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A Preference-based Resource Allocation In Cloud Computing Systems

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Abstract

The 'Pay-as-per-use' model of cloud computing requires a user to rent cloud resources or services for a period of time and make payment accordingly. This paper proposes a demand-based preferential resource allocation technique that designs a market-driven auction mechanism to identify users for resource allocation based on their payment capacities and implements a payment strategy based on a buyer's service preferences. A comparison is drawn between the proposed allocation technique and the famous off-line VCG auction mechanism and results show a performance benefit in revenues to service provider, payments of cloud users besides ensuring an optimum resources use.

1. Introduction

Efficient resource allocation is a major concern in utility-based systems such as cloud computing. Cloud users approach a cloud service provider to execute their tasks which require cloud resources in various measures. In return, users pay for the resources utilized by them. Resources, such as, memory, processing units, network bandwidth, are embodied in virtual machines (VMs). Resources and VMs are used interchangeably in this paper. In order to service multiple users simultaneously with varied resource requirements, a cloud provider needs to implement an efficient resource allocation technique which reflects the demand-supply scenario of the market along with being beneficial to both the service provider and the service user. Offline and online auctions are considered one of the best ways of resource allocation where a user is selected based on his/her payment capacity and resource requirements [11]. In cloud computing, auction is a method of selling cloud resources in a public forum through competitive bidding. In general, a cloud auction requires cloud users to bid for their required resources. Bid prices usually reflect the resource needs and paying capacity of a user. Cloud service provider will select user(s) based on their bids and allocate resources to them in such a manner that profit to service provider is ensured as well as Quality of Service parameters to auction winners is also maintained. These selected users will be termed as 'winners' in the present work
A resource auction in cloud datacenters can be 'open' or 'closed' [7]. Once winners are determined, service provider allocates resources and charges the winners for the same based on their bidding price, task duration, preferences, etc.

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This paper presents a resource allocation technique which is divided into two steps, a market-driven auction process which ensures truthfulness and profit to the service provider and a preference- driven payment process which ensure that the winner pays lesser amount than his bid value provided that his bidding reflects his best paying capacity.

Further, performance of this proposed resource allocation technique is compared with the well known off-line VCG mechanism [20] and obtained results verify that the presented technique is better than VCG in terms of efficient resource allocation, fairness to user and improved revenue to a cloud service provider.

2.　Related Work

The 'pay-as-you-use' model of computing attracts many users due to its various benefits like cost-efficiency, low maintenance, improved flexibility, etc. However, it also highlights an important issue of effective resource management which must ensure a truthful, fair and profit-reaping allocation of resources to users. A dynamic resource allocation scheme is proposed in [1] which introduces a non pre-emptive pricing and allocation scheme for batch jobs with the aim of maintaining social welfare. A performance comparison between offline VCG mechanism and a proposed truthful auction mechanism is shown in [2] which guarantee profit to a service provider. A three tier admission control and scheduling mechanism is presented in [3] which involves cloud users, a SaaS provider and a public IaaS to minimize a provider's expenditure and improve a user's experience in utilizing cloud resources. QoS based resource provisioning mechanism is given in [4] based on the Dirichlet multinomial model claiming to reduce cost of computing resources. Other allocation technique based on QoS is given in [5] which considers allocation as an optimization problem and uses a combination of batch matching and reverse auction to deal with it. Combinatorial auctions are proposed in [6] which guarantee cost efficiency and truthfulness by using price vector space. Another variation of resource auction in cloud computing is presented in [7] using Continuous Double Auction mechanism for order matching and trading.. SLA-based resource management and allocation strategy is studied in [12] presenting its elements and various challenges. Reservation-based and ad-hoc pricing of resources is discussed in [8] and a deterministic equivalent formulation is designed using sample-average approximation and Bender's decomposition for the same. Resource allocation strategies given in [9, 17] use periodic auction of computing resources to match with the dynamic user requirements.

