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Structural assessment under uncertain parameters via interval analysis

Oscar García[a,*], Josep Vehí[a], Jose Campos e Matos[b], António Abel Henriques[b], Joan Ramon Casas[c]

[a] *MiceLab - Modal Intervals and Control Engineering Laboratory, Department of Electronics, Informatic and Automation, University of Girona, Av. Lluís Santaló s/n, 17071 Girona, Spain*
[b] *Civil Engineering Department, Faculty of Engineering, University of Porto, Rua Dr. Roberto Frias s/n, 4200-465 Porto, Portugal*
[c] *School of Civil Engineering, Technical University of Catalonia, C/ Jordi Girona 1-3, Campus Norte, Edificio C1, 08034 Barcelona, Spain*

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Abstract

An efficient health monitoring system for damage detection in civil engineering structures using on-line monitoring data is being developed to identify any possible damage in short time. The present work is based on the treatment of uncertainties, which is one of the basic common difficulties faced when modelling structures. A methodology, based on interval analysis (IA) theory [R.E. Moore, Interval Analysis, Prentice-Hall, Englewood Cliffs, NJ, 1966] applied to a numerical constraint satisfaction problem (CSP) [J.R. Casas, J.C. Matos, J.A. Figueiras, J. Vehí, O. García, P. Herrero, Bridge monitoring and assessment under uncertainty via interval analysis, in: Ninth International Conference On Structural Safety And Reliability—ICOSSAR2005, 2005. pp. 487–494], is implemented in the damage detection [J.R. Casas, J.C. Matos, J.A. Figueiras, J. Vehí, O. García, P. Herrero, Bridge monitoring and assessment under uncertainty via interval analysis, in: Ninth International Conference On Structural Safety And Reliability—ICOSSAR2005, 2005. pp. 487–494] and modelling system of a long-term monitoring project in order to achieve such an objective. An algorithm is being developed for using such methodology with the obtained data.

Such methodology has been first checked in the laboratory with a simple reinforced concrete structure (loaded up to failure). The obtained results are useful for identifying the load where the structure presents changes in its behaviour. The majority of the structures present a linear elastic behaviour throughout their life. However, they tend to deteriorate; such degradation reflects on results obtained from the long term monitoring system. Structural assessment was successfully performed in this case, enabling its application to real structures.

MSC: 65G30

Keywords: Uncertainty; Interval analysis; Constraint satisfaction problem; Structural assessment

1. Introduction

Development of structural health monitoring systems (HMS) has been a subject of increasing activity in recent years. One of the main problems in the structural assessment is the treatment of uncertainty mainly present in numerical models,

* Corresponding author.
E-mail address: odgarcia@eia.udg.es (O. García).

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physical and geometrical parameters such as loading, Young modulus, inertia, and so on, and in measured variables such as displacements, strains and rotations. One of the main issues while considering the various sources of uncertainty is how to define objective and reliable criteria for distinguishing between an abnormal behaviour (differences between measured values and those predicted by the model) due to the presence of damages, and the differences between measured and calculated results because of the uncertainty and randomness present in the experimental data, models and physical parameters. When performing long term structural assessment, methodologies that take into account these uncertainties should be implemented in an efficient, fast and user friendly way. In this paper a methodology that considers uncertainty both in the model and in recorded data is presented.

2. Treatment of uncertainty

There exist different techniques for consideration of uncertainties in numerical models and measured variables. In the present paper we focus on the use of the interval analysis (IA). A brief summary of this subject is presented and their application to the management of uncertainties in structural assessment.

