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SYNCHRONIZATION AND ANTI-SYNCHRONIZATION BETWEEN A CLASS OF FRACTIONAL-ORDER AND INTEGER-ORDER CHAOTIC SYSTEMS WITH ONLY ONE CONTROLLER TERM

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ABSTRACT

In order to bridge between fractional-order and integer order nonlinear dynamical system, in this letter, we brings attention to synchronization and anti-synchronization between fractional-order chaotic system and integer-order chaotic system by using back-stepping method. And the sufficient conditions for achieving the synchronization and anti-synchronization of a class of fractional-order nonlinear system and integer-order nonlinear system are derived based on Lyapunov stability theory. Moreover, a new back-stepping control law with only one term is introduced such that a typical example can be realized successfully, which is easier to be applied to industry for its only one controller term. Finally, numerical simulations are provided to verify the effectiveness and feasibility of the proposed control scheme, which are in agreement with theoretical analysis.

Keywords: *Synchronization, Fractional Order, Back-stepping Control*

1. INTRODUCTION

Fractional calculus is a much older classical mathematical notion with the same three-hundred year history as integer calculus. In recent years, it, however, has found application in many areas of physics and engineering [1-2]. Fractional-order chaotic systems are an extension of integer-order chaotic systems developed by mathematicians, and are more universal. As for synchronization, it is based on the closeness of the frequencies of periodic oscillations in two systems, one of which is the drive system, and the other is the response system. Since Chen et al. [3] pioneered that synchronization is a generalized concept of control, study of synchronization is more valuable. Now, the concept of synchronization has been extending to the scope. Anti phase synchronization (APS) [4] is one of the most important conceptions, which can also be interpreted as anti-synchronization (AS). It is a phenomenon that the state vectors of the slave system and the master system are expected to converge to zero when AS appears [5]. In other words, it is the vanishing the sum of the two relevant state variables of the drive system and response system and prevails in symmetrical oscillators. Therefore, it is also challenging and attractive to realize anti-synchronization of two chaotic systems. And some papers have been published [6-9].

Examples of the synchronization of integer-order chaotic systems and the synchronization of fractional-order chaotic systems have been widely reported. Many methods have been used to synchronize chaotic systems including sliding mode control [10-12], linear feedback control [13-14], adaptive control theory, back-stepping control [15-16], active control [17-18], and fuzzy control [19-20]. More specially, some remarkable contributions should be presented again here. For example, Chen et al. [21] investigated the control of a class of four-dimensional chaotic systems with system parameters varying randomly under the condition of noise, which is a typical master work of the control of a class of nonlinear systems. His team also unified the concept of synchronization and anti-synchronization together for a novel class of chaotic systems with different structure and dimensions, although it is suitable for only integer order nonlinear system [22]. Similar result is that Zhou et al. [23] resolve this problem via a novel passive control technique. There are, however, few results on the synchronization between a fractional-order chaotic system and an integer-order chaotic system [24], to our best knowledge.

Motivated by the above discussion, this paper focuses on the synchronization and anti-synchronization between a class of fractional-order chaotic system and integer-order chaotic systems to expand the applicability of the theory. A new back

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stepping control method with only one controller is proposed to illustrate the effectiveness of the scheme. We report results from numerical computations and theoretical analysis which are a perfect bridge between fractional-order chaotic systems and integer-order chaotic systems. As the synchronization of integer-order chaotic systems and fractional-order chaotic systems are employed extensively in research and engineering applications, we expect our theory to be potentially useful.

2. PRIMARY

The popular definition of fractional derivatives is given by

$$D_*^{\alpha}x(t)=J^{n-\alpha}x^{(n)}(t)$$

where $n:=[\alpha]$ is the first integer which is not less than α and $\alpha>0$ but not necessarily $\alpha\in N$, $x^{(n)}(t)$ is the ordinary nth derivative of $x(t)$.

