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Dynamics of effortful touch and interlimb coordination

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Abstract

In biological movement systems, the level of muscular-articular links is responsible for assembling highly reliable, stable, and reproducible coordination patterns involving very many joints and muscles. Research shows that the important perceptual capabilities of this level arise from the bulk sensitivity of muscles and tendons, so-called effortful or dynamic touch, to the quantities of rotational dynamics that remain invariant (such as the inertia tensor) over variations in rotational forces and motions. The power laws characterizing this sensitivity point to underlying fractal (self-similar) processes. Other research shows that the hallmark ability of this level to produce repetitive interlimb coordinations can be addressed through a dynamics of coordination in which equations express the time-evolution of collective neuromuscular states. This research also suggests that the assembled rhythms exploit the unique blend of stability and variability characteristic of low-dimensional chaotic motion on strange attractors. In overview, research into the capabilities of the level of muscular-articular links highlights the importance of applying classical and modern (nonlinear) dynamics to understanding the assembly and perceptual control of biological movements. © 1998 Elsevier Science Ltd. All rights reserved.

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1. Introduction

In many respects, biomechanics may be viewed as that branch of science that takes as its major focus the functional level of biological movement systems referred to by Bernstein (1996) as the *level of muscular-articular links or synergies*. Whereas other functional levels of the nervous system take as their starting points the intentional aspects of movements, or the perceived aspects of the environment in which the movements are situated, the level of muscular articular links ‘starts from the biomechanical aspects of the movements (Bernstein, 1996, p. 138)’. This functional level assumes the major responsibility for establishing spatio-temporal relations among body segments, maintaining them in the face of perturbations and reproducing them in varied circumstances. Behind the realization of this responsibility is information about the states of muscles and limb segments provided by the patterned activity of mechanoreceptors in response to the

stretching, compressing, twisting, shearing, and bending of the body’s tissues due to bodily movement and environmentally imposed contact forces. In Bernstein’s (1996) view, it is the thoroughness of this information that places the level of muscular-articular links in a unique position — only this functional level possesses the facility to control the large-scale movement patterns (e.g. running). Other functional levels, in which intentions and visual and auditory perceptions of the surroundings play leading roles, must rely on the level of muscular-articular links’ ability when many muscles are involved.

The present article provides a partial summary of contemporary investigations into two major achievements of the level of muscular-articular links. One major achievement is the nonvisible perceptions of the properties of handheld objects and of the directions of limbs and limb segments by so-called effortful or dynamic touch (Gibson, 1966). These perceptions are essential to the on-line tuning of muscular synergies during activities with implements and tools and, most generally, to the nonvisual control of postures and their transformations both with and without attachments to the body. The other major achievement of concern is the rhythmic

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movements that typify locomotion by fins, wings and limbs. The rhythms of single and multiple body segments are a primary expression of how the nervous system organizes movements in space and time, how it achieves highly precise and reproducible patterns, and how it resolves issues of efficiency of motion. Rhythmic movements of articulated bodies are to be included among biology’s earliest and best models of how to generate simple, large-scale behavior from a host of components that are rich in function and diverse in structure. Despite the great complexity of the parts and subparts that are involved, rhythmic movements seem to abide by simple, elegant rules. Presumably, these rules reflect, in part, the resolution of ‘Bernstein’s problem’ (e.g. Turvey, 1977, 1990) — roughly, how the many degrees of freedom involved in a coordinated activity are rendered controllable (Bernstein, 1935/1967, 1996).

As the research summarized below makes clear, understanding the bases for these perceptual and coordinational achievements of the level of muscular-articular links requires the full complement of modern dynamics – the principles and methods originating with Newton together with the principles and methods originating with Poincaré.

2. Haptic perception and peripheral neuropathy

The level of muscular-articular links is a collection of closely connected actuators (muscles) and meters (mechanoreceptors). The collective metering or measuring defines the haptic perceptual system — the system by which one knows the body, and objects adjacent to or attached to the body, by means of the body (Gibson, 1966). Although the effects of losing one’s sight or one’s hearing are easily imagined and simulated, the effects of losing haptic perception are not. Peripheral neuropathy is the absence of afferentation from the large myelinated fibers (originating from skin, muscle, and tendon receptors) with other neural functions essentially intact (e.g.

motor nerves are normal). One major documented case has no sense of the body and attachments to the body below the collarbone (Cole, 1995; Cole and Paillard, 1995). His purposeful movements rely on vision and require considerable concentration and intellectual effort. Incoordination accompanies a momentary loss of concentration or an ancillary intellectual effort (e.g. sneezing when standing, and note-taking while sitting, produce severe imbalance). The neuropathy has compromised the level of muscular-articular links (Bernstein, 1996). Because of the intimacy of haptic perception and movement that defines this level, it alone can ensure movements of many degrees of freedom, the enternal coherence and harmony of coordinations, and the persistence of coordinations against perturbations. For a person with severe peripheral neuropathy, standing, walking, reaching and manipulating are challenging if not impossible tasks and the ability to perceive by eye proves to be a poor substitute for the inability to perceive by muscle (e.g. Cole (1995), see also Simoneau et al. (1995)). But what precisely does a system based in the receptive function of muscle perceive? And what variables is it sensitive to? Given the interconnectedness of sensory and motor processes at the level of muscular-articular links, the answers to these perceptual questions must be tied to the fundamental feature of limb movements, namely, that they are rotational.