2.1. Interval analysis

The initial idea of IA [9,10] is to enclose real numbers in intervals and real vectors in boxes as a method of considering the imprecision of representing real numbers by finite digits in numerical computers. The variables are not deterministic, but take any value between the lower and the upper limits of an interval. The variables are represented by a uniform variation and the probability distribution function is not necessary. IA has become a fundamental nonlinear numerical tool for representing uncertainties or errors, proving properties of sets, solving sets of equations or inequalities and optimizing globally via interval arithmetic [5,7]. A classic interval number is a closed set that includes the possible range of an unknown number. Thus, instead of considering a fixed value a, the following representation is adopted:

$$A' = [\underline{a}, \overline{a}] := \{x \varepsilon \mathbb{R} | \underline{a} \leqslant x \leqslant \overline{a}\}, \tag{1}$$

where $\underline{a}$ is the infimum and $\overline{a}$ the supremum of the interval. The four elementary arithmetic operations $(+, -, \times, \div)$ are extended to intervals. If op denotes an arithmetic operation for real numbers, the corresponding interval arithmetic operation is

$$C = A \operatorname{op} B = \{a \operatorname{op} b | a \varepsilon A,\ \ b \varepsilon B\}. \tag{2}$$

2.1.1. Modal intervals

Modal interval analysis (MIA) is a natural extension of classical interval analysis, where the concept of interval is widened by the set of predicates that are fulfilled by the real numbers [4,12].

Physical intervals have two modalities for practical problems: there exists a value in $[a, b]'(a \leqslant b)$, which satisfies a predicate or some predicates concerned, and for all values in $[a, b]'(a \leqslant b)$, they satisfy a predicate or some predicates concerned. Classical interval analysis cannot distinguish these two types of physical intervals and denotes them as $[a, b]'(a \leqslant b)$ uniformly. MIA does distinguish these two types of physical intervals by denoting them differently, i.e., $[a, b](a \leqslant b)$ for those proper intervals that only require the existence of a value in the domain of $a \leqslant x \leqslant b$ to satisfy a predicate or some predicates concerned and $[b, a](a \leqslant b)$ for those improper intervals that require all the values in the domain of $a \leqslant x \leqslant b$ to satisfy a predicate or some predicates concerned, with $[a, b]$ and $[b, a]$ being denoted as modal intervals henceforth on. The concept of modal intervals is similar to the concept of objects in C + + since it contains not only a purely numerical interval, but also a physical modality of the numerical interval.

A modal interval X is defined as a couple $X = (X', \forall)$ or $X = (X', \exists)$, where X' is its classical interval domain and the quantifiers $\forall$ (universal) and $\exists$ (existential) are a modality selection. Modal intervals of type $X = (X', \exists)$ are defined as *proper intervals*, while intervals of type $X = (X', \forall)$ are designated by *improper intervals*. A modal interval can be represented using its canonical coordinates in the form

$$X = [a, b] = \begin{cases} ([a, b]', \exists) & \text{if } a \leqslant b, \\ ([b, a]', \forall) & \text{if } a \geqslant b. \end{cases} \tag{3}$$

For example, the interval [2,5] is equal to ([2, 5]′∃) and the interval [8,4] is equal to ([4, 8]′∀). For an interval X = [a,b], the operator Dual is defined by

$$\mathrm{Dual}([a, b]) = [b, a]. \tag{4}$$

A complete introduction to MIA and several examples can be found in [12]. Efficient tools to compute the range of modal interval functions as well as many other tools can be found in http://mice.udg.es/fstar.

2.1.2. Computational implementation

In this study, the uncertainty is included in the mathematical model of the structure by replacing the uncertain parameters by intervals. The uncertain parameter can take any value within the limits of the interval. The result of this process is an interval value, obtained once the structural equations have been analysed with theorems of MIA. In interval arithmetic, overestimation is one of the main drawbacks because the range of uncertainty is much larger than the range introduced by round off error. Overestimation is due to dependency and failure of some algebraic laws that are valid in real arithmetic. Such overestimation produces extreme and sometimes meaningless results. Interval implementation with finite element method presents a sharp bound on possible nodal displacement for treatment of uncertainty [3]. The system of interval equations can be written as

$$K \cdot q = P, \tag{5}$$

where K is the global interval matrix of the structure, p the applied interval load vector and q the unknown interval displacement vector.