Here, we consider the fractional differential equation with initial conditions

$$\begin{cases} D_*^{\alpha}x(t)=f(t,x(t)), & 0\le t<T \\ x^{(k)}(0)=x_0^k, & k=0,1,2,...,n-1 \end{cases}$$

It is equivalent to the Volterra integral equation

$$x(t)=\sum_{k=0}^{[\alpha]-1}x_0^k\frac{t^k}{k!}+\frac{1}{\Gamma(\alpha)}\int_0^t(t-\tau)^{\alpha-1}f\left(\tau,x(t)\right)d\tau$$

Set $h=\frac{T}{N}$, $t_j=jh$, $j=0,1,...,N\in Z^+$. Then this equation can be discredited as follows

$$x_h(t_{n+1})=\sum_{k=0}^{[\alpha]-1}x_0^{(k)}\frac{t_{n+1}^k}{k!}+\frac{h^{\alpha}}{\Gamma(\alpha+2)}f(t_{n+1},x_h^p(t_{n+1}))+\frac{h^{\alpha}}{\Gamma(\alpha+2)}\sum_{j=0}^{n}a_{j,n+1}f\left(t_j,x_h(t_j)\right)$$

where

$$a_{j,n+1}=\begin{cases} n^{\alpha+1}-(n-\alpha)(n+1)^{\alpha}, & j=0 \\ (n-j+2)^{\alpha+1}+(n-j)^{\alpha+1}-2\left(n-j+1\right)^{\alpha+1}, & 1\le j\le n \\ 1, & j=n+1 \end{cases}$$

$$x_h^p(t_{n+1})=\sum_{k=0}^{[\alpha]-1}x_0^{(k)}\frac{t_{n+1}^k}{k!}+\frac{1}{\Gamma(\alpha)}\sum_{j=0}b_{i,n+1}f(t_j,x_h(t_j))$$

$$b_{j,n+1}=\frac{h^{\alpha}}{\alpha}((n-j+1)^{\alpha}-(n-j)^{\alpha}),\quad 0\le j\le n$$

This is called the Adams-Bashforth-Moulton predictor-corrector scheme, which is a time-domain approach, and is effective for investigating the dynamics of fractional-order systems.

3. MAIN RESULTS

In this letter, we consider a class of three-dimensional fractional-order chaotic drive system, which is described as follows:

$$\begin{cases} \frac{d^qx_1}{dt^q}=x_2\cdot f(x_1,x_2,x_3)-\alpha x_1 \\ \frac{d^qx_2}{dt^q}=g(x_1,x_2,x_3)-\beta x_2 \\ \frac{d^qx_3}{dt^q}=x_2\cdot h(x_1,x_2,x_3)-\gamma x_3 \end{cases},\qquad (1)$$

where q is fractional order satisfying 0<q<1; x_1, x_2 and x_3 are state variables. And α is any negative or positive value; β and γ are non-negative constants. The three functions ($f(\cdot)$, $g(\cdot)$ and $h(\cdot)$) are regarded as smooth functions which all belong to $R^3\to R$ space.

Remark 1. Note that many fractional-order chaotic systems belong to the class of system characterized by Eq. (1). Examples include the unified chaotic system of fractional-order version and fractional-order Qi system, and so on.

And the response system can be described as

$$\begin{cases} \frac{d^{q_r}y_1}{dt^{q_r}}=y_2\cdot f(y_1,y_2,y_3)-\alpha y_1 \\ \frac{d^{q_r}y_2}{dt^{q_r}}=g(y_1,y_2,y_3)-\beta y_2 \\ \frac{d^{q_r}y_3}{dt^{q_r}}=y_2\cdot h(y_1,y_2,y_3)-\gamma y_3 \end{cases}\qquad (2)$$

Here, if q_r =1, system (2) is an integer-order system. Otherwise, if 0< q_r <1, system (2) is a fractional-order system.

Remark 2. If system (2) is an integer-order system, synchronization between a novel class of fractional-order system and its integer-order system is realized. On the other hand, if system (2) is a fractional-order system, synchronization between fractional-order chaotic systems can be also realized.

