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The Capacity of Three-Dimensional Wireless Ad Hoc Networks

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***Abstract*—Network capacity investigation has been intensive in the past few years. A large body of work has appeared in the literature. However, so far most of the effort has been made on two-dimensional wireless networks only. With the great development of wireless technologies, wireless networks are envisioned to extend from two-dimensional space to three-dimensional space. In this paper, we investigate for the first time the throughput capacity of 3D regular ad hoc networks (RANETs) and of 3D heterogeneous ad hoc networks (HANETs), respectively, by employing a generalized physical model. In 3D RANETs, we assume that the nodes are regularly placed, while in 3D HANETs, we consider that the nodes are distributed according to a general Nonhomogeneous Poisson Process (NPP). We find both lower and upper bounds in both types of networks in a broad power propagation regime, i.e., when the path loss exponent is no less than 2.**

I. Introduction

Network capacity investigation has been intensive in the past few years. A big chunk of work exploring the capacity of wireless networks has appeared in the literature. When we ask ourselves why we should engage in this pursuit, two reasons should be obvious. First, network capacity is the asymptotic property of network performance. In face of the emerging large-scale networks of a large number of connected objects, asymptotic capacity is no longer a cliché and becomes even more critical. Second, network capacity predicts network performance in case of different paradigms as a function of the number of nodes in the network, regardless of detailed protocol design. In contrast, as an alternative way to evaluate the network performance, simulation or numerical results can only be obtained for a certain number of nodes and are hence deterministic. Besides, these results are only available after designing all the protocols considering every detail, and may also require a lot of computing resource and time for a large network. Therefore, capacity investigation is interesting and important in wireless networks. However, it is also a very challenging task.

Gupta and Kumar [11] initiate the study on the capacity of wireless networks and show that the per-node throughput capacity (with unit bits per second) is $\Theta(1/\sqrt{n\log n})$ in random ad hoc networks and the per-node transport capacity (with unit bit-meters per second) is $\Theta(1/\sqrt{n})$ in arbitrary ad hoc networks, where n is the number of nodes in the network. A large body of work (e.g., [1], [2], [4], [6]–[8], [16], [22]–[24]) continues to study the capacity of static ad hoc networks with different network settings, while a tremendous amount of effort (e.g., [3], [10], [13], [17], [20], [28]) is also made on the capacity of mobile ad hoc networks, showing that mobility can significantly improve network capacity. The bulk of work on the capacity of hybrid wireless networks, such as [15], [18], [19], [21], [29], [30], proposes to place base stations in wireless networks and finds that network capacity can also be boosted.

However, all the aforementioned work is conducted on two-dimensional networks only. With the great development of wireless technologies, wireless networks are envisioned to extend from two-dimensional space to three-dimensional space, connecting all kinds of objects such as computers, sensors, actuators, mobile phones, TVs, refrigerators, washers/driers, smart power meters, clothes, food, medicines, and cars. The future three-dimensional (3D) wireless networks will be a fusion of the digital world and the physical world and bring together everything from individuals to objects, from data to services, etc. Only a couple of papers like [12] tentatively study the capacity of 3D wireless networks. In particular, [12] explores the transport capacity in 3D arbitrary ad hoc networks and the throughput capacity in 3D random ad hoc networks, using both Protocol Model and Physical Model. In this paper, we investigate for the first time the throughput capacity of 3D regular ad hoc networks and of 3D heterogeneous ad hoc networks, respectively, by employing a generalized physical model.

More specifically, we consider a network with n nodes distributed in a three-dimensional cube with edge L, where $L = n^{\alpha/3}$ ($0 \le \alpha \le 1$), and the network volume $|\mathcal{V}| = L^3$. In 3D regular ad hoc networks (RANETs), assuming that the n nodes are regularly placed, we find that the throughput capacity is lower bounded by $n^{(\gamma-4)/3}$ when $2 \le \gamma < 3$, by $n^{-\frac{1}{3}}/\ln n$ when $\gamma = 3$, and by $n^{-\frac{1}{3}}$ when $\gamma > 3$, where γ is the path loss exponent, and is upper bounded by $\frac{P_{max}}{P_{min}} n^{-\frac{1}{3}}$ when the transmission power of the nodes can be tuned between P_{min} and P_{max} with $0 < P_{min} \le P_{max}$. In 3D heterogeneous ad hoc networks (HANETs), we assume that the n nodes are distributed according to a general Nonhomogeneous Poisson Process (NPP), with the local intensity at point ξ in the network denoted by $\Psi(\xi)$, and $\int_{\mathcal{V}} \Psi(\xi)\mathrm{d}\xi = n$.

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The minimum and the maximum of $\Psi(\xi)$ are denoted by $\underline{\Psi}$ and $\overline{\Psi}$, respectively, which both scale with n. We show that the throughput capacity is lower bounded by $\frac{\underline{\Psi}^2}{\overline{\Psi}^2 \ln n}(\frac{\ln n}{\underline{\Psi} n^\alpha})^{\frac{4-\gamma}{3}}$ when $2 \le \gamma < 3$, by $\frac{\underline{\Psi}^2}{\overline{\Psi}^2 (\ln n) \ln(\frac{\underline{\Psi} n^\alpha}{\ln n})}(\frac{\ln n}{\underline{\Psi} n^\alpha})^{\frac{1}{3}}$ when $\gamma = 3$, and by $\frac{\underline{\Psi}^2}{\overline{\Psi}^2 \ln n}(\frac{\ln n}{\underline{\Psi} n^\alpha})^{\frac{1}{3}}$ when $\gamma > 3$, and is upper bounded by $\min\left\{\frac{P_{max}}{P_{min}}, \frac{P_{max}}{P_{min}}(\overline{\Psi}^2 n^{2\alpha-3})^{\frac{1}{3}}\right\}$ when the nodes' transmission powers fall into the interval $[P_{min}, P_{max}]$.

The rest of this paper is organized as follows. In Section II we introduce some definitions and models we will use throughout this paper. Section III and Section IV present the capacity of 3D regular ad hoc networks and of 3D heterogeneous ad hoc networks, respectively. We finally conclude this paper in Section V.

II. Definitions and Models

In this section, we introduce the definitions and models we will use in this study.

A. Definitions

Throughput: As defined in the usual way, the time average of the number of bits per second that can be transmitted by each node to its destination is called the *per-node throughput*. The sum of per-node throughput over all the nodes in a network is called the *throughput of the network*.