MIA can also be implemented in a HMS in connection with a constraint satisfaction problem (CSP). In this case, the problem of damage detection in a structure can be stated as a numerical CSP for which its inconsistence has to be proved [1].

3. Assessment methodology

Deterioration is identified as a change into a system, which affects its current or future performance. However, the concept of damage is not meaningful without a comparison between the two different states of the system, one of them representing the undamaged one. Methods for damage identification, also identified as structural assessment techniques, can be classified as dynamic or static-based [2]. Static-based techniques only use static excitations, where the response (displacement or strain) is measured at one or more locations. Present methodology is based on the application of a static load. In this case, the behaviour of a structure when submitted to a rupture test is analysed.

3.1. Constraint satisfaction problem (CSP)

A numerical CSP, as defined in [11], is a triple CSP = $(\boldsymbol{x}, \boldsymbol{D}, \boldsymbol{\mathscr{C}}(\boldsymbol{x}))$ defined by

(i) a set of numeric variables $\boldsymbol{x} = \{x_1, \ldots, x_n\}$,
(ii) a set of domains $\boldsymbol{D} = \{D_1, \ldots, D_n\}$ where D_i, a set of numeric values, is the domain associated with the variable x_i.
(iii) a set of constraints $\boldsymbol{\mathscr{C}}(\boldsymbol{x}) = \{\mathscr{C}_1(\boldsymbol{x}), \ldots, \mathscr{C}_m(\boldsymbol{x})\}$ where a constraint $\mathscr{C}_i(\boldsymbol{x})$ is determined by any numeric relation (equation, inequality, inclusion, etc.) linking a set of variables under consideration.

A CSP is said to be inconsistent if one or more of its constraints are inconsistent, what is expressed by the following expression:

$$\text{CSP is inconsistent if } (\forall \boldsymbol{x} \in \boldsymbol{X})\{\neg(\mathscr{C}_1(\boldsymbol{x})) \vee \cdots \vee \neg(\mathscr{C}_m(\boldsymbol{x}))\}. \tag{6}$$

Let us consider the case when the constraints are in the form $\boldsymbol{\mathscr{C}}(\boldsymbol{x}) := f(\boldsymbol{x}) = 0$, with f a continuous function from $\mathbb{R}^n$ to $\mathbb{R}$. The logic formulation needed for proving the inconsistence of a constraint $\mathscr{C}_i(\boldsymbol{x})$ is as follows:

$$(\forall \boldsymbol{x} \in \boldsymbol{X})\neg(f_i(\boldsymbol{x}) = 0). \tag{7}$$

In order to prove this logic formulation, (IA) [9] is proposed. IA is a powerful mathematical tool which allows the evaluation of constraints over the reals by means of interval computations. Then, the evaluation of formula (7) is equivalent to proving the following interval exclusion: $0 \nsubseteq \mathrm{Out}(f^*(\boldsymbol{X}))$, where $\mathrm{Out}(f^*(\boldsymbol{X}))$ is an outer approximation of the range of the continuous function f.

Proving the inconsistence of a CSP have been reduced to proving the exclusion of zero from the range of a set of continuous functions. However, computing the range of a continuous function f by means of the rational extensions given by IA provokes an overestimation of the interval evaluation, due to the possible multi-occurrences of some variables and when the rational computations are not optimal. An algorithm based on results of MIA and branch-and-bound techniques which allows to efficiently compute an inner and an outer approximation of f^* has been built [6].

4. Application

In order to determine whether a structural system is under normal or abnormal behaviour under the presence of uncertain parameters, the structural assessment is made based on a numerical CSP, for this we must know all the parameters of the system. Thus, MIA is used to simulate the unknown variables by solving the static equilibrium Eq. (5) with deterministic and uncertain variables. Once all parameters are known we can define the CSP. Then we put all the known data into the algorithm based on MIA and branch-and-bound techniques to prove the inconsistence of the defined constraints. If there is an inconsistence we can determine the presence of an abnormal behaviour. To demonstrate the methodology we have studied a concrete beam by means of three methods, the first and second case are based on known variables and simulation of the unknown (i.e., rotations are simulated by solving Eq. (5)). The third case is based only on the known variables, this is possible applying analytical redundancy reduction.

