_x(i+1)^H = \mathbf{H}\mathbf{I}_{(i+1)N}\mathbf{H}^H \tag{3.12}$$

Then we can form another Hermitian matrix from

$$\Delta\mathbf{D}_i \triangleq \mathbf{D}_i - \mathbf{D}_{i+1} = \mathbf{H}(\mathbf{I}_{iN} - \mathbf{I}_{(i+1)N})\mathbf{H}^H \tag{3.13}$$
$$= \mathbf{H}\Delta\mathbf{I}_{iN}\mathbf{H}^H$$
$$= \mathbf{H}(i)\mathbf{H}(i)^H \tag{3.14}$$

It is readily seen that matrix $\Delta\mathbf{D}_i$ has rank N.

3.3.2 Method B

Notice that

$$\mathbf{R}_x(i+1) = \mathbf{H}\mathbf{J}^{(i+1)N}\mathbf{H}^H$$
$$= \mathbf{R}_x(1)\mathbf{R}_x(0)^{\#}\mathbf{R}_x(i). \tag{3.15}$$

Similarly, $\mathbf{R}_x(i+1) = \mathbf{R}_x(i)\mathbf{R}_x(0)^{\#}\mathbf{R}_x(1)$. From this relationship, we get another equivalent formula for $\Delta\mathbf{D}_i$ as follows

$$\Delta\mathbf{D}_i = \mathbf{R}_x(i)\mathbf{R}_x(0)^{\#}[\mathbf{R}_x(0) - \mathbf{R}_x(1)\mathbf{R}_x(0)^{\#}\mathbf{R}_x(1)^H]$$
$$[\mathbf{R}_x(i)\mathbf{R}_x(0)^{\#}]^H, \tag{3.16}$$

4 BLIND ZERO-FORCING EQUALIZATION

Recall from (2.5) that $\mathbf{x}[k] = \mathbf{H}\mathbf{s}[k] + \mathbf{w}[k]$. Denote the zero-forcing equalizer as $\mathbf{G} = [\mathbf{g}_1 \quad \mathbf{g}_2 \quad \cdots \quad \mathbf{g}_N]$ which satisfies

$$\begin{aligned}\mathbf{G}^H\mathbf{x}[k] &= \mathbf{G}^H\mathbf{H}\mathbf{s}[k] + \mathbf{G}^H\mathbf{w}[k] \\ &= \mathbf{s}_{k-i} + \mathbf{G}^H\mathbf{w}[k]. \end{aligned} \tag{4.1}$$

(4.1) implies that the zero-forcing equalizer $\mathbf{G}$ should satisfy $\mathbf{G}^H\mathbf{H} = [\mathbf{0} \ \ \mathbf{0} \ \ \ldots \ \ \mathbf{0} \ \ \mathbf{I} \ \ \mathbf{0} \ \ \ldots \ \ \mathbf{0}]$, where $\mathbf{0}$ is an $N \times N$ all zero matrix and $\mathbf{I}$ is an $N \times N$ identity matrix and is on the i-th block column position. To find $\mathbf{G}$, it is equivalent to

$$\max_{\mathbf{G}} \mathbf{G}^H \Delta\mathbf{D}_i \mathbf{G} \quad \text{subject to} \quad \mathbf{G}^H\mathbf{R}_x(0)\mathbf{G} = \mathbf{I}. \tag{4.2}$$

The above maximization problem is the generalized eigen problem [9] where we need to solve for the N eigenvectors of $\Delta\mathbf{D}_i\mathbf{G} = \lambda\mathbf{R}_x(0)\mathbf{G}$ corresponding to the N largest generalized eigenvalues of λ.

Like other SOS based equalizers, the proposed zero-forcing equalizer generates a memoryless mixture of N user signals that need further separation. The actual zero-forcing equalizer $\widehat{\mathbf{G}}$ obtained from (4.2) is the true $\mathbf{G}$ with a matrix ambiguity, i.e.,

$$\widehat{\mathbf{G}} = \mathbf{G}\mathbf{Q}^H, \tag{4.3}$$

in which $\mathbf{Q} = [\mathbf{q}_1 \quad \mathbf{q}_2 \quad \cdots \quad \mathbf{q}_N]$ is an $N \times N$ unitary matrix. As a result, the output of the blind equalizer in the absence of channel noise is simply $\widehat{\mathbf{z}}_{k-i} = \widehat{\mathbf{G}}^H\mathbf{x}[k] = \mathbf{Q}\mathbf{G}^H\mathbf{x}[k] = \mathbf{Q}\mathbf{s}_{k-i}$. This memoryless mixing matrix is intrinsic to the multiple input system [8] and can not be resolved unless additional information is available.

4.1 Performance Analysis and Optimum Delay Selection

It should be noted that the zero-forcing equalizer is designed for mixture signal estimate $\mathbf{s}_{k-i}$ at a specific delay i. Thus, different delay can result in different MSE. For practical purposes, the minimum MSE criterion can be utilized to select the best delay i for the equalizers.