Feasible Throughput: We say that a per-node throughput, denoted by $\lambda(n)$, is *feasible* if there exists a spatial and temporal scheduling scheme that yields a per-node throughput of $\lambda(n)$ bits per second. Let $\lambda_i(n)$ denote the throughput of node i. We say that a per-node throughput, denoted by $\lambda(n)$, is *feasible by all nodes* if there exists a spatial and temporal scheduling scheme such that $\lambda_i(n) \ge \lambda(n)$ for all $i \in [1, n]$, and is *feasible on average* if there exists a spatial and temporal scheduling scheme such that $\frac{1}{n}\sum_{i=1}^{n} \lambda_i(n) \ge \lambda(n)$. In this paper, we will derive a per-node throughput feasible on average unless otherwise specified, which we call "per-node throughput" for simplicity.

Per-node Throughput Capacity: We say that the per-node throughput capacity in the network is of order $\mathrm{O}(f(n))$ bits/sec if there is a deterministic constant $0 < c_1 < +\infty$ such that

$$\liminf_{n\to+\infty} \mathrm{P}(\lambda(n) = c_1 f(n) \text{ is feasible}) < 1,$$

and is of order $\Theta(f(n))$ bits/sec if there are deterministic constants $0 < c_2 < c_3 < +\infty$ such that

$$\liminf_{n\to+\infty} \mathrm{P}(\lambda(n) = c_2 f(n) \text{ is feasible}) = 1,$$
$$\liminf_{n\to+\infty} \mathrm{P}(\lambda(n) = c_3 f(n) \text{ is feasible}) < 1.$$

B. Network Model

We consider a network with n nodes distributed in a three-dimensional cube with edge L, where $L = n^{\alpha/3}$ ($0 \le \alpha \le 1$), and the network volume $|\mathcal{A}| = L^3$. Thus, we can model all kinds of networks including *dense networks* ($\alpha = 0$) in [19], *extended networks* ($\alpha = 1$) in [15], and *semi-extended networks* ($0 < \alpha < 1$) in [7]. We also assume that the network nodes can have transmission powers ranging from P_{min} to P_{max} with $0 < P_{min} \le P_{max}$.

3D Regular Ad Hoc Networks (RANETs): We first assume that the n nodes are regularly placed in the network. An example is shown in Fig. 1(a), in which there is one node at the center of each cubelet.

3D Heterogeneous Ad Hoc Networks (HANETs): We then extend our study to the case in which the n nodes are distributed according to a general Nonhomogeneous Poisson Process (NPP), with the local intensity at point ξ in the network denoted by $\Psi(\xi)$, and $\int_{\mathcal{A}} \Psi(\xi)\mathrm{d}\xi = n$. The minimum and the maximum of $\Psi(\xi)$ are denoted by $\underline{\Psi}$ and $\overline{\Psi}$, respectively, which both scale with n. We also assume $\underline{\Psi} n^\alpha = \omega(\ln n)$. Fig. 1(b) shows one example of 3D HANETs.

Moreover, we follow the process in [10] to choose random sender-receiver pairs so that each node is a source node for one flow and a destination node for at most O(1) flows.

C. Channel Capacity Model

Let d_{ij} denote the distance between a node i and another node j. The reception power at node j of the signal from node i, denoted by P_{ij}, follows the power propagation model described in [25], i.e.,

$$P_{ij} = C\frac{P_i}{d_{ij}^\gamma}, \tag{1}$$

where P_i is the transmission power of node i, γ is the path loss exponent, and C is a constant related to the antenna profiles of the transmitter and the receiver, wavelength, and so on. Note that $\gamma \ge 2$.

We consider the Shannon Capacity as the channel capacity between two nodes. Specifically, a transmission from node i to node j can have channel capacity, R_{ij}, which is calculated as follows:

$$R_{ij} = B\log_2(1 + SINR_{ij}), \tag{2}$$

where B is the channel bandwidth, and $SINR_{ij}$ is the SINR (Signal-to-Interference plus Noise Ratio) of the signal from node i to node j. In this study, we consider the channel bandwidth B to be a constant.

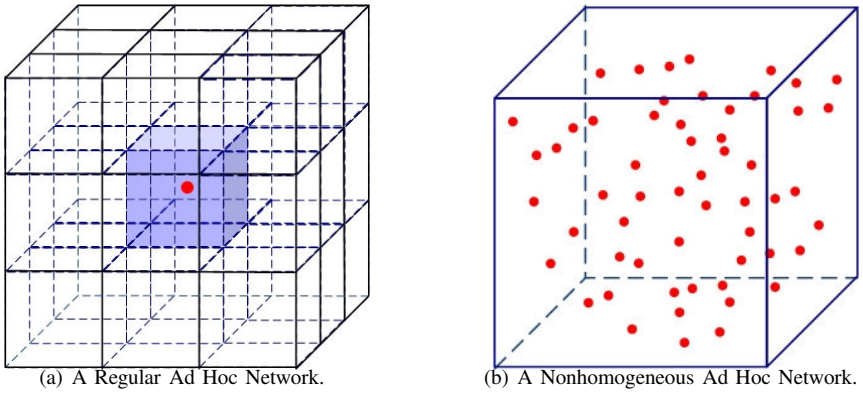
III. Capacity of Three-Dimensional Regular Ad Hoc Networks

In this section, we investigate the capacity of three-dimensional regular ad hoc networks (RANETs). Both a lower bound and an upper bound on the capacity will be presented, respectively.

A. A Lower Bound on Capacity

We first find a lower bound on capacity by obtaining an achievable throughput. As mentioned in Section II-B, a 3D RANET can be divided into cubelets with an edge length of $l = n^{(\alpha-1)/3}$ and a node at the center. We let all nodes employ the same transmission power so that the transmission range is l, the distance between two neighboring nodes.

We divide a 3D RANET into groups each of which contains twenty-seven cubelets, as shown in Fig. 1(a). The twenty-seven



(a) A Regular Ad Hoc Network. (b) A Nonhomogeneous Ad Hoc Network.

Fig. 1. Examples of 3D ad hoc networks.

cubelets in each group are numbered from 1 to 27 in the same way. We further divide time into sequences of successive slots, denoted by t ($t = 0, 1, 2, 3, ...$). During a time slot t, all nodes in cubelets that are numbered $(t \mod 27) + 1$ are allowed to transmit packets.