4.1. Concrete beam

A reinforced concrete beam with a square section of 0.15 m (b)×0.15 m(h) and a total length of 2.10 m was tested in the laboratory up to failure (Fig. 1). The beam, clamped on both extremities, is loaded by two punctual forces F applied at third parts of the span (Fig. 2). Such forces are applied by an actuator. The error in the measurement of applied load can be modelled as a random variable with a mean equivalent to the load value and a standard deviation equal to 5% of it. Compressive tests were executed in some samples during the pouring of the beam to determine the concrete elasticity modulus. From the samples, a mean value of 32.6 GPa and a standard deviation of 1.94 GPa ($E_{\mathrm{cm}} = 32.6$ GPa; $\delta E_{\mathrm{c}} = 1.94$ GPa) are estimated.

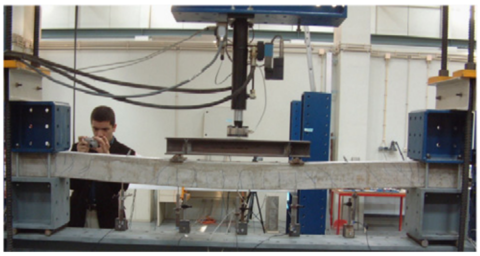

Fig. 1. Laboratory test of concrete beam.

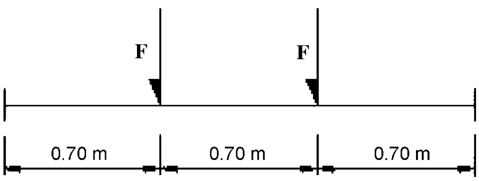

Fig. 2. Concrete beam model.

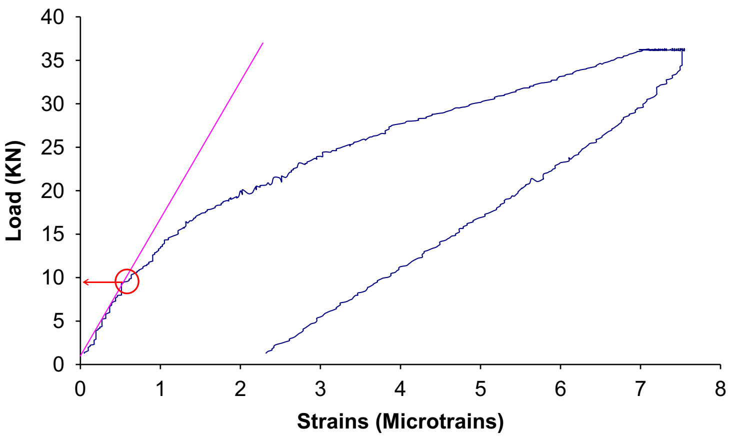

Fig. 3. Obtained load–strain diagram.

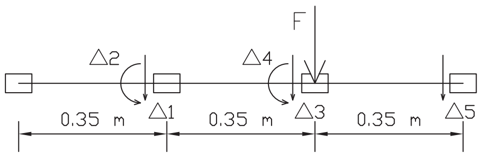

Fig. 4. Simplified numerical model (5 constraints).

A numerical model of this beam was considered in order to analyse its behaviour (see Figs. 4 and 5). The model is constituted of one-dimensional Euler–Bernoulli beam elements and two displacements at each node. In both ends the beam model is simply supported, the rotational displacements also being partially restrained due to the test set-up of the supports (Fig. 1). The uncertainty is considered in the elasticity modulus (E), applied load (F), the obtained values ($\triangle_{(i)}$) from displacement transducers attached to the studied structure (Linear variable displacement transducer, LVDT) and in the spring stiffness (k) (for Case 2). Uniform distributions were considered for all uncertain variables. Other variables are considered to be deterministic (length $L = 0.35$ m; inertia $I = 4.21875 \times 10^{-5}$ m^4). Fig. 3 represents the load–strain diagram obtained from this test. The force where a possible damage is detected is around 10 KN. In this case, strains are measured at mid-span section, bottom fibre, by an electric strain gauge (tension strains).

