A Variational Message Passing Algorithm for Sensor Self-Localization in Wireless Networks

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A Variational Message Passing Algorithm for Sensor Self-Localization in Wireless Networks

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***Abstract*—We propose a novel algorithm for sensor self-localization in cooperative wireless networks where observations of relative sensor distances are available. The variational message passing (VMP) algorithm is used to implement a mean field solution to the estimation of the posterior probabilities of the sensor positions in an $\mathbb{R}^2$ scenario. Extension to $\mathbb{R}^3$ is straightforward. Compared to non-parametric methods based on belief propagation, the VMP algorithm features significantly lower communication overhead between sensors. This is supported by performance simulations which show that the estimated mean localization error of the algorithm stabilizes after approximately 30 iterations.**

I. Introduction

Information collected and communicated by a wireless sensor in a wireless sensor network (WSN) is often only valuable if the location of the wireless sensor is known [1], [2]. Manually supplying wireless sensors with their positions is cumbersome or impossible and equipping wireless sensors with a global positioning system (GPS) receiver may be cost and energy prohibitive [1], [3]. Furthermore, GPS signals have poor building penetration properties and receiving these signals indoors or in urban areas surrounded by tall buildings may be difficult or impossible. Consequently, the position information is inadequate or erroneous [2]. To meet the challenge of providing position information in wireless networks, reliable methods for self-localization of wireless sensors are in demand.

In cooperative localization, sensors in a network estimate their own positions by exploiting relative position information obtained from measurements with neighbour sensors and/or absolute reference locations available from anchor sensors [3]. In order to estimate its own position from the information obtained from other sensors, each sensor needs a processing unit and an algorithm for self-localization.

Self-localization algorithms based on geometric and probabilistic methods have been considered previously. In [4] and [5], sensor localization methods based on convex optimization and semidefinite programming are considered. Probabilistic localization methods based on belief propagation (BP) in factor graphs, and its sum-product (SP) implementation, are proposed in [2] and [3]. The BP methods in these contributions yield accurate results at the expense of large communication overhead due to the use of a large number (typically hundreds) of samples (particles) to represent the messages. The authors of [6] propose an expectation-propagation based localization algorithm which uses Gaussian estimates instead of particles to represent messages. Variational Bayesian methods, and their variational message passing (VMP) implementation, complement BP methods for probabilistic inference on factor graphs [7], [8]. In the particular application context of localization considered in this paper, the VMP algorithm allows for simpler message representations than the SP algorithm. This translates into lower communication overheads between nodes.

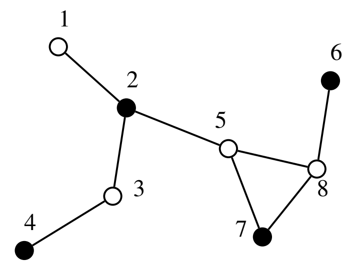

Figure 1. Example network with anchor sensors (black vertices), mobile sensors (white vertices) and their communication links (edges).

In this contribution, we apply the VMP algorithm to distributed, iterative self-localization of sensors in cooperative wireless networks. We present a probabilistic model for the joint probability density of sensor positions and relative sensor distance observations in a WSN. We contrast the structures of the SP and VMP algorithms in this particular application context and show that compared to particle based BP methods the communication overhead of VMP can be drastically reduced by approximating the posterior densities of the sensor positions with circular symmetric Gaussian densities. The resulting scheme features a simple representation of the messages broadcast by the nodes. For instance, in an $\mathbb{R}^2$ localization scenario, the mobile nodes need pass only three real values (the mean, and the standard deviation of the Gaussian pdf approximating their position) at each iteration. We investigate the performance of the VMP algorithm in a static scenario containing 100 mobile and 13 anchor sensors by means of Monte Carlo simulations. Finally, we present our concluding remarks.