3. The inertia tensor and its role in perceiving spatial properties of handheld objects

Wielding an object entails time-variations in torque, angular velocity and angular acceleration together with variations, relative to gravity, in the planes of wielding motions. Experiments suggest that the basis for non-visual spatial perception connected with wielded handheld objects is the time-independent and gravity-independent inertia tensor I_{ij} (Fig. 1c). This tensor quantifies an object’s invariant resistance to variable

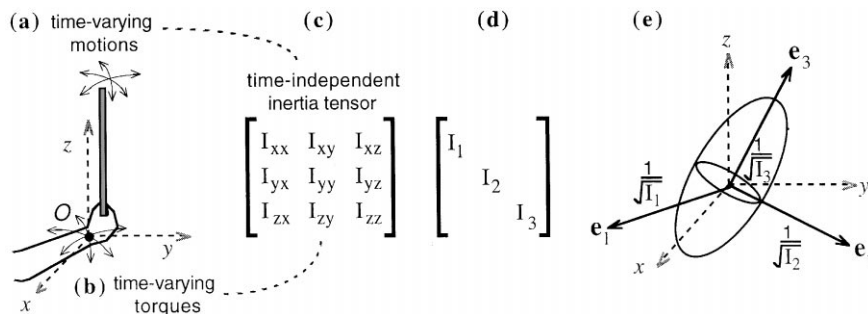


Fig. 1. When an object is gripped tightly and wielded, the angular accelerations (a) and torques (b) that vary in the course of the three-dimensional wielding activity are coupled through the time-invariant inertia tensor (c) at a point O in the wrist. The diagonalized form of the tensor (d) consists of the major (I_1), minor (I_3) and intermediate (I_2) resistances to rotational acceleration about the directions of the eigenvectors (e_1, e_2, e_3). The eigenvectors define the axes of the ellipsoid of inertia, with semilengths given by the inverses of the square roots of the corresponding eigenvalues. Experiments show that the nonvisible perceptions of a handheld object’s properties are tied to the inertia ellipsoid.

three-dimensional rotational accelerations about a fixed point O (Fig. 1a and b). When I_{ij} is rendered in diagonal form (Fig. 1d), the three components are the principal moments or eigenvalues I_k (where $k = 1, 2, 3$, identifying the major, intermediate, and minor eigenvalues, respectively). Diagonalization refers the rotational inertia relative to O to the principal axes or eigenvectors e_k through O about which the off-diagonal terms disappear (e.g. Goldstein, 1980). One obvious significance of e_k is the elimination of a ‘decision’ at the level of muscular-articular links about which coordinate system to use. The eigenvectors e_k constitute the only nonarbitrary coordinate system at O , one that is physically determined by the object itself. Any handheld object possesses an intrinsic reference frame (denoted by brackets), $\{e_k\}$.

It is useful to express the inertia tensor geometrically by the *ellipsoid of inertia* with axes e_k (Fig. 1e) (e.g. Goldstein, 1980). Research investigating effortful touch’s dependency on the inertial ellipsoid has involved the wielding of occluded handheld objects in simple variants of the magnitude estimation paradigm developed by Stevens (1961, 1962), as shown in Fig. 2a. The upshot of this research (summarized in Turvey (1996) and Turvey and Carello (1995)) is that the perception of spatial properties such as length, width, and shape, are functions of the magnitudes of e_k , namely the eigenvalues I_k , and the perception of object-to-hand relations (e.g. ‘how is the object oriented in the hand?’) and hand-to-object relations (e.g. ‘where is the grasp relative to the object?’) are functions of the directions of e_k .

Because of its basis in an invariant mechanical quantity, the scaling of perceived spatial magnitudes and directions to actual spatial magnitudes and directions tends to

be neither absolute (meaning a perfect match) nor relative (meaning that the perceived magnitudes are properly ordered but arbitrary) (Gogel, 1977). The scaling is, nonetheless, *definite* (Bingham, 1993) meaning that the perceived magnitudes tend to be both properly ordered and within a marginal tolerance of the actual magnitudes. For example, the specificity of length perception to the magnitudes of e_k takes the form of a power-law with an approximately 1/3 scaling on the major eigenvalue I_1 . This evidence for power-law behavior has important implications, most notably, that perception by effortful touch is a self-similar or fractal process (e.g. Peitgen et al., 1992; Schroeder, 1991; West and Deering, 1997). That is, it depends in the same way on I_1 at all object sizes or scales (e.g. consider very small objects with little resistance to being rotationally accelerated, such as sewing needles, and very large objects with great resistance, such as mallets), within largest and smallest scale sizes that have yet to be identified (Turvey, 1996; Turvey et al. 1998a).