A detailed survey on various QoS parameters in cloud computing is done in [10] outlining their strengths and limitations. Amazon's Spot-market auction mechanism is studied in [11] and a dynamic resource allocation algorithm is developed to increase a service provider's revenue. A threshold-driven learning based algorithm is presented in [13] which admit users based on a service provider's overload threshold. Resource management problem is considered in [14, 15] where the main revenue maximization is considered as the main objective. Additionally, negotiations of resource leasing contracts between a service provider and users are explored in [15] to improve their proposed allocation strategy. Benders decomposition and sample-average approximation methods are implemented in [16] to solve reservation-based resource allocation problems. Further, in [18] a multi-time period optimization model is designed to reduce expenditures of a service provider by turning off servers over a time period and adjusting their work-loads. A truthful and dynamic online resource auction is presented in [19] which avoids false bidding behaviors of users and reflects the supply-demand curve of various resource types. The much celebrated offline VCG auction mechanism along with its variants is explained in [20]. Most of the work related to resource distribution in cloud computing favors an online truthful auction mechanism with the aim of either maximizing service provider's revenue or minimizing their operational cost. The resource allocation technique proposed in the present work ensures fair payment strategy to selected users and revenue maximization to cloud service provider, along with reflecting the demands of every resource type. Further, comparison of the presented technique with offline VCG auction mechanism shows a considerable increase in provider's revenue and decrease in a user's payment amount.

3.　System Model

The proposed resource allocation technique is divided into two phases- an open market-driven auction process followed by preference-driven payment process. Main entities involved in the resource allocation process are a single cloud service provider and N cloud users. Cloud service provider maintains multiple heterogeneous resources such as memory, processing units,

network bandwidth, etc. in the form of virtual machines (VMs). When these VMs are requested by users to accomplish their tasks, cloud provider allocates them depending on user's payment capacity and preferences. In return the user pays for these VMs as per the quantity and duration used.

Keeping in mind the varieties of users' tasks such as input-output intensive, processor-intensive or memory-intensive, our proposed resource allocation model considers three types of VMs (denoted by K), namely, communication-driven, processing-driven and storage-driven VMs. Hence, the value of K (VM type) is taken to be 3 as K1 (communication-driven VMs), K2 (processing-driven VMs) and K3 (storage-driven VMs). Total expenditure incurred by a cloud provider for resource allocation includes VM initiation cost (IC_{VMtype}), task processing cost (TPC), data transfer cost (DTC) and data storage cost (DSC). Cloud provider adds these costs as additional cost (AC). On the other hand, a Cloud user maintains his unique id and declares his task arrival time (AT) along with bid-prices of VM instances at the commencement of auction process. Bid prices are indicated by BP_{VMtype}. Table 1, given below, summarizes the symbols used in the paper along with their meanings.

Table 1: Summary of symbols used and their meanings

Symbol	Meaning	Symbol	Meaning
N	Number of Cloud Users denoted as $n_1, n_2 \ldots n_N$	SP_{K1}	Starting Price of K1 VM instance
K	Types of VMs considered as K1, K2, K3	SP_{K2}	Starting Price of K2 VM instance
BP_{K1}	User's Best-Bid price for K1 VM instance	SP_{K3}	Starting Price of K3 VM instance
BP_{K2}	User's Best-Bid price for K2 VM instance	R_{K1}	Number of K1 instances required by a user
BP_{K3}	User's Best-Bid price for K3 VM instance	R_{K2}	Number of K2 instances required by a user
RAU	Resource Allocation Unit	R_{K3}	Number of K3 instances required by a user
IC_{K1}	Initiation Cost of K1 VM instance	DSC	Data Storage Cost
IC_{K2}	Initiation Cost of K2 VM instance	DTC	Data Transfer Cost
IC_{K3}	Initiation Cost of K3 VM instance	AC	cloud service provider's Additional Cost
TPC	Task Processing Cost	TBP	Total Bid Price of a user
AT	Task Arrival Time	BPP	Bid-Price Payment
T	Duration of one allocation round		

The proposed resource allocation system model is shown in figure 1. There are N users, denoted as $n_1, n_2, n_3, \ldots$ and n_N. A single cloud provider manages three types of VMs as K1, K1 and K3. Service provider maintains updated information about the availability and demands of these VMs. It helps the provider in determining the starting price (SP_{VMtype}) of each VM type at the start of auction.

The Resource Allocation Unit (RAU) is divided into two sub-units as-

1. Auction Sub-unit
2. Payment Sub-unit

The purpose of auction sub-unit is to determine winners among bidders and payment sub-unit charges winners according to their preferences and bid prices. Detailed functioning of these two sub-units is explained in the next section.
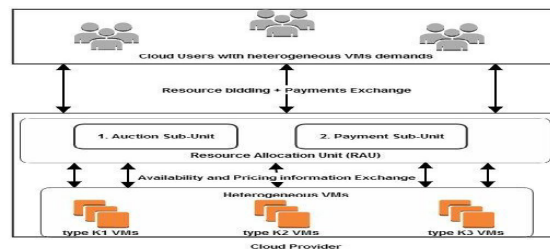

Figure 1: Resource Allocation System Model

4. Proposed Resource Allocation Technique

The proposed resource allocation technique is a periodic process where a cloud service provider auctions cloud resources (VMs) to users and charges them for the quantity of resources used. The duration of a single round of allocation is set as T and each round consists of three steps. The current allocation round is denoted as i. Step 1 describes a pre-auction process followed by a market-driven open auction in step 2 and finally in step 3 a user's payment mechanism is explained. Detailed description of each step in a single round of the proposed allocation technique is given below.