Consider a time slot when the node T_i in cubelet C_i is allowed to transmit to another node R_i. Denote by P_t the nodes' transmission power. The reception power level at R_i, denoted by $P_{r,i}$, is thus

$$P_{r,i} = \frac{CP_t}{l^\gamma}.$$

Besides, those nodes that may interfere with the transmission of T_i are located on the sides of concentric cubelets centered at C_i. At the jth tier, the number of interfering nodes, denoted by N_j, is

$$\begin{aligned} N_j &= (2j+1)^3 - (2j-1)^3 \\ &= 2 \times (12j^2 + 1) \\ &\leq 26j^2, \end{aligned}$$

and the distance from an interfering transmitter to R_i, denoted by l_j, satisfies

$$l_j \geq (3j-1) \cdot l.$$

Thus, according to the power propagation model in (1), the cumulative interference at R_i, denoted by I_i, can be calculated as follows:

$$\begin{aligned} I_i &\leq \sum_{j=1}^{N_{max}} 26j^2 \times \frac{CP_t}{[(3j-1)l]^\gamma} \\ &\leq \frac{26CP_t}{l^\gamma}\left[1 + \sum_{j=2}^{N_{max}} (3j-1)^{2-\gamma}\right] \\ &= \frac{26CP_t}{l^\gamma}\left[1 + \sum_{j=1}^{N_{max}-1} (3j+2)^{2-\gamma}\right] \end{aligned}$$

where N_{max} is the maximum number of tiers. Obviously, we have $N_{max} = n^{\frac{\alpha}{3}}/n^{\frac{\alpha-1}{3}} = n^{\frac{1}{3}}$.

Case I: $\gamma = 2$.

When $\gamma = 2$, the cumulative interference is

$$\begin{aligned} I_i &\leq \frac{26CP_t}{l^\gamma}[1 + \sum_{j=1}^{N_{max}-1} 1] \\ &= \frac{26CP_t}{l^\gamma} \cdot n^{\frac{1}{3}}. \end{aligned}$$

Like in [14] [27], we consider an interference dominated environment where noise can be ignored. Thus, the SINR at the receiver R_i, denoted by $SINR_i$, is

$$SINR_i \geq \frac{\frac{CP_t}{l^\gamma}}{\frac{26CP_t}{l^\gamma} \cdot n^{\frac{1}{3}}} = \frac{1}{26} n^{-\frac{1}{3}}.$$

According to the Shannon Capacity, the transmission rate from T_i to R_i, denoted by R_C^i, is

$$R_C^i \geq B \log_2 \left(1 + \frac{1}{26} n^{-\frac{1}{3}}\right) \approx \frac{B}{26} n^{-\frac{1}{3}}.$$

Case II: $2 < \gamma < 3$.

In this case, we get

$$\begin{aligned} I_i &\leq \frac{26CP_t}{l^\gamma}\left[1 + \int_0^{N_{max}} (3j+2)^{2-\gamma} \mathbf{d}j\right] \\ &= \frac{26CP_t}{l^\gamma}\left[1 + \frac{(3j+2)^{3-\gamma}}{3(3-\gamma)}\Big|_0^{N_{max}}\right] \\ &\leq \frac{26CP_t}{l^\gamma}\left[1 + \frac{(4N_{max})^{3-\gamma}}{3(3-\gamma)}\right] \\ &\leq \frac{52 \cdot 4^{3-\gamma}}{3(3-\gamma)} \cdot \frac{CP_t N_{max}^{3-\gamma}}{l^\gamma}, \end{aligned}$$

and hence

$$SINR_i \geq \frac{\frac{CP_t}{l^\gamma}}{\frac{52 \cdot 4^{3-\gamma}}{3(3-\gamma)} \cdot \frac{CP_t N_{max}^{3-\gamma}}{l^\gamma}} = \frac{3(3-\gamma)}{52 \cdot 4^{3-\gamma}} n^{\frac{1}{3}(\gamma-3)}.$$

Since $\gamma < 3$, we can obtain that

$$\begin{aligned} R_C^i &\geq B \log_2 \left(1 + \frac{3(3-\gamma)}{52 \cdot 4^{3-\gamma}} n^{\frac{1}{3}(\gamma-3)}\right) \\ &\approx \frac{3(3-\gamma)B}{52 \cdot 4^{3-\gamma}} n^{\frac{1}{3}(\gamma-3)}. \end{aligned}$$

Case III: $\gamma = 3$.

When $\gamma = 3$, we can get that

$$\begin{aligned} I_i &\leq \frac{26CP_t}{l^\gamma}\left[1+\int_0^{N_{max}} \frac{1}{3j+2}\mathbf{d}j\right] \\ &= \frac{26CP_t}{l^\gamma}\left[1+\frac{1}{3}\ln(3N_{max}+2)-\frac{1}{3}\ln 2\right] \\ &\leq \frac{26CP_t}{l^\gamma}\left[\frac{1}{3}\ln(4N_{max})+1-\frac{1}{3}\ln 2\right] \\ &= \frac{26CP_t}{l^\gamma}\left[\frac{1}{3}\ln N_{max}+1+\frac{1}{3}\ln 2\right] \\ &\leq \frac{26CP_t}{l^\gamma}\ln N_{max}. \end{aligned}$$

As a result, the SINR of the transmission from T_i to R_i is

$$SINR_i \geq \frac{\frac{CP_t}{l^\gamma}}{\frac{26CP_t}{l^\gamma}\ln N_{max}} = \frac{3}{26\ln n},$$

and

$$R_C^i \geq B\log_2\left(1+\frac{3}{26\ln n}\right) \approx \frac{3B}{26\ln n}.$$

Case IV: $\gamma > 3$.

In this case, the cumulative interference can be calculated as

$$\begin{aligned} I_i &\leq \frac{26CP_t}{l^\gamma}\left[1+\int_0^{N_{max}} (3j+2)^{2-\gamma}\mathbf{d}j\right] \\ &= \frac{26CP_t}{l^\gamma}\left[1+\frac{2^{3-\gamma}}{3(\gamma-3)}\right] \\ &< \frac{26CP_t}{l^\gamma}\left[1+\frac{1}{3(\gamma-3)}\right] \\ &= \frac{26CP_t}{l^\gamma}\cdot\frac{3\gamma-8}{3\gamma-9}. \end{aligned}$$

Thus, we can have that

$$SINR_i \geq \frac{\frac{CP_t}{l^\gamma}}{\frac{26CP_t}{l^\gamma}\cdot\frac{3\gamma-8}{3\gamma-9}} = \frac{3\gamma-9}{26(3\gamma-8)},$$

and

$$R_C^i \geq B\log_2\left(1+\frac{3\gamma-9}{26(3\gamma-8)}\right),$$

which can be lower bounded by a constant.

Note that every node transmits every 27 time slots. The final results follow subsequently.

Lemma 1: In 3D RANETs, the transmission rate of each node, denoted by R_C, is

$$R_C \geq \begin{cases} \frac{B}{c_1}n^{-\frac{1}{3}} & \text{if } \gamma = 2 \\ \frac{B}{c_2}n^{\frac{1}{3}(\gamma-3)} & \text{if } 2<\gamma<3 \\ \frac{B}{c_3\ln n} & \text{if } \gamma = 3 \\ \frac{B}{c_4} & \text{if } \gamma > 3 \end{cases} \quad (3)$$

where $c_1 = 702$, $c_2 = 468\cdot 4^{3-\gamma}/(3-\gamma)$, $c_3 = 234$, $c_4 = 27\log_2^{-1}\left(1+\frac{3\gamma-9}{26(3\gamma-8)}\right)$.