As we can see in Fig. 3, the behaviour of the structure is divided into two parts, elastic behaviour at beginning and nonlinear behaviour when a structural damage starts to appear. Application of IA in a CSP was used to determine the load where the structure changes from elastic behaviour to nonlinear behaviour and to perform the structural assessment. To formulate the CSP, simplified models that consider the existent symmetry and one-dimensional Euler–Bernoulli beam elements were used. From the established Eq. (5), the respective constraints were obtained. Those equations depend on uncertain variables such as the applied load (F), the Young modulus (E), and the obtained vertical displacements from LVDT ($\triangle_{(i)}$). Taking into account the uncertainty due to material properties, the inconsistence test is made with $EI = [1\,293\,468.75,\ 1\,457\,156.25]$ N m^2. The load (F) varies between 1271 and 26 279 N and is affected by the actuator uncertainty, which is 5% of the applied value. During CSP analysis rotations ($\triangle_{(j)}$) were considered to be deterministic. The used vertical displacements are the measured values affected by the sensor sensitivity. Two cases, as showed in Figs. 4 and 5, have been studied to develop the numerical model of the beam for this structural assessment.

4.1.1. Case 1

The simplified model has been developed taking into account the symmetry of the beam (Fig. 4). Values obtained from the sensors (vertical displacements) are represented by the variables $\triangle_i : \triangle_1$, $\triangle_3$ and $\triangle_5$. Rotations are assumed solving Eq. (5). F, E, L and I are simulated with the previously mentioned values. The following constraints are defined

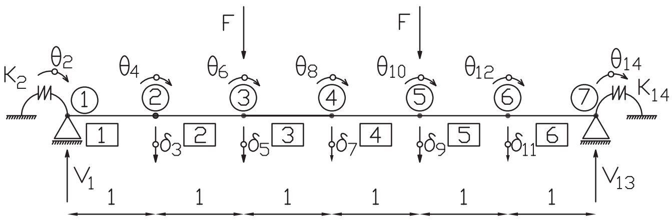

Fig. 5. Simplified numerical model (14 constraints).

Table 1
Results for structural assessment with five constraints

No.	$F(N)$	C_1	C_2	C_3	C_4	C_5
1	8439	True	False	True	True	True
2	9195	False	False	True	True	True

for this structural assessment:

$$C_1 : \frac{24EI}{L^3}\Delta_1 + \left(\frac{6EI}{L^2} - \frac{6EI}{L^2}\right)\Delta_2 - \frac{12EI}{L^3}\Delta_3 - \frac{6EI}{L^2}\Delta_4 \subseteq 0, \tag{8}$$

$$C_2 : \left(\frac{6EI}{L^2} - \frac{6EI}{L^2}\right)\Delta_1 + \frac{8EI}{L}\Delta_2 + \frac{6EI}{L^2}\Delta_3 + \frac{2EI}{L}\Delta_4 \subseteq 0, \tag{9}$$

$$C_3 : -\frac{12EI}{L^3}\Delta_1 + \frac{6EI}{L^2}\Delta_2 + \frac{24EI}{L^3}\Delta_3 + \left(\frac{6EI}{L^2} - \frac{6EI}{L^2}\right)\Delta_4 - \frac{12EI}{L^3}\Delta_5 - F \subseteq 0, \tag{10}$$

$$C_4 : -\frac{6EI}{L^2}\Delta_1 + \frac{2EI}{L}\Delta_2 + \left(\frac{6EI}{L^2} - \frac{6EI}{L^2}\right)\Delta_3 + \frac{8EI}{L}\Delta_4 + \frac{6EI}{L^2}\Delta_5 \subseteq 0, \tag{11}$$

$$C_5 : -\frac{12EI}{L^3}\Delta_3 + \frac{6EI}{L^2}\Delta_4 + \frac{12EI}{L^3}\Delta_5 \subseteq 0. \tag{12}$$