Two additional observations are worth noting. The first is that a point O in the wrist proves to be the relevant fixed point for mechanical analyses. Although wielding an object held firmly in the hand can involve any of the arm’s joints, singly or in combination, the wrist joint is always involved and only I_{ij} defined about this joint is time independent. A handheld object in free wielding continually changes its distance from the elbow and shoulder but not from the wrist. Experiments have shown that it is I_{ij} defined about the wrist that constrains perception by manual effortful or dynamic touch (Pagano et al., 1993). The second notable observation, depicted in Fig. 3, is that a nonvisible property of significance to controlling successful striking of a ball with a bat, racket or club, namely, the center of percussion or ‘sweet spot’ defined by the ratio of the second to first moments, is perceptible in the act of wielding. Contact and impulse dynamics are not needed for the perception (Carello et al., submitted).

4. Heaviness and dynamical symmetry

The significance of the inertia tensor to effortful or dynamic touch has warranted the development of methods by which the directions and magnitudes of e_k can be manipulated directly. Objects of the kind shown in Fig. 2b, referred to as *tensor objects*, allow strict manipulation of the inertial structure, I_{ij} . Specifically, they permit the conduct of experiments designed to elicit, in Gibson’s (1979, pp. 153, 154) terms, ‘a synthetic perception’ by ‘synthesizing the information’. Investigations of the haptic size-weight illusion provide an exemplary case (Amazeen and Turvey, 1996). In the size-weight illusion, bigger versions of an object of fixed mass are perceived as lighter (Fig. 4a). The set of objects that typically give rise

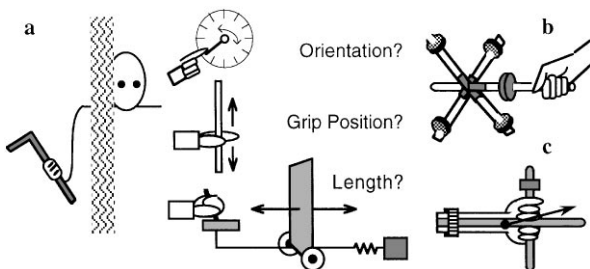


Fig. 2. The basic method for investigating effortful or dynamic touch involves a participant wielding a handheld object out of sight and adjusting a visible report apparatus to indicate the perceived object property. In panel (a), an L-shaped rod is wielded and the participant adjusts a pointer on an unmarked dial to indicate the felt orientation of the rod in the hand (‘Where is the crossbar pointing relative to the thumb?’), or places the left hand on a visible vertical rod to indicate the felt location of the right hand on the wielded L-shaped rod, or adjusts the distance of a visible board to match felt length of the L-shaped rod. Panel (b) shows a so-called tensor object, an arrangement of rods with attached metal rings that can be manipulated in either location or magnitude or both to produce different inertia tensors about the rotation point in the wrist. Panel (c) shows how splints can be attached to the forearm so as to change the eigendirections of the forearm’s inertia tensor.

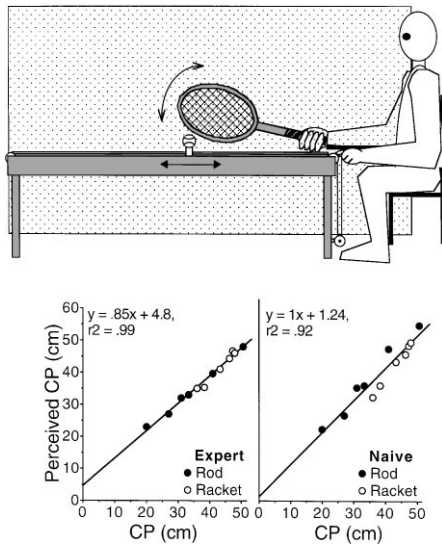


Fig. 3. The two lower panels summarize the results of an experiment in which varsity tennis players (*expert participants*) and randomly selected undergraduates (*naive participants*) attempted to perceive, on the basis of wielding, the location of the center of percussion (CP) or sweet spot of a tennis racket or a wooden rod. The rackets were of six different sizes and the rods were of three different sizes with metal rings attached either close to or far from the hand. The upper panel shows the experimental arrangement for judging the sweet spot. The participant wielded the occluded racket (or rod) and adjusted the position of the visible ball until it coincided with that location on the racket (or rod) at which the participant felt that he would want to make contact with the ball. As the results indicate, the mean perceptions of both the expert and naive participants closely matched the actual CPs [adapted from Carello et al., submitted].

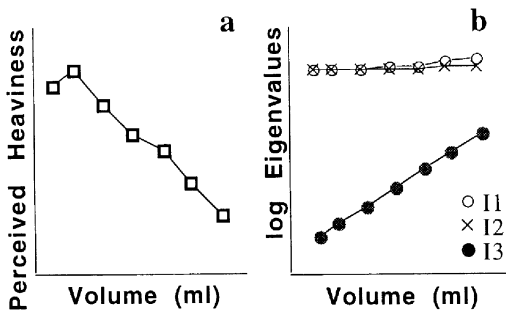


Fig. 4. Panel a: The size-weight illusion is evidenced by a decreasing impression of a handheld object’s heaviness as the volume of the object increases with mass held constant. Panel b: The set of objects that produce the illusion are characterized by a particular change in the inertia tensor about the rotation point. The major and intermediate eigenvalues remain fairly constant across the set of volume-increasing objects of constant mass but the minor eigenvalue becomes increasingly larger and more similar to the major and intermediate eigenvalues (and the ellipsoid of inertia, therefore, increasingly more spherical). Tensor objects of Fig. 2b constructed to conform to the changing eigenvalue pattern of panel b yield a decreasing impression of heaviness (see text for details).