Besides, we use the following routing strategy to relay the packets. Specifically, as shown in Fig. 2, assume that a source node S is located at (x_s, y_s, z_s) and its destination node D is located at (x_d, y_d, z_d). Packets from this source node are firstly relayed from (x_s, y_s, z_s) to (x_s, y_s, z_d), then to (x_s, y_d, z_d), and finally to (x_d, y_d, z_d).

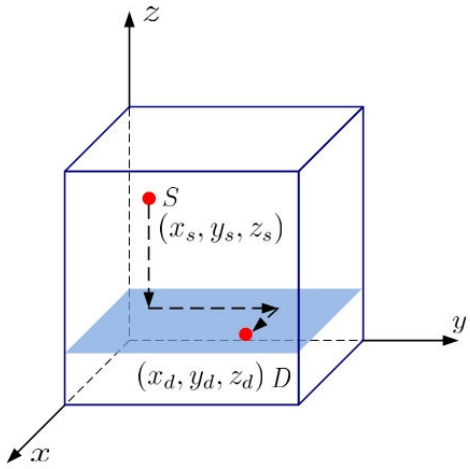


Fig. 2. The routing strategy used for ad hoc mode transmissions.

Denote the average distance between a source-destination pair by $\bar{L}$. We can get that [5]

$$\bar{L} \leq \frac{1}{\sqrt{6}}\left(\frac{1+2\sqrt{1-1/5}}{3}\right)^{1/2}\cdot\sqrt{3}n^{\alpha/3} \approx 0.68n^{\alpha/3}.$$

Let $c_5 = 0.68$. Then, $\bar{L} \leq c_5 n^{\alpha/3}$ and the average number of hops, denoted by $\bar{H}$, is

$$\begin{aligned} \bar{H} &= \frac{||z_s-z_d||}{l}+\frac{||y_s-y_d||}{l}+\frac{||x_s-x_d||}{l} \\ &\leq \frac{3\bar{L}}{l}. \end{aligned}$$

Thus, a per-node throughput, denoted by $\lambda(n)$, is feasible if

$$\lambda(n)\cdot\bar{H} \leq R_C,$$

i.e.,

$$\lambda(n) \leq \frac{1}{3c_5 n^{1/3}}R_C,$$

which leads to the following theorem.

Theorem 1: A lower bound on the capacity of 3D RANETs is

$$\lambda(n) = \Omega\begin{cases} \frac{B}{3c_1c_5}n^{-\frac{2}{3}} & \text{if } \gamma = 2, \\ \frac{B}{3c_2c_5}n^{\frac{\gamma-4}{3}} & \text{if } 2<\gamma<3, \\ \frac{B}{3c_3c_5\ln n}n^{-\frac{1}{3}} & \text{if } \gamma = 3, \\ \frac{B}{3c_4c_5}n^{-\frac{1}{3}} & \text{if } \gamma > 3. \end{cases}$$

B. An Upper Bound on Capacity

Next, we investigate the upper bound on the network capacity by finding the maximum amount of information that passes a cut plane in a 3D RANET.

As shown in Fig. 3, a cut plane is chosen such that the network space is divided into two parts each of which contains $n/2$ nodes. The transmitters are on the left side and the receivers are on the right side. We divide the left part of the

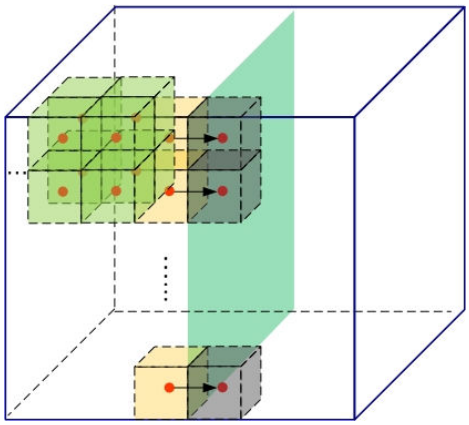


Fig. 3. Finding an upper bound on capacity using a cut plane.

network into tiers each of which are composed of *blocks* of different sizes. Specifically, at tier j ($j \geq 1$), the *blocks* have an edge length of $2^{j-1} \cdot l$. We now show that each *block* as a whole can have at most a constant transmission rate.

Recall that the transmission power of each node falls into the interval $[P_{min}, P_{max}]$ with $P_{min} \leq P_{max}$. We use P_r^j, I^j, and $SINR^j$ to denote the reception power, interference suffered at receiver, and the SINR of transmissions originated from transmitters in tier j blocks, respectively. We also denote the total transmission rate of all nodes in a tier j block by R_C^j. Then, at tier 1, since the minimum distance between a transmitter and a receiver is l, we have

$$P_r^1 \leq \frac{P_{max}}{l^\gamma}.$$

Besides, the minimum interference is observed when there is only one other transmitter right next to the current one with the minimum transmission power P_{min}. Thus, we can get

$$I^1 \geq \frac{P_{min}}{\left(\sqrt{2}l\right)^\gamma}.$$

As a result, neglecting the noise, we can have

$$SINR^1 \leq 2^{\frac{\gamma}{2}} \frac{P_{max}}{P_{min}},$$

and hence

$$R_C^1 \leq B \log_2(1 + SINR^1) \leq 2^{\frac{\gamma}{2}} \frac{P_{max}}{P_{min}} B$$

due to the fact that $\log_2(1+x) < x$ for $x > 0$.