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One adds the controller $u(t)\in R$ into the integer-order system, which is given by

$$\begin{cases} \dfrac{dy_1}{dt}=y_2\cdot f(y_1,y_2,y_3)-\alpha y_1+u_{1y_1}(t)+u_{2y_1}(t) \\ \dfrac{dy_2}{dt}=g(y_1,y_2,y_3)-\beta y_2+u_{1y_2}(t) \\ \dfrac{dy_3}{dt}=y_2\cdot h(y_1,y_2,y_3)-\gamma y_3+u_{1y_3}(t) \end{cases} \tag{3}$$

where $u_2(t)\in R$ is a vector control function that will be designed later. The $u_1(t)\in R^n$ is a compensation controller, which is

$$\begin{cases} u_{1y_1}(t)=\dfrac{dy_1}{dt}-\dfrac{d^q y_1}{dt^q} \\ u_{1y_2}(t)=\dfrac{dy_2}{dt}-\dfrac{d^q y_2}{dt^q} \\ u_{1y_3}(t)=\dfrac{dy_3}{dt}-\dfrac{d^q y_3}{dt^q} \end{cases}. \tag{4}$$

Moreover, $u_1(t)$ and $u_2(t)$ are independent with each other. To achieve the control law $u_2(t)$.

3.1 Synchronization

First, we bring attention to the synchronization in this subsection. Therefore, the synchronization error is usually defined as

$$e_i=y_i-x_i\,. \tag{5}$$

According to Eq. (4), the response system (3) can be rewritten as

$$\begin{cases} \dfrac{d^q e_1}{dt^q}=\phi(e_1,e_2,e_3)-\alpha e_1+u_{2y_1}(t) \\ \dfrac{d^q e_2}{dt^q}=p(e_1,e_2,e_3)-\beta e_2 \\ \dfrac{d^q e_3}{dt^q}=q(e_1,e_2,e_3)-\gamma e_3 \end{cases} \tag{6}$$

where $\phi(\cdot)$, $p(\cdot)$ and $q(\cdot)$ are regarded as smooth functions which belong to $R^3\to R$ space.

Theorem 1. As to the fractional-order system, if there is a real symmetric positive definite matrix P satisfying the equation $J=y^T P\dfrac{d^q y}{dt^q}\le 0$ with any state variables y ($y=(y_1,y_2,y_3)^T$), the system is asymptotically stable.

Theorem 2. According to the back-stepping method, the error dynamics on is asymptotically stable, as long as the vector controller is designed as

$$u_{2y_1}(t)=(\alpha-1)e_1-\phi(e_1,e_2,e_3)$$
$$-e_1^{-1}e_2\cdot p(e_1,e_2,e_3)-e_1^{-1}e_3\cdot q(e_1,e_2,e_3)$$

Proof . Based on back-stepping method, function J is designed as follows

Step 1: Define $s_1=e_1$, one has

$$\begin{aligned} J_1&=s_1\frac{d^q s_1}{dt^q}=e_1\frac{d^q e_1}{dt^q} \\ &=e_1\cdot\phi(e_1,e_2,e_3)-\alpha e_1^{\,2}+e_1u_{2y_1}(t) \end{aligned} \tag{7}$$

Step 2: Define $s_2=e_2$, one has

$$\begin{aligned} J_2&=s_1\frac{d^q s_1}{dt^q}+s_2\frac{d^q s_2}{dt^q} \\ &=e_1\cdot\phi(e_1,e_2,e_3)-\alpha e_1^{\,2}+e_1u_{2y_1}(t) \\ &+e_2\cdot p(e_1,e_2,e_3)-\beta e_2^{\,2} \end{aligned} \tag{8}$$

Step 3: Define $s_3=e_3$, one has

$$\begin{aligned} J_3&=s_1\frac{d^q s_1}{dt^q}+s_2\frac{d^q s_2}{dt^q}+s_3\frac{d^q s_3}{dt^q} \\ &=e_1\cdot\phi(e_1,e_2,e_3)-\alpha e_1^{\,2}+e_1u_{2y_1}(t) \\ &+e_2\cdot p(e_1,e_2,e_3)-\beta e_2^{\,2}+e_3\cdot q(e_1,e_2,e_3)-\gamma e_3^{\,2} \\ &=-\alpha e_1^{\,2}-\beta e_2^{\,2}-\gamma e_3^{\,2}+e_1\cdot\phi(e_1,e_2,e_3) \\ &+e_2\cdot p(e_1,e_2,e_3)+e_3\cdot q(e_1,e_2,e_3)+e_1u_{2y_1}(t) \end{aligned} \tag{9}$$