From the CSP analysis, the first inconsistence appeared in C_2 for the value of $F = 8439\,\text{N}$. It seems to be the load where the structural behaviour changes from elastic behaviour to nonlinear behaviour. Secondly, for $F = 9195\,\text{N}$ the constraint C_1 becomes inconsistent too, confirming the appearance of a possible damage on the support region (Table 1). CSP detects the appearance of this change near the experimental results. This is due to the fact that the used rotations, deterministic values, were obtained by a previous elastic analysis of the structure.

4.1.2. Case 2

To simulate the clamped-clamped boundary conditions, the simplified model has been developed taking into account a rotational spring in the model ($K = K_2 = K_{14} = 6000\,\text{KN m/rad}$; $\delta k = 60\,\text{KN m/rad}$). During the test, displacement transducers control the beam deflection ($\triangle_i$), namely δ_3, δ_5, δ_7, δ_9 and δ_{11} (Fig. 5). Rotations are assumed solving Eq. (5). F, E, L and I are simulated with the previously mentioned values. The following constraints are defined for this structural assessment:

$$C_1 : F + \frac{6EI}{L^2}\Delta_2 - \frac{12EI}{L^3}\Delta_3 + \frac{6EI}{L^2}\Delta_4 \subseteq 0, \tag{13}$$

$$C_2 : \left(\frac{4EI}{L} + K_2\right)\Delta_2 - \frac{6EI}{L}\Delta_3 \subseteq 0, \tag{14}$$

$$C_3: -\frac{6EI}{L^2}\Delta_2 + \left(\frac{12EI}{L^3} + \frac{12EI}{L^3}\right)\Delta_3 - \frac{12EI}{L^3}\Delta_5 + \frac{6EI}{L^2}\Delta_6 \subseteq 0, \quad (15)$$

$$C_4: \frac{2EI}{L}\Delta_2 + \left(\frac{4EI}{L} + \frac{4EI}{L}\right)\Delta_4 - \frac{6EI}{L^2}\Delta_5 + \frac{2EI}{L}\Delta_6 \subseteq 0, \quad (16)$$

$$C_5: -\frac{12EI}{L^3}\Delta_3 - \frac{6EI}{L^2}\Delta_4 + \left(\frac{12EI}{L^3} + \frac{12EI}{L^3}\right)\Delta_5 - \frac{12EI}{L^3}\Delta_7 + \frac{6EI}{L^2}\Delta 8 - F \subseteq 0, \quad (17)$$

$$C_6: \frac{6EI}{L^2}\Delta_3 + \frac{2EI}{L}\Delta_4 + \left(\frac{4EI}{L} + \frac{4EI}{L}\right)\Delta_6 - \frac{6EI}{L^2})\Delta_7 + \frac{2EI}{L}\Delta 8 \subseteq 0, \quad (18)$$

$$C_7: -\frac{12EI}{L^3}\Delta_5 - \frac{6EI}{L^2}\Delta_6 + \left(\frac{12EI}{L^3} + \frac{12EI}{L^3}\right)\Delta_7 - \frac{12EI}{L^3}\Delta_9 + \frac{6EI}{L^2}\Delta 10 \subseteq 0, \quad (19)$$

$$C_8: \frac{6EI}{L^2}\Delta_5 + \frac{2EI}{L}\Delta_6 + \left(\frac{4EI}{L} + \frac{4EI}{L}\right)\Delta_8 - \frac{6EI}{L^2}\Delta_9 + \frac{2EI}{L}\Delta 10 \subseteq 0, \quad (20)$$