Similarly, at tier j, the minimum distance between a transmitter and a receiver, denoted by d_{min}^j, is

$$d_{min}^j = (1 + 2 + ... + 2^{j-2} + 1) \cdot l = 2^{j-1} l,$$

and

$$P_r^j \leq \frac{P_{max}}{\left(2^{j-1}l\right)^\gamma}.$$

Denote by d_{max}^j the maximum distance between another transmitter in the same block and the receiver. Then,

$$d_{max}^j = \sqrt{\left[2^{j-1}l + (2^{j-1}-1)l\right]^2 + \left[(2^{j-1}-1)l\right]^2 \times 2}.$$

Let n_j denote the number of nodes in a block at tier j. So we get

$$\begin{aligned} I^j &\geq n_j \frac{P_{min}}{\left(d_{max}^j\right)^\gamma} \\ &\geq \frac{n_j P_{min}}{\left(\sqrt{(2^j l)^2 + 2(2^{j-1}l)^2}\right)^\gamma} \\ &= \frac{n_j P_{min}}{\left(\sqrt{6} \cdot 2^{j-1} \cdot l\right)^\gamma}. \end{aligned}$$

As a result, we can obtain

$$SINR^j \leq \frac{6^{\frac{\gamma}{2}}}{n_j} \frac{P_{max}}{P_{min}},$$

and hence

$$\begin{aligned} R_C^j &\leq n_j B \log_2 \left(1 + \frac{6^{\frac{\gamma}{2}}}{n_j} \frac{P_{max}}{P_{min}}\right) \\ &\leq 6^{\frac{\gamma}{2}} \frac{P_{max}}{P_{min}} B. \end{aligned}$$

We further denote the number of blocks at tier j by X_j. Then, we have

$$X_j = \frac{L^2}{l^2} \cdot \left(\frac{1}{4}\right)^{j-1}.$$

Thus, the per-node throughput, i.e., $\lambda(n)$, satisfies

$$\begin{aligned} \lambda(n) &= \frac{\sum_j X_j R_C^j}{n/2} \\ &\leq \frac{\frac{L^2}{l^2}(1 + \frac{1}{4} + \frac{1}{16} + ...) 6^{\frac{\gamma}{2}} \frac{P_{max}}{P_{min}} B}{n/2} \\ &= 6^{\frac{\gamma}{2}} \frac{8P_{max}}{3P_{min}} B n^{-\frac{1}{3}}. \end{aligned}$$

We can thus have the following theorem.

***Theorem** 2:* An upper bound on the capacity of 3D RANETs is

$$\lambda(n) = \mathrm{O}\left(6^{\frac{\gamma}{2}} \frac{8P_{max}}{3P_{min}} B n^{-\frac{1}{3}}\right).$$

IV. Capacity of Three-Dimensional Heterogeneous Ad Hoc Networks

In this section, we explore the capacity of three-dimensional heterogeneous ad hoc networks (HANETs), where the distribution of the n nodes follows a general NPP.

A. A Lower Bound on Capacity

We divide the network space into cubelets with an edge length of $l' = \left(\frac{c' \ln n}{\underline{\Psi}}\right)^{1/3}$, where $c'(c' > 2)$ is a constant. Since $\underline{\Psi} n^\alpha = \omega(\ln n)$, we have $l' = o(n^{\alpha/3}) = o(L)$. Then, we have the following lemma.

***Lemma** 2:* No cubelet is empty with high probability (w.h.p.).

Proof: For a cubelet C_b, the probability that there is no node in it, denoted by P_e, is

$$P_e = e^{-\int_{C_b} \Psi(\xi) d\xi} \leq e^{-\underline{\Psi}(l')^3} = \frac{1}{n^{c'}}.$$

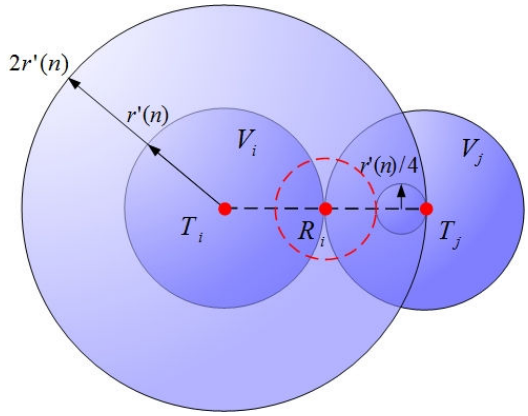


Fig. 4. Disjoint balls in a three-dimensional ad hoc networks.

So, $P_e \to 0$ as $n \to \infty$. Moreover, let n_c be the number of cubelets in the network. We have $n_c = n^\alpha/(l')^3$. Then, the probability that at least one cubelet has no node in it, denoted by P_E, is

$$P_E \le n_c \cdot P_e \le \frac{n^\alpha}{(l')^3} \cdot \frac{1}{n^{c'}} = \frac{\underline{\Psi} n^\alpha}{(c' \ln n) n^{c'}}.$$

Since $\underline{\Psi} n^\alpha \le n$ and $c' > 2$, we can get that

$$P_E \le \frac{1}{(c' \ln n) n^{c'-1}} \to 0,$$

i.e., no cubelet is empty w.h.p.. ■

We let all nodes employ the same transmission power P_t' so that the transmission range is $r'(n) = \sqrt{6} l'$. Thus, we can enable the transmissions between any two nodes located in two neighboring cubelets. We also choose the physical carrier sensing range to be $2r'(n)$. Then, as shown in Fig. 4, the balls centered at the transmitters transmitting at the same time with radius $r'(n)$ are disjoint. So are the inside balls with radius $r'(n)/4$ and a transmitter on the boundary.

Consider a transmission from a transmitter T_i to a receiver R_i located at point $\xi_0 \in \mathbb{R}^3$. The reception power level at ξ_0, denoted by $P_r(\xi_0)$, is

$$P_r(\xi_0) \ge \frac{CP_t'}{\big(r'(n)\big)^\gamma}.$$

Let $\mathbb{T} = \{x_k\}$ denote the set of transmitters transmitting at the same time as T_i, where x_k also stands for the position of a node. We also denote the transmission volume by $|V_t|$. Then, the cumulative interference suffered at ξ_0, denoted by $I(\xi_0)$, is

$$\begin{aligned} I(\xi_0) &= \sum_{x_k \in \mathbb{T}} \frac{CP_t'}{||x_k - \xi_0||^\gamma} \\ &= CP_t' \sum_{x_k \in \mathbb{T}} \frac{1}{||x_k - \xi_0||^\gamma} \cdot \frac{1}{|V_t|/64} \cdot (|V_t|/64) \\ &\le \frac{64CP_t'}{|V_t|} \int_0^{2\pi} \int_0^{\pi} \int_{r'(n)/2}^{\sqrt{3}L} \frac{1}{\rho^\gamma} \cdot \rho^2 \sin\phi \\ &\quad \mathbf{d}\rho\mathbf{d}\phi\mathbf{d}\theta \\ &= \frac{256\pi CP_t'}{|V_t|} \int_{r'(n)/2}^{\sqrt{3}L} \rho^{2-\gamma} \mathbf{d}\rho. \end{aligned}$$

Case I: $\gamma = 2$.