to the size-weight illusion (e.g. Charpentier, 1891; Stevens and Ruben, 1970) prove to be characterized by a particular pattern of I_k . As the objects in the set become larger with no change in mass, I_3 increases and I_1 and I_2 (where

$I_1 \approx I_2$) are relatively constant (Fig. 4b). In contrast, the set of objects showing heaviness perception increasing with weight (namely, objects that increase in mass) are characterized by an increase in all three eigenvalues. These inertial patterns were simulated across tensor objects that were otherwise identical in linear dimensions and mass. Experiments on the wielding of such objects showed (a) a decrease in perceived heaviness when I_3 increased relative to constant I_1 and I_2 (the classic size-weight illusion) and (b) an increase in perceived heaviness when all eigenvalues increased (a new size-weight illusion), with overall perceived heaviness a function of I_1 to a positive power times I_3 to a negative power.

There are some valuable lessons to be learned from the reviewed experiments with tensor objects of constant mass. The experiments have shown that (a) a decrease in perceived heaviness when I_3 increased relative to constant I_1 and I_2 and (b) an increase in perceived heaviness when all eigenvalues increased, with overall perceived heaviness a function of I_1 to a positive power times I_3 to a negative power. Outcome (a) suggests that for objects of the same mass, as their corresponding inertia ellipsoids approximate a sphere ($I_1 = I_2 = I_3$), they feel lighter. That is, an object that is dynamically centrosymmetric (Hestenes, 1986) about the rotation point will feel lighter than an object of the same mass that is dynamically asymmetrical ($I_1 > I_2 > I_3$) or possesses dynamical axial symmetry (e.g. $I_1 = I_2 > I_3$). Said differently, an object that offers (to muscular exertion) equal resistances to rotational acceleration in all directions feels lighter than an object of the same mass that offers unequal resistances. Outcome (b) then suggests that, when the resistances are disparate, the larger the values of the resistances for fixed object mass, the heavier is one’s perception of the object. In sum, the research of Amazeen and Turvey (1996) moves the ‘natural kind’ for heaviness (a property or kind that could support a generalization of facts about heaviness) in the direction of dynamical symmetry taken in relation to the biological movement system that patterns the forces applied to handheld objects.

5. Perceiving limb directions

The lessons learned from wielding attachments to the body (handheld objects) apply to the body. Moving an arm about the shoulder can be equated with wielding a cylindrical object about a fixed rotation point. Given that I_{ij} is defined at a point of rotation, each joint defines the position of a tensor and, therefore, inertia ellipsoid. Accordingly, the nested body segments can be interpreted as a nesting of inertia ellipsoids. The body is a tensor field. As such, stored knowledge of the body’s dimensions may not be the basis for the control of movements as often argued (e.g. Gurfinkel and Levik, 1991);

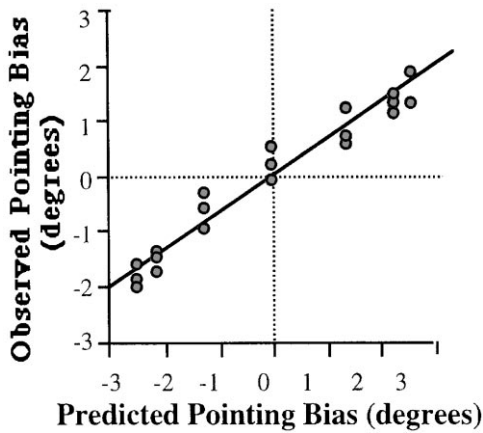


Fig. 5. The right arm's $\{e_k\}$ is rotated by a few degrees relative to the arm's spatial axes by means of an attached splint. If a person then tries to point at visible targets with the right arm occluded, their pointing is systematically biased by the rotation of $\{e_k\}$. It seems that one points with the arm's inertia ellipsoid rather than the arm's longitudinal axis.

the lengths and directions of limb segments could be specified on-line by the segmental inertia tensors. As shown in Fig. 5, if the normal relation between an arm's spatial axes and the arm's $\{e_k\}$ is broken by means of an attached splint (e.g. Fig. 2c), the positioning of the occluded arm is systematically altered relative to visual targets (Pagano and Turvey, 1995). Participants point with the arm's inertia ellipsoid rather than the arm's longitudinal axis. When the position of one forearm is matched nonvisually with that of the other, under conditions in which splints have rotated $\{e_k\}$ relative to the limb segment's spatial axes, the matching is in terms of the respective forearm eigenvectors rather than elbow angles (Pagano et al., 1996). Similarly, when subjects are required to orient the forearm parallel to, or at 45° to, the environmental horizontal, they produce limb orientations that are systematically deflected from the forearm's longitudinal spatial axis in the direction of the forearm's $\{e_k\}$ manipulated through attached splints (Garret et al., in press). Among other things, the preceding experiments reinforce the impression that, despite the prominence of joint angle in theoretical discussions of perceiving limb position and controlling limb trajectories (e.g. Hollerbach, 1990), its status as the relevant perceptual and control variable is questionable (Soechting and Ross, 1984; Worringham et al., 1987). In perceiving and controlling limb directions at the level of muscular-articular links, the configuration of an arm is not in terms of joint angles but of relative directions of segmental inertial ellipsoids or $\{e_k\}$ s.