II. Models

Consider a graph defined by a set of vertices $\mathcal{V}$ and a set of edges $\mathcal{E}$ (cf. Figure 1). Each vertex $v \in \mathcal{V}$ represents a wireless sensor placed randomly in the plane and each edge $(r,t) \in \mathcal{E}$ represents a communication link between sensors r and t, where sensor r receives a signal transmitted from a neighbouring sensor t. The set $\mathcal{V}$ of sensors is divided into a set of anchor sensors $\mathcal{V}_A$ at known, fixed positions and a set of mobile sensors $\mathcal{V}_M$ at unknown positions. The position of sensor v is given by the vector $\boldsymbol{x}_v \in \mathbb{R}^2$.

We describe sensor v's prior knowledge of its position by a circular symmetric Gaussian pdf $p_v(\boldsymbol{x}_v)$ in $\mathbb{R}^2$ with mean $\boldsymbol{\mu}_v = \mathrm{E}_{p_v(\boldsymbol{x}_v)}[\boldsymbol{x}_v]$ and variance $\sigma_v^2 = \frac{1}{2}\mathrm{E}_{p_v(\boldsymbol{x}_v)}\left[\|\boldsymbol{x}_v - \boldsymbol{\mu}_v\|^2\right]$, where $\mathrm{E}_p[\cdot]$ denotes expectation with respect to the pdf p, and $\|\cdot\|$ is the Euclidean norm. In the special case when $v \in \mathcal{V}_A$, $\sigma_v^2 = 0$ and $p_v(\boldsymbol{x}_v)$ reduces to a Dirac's delta function localized at $\boldsymbol{\mu}_v$ in $\mathbb{R}^2$.

If $(r,t) \in \mathcal{E}$, sensor r can obtain sensor t's current position information and a noisy measurement of the distance $d_{r,t}$ between r and t:

$$d_{r,t} = \|\boldsymbol{x}_r - \boldsymbol{x}_t\| + w_{r,t}, \tag{1}$$

where $w_{r,t}$ represents observation noise. In this work, $w_{r,t}$ is a zero-mean Gaussian random variable with variance $\sigma_{r,t}^2$.

Given a network of N sensors, let $\mathcal{X} = \{\boldsymbol{x}_i : i \in \mathcal{V}_M\}$ denote the set of unknown sensor positions. The set $\mathcal{E}$ is obtained as follows: for any $r,t \in \mathcal{V}$, $(r,t) \in \mathcal{E}$ if, and only if, $\|\boldsymbol{x}_r - \boldsymbol{x}_t\| \leq R$. Thus, any two sensors in $\mathcal{V}$ are connected if, and only if, their distance is not larger than a given coverage radius R. The set $\mathcal{D} = \{d_{r,t} \,|\, (r,t) \in \mathcal{E}\}$ contains the distance observations between the connected sensors. In the considered decentralized scheme each mobile sensor only utilizes the distance measurements from the sensors with which it is connected. Notice that, if each sensor in addition has access to information on the network topology, e.g. to know the positions of the sensors connected to its neighbours with which it is connected, a more sophisticated scheme would also exploit the position information inherent to the knowledge of absence of connection [3], e.g. to these neighbours' neighbours.

The joint pdf describing the probabilistic model for the considered scenario reads

$$p(\mathcal{X},\mathcal{D}) = p(\mathcal{D}|\mathcal{X})\,p(\mathcal{X}) \tag{2}$$

$$= \left(\prod_{(r,t)\in\mathcal{E}} p(d_{r,t} \,|\, \boldsymbol{x}_r, \boldsymbol{x}_t)\right)\left(\prod_{v\in\mathcal{V}_M} p_v(\boldsymbol{x}_v)\right), \tag{3}$$

where $p(d_{r,t} \,|\, \boldsymbol{x}_r, \boldsymbol{x}_t)$ is the pdf of the observation $d_{r,t}$ conditioned on the positions of sensors r and t.