When $\gamma = 2$, the cumulative interference can be calculated as:

$$\begin{aligned} I(\xi_0) &\le \frac{256\pi CP_t'}{|V_t|} \int_{r'(n)/2}^{\sqrt{3}L} 1 \mathbf{d}\rho \\ &\le \frac{256\pi CP_t'}{\frac{4}{3}\pi\big(r'(n)\big)^3} \cdot \sqrt{3}L \\ &= \frac{192\sqrt{3}CP_t'L}{\big(r'(n)\big)^3}. \end{aligned}$$

As mentioned before, we consider an interference dominated environment. Thus, the SINR suffered by the receiver at ξ_0, denoted by $SINR(\xi_0)$, is

$$SINR(\xi_0) \ge \frac{CP_t'/\big(r'(n)\big)^2}{192\sqrt{3}CP_t'L/\big(r'(n)\big)^3}.$$

Recall that $r'(n) = \sqrt{6}\big(\frac{c' \ln n}{\underline{\Psi}}\big)^{1/3}$ and $L = n^{\alpha/3}$. We can get

$$SINR(\xi_0) \ge \frac{1}{96\sqrt{2}} \cdot \left(\frac{c' \ln n}{\underline{\Psi} n^\alpha}\right)^{\frac{1}{3}}.$$

Thus, the transmission rate from T_i to R_i, denoted by R_C^i, is

$$R_C^i \ge B \log_2 \left(1 + \frac{1}{96\sqrt{2}} \cdot \left(\frac{c' \ln n}{\underline{\Psi} n^\alpha}\right)^{\frac{1}{3}}\right).$$

Let $c_1' = \frac{1}{96\sqrt{2}}$. Since $\underline{\Psi} n^\alpha = \omega\big(\ln n\big)$, then $\ln n/(\underline{\Psi} n^\alpha) \to 0$, and hence

$$R_C^i \ge c_1' B \left(\frac{c' \ln n}{\underline{\Psi} n^\alpha}\right)^{\frac{1}{3}}.$$

Case II: $2 < \gamma < 3$.

When $2 < \gamma < 3$, we can get

$$\begin{aligned} I(\xi_0) &\le \frac{256\pi CP_t'}{|V_t|} \int_{r'(n)/2}^{\sqrt{3}L} \rho^{2-\gamma} \mathbf{d}\rho \\ &\le \frac{256\pi CP_t'}{\frac{4}{3}\pi\big(r'(n)\big)^3} \cdot \frac{(\sqrt{3}L)^{3-\gamma}}{3-\gamma} \\ &= \frac{192CP_t'(\sqrt{3}L)^{3-\gamma}}{(3-\gamma)\big(r'(n)\big)^3}, \end{aligned}$$

and hence

$$\begin{aligned} SINR(\xi_0) &\ge \frac{CP_t'/\big(r'(n)\big)^\gamma}{\frac{192CP_t'(\sqrt{3}L)^{3-\gamma}}{(3-\gamma)(r'(n))^3}} \\ &= \frac{(3-\gamma)2^{\frac{3-\gamma}{2}}}{192} \cdot \left(\frac{c' \ln n}{\underline{\Psi} n^\alpha}\right)^{\frac{3-\gamma}{3}}. \end{aligned}$$

Thus, the transmission rate from T_i to R_i, i.e., R_C^i, can be obtained by

$$R_C^i \ge B \log_2 \left(1 + \frac{(3-\gamma)2^{\frac{3-\gamma}{2}}}{192} \cdot \left(\frac{c' \ln n}{\underline{\Psi} n^\alpha}\right)^{\frac{3-\gamma}{3}}\right).$$

Let $c_2' = \frac{(3-\gamma)2^{\frac{3-\gamma}{2}}}{192}$. Then we can get

$$R_C^i \geq c_2' B\left(\frac{c' \ln n}{\underline{\Psi} n^\alpha}\right)^{\frac{3-\gamma}{3}}.$$

Case III: $\gamma = 3$.

When $\gamma = 3$, we can obtain that

$$\begin{aligned} I(\xi_0) &\leq \frac{256\pi C P_t'}{|V_t|} \int_{r'(n)/2}^{\sqrt{3}L} \rho^{-1} \mathrm{d}\rho \\ &= \frac{256\pi C P_t'}{\frac{4}{3}\pi \big(r'(n)\big)^3} \cdot \left[\ln \sqrt{3}L - \ln \frac{r'(n)}{2}\right] \\ &= \frac{192 C P_t'}{\big(r'(n)\big)^3} \ln \frac{2\sqrt{3}L}{r'(n)}, \end{aligned}$$

As a result, the SINR of the transmission from T_i to R_i located at ξ_0, is

$$\begin{aligned} SINR(\xi_0) &\geq \frac{C P_t' / \big(r'(n)\big)^3}{\frac{192 C P_t'}{(r'(n))^3} \ln \frac{2\sqrt{3}L}{r'(n)}} \\ &= \frac{1}{192 \ln \frac{2\sqrt{3} n^{\alpha/3}}{\sqrt{6}(\frac{c' \ln n}{\underline{\Psi}})^{1/3}}} \\ &= \frac{1}{192\big(\ln \sqrt{2} + \frac{1}{3} \ln \frac{\underline{\Psi} n^\alpha}{c' \ln n}\big)} \\ &\geq \frac{1}{256 \ln \frac{\underline{\Psi} n^\alpha}{c' \ln n}}. \end{aligned}$$

The last step is due to the fact that $\ln \sqrt{2} < \ln \frac{\underline{\Psi} n^\alpha}{c' \ln n}$. Thus, the transmission rate from T_i to R_i, i.e., R_C^i, can be calculated as

$$R_C^i \geq B \log_2 \left(1 + \frac{1}{256 \ln \frac{\underline{\Psi} n^\alpha}{c' \ln n}}\right) \approx c_3' B \ln^{-1} \left(\frac{\underline{\Psi} n^\alpha}{c' \ln n}\right).$$

where $c_3' = 1/256$.

Case IV: $\gamma > 3$.

When $\gamma > 3$, the cumulative interference can be calculated as

$$\begin{aligned} I(\xi_0) &\leq \frac{256\pi C P_t'}{|V_t|} \int_{r'(n)/2}^{\sqrt{3}L} \rho^{2-\gamma} \mathrm{d}\rho \\ &= \frac{256\pi C P_t'}{\frac{4}{3}\pi \big(r'(n)\big)^3} \cdot \frac{\rho^{3-\gamma}}{3-\gamma}\Big|_{r'(n)/2}^{\sqrt{3}L} \\ &\leq \frac{192 C P_t'}{(\gamma-3)2^{3-\gamma}} \big(r'(n)\big)^{-\gamma}. \end{aligned}$$

Thus, the SINR of the transmission from T_i to R_i, located at ξ_0, is

$$\begin{aligned} SINR(\xi_0) &\geq \frac{C P_t' / \big(r'(n)\big)^\gamma}{\frac{192 C P_t'}{(\gamma-3)2^{3-\gamma}} \big(r'(n)\big)^{-\gamma}} \\ &= \frac{(\gamma-3)2^{3-\gamma}}{192}, \end{aligned}$$

which can be lower bounded by a constant.