According to theorem 2, the vector controller is designed as

$$\begin{aligned} &u_{2y_1}(t)=(\alpha-1)e_1-\phi(e_1,e_2,e_3) \\ &-e_1^{-1}e_2\cdot p(e_1,e_2,e_3)-e_1^{-1}e_3\cdot q(e_1,e_2,e_3) \end{aligned} \tag{10}$$

From the Eq.(9) and (10), one has

$$\begin{aligned} J_3&=s_1\frac{d^q s_1}{dt^q}+s_2\frac{d^q s_2}{dt^q}+s_3\frac{d^q s_3}{dt^q} \\ &=-e_1^{\,2}-\beta e_2^{\,2}-\gamma e_3^{\,2}\le 0 \end{aligned} \tag{11}$$

Therefore, the synchronization between system (2) and system (1) is realized.

In the follow-up content, we present an illustrative example to verify and demonstrate the effectiveness of the proposed control scheme. The simulation results are carried out using Adams-Bashforth-Moulton scheme with time step size 0.01.

Case: Synchronization between fractional-order and integer-order Lorenz systems

The fractional-order Lorenz system, which is shown in Fig. 1a, is described as

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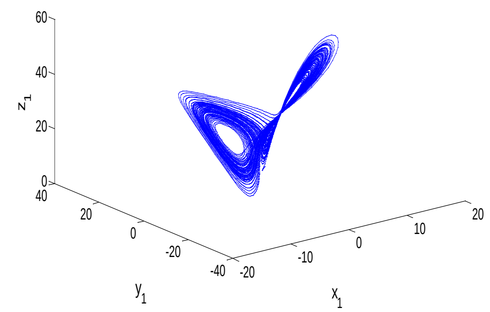
$$\begin{cases} \dfrac{d^q x_1}{dt^q} = a(x_2 - x_1) \\ \dfrac{d^q x_2}{dt^q} = x_1(b - x_3) - x_2 \\ \dfrac{d^q x_3}{dt^q} = x_1 x_2 - c x_3 \end{cases} \tag{12}$$

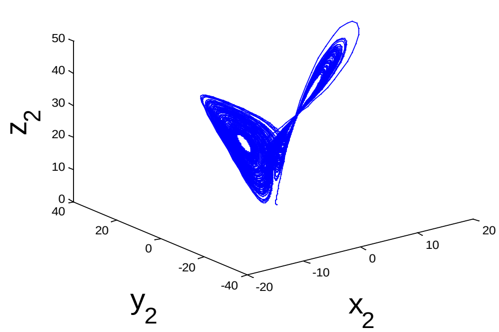
The integer-order Lorenz system, which is shown in Fig. 1b, with controllers are described as

$$\begin{cases} \dfrac{dy_1}{dt} = a(y_2 - y_1) + u_{1y_1}(t) + u_{2y_1}(t) \\ \dfrac{dy_2}{dt} = y_1(b - y_3) - y_2 + u_{1y_2}(t) \\ \dfrac{dy_3}{dt} = y_1 y_2 - c y_3 + u_{1y_3}(t) \end{cases} \tag{13}$$

where y_1, y_2 and y_3 are state variables, and b, c are non-negative constants. With the parameters a=10, b=28, c=8/3 and q=0.9, the fractional-order system exhibits chaotic attractors.



a. Integer-order chaotic system.



b. Fractional-order system

Fig. 5. Chaotic attractors of integer-order and fractional-order chaotic Lorenz systems with initial conditions (x1, y1, z1) = (1, 0, 9) and (x2, y2, z2) = (1, 1, 1) respectively.