III. Message Passing for Localization

A. Message Passing on Factor Graphs

The joint pdf in (3) is representable by a factor graph [9] with local factors

$$f_v(\boldsymbol{x}_v) = p_v(\boldsymbol{x}_v), \tag{4}$$

$$g_{r,t}(\boldsymbol{x}_r, \boldsymbol{x}_t) = p(d_{r,t} \,|\, \boldsymbol{x}_r, \boldsymbol{x}_t). \tag{5}$$

We abbreviate the notation as f_v and $g_{r,t}$ for convenience. For each sensor v, we draw a variable node, representing the sensor's position $\boldsymbol{x}_v$. We connect each $\boldsymbol{x}_v$, $v \in \mathcal{V}_M$ to a factor node f_v, representing the prior position pdf. For each pair of sensors (r,t), for which a distance observation is available, we draw a factor $g_{r,t}$ and connect the variable nodes $\boldsymbol{x}_r$ and $\boldsymbol{x}_t$ to it. In this step, we make the following two assumptions: *a*) For sensors r and t, $(r,t) \in \mathcal{E} \Leftrightarrow (t,r) \in \mathcal{E}$; *b*) Anchor sensors have known positions. Thus, the variable node of an anchor sensor a is only connected to the variable node of a mobile sensor m via $g_{m,a}$ because anchors need not estimate their positions and therefore may disregard messages from neighbour sensors. The result is an undirected graph. As an example, the factor graph depicted in Figure 2 corresponds to the WSN topology of Figure 1.

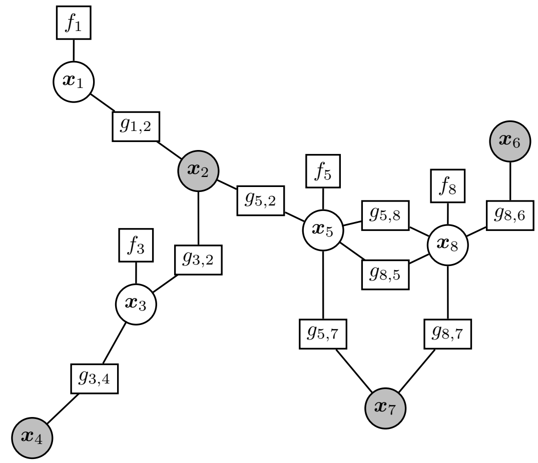

Figure 2. The factor graph that represents the WSN topology in Figure 1. Anchor variable nodes are gray.

The position posterior pdf $p(\boldsymbol{x}_r|\mathcal{D})$ of any mobile sensor $r \in \mathcal{V}_M$ can now be estimated via message passing methods [7]–[14]. Two common message passing methods adapted to graphs as depicted in Figure 2 are displayed in Figure 3: the SP algorithm, which implements BP [9], and VMP, which implements the variational Bayesian method [12].

B. The Sum-Product Algorithm

For continuous hidden variables, evaluation of (6), (8) and (9) (see Figure 3a) can become arbitrarily complex [3], [7], [12], [13]. A way to control this is to restrict the messages passed between the nodes to be Gaussian [7]. Nevertheless, Gaussian SP remains unattractive for the problem of localization adressed in this contribution because the nonlinear sensor relationship in the observation model (1) leads to unwieldy integrals in (8). This difficulty can be remedied via the use of particle based methods, e.g. nonparametric belief propagation [3], [12], [13]. In this approach, messages (6) – (8) in Figure 3a are represented by typically hundreds of real-valued samples. Transmission of such messages imposes substantial communication overhead and collecting the incoming messages require that the sensors be equipped with ample memory hardware.

C. The Variational Message Passing Algorithm

Variational methods aim at approximating a complex or intractable pdf by a simpler pdf [7], [12]. That is, using the notation in Section II, given the set $\mathcal{X}$ of unknown positions $\boldsymbol{x}_i, i \in \mathcal{V}_M$ and the set $\mathcal{D}$ of distance measurements, the posterior pdf $p(\mathcal{X}|\mathcal{D})$ is approximated by a pdf that belongs to a certain family of pdfs satisfying certain constraints that make their computation tractable. The selected pdf $q(\mathcal{X})$ is the one in the family for which the Kullback-Leibler divergence

$$\mathrm{KL}(q(\mathcal{X}) \,\|\, p(\mathcal{X}|\mathcal{D})) = \int_{\mathcal{X}} q(\mathcal{X}) \ln \frac{q(\mathcal{X})}{p(\mathcal{X}|\mathcal{D})}\, d\mathcal{X} \tag{14}$$

a) The sum-product algorithm.