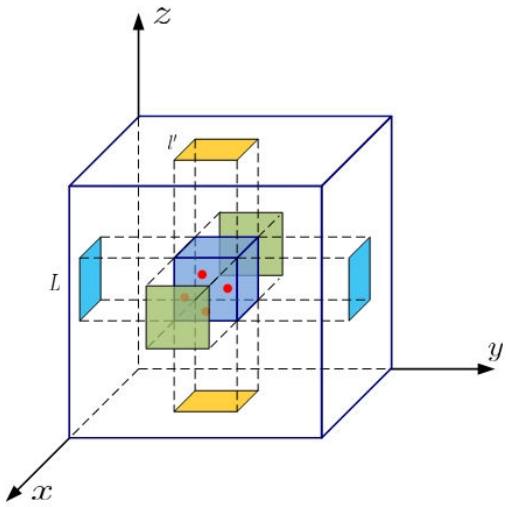


Fig. 5. The routing strategy used in nonhomogeneous three-dimensional ad hoc networks.

As a result, by letting $c_4' = \frac{(\gamma-3)2^{3-\gamma}}{192}$, the transmission rate from T_i to R_i is

$$R_C^i \geq B \log_2 \big(1 + c_4'\big).$$

Thus, we have the following lemma.

Lemma 3: In 3D HANETs, the data rate of each transmission, denoted by R_C', is:

$$R_C' \geq \begin{cases} c_1' B \big(\frac{c' \ln n}{\underline{\Psi} n^\alpha}\big)^{\frac{1}{3}} & \text{if } \gamma = 2, \\ c_2' B \big(\frac{c' \ln n}{\underline{\Psi} n^\alpha}\big)^{\frac{3-\gamma}{3}} & \text{if } 2 < \gamma < 3, \\ c_3' B \ln^{-1} \big(\frac{\underline{\Psi} n^\alpha}{c' \ln n}\big) & \text{if } \gamma = 3, \\ B \log_2(1 + c_4') & \text{if } \gamma > 3. \end{cases}$$

Notice that nodes' transmission range $r'(n)$ is chosen to be

$$r'(n) = \sqrt{6} l' = \sqrt{6}\left(\frac{c' \ln n}{\underline{\Psi}}\right)^{\frac{1}{3}}.$$

So the maximum number of nodes that share the transmission rate R_C', denoted by N_n, is

$$N_n = \overline{\Psi} \cdot 8|V_t| = \overline{\Psi} \cdot \frac{32}{3}\pi \big(r'(n)\big)^3 = 64\sqrt{6}\pi c' \frac{\overline{\Psi} \ln n}{\underline{\Psi}}.$$

Besides, we employ a routing strategy similar to that in Section III-A. As shown in Fig. 5, the maximum number of nodes that each cubelet relays packets for, denoted by N_r, can be obtained by $N_r = 3(l')^2 L \cdot \overline{\Psi}$. Since the minimum number of nodes in each cubelet, denoted by n_c, is $n_c = \underline{\Psi}(l')^3$, the maximum average traffic load for each node, denoted by $\bar{H}'$, is

$$\bar{H}' = \frac{N_r}{n_c} = \frac{3L\overline{\Psi}}{\underline{\Psi} l'} = \frac{3\overline{\Psi}}{\underline{\Psi}}\left(\frac{\underline{\Psi} n^\alpha}{c' \ln n}\right)^{\frac{1}{3}}.$$

As a result, a per-node throughput, denoted by $\lambda'(n)$, is feasible if the following holds:

$$\lambda'(n) \bar{H}' \leq \frac{R_C'}{N_n},$$

i.e.,

$$\lambda'(n) \leq \frac{1}{192\sqrt{6}\pi c'} \cdot \frac{\underline{\Psi}^2}{\overline{\Psi}^2 \ln n} \cdot \left(\frac{c' \ln n}{\underline{\Psi} n^{\alpha}}\right)^{\frac{1}{3}} \cdot R'_C.$$

We can thus have the following theorem.

***Theorem** 3:* A lower bound on the capacity of 3D HANETs is

$$\lambda'(n) = \Omega \begin{cases} \frac{c'_1 B}{192\sqrt{6}\pi c'} \frac{\underline{\Psi}^2}{\overline{\Psi}^2 \ln n} \left(\frac{c' \ln n}{\underline{\Psi} n^{\alpha}}\right)^{\frac{2}{3}} & \text{if } \gamma = 2, \\ \frac{c'_2 B}{192\sqrt{6}\pi c'} \frac{\underline{\Psi}^2}{\overline{\Psi}^2 \ln n} \left(\frac{c' \ln n}{\underline{\Psi} n^{\alpha}}\right)^{\frac{4-\gamma}{3}} & \text{if } 2 < \gamma < 3, \\ \frac{c'_3 B}{192\sqrt{6}\pi c'} \frac{\underline{\Psi}^2}{\overline{\Psi}^2 (\ln n) \ln(\frac{\underline{\Psi} n^{\alpha}}{c' \ln n})} \left(\frac{c' \ln n}{\underline{\Psi} n^{\alpha}}\right)^{\frac{1}{3}} & \text{if } \gamma = 3, \\ \frac{B \log_2(1+c'_4)}{192\sqrt{6}\pi c'} \frac{\underline{\Psi}^2}{\overline{\Psi}^2 \ln n} \left(\frac{c' \ln n}{\underline{\Psi} n^{\alpha}}\right)^{\frac{1}{3}} & \text{if } \gamma > 3. \end{cases}$$

B. An Upper Bound on Capacity

We then use percolation theory [9], [26] to find an upper bound on the capacity of 3D heterogeneous ad hoc networks.

We divide the network space into cubelets with an edge length of l'_c. Then, the 3D network space can be decomposed into L/l'_c 2D *planar* areas each of which contains $(L/l'_c) \times (L/l'_c)$ cubelets. Thus, the capacity of the 3D network can be upper bounded by the sum of the capacity of each individual 2D *planar* network with sources and destinations on the same *plane*.

Let $0 < p < 1$ be a constant. Choosing

$$\left(l'_c\right)^3 = -\frac{\ln p}{\overline{\Psi}},$$

we know that the probability that a cubelet C_b is empty, denoted by P_e, is

$$P_e = e^{-\int_{C_b} \Psi(\xi) d\xi} \geq e^{-\overline{\Psi}(l'_c)^3} = p.$$

Since $p_c \approx 0.59$ is the critical probability of independent site percolation in a square lattice, choosing $p > p_c$ can make all L/l'_c 2D *planar* areas percolated. Moreover, in each 2D *planar* network, inside any *rectangle* of size $L \times (\kappa \log n) l'_c$ ($\kappa > 0$), there exists at least one path composed of $\Theta(L/l'_c)$ empty cublets connecting the top side with the bottom side of the network.