Regarding (10), the control law is given as follows

$$u_{2y_1}(t) = (a-1)\cdot e_1 - (a + b - y_3)\cdot e_2 - y_2 e_3 \tag{14}$$

One has

$$J_3 = s_1 \frac{d^q s_1}{dt^q} + s_2 \frac{d^q s_2}{dt^q} + s_3 \frac{d^q s_3}{dt^q} = -e_1^2 - e_2^2 - c e_3^2 \le 0 \tag{15}$$

The error dynamics under the controller (14) can be written as

$$\begin{cases} \dfrac{d^q e_1}{dt^q} = -e_1 - (b - y_3)\cdot e_2 - y_2 e_3 \\ \dfrac{d^q e_2}{dt^q} = (b - y_3)\cdot e_1 - e_2 - x_1 e_3 \\ \dfrac{d^q e_3}{dt^q} = y_2 e_1 + x_1 e_2 - c e_3 \end{cases} \tag{16}$$

Without loss of generality, in the simulation we choose the initial conditions of the system [0.1, 0.1, 0.1]. The error dynamics (16) are illustrated in Fig. 2, which shows that the control law guarantees finally synchronization and stabilization. Similarly, we can also get the phase trajectories of synchronization, which is in detail shown in Fig. 3.

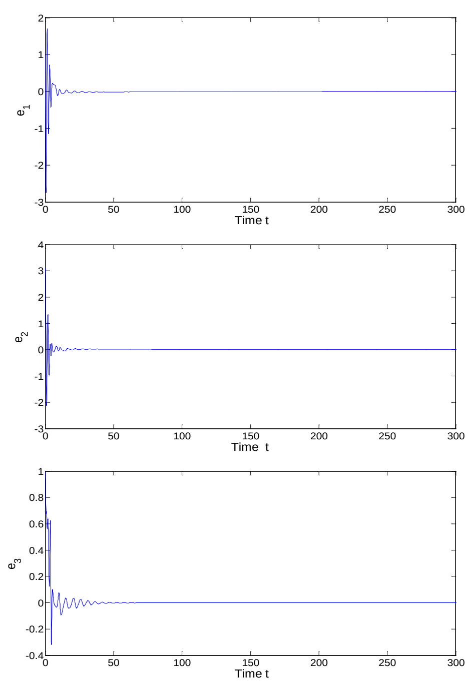


Fig. 2. Synchronization Errors Between Fractional-Order Lorenz Chaotic System And Integer-Order Lorenz Chaotic System.

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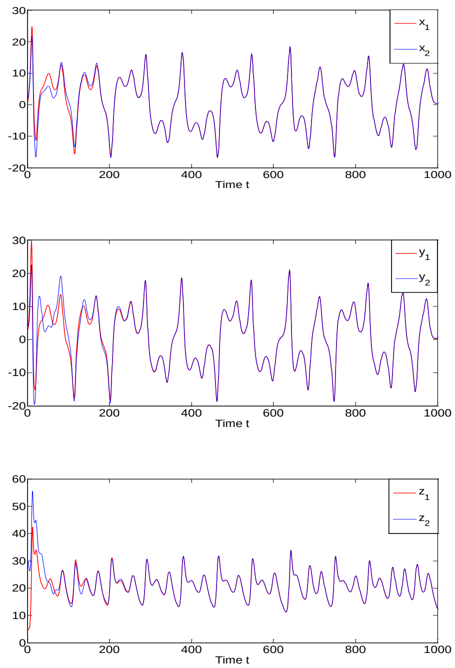

Fig. 3. Trajectories Of The State Variables In Drive And Response Systems In Process Of Synchronization.