6. A motion equation in a collective variable

Let us now turn to the coordination capabilities of the level of muscular-articular links. At issue here is the kind

of dynamics that will successfully express these capabilities. There is growing awareness that a dynamics of coordination, understood as the time-evolution of states of the central nervous system, is both required and feasible (Kelso, 1995; Saltzman and Munhall, 1992; Schöner, 1994; Turvey, 1990). Consider riding a bicycle, in particular the basic challenge of keeping the frame of the bicycle and oneself upright (Mitra et al., 1998). A bicycle has five main moving parts (handlebars, front wheel, crank-chain-rear wheel assembly, two pedals) resulting in 10 degrees of freedom (each part requires for its description one position and one velocity coordinate). In order to ride a bicycle it has been suggested that 'you must gain intuition about the motion of a point in 10-space (Stewart, 1989, p. 91)'. The bicycle's behavior can be represented by the time variations of a single (10 dimensional) point that defines all of the bicycle's 10 variables simultaneously. At one level of description, the would-be-bicycle rider is confronted by all of the force and motion complexity of the bicycle and its parts (torques on handlebars, pumping motions of the pedals, and so on). At another level, the rider is confronted, perhaps, by one point. At this alternative level, the question becomes: Is there an equation of motion for this one point, an equation that expresses all possible motions of the point? If there is, then it would be the 'intuition' that the would-be-bicycle rider must 'gain'. The requisite 'intuition' is both simplified and made more profound by (a) the possibility that the behavior of the 10 variables is fully accommodated by the behavior of one variable (or, at most, a few variables) composed in some way from the initial 10 but differing markedly from them in the relative slowness of its changes and (b) the likely presence of stabilities, that is, preferred locations and paths to which the behavior is drawn and to which it tends to return if momentarily displaced. Feature (a) expresses the idea of a collective variable or order parameter (Haken, 1977, 1983, 1996) that captures the relative variations of the bicycle's 10 degrees of freedom and feature (b) expresses — in the spirit of Poincaré's strategy for addressing complexity — the geometry or essential qualitative aspects of the bicycle's dynamics. Bicycle riding, therefore, can be expressed as a motion equation expressing the time-evolution of a collective variable. The solutions of this equation define the stable and unstable organizational or coordinational states, that is, the attractors and repellers of the task.

The above notions find expression in the analysis of rhythmically synchronizing two body segments. As developed by Kelso and colleagues (see summaries in Kelso, 1995), 1:1 rhythmic coordination can be modeled by a motion equation in the collective variable of relative phase ϕ — the difference in the phase angles of the two limbs. Thus,

$$\dot{\phi} = \Delta\omega - a \sin \phi - 2b \sin 2\phi + \sqrt{Q}\zeta_t \quad (1)$$

The overdot signifies the first time derivative, $\Delta\omega$ is a detuning term that can be equated with the difference ($\omega_2 - \omega_1$) between the uncoupled (preferred) frequencies of the limbs, and a and b are coefficients that determine the relative strengths of attractors for the coordination at or in the vicinity of the values $\phi = 0$ and $\phi = \pi$. The faster interactions among the internal subsystems are identified with ζ_t , a Gaussian white noise process of strength $Q > 0$. The predictive successes of Eq. (1) (summarized in Amazeen et al., in press; Kelso, 1994, 1995) indicate the intimate connection between motor timing and stability and the significance of symmetry breaking ($\Delta\omega \neq 0$) (see Fig. 5). Among the nonobvious and experimentally confirmed predictions are: (a) the invariance of mean relative phase over variations in movement speed when $\Delta\omega = 0$ but not when $\Delta\omega \neq 0$, (b) increased variability in relative phase with increased movement speed when $\Delta\omega = 0$ despite constancy of mean relative phase, (c) greater change in mean relative phase per change in $\Delta\omega$ for intended antiphase than intended inphase coordination, and (d) a discontinuous transition from antiphase to inphase at high movement rates (a pitchfork bifurcation) when $\Delta\omega = 0$ but, possibly, a gradual transition (a saddle-node bifurcation) when $\Delta\omega \neq 0$ (Amazeen et al., 1996; Kelso, 1984, 1995; Mitra et al., 1997a; Schmidt et al., 1993; Serrien and Swinnen, 1997; Sternad et al., 1992, 1996). In these experiments, movement rate was manipulated by a metronome and $\Delta\omega$ was manipulated through pendulums, one held in the right hand and one held in the left hand, functioning as virtual limbs that could differ in length and, therefore, uncoupled frequency.