Messages from variable $\boldsymbol{x}_t$ to local factor $g_{r,t}(\boldsymbol{x}_r, \boldsymbol{x}_t)$

$$m_{\boldsymbol{x}_t \to g_{r,t}}(\boldsymbol{x}_t) = \prod_{h \in \mathcal{N}(\boldsymbol{x}_t) \setminus \{g_{r,t}\}} m_{h \to \boldsymbol{x}_t}(\boldsymbol{x}_t), \tag{6}$$

Messages from local factors to variable $\boldsymbol{x}_r$

$$m_{f_r \to \boldsymbol{x}_r}(\boldsymbol{x}_r) = p_r(\boldsymbol{x}_r) \tag{7}$$

$$m_{g_{r,t} \to \boldsymbol{x}_r}(\boldsymbol{x}_r) = \int_{\boldsymbol{x}_t} m_{\boldsymbol{x}_t \to g_{r,t}}(\boldsymbol{x}_t)\; g_{r,t}(\boldsymbol{x}_r, \boldsymbol{x}_t)\; d\boldsymbol{x}_t, \tag{8}$$

Marginal update of the pdf estimate of $\boldsymbol{x}_r$

$$q_r(\boldsymbol{x}_r) = \prod_{h \in \mathcal{N}(\boldsymbol{x}_r)} m_{h \to \boldsymbol{x}_r}(\boldsymbol{x}_r). \tag{9}$$

b) The variational message passing algorithm.

Messages from $\boldsymbol{x}_t$ to $g_{r,t}(\boldsymbol{x}_r, \boldsymbol{x}_t) \in \mathcal{N}(\boldsymbol{x}_t)$

$$m_{\boldsymbol{x}_t \to \mathcal{N}(\boldsymbol{x}_t)}(\boldsymbol{x}_t) = \tfrac{1}{Z} \prod_{h \in \mathcal{N}(\boldsymbol{x}_t)} m_{h \to \boldsymbol{x}_t}(\boldsymbol{x}_t), \tag{10}$$

Messages from local factors to variable $\boldsymbol{x}_r$

$$m_{f_r \to \boldsymbol{x}_r}(\boldsymbol{x}_r) = p_r(\boldsymbol{x}_r) \tag{11}$$

$$m_{g_{r,t} \to \boldsymbol{x}_r}(\boldsymbol{x}_r) = \exp\left(\int_{\boldsymbol{x}_t} m_{\boldsymbol{x}_t \to g_{r,t}}(\boldsymbol{x}_t)\; \ln g_{r,t}(\boldsymbol{x}_r, \boldsymbol{x}_t)\; d\boldsymbol{x}_t \right), \tag{12}$$

Marginal update of the pdf estimate of $\boldsymbol{x}_r$

$$q_r(\boldsymbol{x}_r) = m_{\boldsymbol{x}_r \to \mathcal{N}(\boldsymbol{x}_r)}(\boldsymbol{x}_r). \tag{13}$$

Figure 3. Two message passing algorithms for unconstrained Bayesian inference in a localization factor graph: $\mathcal{N}(\boldsymbol{x}_t)$ denotes the set of factor nodes neighbouring the node $\boldsymbol{x}_t$ and Z is the normalization constant defined in (17).

is minimum. A well-known variant of variational methods is the mean field approximation framework from statistical physics where $q(\mathcal{X})$ is assumed to factorize as $q(\mathcal{X}) = \prod_{\boldsymbol{x}_i \in \mathcal{X}} q_i(\boldsymbol{x}_i)$ [7], [8], [12]. The mean field approximation yields an iterative algorithm that approximates $p(\mathcal{X}|\mathcal{D})$ by separately updating the factors $q_i(\boldsymbol{x}_i)$ in a sequential manner. Note that this factorization tends to produce overly confident marginals in the approximation of the posterior pdf [15, Section 10.1.2]. A message passing interpretation of this algorithm, which we refer to as variational message passing (VMP), is provided in [12].