3.2 Anti-synchronization

In this subsection, we mainly concern anti-synchronization. Actually, the main difference between synchronization and anti-synchronization is the error system. Therefore, the error system of anti-synchronization can be defined as follows, according the definition of anti-synchronization.

$$e_i = y_i + x_i \tag{17}$$

Thus, the anti-synchronization between the response system and drive system can be converted to the stability of error system. In other words, we should design a suitable controller $u_{2y_1}(t)$ to control the error system to (0, 0, 0).

$$\begin{cases} \dfrac{d^q e_1}{dt^q} = \phi(e_1,e_2,e_3) - \alpha e_1 + u_{2y_1}(t) \\ \dfrac{d^q e_2}{dt^q} = p(e_1,e_2,e_3) - \beta e_2 \\ \dfrac{d^q e_3}{dt^q} = q(e_1,e_2,e_3) - \gamma e_3 \end{cases} \tag{18}$$

According to **Theorem 1 and Theorem 2**, we can easily get the generalized controller for anti-synchronization of this class of chaotic system described in Eq. (1), which is

$$u_{2y_1}(t) = (\alpha - 1)e_1 - \phi(e_1,e_2,e_3)$$
$$-e_1^{-1}e_2 \cdot p(e_1,e_2,e_3) - e_1^{-1}e_3 \cdot q(e_1,e_2,e_3)$$

And similar proof process, we can also get the conclusion

$$J_3 = s_1 \frac{d^q s_1}{dt^q} + s_2 \frac{d^q s_2}{dt^q} + s_3 \frac{d^q s_3}{dt^q}$$
$$= -e_1^{\,2} - \beta e_2^{\,2} - \gamma e_3^{\,2} \le 0$$

Therefore, the anti-synchronization between system (2) and system (1) is strictly realized.

For a specific example, anti-synchronization between fractional-order and integer-order Lorenz systems, the control law is given as follows

$$u_{2y_1}(t) = (a-1)\cdot e_1 - (a+b-y_3)\cdot e_2 - y_2 e_3 \tag{11}$$

The numerical results can be easily got, which is shown in Fig. 4 and Fig. 5. From Fig. 4, the Trajectories of anti-synchronization of the state variables in drive and response systems can effectively prove the validity of the proposed control method for anti-synchronization between fractional-order and integer order chaotic system. Furthermore, Fig. 5 is a perfect complement.

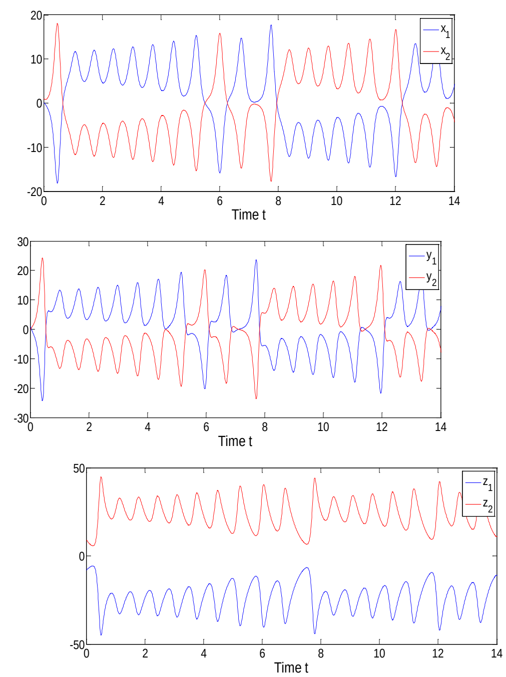

Fig. 4. Trajectories Of Anti-Synchronization Of The State Variables In Drive And Response Systems.

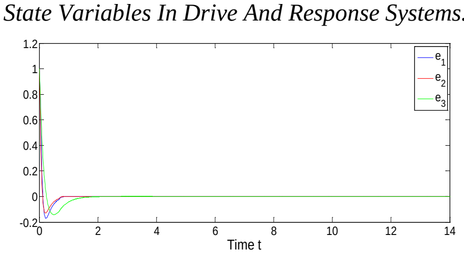

Fig.5. Anti-Synchronization Errors Between Integer-Order And Fractional-Order Chaotic Lorenz Systems.

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4 CONCLUSION

In summary, the synchronization and anti-synchronization of a class of fractional-order systems and integer-order systems is investigated. And a new back-stepping method with only one term is presented. More importantly, it is rigorously proven that the proposed synchronization approach can be achieved between fractional-order and integer-order systems with only one controller. Numerical simulations are presented to show the applicability and feasibility of the proposed scheme.

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