Consider one of the 2D *planar* networks as shown in Fig. 6. We choose a *rectangle* to the right of $y = y_0$ with width $h = (\kappa \log n) l'_c$ so that on the left side of $y = y_0$ there are $n/2$ nodes in the 3D cube. Then, inside this *rectangle* there is at least one crossing path composed of $\Theta(L/l'_c)$ empty cubelets. Our objective is to find an upper bound on the amount of information $\mathcal{I}$ that can traverse from left to right through the cut planes in the 3D network. Since there are at least $n/2$ end-to-end data flows going through the cut planes, the per-node throughput can be upper bounded by $\mathcal{I}/(n/2)$.

We further divide the left part of the cutting plane (as shown in Fig. 6) into tiers of *blocks*. *Blocks* at different tiers are of different sizes. Specifically, the tier j ($j \geq 1$) blocks are of size $(2^{j-1} l'_c) \times (2^{j-1} l'_c) \times l'_c$. Besides, Tier 1 blocks are those right next to the rightmost empty cubelets along the crossing path (as shown in Fig. 6). Tier 2 blocks are next to Tier 1 blocks, and so on and so forth.

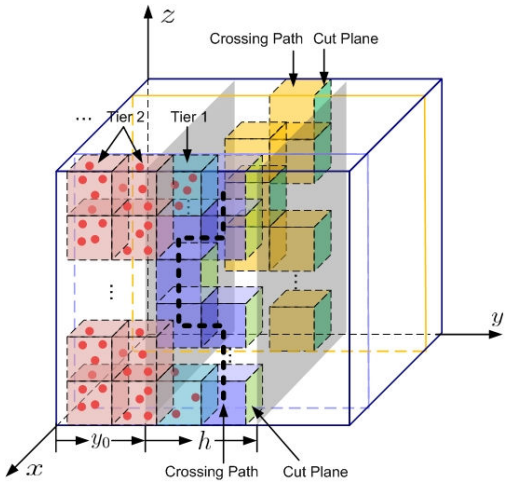


Fig. 6. Finding an upper bound by decomposing a 3D network into 2D networks.

As a result, the minimum distance between a transmitter in a tier j block and its corresponding receiver, denoted by d^j_{min}, is

$$d^j_{min} = (1 + 1 + 2 + ... + 2^{j-2}) \cdot l'_c = 2^{j-1} l'_c,$$

and since the transmission power of the nodes ranges from P_{min} to P_{max}, the reception power at the receiver, denoted by P^j_r, is

$$P^j_r \leq \frac{C P_{max}}{\left(2^{j-1} l'_c\right)^{\gamma}}.$$

Denote by d^j_{max} the maximum distance between another transmitter in the same block and the receiver. Then,

$$d^j_{max} = \sqrt{\left(2^{j-1} + 2^{j-1}\right)^2 + \left(2^{j-1}\right)^2 + 1} \cdot l'_c.$$

Let n_j denote the number of nodes in a block at tier j. The cumulative interference suffered at the receiver, denoted by I^j, can be obtained as

$$I^j \geq n_j \frac{C P_{min}}{\left(d^j_{max}\right)^{\gamma}} \geq \frac{n_j C P_{min}}{\left(\sqrt{3} \cdot 2^j l'_c\right)^{\gamma}}.$$

As a result, we can get

$$SINR^j \leq \frac{(2\sqrt{3})^{\gamma}}{n_j} \frac{P_{max}}{P_{min}},$$

and hence

$$\begin{aligned} R^j_C &\leq n_j B \log_2 \left(1 + \frac{(2\sqrt{3})^{\gamma}}{n_j} \frac{P_{max}}{P_{min}}\right) \\ &\leq (2\sqrt{3})^{\gamma} \frac{P_{max}}{P_{min}} B. \end{aligned}$$

We denote the number of non-empty tier j blocks in each 2D *planar* network by Y_j. Since as shown in Fig. 6 the crossing path might go through some of the blocks, some blocks may be empty. Thus, we have

$$Y_j \leq \frac{L}{l'_c} \cdot \left(\frac{1}{2}\right)^{j-1}.$$

So, the per-node throughput, i.e., $\lambda'(n)$, satisfies

$$\begin{aligned}\lambda'(n) &= \frac{(L/l'_c)\cdot\sum_j Y_j R_C^j}{n/2}\\ &\le \frac{\frac{L^2}{(l'_c)^2}(1+\frac{1}{2}+\frac{1}{4}+...)(2\sqrt{3})^\gamma\frac{P_{max}}{P_{min}}B}{n/2}\\ &= (2\sqrt{3})^\gamma\frac{4P_{max}}{P_{min}}B\left(\frac{\overline{\Psi}^2 n^{2\alpha-3}}{\ln^2 p}\right)^{\frac{1}{3}}.\end{aligned}$$

Moreover, since in the network there are at most n non-empty blocks, we also have

$$\lambda'(n) \le \frac{n\cdot R_C^j}{n/2} \le \big(2\sqrt{3}\big)^\gamma\frac{2P_{max}}{P_{min}}B.$$

We can thus have the following theorem.

Theorem 4: An upper bound on the capacity of 3D HANETs is

$$\begin{aligned}\lambda'(n) &= \mathrm{O}\Bigg(\min\Bigg\{(2\sqrt{3})^\gamma\frac{2P_{max}}{P_{min}}B,\\ &\qquad \big(2\sqrt{3}\big)^\gamma\frac{2P_{max}}{P_{min}}B\left(\frac{\overline{\Psi}^2 n^{2\alpha-3}}{\ln^2 p}\right)^{\frac{1}{3}}\Bigg\}\Bigg).\end{aligned}$$

Considering a special case when $\overline{\Psi}=\Theta(\underline{\Psi})$, we get $\overline{\Psi}n^\alpha = \underline{\Psi}n^\alpha = n$. Therefore, we can have $\lambda'(n)=\mathrm{O}\big(n^{-\frac{1}{3}}\big)$.

V. Conclusion

In this paper, we have explored the capacity of three-dimensional wireless ad hoc networks, including 3D regular ad hoc networks (RANETs) and 3D heterogeneous ad hoc networks (HANETs). Both lower and upper bounds have been obtained for the two types of networks when the path loss exponent is no less than 2. We find that lower bounds in the networks are dependent on the power propagation environment, i.e., the path loss exponent, while upper bounds are not. In our future work, we will investigate how to further bridge the gap between the lower bounds and the upper bounds on the capacity of 3D ad hoc networks.

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