Similar to BP, the equations (10), (12) and (13) (see Figure 3b) of unconstrained VMP can become arbitrarily complex for continuous hidden variables. A method to harness the complexity is to restrict the messages passed by variable nodes to belong to the family of exponential pdfs [7]. In our work, we restrict these messages to be circular symmetric Gaussian pdfs and demonstrate that, contrary to Gaussian SP, this so-called Gaussian VMP leads to a tractable iterative scheme for distributed localization.

When the message $m_{\boldsymbol{x}_t \to g_{r,t}}(\boldsymbol{x}_t)$ in (12) is restricted to be a Gaussian pdf, the $\ln g_{r,t}$ term in this equation yields the exponent of a Gaussian, and the integral can be computed analytically despite the nonlinear sensor relationships in (1). In this way all messages from factor nodes to variable nodes are given in closed form. Each variable node computes its corresponding product in (10), approximates this product by a circular symmetric Gaussian pdf, and passes the parameters of this pdf as the message to its neighbour factor nodes. For localization in $\mathbb{R}^2$, this amounts to three real values which are broadcast to all neighbouring factor nodes (see (10)). Compared to particle based BP methods, Gaussian VMP messages impose significantly smaller requirements on message communication overhead and sensor memory hardware.

IV. The Gaussian VMP Localization Algorithm

The unconstrained VMP algorithm listed in Figure 3b imposes no restrictions on the messages passed between the nodes in the factor graph. To develop a tractable mean field localization algorithm, we restrict the messages from variable nodes to factor nodes to be in the family $\mathcal{G}$ of circular symmetric Gaussians with mean $\hat{\boldsymbol{x}}_i$ and variance $\hat{\sigma}_i^2$ for the i^{th} node. As a result of this constraint, equations (10) and (13) in Figure 3b must be modified according to (superscript G indicates Gaussian restriction)

$$m^G_{\boldsymbol{x}_t \to \mathcal{N}(\boldsymbol{x}_r)}(\boldsymbol{x}_r) = \underset{q'_r(\boldsymbol{x}_r) \in \mathcal{G}}{\arg\min}\ \mathrm{KL}(q'_r(\boldsymbol{x}_r) \,\|\, \tilde{p}_r(\boldsymbol{x}_r)) \tag{15}$$

with

$$\tilde{p}_r(\boldsymbol{x}_r) = \tfrac{1}{Z} \prod_{h \in \mathcal{N}(\boldsymbol{x}_r)} m_{h \to \boldsymbol{x}_r}(\boldsymbol{x}_r), \tag{16}$$

where Z is the normalization constant

$$Z = \int_{\boldsymbol{x}_r} \prod_{h \in \mathcal{N}(\boldsymbol{x}_r)} m_{h \to \boldsymbol{x}_r}(\boldsymbol{x}_r)\; d\boldsymbol{x}_r \tag{17}$$

and

$$q^G_r(\boldsymbol{x}_r) = m^G_{\boldsymbol{x}_r \to \mathcal{N}(\boldsymbol{x}_r)}(\boldsymbol{x}_r) \tag{18}$$

respectively. The solution to (15) is obtained by finding the postion and variance estimates $\hat{\boldsymbol{x}}_r$ and $\hat{\sigma}_r^2$ of $q'_r(\boldsymbol{x}_r) \in \mathcal{G}$ minimizing $\mathrm{KL}(q'_r(\boldsymbol{x}_r) \,\|\, \tilde{p}_r(\boldsymbol{x}_r))$. These estimates can be obtained by solving the minimization problem using numerical methods.