$$C_9: -\frac{12EI}{L^3}\Delta_7 + \frac{6EI}{L^2}\Delta_8 + \left(\frac{12EI}{L^3} + \frac{12EI}{L^3}\right)\Delta_9 - \frac{12EI}{L^3}\Delta_{11} + \frac{6EI}{L^2}\Delta 12 - F \subseteq 0, \quad (21)$$

$$C_{10}: \frac{6EI}{L^2}\Delta_7 + \frac{2EI}{L}\Delta_8 + \left(\frac{4EI}{L} + \frac{4EI}{L}\right)\Delta_{10} - \frac{6EI}{L^2}\Delta_{11} + \frac{2EI}{L}\Delta 12 \subseteq 0, \quad (22)$$

$$C_{11}: -\frac{12EI}{L^3}\Delta_9 - \frac{6EI}{L^2}\Delta_{10} + \left(\frac{12EI}{L^3} + \frac{12EI}{L^3}\right)\Delta_{11} + \frac{6EI}{L^2}\Delta 14 \subseteq 0, \quad (23)$$

$$C_{12}: \frac{6EI}{L^2}\Delta_9 + \frac{2EI}{L}\Delta_{10} + \left(\frac{4EI}{L} + \frac{4EI}{L}\right)\Delta_{12} + \frac{2EI}{L}\Delta 14 \subseteq 0, \quad (24)$$

$$C_{13}: -\frac{12EI}{L^3}\Delta_{11} - \frac{6EI}{L^2}\Delta_{12} + F - \frac{6EI}{L^2}\Delta 14 \subseteq 0, \quad (25)$$

$$C_{14}: \frac{6EI}{L^2}\Delta_{11} + \frac{2EI}{L}\Delta_{12} + \left(\frac{4EI}{L} + K_{14}\right)\Delta_{12} + \frac{2EI}{L}\Delta 14 \subseteq 0. \quad (26)$$

For this numerical model first CSP analysis results were not expected because of the huge number of variables involved. Two events were considered, first with the original experimental results and then with a signal treatment because of the noise and delay present in them. Structural behaviour for Case 2 is improved after the signal treatment as seen in Table 2. Inconsistence appeared in C_{13} and C_{14} for the value of $F = 4085$ N. Simulation of a big number of constraints does not warranty a better solution.

4.1.3. Case 3: analytical redundancy reduction (ARR)

In order to minimize uncertainty we have made a variable reduction based on ARR. There exists analytical redundancy if there exists two or more different ways to determine a variable. A constraint that applies to only known variables and parameters constitutes an ARR and it can be evaluated from only observed variables in order to be used in damage detection. Based on the simplified numerical model of five constraints (Eqs. (8) to (12)) the following ARR's are defined:

$$\mathrm{ARR}_1: \left(\frac{24EI}{L^3}\right)\Delta_1 - \left(\frac{24EI}{L^3}\right)\Delta_3 + \left(\frac{12EI}{L^3}\right)\Delta_5 \subseteq 0, \quad (27)$$

$$\mathrm{ARR}_2: 3EI(12\Delta_1 - 15\Delta_3 + 8\Delta_5) + 2FL^3 \subseteq 0, \quad (28)$$

$$\mathrm{ARR}_3: \frac{6EI(15\Delta_1 - 12\Delta_3 + 5\Delta_5) + FL^3}{4L^3} \subseteq 0. \quad (29)$$