A potentially important lesson from the research conducted on Eq. (1) is that for symmetrical limbs ($\Delta\omega = 0$), increasing speed weakens the attractors at relative phases of 0 and π whereas for asymmetrical limbs ($\Delta\omega \neq 0$), increasing speed both weakens and shifts the attractors at relative phases of 0 and π (see Fig. 6). A common assumption in theories of motor control is the factorization of energy and pattern, or power and timing, in coordinated movements (e.g. Bullock and Grossberg, 1988; Schmidt, 1988). Thus, an assumption of the popular idea of generalized motor programs is that faster and slower (more and less powerful, more and less energetic) variants of a given skill are executed with the same relative timing (or pattern) of muscle activations. For interlimb rhythmic coordination, this assumption appears to hold only when the coordination dynamics are symmetric. It does not hold, apparently, for the more general asymmetric case when the component limbs participating in the coordination contribute unequally to its temporal evolution and fixed point properties.

7. Movement chaos

A reproducible, stable coordination pattern implies an attractor — a geometric object of dimension d_A to which

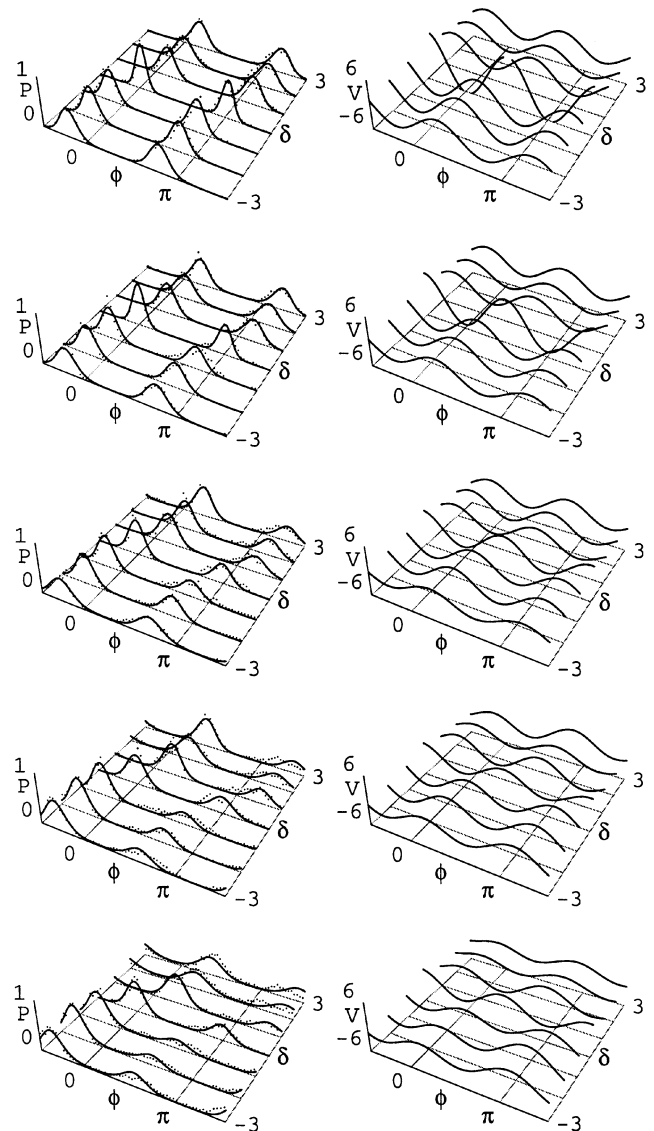


Fig. 6. Each panel in the left column shows the changing probability mass distributions (P) of relative phase ϕ in an interlimb rhythmic coordination task as a function of detuning δ (corresponding to $\Delta\omega$ in Eq. (1)). On a trial in the experiment, the participant oscillates simultaneously a pendulum held in the left hand and a pendulum held in the right hand, with the oscillations occurring about the wrists in planes parallel to the body's coronal plane. The required frequency ratio is always 1:1 and the required phase relation is either inphase ($\phi = 0$) or antiphase ($\phi = \pi$) depending on the trial. The detuning is achieved by manipulating the uncoupled frequencies of the handheld pendulums; $\delta = 0$ means they are identical and $\delta > 1$ means that the left is of higher uncoupled frequency than the right. The change in $P(\phi)$ from the top panel in the left column to the bottom panel in the left column is due to the speed or coupled frequency of the interlimb rhythmic movement interacting with δ and with required phase as expected from Eq. (1). The dotted curves are estimates of $P(\phi)$ from the experimental data; the solid curves are the best fitting nonlinear regressions. The panels in the right column display the corresponding potential functions. Given that Eq. (1) is expressible as the negative derivative of a potential function V (Haken et al. 1985), the changing parameters of V can be estimated from $P(\phi)$ by means of the Fokker-Planck equation (e.g. Collins and Turvey, 1997). See text for summary of the data patterns. [Figure is reproduced with permission from Park and Collins, (1997).]