To compute $\tilde{p}_r(\boldsymbol{x}_r)$, let $\mathcal{V}_r = \{t \in \mathcal{V} : (t, r) \in \mathcal{E}\}$ and recast (16) as

$$\tilde{p}_r(\boldsymbol{x}_r) = \tfrac{1}{Z} m_{f_r \to \boldsymbol{x}_r}(\boldsymbol{x}_r) \prod_{t \in \mathcal{V}_r} m_{g_{r,t} \to \boldsymbol{x}_r}(\boldsymbol{x}_r). \tag{19}$$

From (11)

$$m_{f_r \to \boldsymbol{x}_r}(\boldsymbol{x}_r) \propto \exp\left(-\tfrac{\|\boldsymbol{x}_r - \boldsymbol{\mu}_r\|^2}{2\sigma_r^2} \right), \tag{20}$$

where $\propto$ denotes proportionality. For $t \in \mathcal{V}_r \cap \mathcal{V}_A$,

$$m_{g_{r,t} \to \boldsymbol{x}_r}(\boldsymbol{x}_r) \propto \exp\left(-\tfrac{1}{2\sigma_{r,t}^2} \left(d_{r,t} - \|\boldsymbol{x}_r - \boldsymbol{\mu}_t\| \right)^2 \right). \tag{21}$$

For $t \in \mathcal{V}_r \cap \mathcal{V}_M$, we first substitute (5) into (12) to get

$$m_{g_{r,t} \to \boldsymbol{x}_r}(\boldsymbol{x}_r) = \exp\left(\int_{\boldsymbol{x}_t} q^G_t(\boldsymbol{x}_t) \ln p(d_{r,t} \,|\, \boldsymbol{x}_r, \boldsymbol{x}_t)\; d\boldsymbol{x}_t \right). \tag{22}$$

Initialization
for all sensors (in parallel) **do**
- I1) Broadcast position information, and collect position information broadcast from neighbouring sensors.
- I2) Obtain distance observations $d_{r,t}$ to neighbouring nodes.

end for

Location estimation
repeat
for all sensors (in parallel) **do**
- L1) Compute $\tilde{p}_r(\boldsymbol{x}_r)$ using (19).
- L2) Compute $q'_r(\boldsymbol{x}_r) \in \mathcal{G}$ that minimizes $\mathrm{KL}(q'_r(\boldsymbol{x}_r) \,\|\, \tilde{p}_r(\boldsymbol{x}_r))$.
- L3) Broadcast $\hat{\boldsymbol{x}}_r$ and $\hat{\sigma}_r^2$ and collect corresponding broadcast from neighbouring sensors.

end for
until stopping criterion is reached.

Figure 4. The Gaussian VMP algorithm for distributed self-localization in cooperative wireless networks.

Inserting $q_t^G(\boldsymbol{x}_t)$ in (22) yields

$$m_{g_{r,t}\to\boldsymbol{x}_r}(\boldsymbol{x}_r) \propto \exp\Bigg(-\frac{1}{2\sigma_{r,t}^2}\Big[d_{r,t}^2 - 2d_{r,t}\hat{\sigma}_t\sqrt{\frac{\pi}{2}}\times {}_1F_1\left(-\frac{1}{2};1;-\frac{\|\boldsymbol{x}_r-\hat{\boldsymbol{x}}_t\|^2}{2\hat{\sigma}_t^2}\right) + \|\boldsymbol{x}_r-\hat{\boldsymbol{x}}_t\|^2 + 2\hat{\sigma}_t^2\Big]\Bigg), \quad (23)$$

where ${}_1F_1(a;b;x)$ is the confluent hypergeometric function of the first kind. In this particular case we can write

$${}_1F_1\left(-\frac{1}{2};1;x\right) = \exp\left(\frac{x}{2}\right)\left[(1-x)\mathcal{I}_0\left(-\frac{x}{2}\right) - x\mathcal{I}_1\left(-\frac{x}{2}\right)\right], \quad (24)$$