Let the estimated input symbol with delay i be $\widehat{\mathbf{s}}_{k-i} = \mathbf{G}^H\mathbf{x}[k]$. The MSE for the zero-forcing equalizer is

$$\begin{aligned} MSE_{zf}(i) &\triangleq \sum_{j=1}^{N} E\{|s_{k-i,j} - \widehat{s}_{k-i,j}|^2\} \\ &= \sum_{j=1}^{N} E\{|s_{k-i,j} - \mathbf{g}_j^H(\mathbf{H}\mathbf{s}[k] + \mathbf{w}[k])|^2\} \\ &= \sigma_w^2 \sum_{j=1}^{N} ||\mathbf{g}_j||^2, \end{aligned} \tag{4.4}$$

The optimum delay can be found by $\arg\min_i \sum_{j=1}^{N} ||\widehat{\mathbf{g}}_j||$.

5 PARTIAL KNOWLEDGE BASED MUI CANCELLATION

The detector obtained from (4.2) cancels ISI. For multiuser system, the MUI still exists. To cancel MUI, notice that in many communication systems, part of the composite signal channel, such as the pulse-shaping filter, or the CDMA spreading code is known to the receiver. We would like to cancel the MUI based on the known filter response $f(t)$ for signal separation. Without loss of generality, we assume the user of interest is user 1. Our task is to find $\mathbf{g}_1$ so that the ISI and MUI contained in $\mathbf{g}_1^H\mathbf{x}[k]$ is minimized.

Let $\mathbf{f}_1 \triangleq [f_{1,1} \quad f_{1,2} \quad \cdots \quad f_{1,n_2}]'$ be the discrete-time representation of $f(t)$. The sampled channel impulse response becomes $h_u[i] = \sum_{k=0}^{i} c_{1,i-k} f_{1,k}$, where $\mathbf{c}_1 \triangleq [c_{1,1} \quad c_{1,2} \quad \cdots \quad c_{1,n_1}]'$ is the unknown channel response. Now define $\mathbf{B}_1 \triangleq \mathcal{Z}\mathcal{F}$ where

$$\mathcal{Z} \triangleq \begin{bmatrix} \mathbf{0} & \ldots & \mathbf{0} & \mathbf{I}_{p\times p} \\ \mathbf{0} & \ldots & \mathbf{I}_{p\times p} & \mathbf{0} \\ \vdots & & \vdots & \vdots \\ \mathbf{I}_{p\times p} & \ldots & \mathbf{0} & \mathbf{0} \end{bmatrix}_{Mp\times Mp} \tag{5.1}$$

and

$$\mathcal{F} \triangleq \begin{bmatrix} f_{1,1} & 0 & \ldots & 0 \\ f_{1,2} & f_{1,1} & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ f_{1,n_2} & \ddots & \ddots & f_{1,1} \\ 0 & f_{1,n_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & f_{1,n_2} \end{bmatrix} \tag{5.2}$$

Notice that the Wiener-Hopf equation for the multiuser system is

$$\mathbf{R}_x(0)\mathbf{g}_1 = \mathbf{H}(i,1) = \mathbf{B}_1\mathbf{c}_1, \tag{5.3}$$

where $\mathbf{H}(i,j)$ denotes the j-th column in the i-th block column of $\mathbf{H}$. Based on this equation, the desired equalizer $\mathbf{g}_1$ can be determined by the following three ways.

5.1 Two-Step Method (TSM)

We've already got $\widehat{\mathbf{G}} = \mathbf{G}\mathbf{Q}^H$ in (4.3). The second step is to determine $\mathbf{q}_1$. Because $\mathbf{B}_1^{\perp}\mathbf{R}_x(0)\mathbf{g}_1 = \mathbf{0}$, where $\mathbf{B}_1^{\perp}$ denotes the null space of $\mathbf{B}_1$. Then $\mathbf{g}_1 = \mathbf{G}\mathbf{Q}^H\mathbf{q}_1 = \widehat{\mathbf{G}}\mathbf{q}_1$. $\mathbf{q}_1$ can be found by solving $\mathbf{B}_1^{\perp}\mathbf{R}_x(0)\widehat{\mathbf{G}}\mathbf{q}_1 = \mathbf{0}$.

5.2 Direct Estimation Method (DEM)

We can also estimate $\mathbf{c}_1$ directly from (4.2) and (5.3). From (5.3), we have $\mathbf{g}_1 = \mathbf{R}_x(0)^{-1}\mathbf{B}_1\mathbf{c}_1$. $\mathbf{c}_1$ can be found by

$$\max_{\mathbf{c}_1} \mathbf{c}_1^H\mathbf{B}_1^H\mathbf{R}_x(0)^{-1}\Delta\mathbf{D}_i\mathbf{R}_x(0)^{-1}\mathbf{B}_1\mathbf{c}_1 \quad \text{subject to} \quad \mathbf{c}_1^H\mathbf{B}_1^H\mathbf{R}_x(0)^{-1}\mathbf{B}_1\mathbf{c}_1 = 1$$

and therefore $\mathbf{g}_1$ can be determined.