the (longer term) motions composing the pattern are confined. Importantly, because all variables are generically connected in a nonlinear process, measurement of a single scalar, such as the amplitude $\theta(t)$ of a rhythmically moving hand, can suffice to reconstruct the vector space of the attractor (e.g. Abarbanel, 1996; Eckmann and Ruelle, 1985). It is generally assumed that the attractor for an individual oscillating limb is a limit cycle, a closed orbit in the phase plane of position by velocity. Fig. 7 (upper panels) shows these orbits to be thick bands raising the contrasting possibilities that (a) the limit cycles are noisy or (b) motions from higher-dimensional phase spaces are being projected (inappropriately) onto the phase plane. Recent research favors possibility (b) with the implication that the level of muscular-articular links is disposed toward the chaotic dynamics of strange attractors (Mitra et al., 1997b). The advantage of such dynamics for biological rhythmic movement is their special blend of stability and variability. The movement trajectories are bound to a region of the phase space but never repeat themselves within this bounded region. Consequently, one can think of a rhythmically moving limb as exploring fully its possible dynamics and garnering, thereby, haptic information about how to respond adaptively to changing conditions.

The essentials of the method for arriving at the preceding conclusions can be summarized briefly (see Abar-

banel (1996) and Mitra et al. (1998) for details). Phase space reconstruction makes time-delayed copies of the observed variable to provide independent coordinates: $\theta(t)$ is embedded in a space of ever more dimensions until points are neighbors for dynamical (deterministic) reasons and not because of too few dimensions. The determination of global false nearest neighbors (FNN) in an embedding space involves taking sets of points nearest to every data point from a series $y(t)$ embedded in d -space, calculating the Euclidean distances, and assessing whether these distances change substantially when the same calculations are made on $y(t)$ embedded in $d + 1$ -space. If distance changes exceed an adopted threshold, then embedding in $d + 1$ -space is taken to have removed FNN that remained in d -space embedding. The number of dimensions beyond which FNN no longer reduces is the embedding dimension d_E in which the attractor is fully unfolded. Determining the number of active degrees of freedom (DF) or local dimension d_L , begins in a working (reconstructed) space of dimension $d_w \geq d_E$. Then, for any point in d_w , a sub-space of dimension $d_L \leq d_E$ is found in which accurate local neighborhood-to-neighborhood maps of the data can be constructed.

Neighborhoods of several sizes are specified by taking sets of n neighbors of a given point $y(t)$, and then a local rule is abstracted for how these points evolve in one time-step into the same n points in the neighborhood of

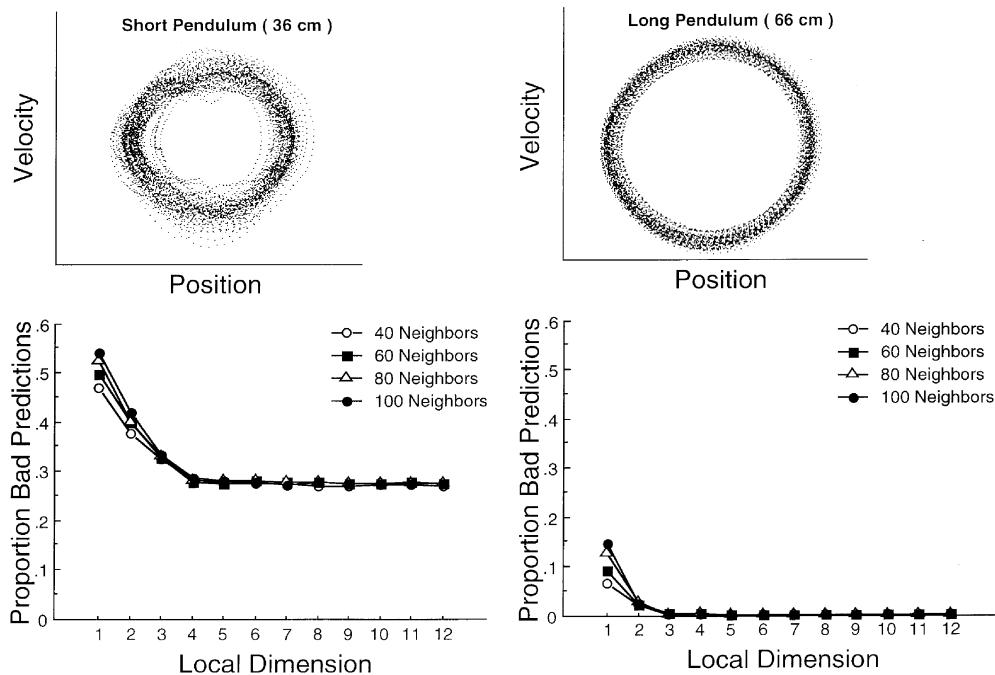


Fig. 7. The left and right top panels show, respectively, the phase plane behaviors of a small virtual limb (a short and light handheld pendulum oscillated by hand motions about the wrist) and a large virtual limb (a long and heavy handheld pendulum oscillated in the same manner). The corresponding lower panels show the %Bad Predictions as a function of local dimension. Whereas three active DFs were required to accommodate the large virtual limb, four active DFs were needed for the short virtual limb. The near 30% residual bad predictions in the case of the short virtual limb (relative to the near 0% in the case of the large virtual limb) is indicative of a substantial amount of local (short time-scale) noise. See text for details. [Figure is adapted from Mitra et al. (1997a).]