where $\mathcal{I}_n(\cdot)$ is the modified Bessel function of order n [16]. We note that when $p(d_{r,t}\,|\,\boldsymbol{x}_r,\boldsymbol{x}_t)$ is Gaussian, (22) is proportional to the expectation of the exponent of $p(d_{r,t}\,|\,\boldsymbol{x}_r,\boldsymbol{x}_t)$ with respect to $q_t^G(\boldsymbol{x}_t)$. Equations (15) and (19) define the Gaussian VMP algorithm for sensor self-localiztion, which we list in Figure 4.

V. Simulations

We verify the performance of the Gaussian VMP algorithm in a scenario similar to the one described in [2] by means of Monte Carlo simulations. In this scenario, static anchors are positioned in a structured manner as depicted in Figure 5. Moreover, a sensor has a communication link to all other sensors within a range of 20 m, i.e. $(r,t) \in \mathcal{E}$ and $(t,r) \in \mathcal{E}$ if, and only if, $\|\boldsymbol{x}_r - \boldsymbol{x}_t\| \leq 20$ m. For simplicity, we assume that sensors r and t make the same distance observation $d_{r,t} = d_{t,r}$ and that the observation noise variance is constant and equal for all sensors, i.e. $\sigma_{r,t}^2 = \sigma_w^2$. In each simulation run, 100 static mobiles are uniformly and independently scattered in the area and each mobile sensor estimates its position with the Gaussian VMP localization algorithm depicted in Figure 4.

Figure 6, depicts the estimated mean localization error of the Gaussian VMP algorithm vs. iteration index with σ_w as a parameter. We see that depending on σ_w, the estimated

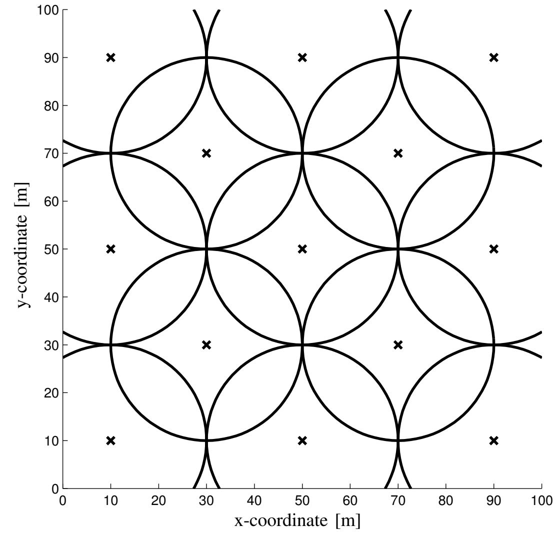

Figure 5. Simulation scenario: A 100 m × 100 m area with 13 anchor sensors (crosses) and their 20 m communication link radii. The scenario is similar to that in [2].

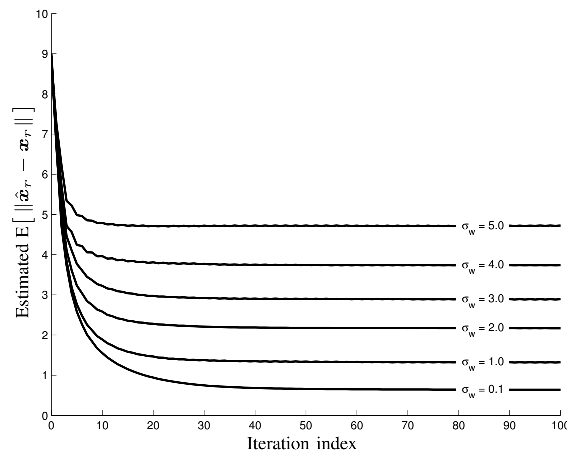

Figure 6. Estimated mean localization error distance with σ_w as a parameter averaged over 100 independent simulation runs.