Table 2
Results for structural assessment with 14 constraints—signal treatment

No.	$F(N)$	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}
1	1271	T	T	T	T	T	T	T	T	T	T	T	T	T	T
2	4085	T	T	T	T	T	T	T	T	T	T	T	T	F	F
3	4184	T	T	T	T	T	T	T	T	T	T	F	T	F	F
4	4688	T	T	T	T	T	T	T	T	T	F	F	T	F	F
5	5794	T	T	T	T	T	T	T	F	T	F	F	T	F	F
6	7989	T	T	T	F	T	T	T	F	T	F	F	T	F	F
7	11 082	T	T	T	F	T	T	T	F	T	F	F	T	F	F
8	11 212	T	T	F	F	F	T	T	F	T	F	F	T	F	F
9	13 674	F	T	F	F	F	T	T	F	T	F	F	T	F	F
10	14 902	F	F	F	F	F	T	T	F	T	F	F	T	F	F
11	16 192	F	F	F	F	F	T	T	F	T	F	F	F	F	F
12	18 365	F	F	F	F	F	T	T	F	F	F	F	F	F	F
13	20 997	F	F	F	F	F	F	T	F	F	F	F	F	F	F
14	25 927	F	F	F	F	F	F	F	F	F	F	F	F	F	F

T = True; F = False.

Table 3
Results for structural assessment with 3 ARRs

No.	$F(N)$	ARR_1	ARR_2	ARR_3
1	10 603	True	True	False
2	11 501	False	True	False
3	10 603	False	False	False

Results (Table 3) based on in the measured values (Δ_1, Δ_2 and Δ_5) give quite an approximation of the experimental results.

5. Structural assessment of real structures

The final objective of developing the damage identification technique, dealing with uncertainties, is their application to real civil engineering structures. To this end, a first interesting application is the long term monitoring and assessment of the Sorraia River Bridge, which is part of a research project—SMARTE—developed in Portugal. This project involves one highway owner (BRISA), responsible for the maintenance of bridges and other infrastructures, and a civil and an electronic engineering research laboratory. Sorraia River Bridge (Fig. 6) is defined as a real prototype where the presented methodology will be tested for the first time. This bridge is a prestressed concrete bridge, with a total length of 270 m, constructed by the cantilever process. The bridge section is a box girder type. The section height varies from 2.55 m at middle span to 6.00 m over the piers.

Fig. 7 shows instrumentation plan of this bridge. The whole sensor network will have to be measured from two accessible locations (Local Stations) [8].

6. Conclusions and future developments

Nowadays bridge monitoring is efficiently done by different institutes and engineers. One of the problems detected in long term monitoring of any bridge is what to do with the obtained amount of data. It is always necessary to keep in mind that the final objective of monitoring is the assessment of bridges helping owners in their maintenance. In order to obtain it, a new methodology based on a combination of IA and constraint propagation techniques was studied and applied to simple examples. From IA it is possible to conclude that it is a methodology which considers the whole uncertainty, warranting that the real answer is within the range of output values. Besides, the connection between IA and (CSP) seems to be very useful in performing damage assessment of any structure given an uncertain model. In such

Fig. 6. Sorraia River Bridge.

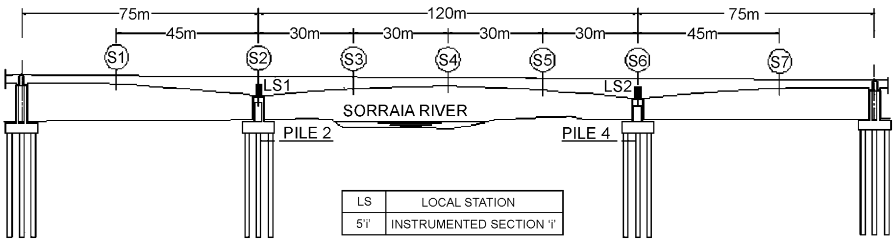

Fig. 7. Instrumentation plan of Sorraia River Bridge.

methodology, the consistency of each constraint of the system is analysed. A motivational example has been solved to show the viability of the method, however, some cases of damage are not found because of the tolerance given to the sensor and to the applied load. To improve it an analytical redundancy reduction is presented; this approach demonstrates a sharper solution based on known variables and parameters.

Based on the advantages and disadvantages of each case presented and taking into account the results obtained in the example, it was decided to use the ARR solution in the structural assessment of the Sorraia River Bridge. During the load test, support rigidity was identified as a critical parameter of the model and therefore is considered as one of the sources of uncertainty in the identification of future damages.

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