$y(t + 1)$. The rule's success is measured as percent bad predictions (%bad). One seeks a value of d_L at which %bad becomes independent of d_L and of the number of neighbors n . This value is the number of active DFs determining the evolution of the dynamics on the attractor. The d_L analysis also provides indications of local, low-amplitude (and/or fast time-scale) noise that may not be detectable in the attractor-level d_E analysis. The d_E analysis fails to remove all FNNs only when the level of noise at the scale of the attractor is high. In the d_L analysis, however, the value of %bad at which it becomes independent of d_L and n can serve as an indicator of noise at finer space-time scales.

Application of the preceding method (Mitra et al., 1997b) yielded three active DFs for larger (virtual) limbs and four active DFs for smaller (virtual) limbs, respectively, as can be seen in Fig. 7 (lower panels). In both cases, related analysis revealed that the dynamics were dissipative (sums of the average local Lyapunovs were negative) but that distances between points in phase space grew exponentially in time (at least one of the Lyapunov exponents was positive), as would be expected of chaos (e.g. Peitgen et al., 1992). Further, the mean Lyapunov dimensions for the two cases were fractal, 2.25 and 3.49, respectively, indicative of strange attractors (e.g. Kaplan and Yorke, 1979).

Beyond their significance for understanding, the types of dynamical systems manifest in biological movements, the preceding analyses bear on a more traditional concern, namely, the nature of neural control variables. Although conventional biomechanical quantities (e.g. joint angle, movement amplitude, torque, stiffness, viscosity) have been traditionally promoted as the likely variables of motor control, there are doubts about their appropriateness for the task (e.g. Bullock and Grossberg, 1986; Feldman and Levin, 1995; Hasan, 1991; Latash, 1993; Loeb, 1987; Zatzorsky, 1997). Indeed, it has been suggested that a desideratum for a neural control variable is *independence* from conventional biomechanical quantities (Feldman, 1986; Feldman and Levin, 1995). In this regard, the active DFs of a rhythmic movement are not ordinary biomechanical quantities but the variables of nonlinear rules that govern a movement's time-evolution on a strange attractor. Although the phase-reconstruction method is moot on the physical or physiological identity of these active DFs, it is worth remarking that more neurally-oriented analyses have identified a minimum of three neural control variables for single-joint rhythmic movement — the r , c , and μ commands (Feldman, 1980). The r and c commands determine, respectively, joint position and joint compliance with the μ command adjusting the r and c commands (precisely, the muscle thresholds that they specify) to movement velocity. The closeness in number of active DFs and neural control variables raises the question for future investigations of whether there is a deeper similarity. At

the very least, it provides encouragement for examining rhythmic movement's neurobiology in the context of rhythmic movement's dynamics.

8. Conclusions

As research progresses along the lines sketched above, one can expect further and, ideally, deeper insights into the achievements of the level of muscular-articular links. There are already hints that quaternion- or spinor-valued quantities may be needed to accommodate the selective capabilities of effortful touch (Turvey et al., 1996; Turvey et al., 1998b), consistent with the hypothesis that limb movements and their perceptual guidance are interpretable in the spinor-based language of the geometric algebra (e.g. Hestenes, 1994a, b). A further example of what might be expected in the future is provided by the equivalence and transfer of movement patterns. These phenomena (of central and traditional concern to the theory of motor learning) may be understood as dynamical phenomena. It is well known that a coordination pattern acquired with one set of effectors (e.g. the fingers) can be transferred (within limits) to others (e.g. arms, feet) without further learning. The implication is that what is learned is abstract and effector independent (Keele et al., 1990). An hypothesis that the learning of movement patterns is solely at an abstract level may be too strong, however. It would exclude, for example, contributions to pattern fluency from the special properties of a given set of effectors (Jordan, 1995). A speculative dynamical perspective on this latter view follows from equating the notion of attractor with the hypothesized abstract level of learning and the notions of active DFs, local Lyapunovs and noise at very small time scales with effector specificity. Thus, the attractor of an acquired coordination may transfer to other effectors but the specifics of local evolution on the attractor may not. Consider a rhythmic coordination pattern acquired with effectors A and then produced with effectors B . If transfer is limited to the attractor, then the phase reconstruction procedure might reveal no difference across A and B in the measures d_E and asymptotic FNN but significant differences in active DFs (d_L), local Lyapunovs and %bad predictions.

The reviewed research and final speculations are based on the assumption that the careful study of coordination patterns and corresponding perceptions that are achieved fluently and reliably will lead to genuine advancement in understanding how movements are assembled and perceptually controlled. Such coordinations and perceptions, which typify the level of muscular-articular links, are fluent and reliable because of 'sound underlying method (Marr, 1981, p. 140)'. A reasonable conclusion from the reviewed research is that general principles of the kind identified in classical and

post-Poincaréan dynamics may well provide the key to understanding the ‘method’ of that functional level of the nervous system that begins, as Bernstein (1996) remarked, with biomechanics.

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