mean localization error does not change significantly after approximately 10–30 iterations. At this point, each sensor has in total broadcast $3 \cdot 30 = 90$ real values in its messages to the neighbouring sensors. Compared to particle based BP methods (e.g. as proposed in [2] and [3]), the Gaussian VMP algorithm for localization has dramatically lower communication requirements. Furthermore, the plot shows that for $\sigma_w \leq 2.0$ the mean localization error stabilizes at a value that is higher than the standard deviation of the noise. This is caused by sensors for which the position cannot be unambiguously determined, due to the fact that their neighbour sensors are too few and/or the topology of these neighbour sensors does not enable an unambiguous determination of the position. E.g. in the ambiguous case when $\tilde{p}_r(\boldsymbol{x}_r)$ is multimodal with equal-mass modes the VMP algorithm will produce a $q_r^G(\boldsymbol{x}_r)$ which approximates one of these modes selected at random [7]. When $\sigma_w > 2.0$, the algorithm exploits the network topology to mitigate the noise impact on the distance observations and the mean localization error reaches a value less than the

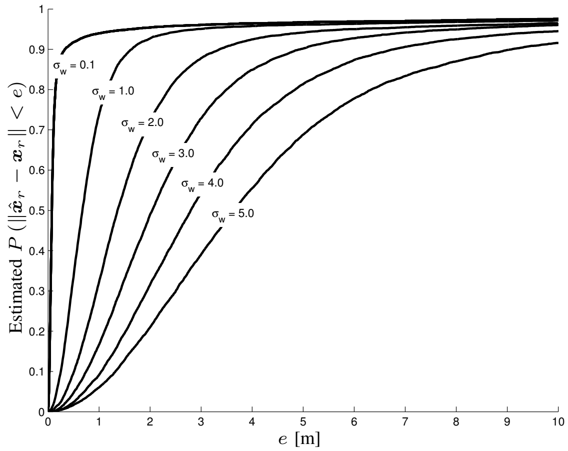

Figure 7. Estimated cumulative localization error probability at iteration 100 with σ_w as a parameter based on 100 independent simulation runs.

standard deviation of the noise.

In Figure 7, we plot the estimated cumulative probability distribution of the localization error. We see that for all noise standard deviations, at least 65 % of the sensors localize with an error less than or equal to the noise standard deviation on average. For an allowable error of 5 m, more than 68 % of the sensors are well localized for all the plotted curves. This percentage increases as σ_w decreases. At $\sigma_w = 0.1$, nearly 97 % of the sensors are localized within 5 m. The fact that the curves in Fig. 7 stabilize to values lower than one results from large errors due to an ambiguity in the estimation of the position of sensors having too few distance observations from their neighbours.

VI. Conclusion

We have proposed a novel low-complexity algorithm for sensor self-localization in cooperative wireless networks. The algorithm is a special implementation of the variational message passing (VMP) method, in which messages from variable nodes to factor nodes are approximated by circular symmetric Gaussian probability densities. Note that in the VMP method these messages coincide with the estimated marginal posterior densities of the node positions. The main virtue of the proposed Gaussian VMP algorithm is a low communication overhead when compared to the corresponding requirements for particle based BP localization schemes. The performance of the algorithm is illustrated in a scenario with static sensors by means of Monte Carlo simulations. When the density of sensors is low, some of them cannot be localized unambiguously as they have too few observations from other sensors. This network topology leads to a non-identifiable estimation problem. In a cooperative setting this problem can be alleviated by exchanging additional information on the network topology, e.g. each sensor gets the positions of sensors connected to its neighbours with which it has connection. The cost of this improvement is a larger communication overhead.

An extension of the Gaussian VMP algorithm to scenarios with moving sensors is currently under investigation. Also, work on comparing the proposed algorithm to particle-based methods is in progress. Further theoretical studies shall be conducted to assess the performance of the algorithm versus network characteristics like link attenuation, standard deviation of the distance measurement, and node density.

VII. Acknowledgment

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