5.3 Parameter Weighting Method (PWM)

From (4.2) and (5.3), we can find $\mathbf{g}_1$ by

$$\min_{\mathbf{g}_1} \mathbf{g}_1^H[\mathbf{R}_x(0) + \lambda\mathbf{R}_x(0)\mathbf{B}_1^{\perp H}\mathbf{B}_1^{\perp}\mathbf{R}_x(0)]\mathbf{g}_1 \quad \text{subject to} \quad \mathbf{g}_1^H\Delta\mathbf{D}_i\mathbf{g}_1 = 1,$$

where λ is an arbitrary weight that can be chosen by the user.

5.4 Conditions for Unique Solution

We have the following proposition on the conditions for unique solution.

Proposition 5.1 *Suppose that in addition to the condition that* $\mathbf{H}$ *has full column rank, the condition that matrix* $[\mathbf{B}_1 \quad \mathbf{H}(i,2) \quad \mathbf{H}(i,3) \quad \cdots \quad \mathbf{H}(i,N)]$ *has full column rank also holds, then* $\mathbf{g}_1$ *can be uniquely determined up to a multiplicative constant* α.

Proof is omitted due to page limit.

□

6 SIMULATION RESULTS

In the simulations, the input data signal is i.i.d., QPSK. The noise is zero mean, white and Gaussian. The BER is averaged over 1500 Monte Carlo runs. First we simulate an SIMO system. We consider a raised-cosine pulse $f(t)$ limited in $6T$ with roll-off factor 0.10 and a two-ray multipath channel $c(t) = \delta(t) - 0.7\delta(t - T/3)$. The over sampling factor is $p = 3$. We also implemented the Subspace Method (SSM) [4] for channel identification, and then apply the MMSE algorithm for channel equalization. As shown in Fig. 1, when the channel order is perfectly known, the proposed methods perform comparably well with the SSM. When the channel order is not exactly known as shown in Fig. 2, the performance of SSM degrades drastically, while the proposed methods are more robust to channel order estimation errors.

Next, we consider an MIMO system. We simulated the CDMA system with 4 users. The spreading codes are

$$\mathbf{F} = \left[\begin{array}{c} ++--++--++--++--++-- \\ ++++----++++----++++ \\ ++++++++--------++-- \\ +--+++----+++--+--++ \end{array} \right]$$

where each row of $\mathbf{F}$ corresponds to one user. The channels for all users are randomly generated as [1 randn(20)]. Equal energy for all users are assumed and λ is selected to be $\lambda = 200$ in PWM. The BER vs. SNR plots for three proposed user separation methods based on method A are shown in Fig. 3 together with the method proposed in [6], where MMSE equalization was applied on the identified channel.

7 CONCLUSIONS

We presented a family of new blind zero-forcing channel equalization algorithms based on the subspace estimation of a preselected channel block column. The algorithms are capable of generating robust equalization results without accurate channel order estimation. They can be applied to SIMO systems directly to remove the ISI. Their applications to MIMO systems are simple and direct. These algorithms are very useful in many communication systems.

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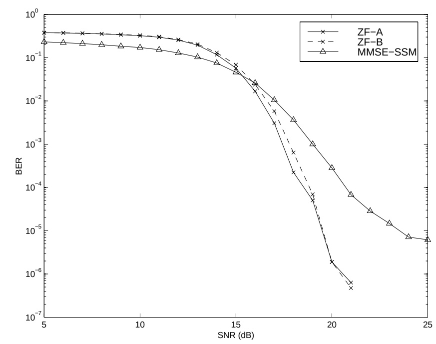

Figure 1. BER vs SNR at the outputs of the different equalizers for SIMO system. Data length is 350 symbols. True channel order assumed.

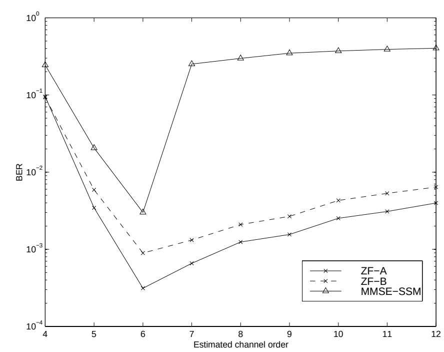

Figure 2. BER vs the estimated channel order. Data length is 350 symbols. SNR=18dB.

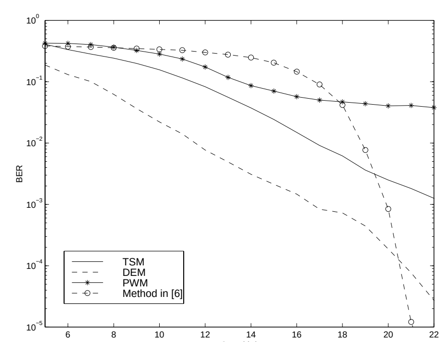

Figure 3. BER vs SNR at the output of different user separation methods for MIMO system. Data length is 350 symbols.