4.1 Pre-auction- An ith round of the resource allocation technique starts at time t=0 when a cloud service provider establishes the starting price of each type of VM instance, i.e. SP_{K1}, SP_{K2}, SP_{K3} using equation 1. Starting price is also considered as reserve price for a resource type in the present work.

$$\left(SP_{VMtype}\right)_i = \left(MeanBP_{VMtype}\right)_{i-1} \qquad \text{... (1)}$$

where $(SP_{VMtype})_i$ = SP_{K1}/ SP_{K2}/ SP_{K3} in the ith round of allocation process and
(Mean $BP_{VMtype})_{i-1}$ = $MeanBP_{K1}$/$MeanBP_{K2}$/$MeanBP_{K3}$ in the $(i-1)^{th}$ round of allocation.
Equation 1 states that the starting price of a VM type instance in the current allocation round will be based on the average bid price of the same VM type instance in the previous allocation round. For example, suppose the mean bid-price for an instance of VM type K3 in the last allocation round was P, then at the commence of next auction round, starting price of K3 type VM instance is set as P. A resource's bid-price reflects its current demand. Hence, this strategy of setting the last mean bid price as the next starting price of a resource instance follows the market supply-demand scenario and justifies a resource's utility. Therefore, the allocation presented in this paper is a 'market-driven' technique. Besides establishing the starting price, a cloud service provider also calculates the additional cost incurred by him during the resource allocation process as-

$$AC = DSC + DTC + TPC + IC_{VMtype} \qquad \text{... (2)}$$

4.2 Market-driven Open Auction- The 2nd step of resource allocation technique is an auction process which is divided into two halves. At t=0, the service provider declares the starting prices of resources (calculated in step 4.1) to the public and invites bid-prices from interested users for each required VM type, such that

$$BP_{VMtype} \geq SP_{VMtype}$$

Bid Price offered by a user indicates his highest payment capacity for a resource type and it will increase his chances of winning the auction. Hence, we also term it as a user's 'best-bid'. Also if a user wins the auction process, then his actual payment will be less than his best bid payment (calculated in the following step 4.3). Interested users or bidders submit their task's VM requirements along with their best bid for each VM type as per the format given below in figure 2. Here, TBP is the calculated sum of BPK1, BPK2 and BPK3.

User Id	AT	RK1	BPK1	RK2	BPK2	RK3	BPK3	TBP

Figure 2: User's bid format

At time t = T/2, i.e. midway in the allocation round, service provider calculates the mean total bid of all the bidders for the duration 0 to T/2, using equation 3 and mean bid price of each VM type using equation 4.

$$MeanTBP = \frac{\sum_{i=1}^{x} TBP_i}{x} \qquad \text{... (3)}$$

$$MeanBP = \frac{\sum_{i=1}^{x} \left(BP_{VMtype}\right)_i}{x} \qquad \text{... (4)}$$

where x is the total number of bidders from time t=0 to t=T/2. Bidders whose total bid price (TBP) is equal to or higher than the mean total bid price (MeanTBP), are considered winners for allocation from t=0 to t=T/2. As an example to illustrate the first half of an auction process, consider 3 bidders with their bid formats as given in figure 3.

n1	1	6	10	3	15	5	8	33

n2	2	3	12	6	14	2	10	36

n3	1	8	11	7	10	2	13	34

Figure 3: Example for winner determination

As shown in figure 3, cloud users n1, n2 and n3 state their individual VM requirements, individual bid prices for each VM type instance and total bid price [for format refer figure 2]. MeanTBP for x as 3 is calculated using eq^n 3 as

$$Mean\ TBP = \frac{\sum_{i=1}^{3} TBP_i}{3} = \frac{33+36+34}{3} = 34$$

Also, 3 as

$$MeanBP_{K1} = \frac{10+12+11}{3} = 11$$

Similarly, $$MeanBP_{K2} = \frac{15+14+10}{3} = 13$$

and $$MeanBP_{K3} = \frac{8+10+13}{3} = 10$$

Users n2 and n3 are selected as winners for first half of the auction as their offered TBP >= MeanTBP.

Unselected users (like n1 in example) are asked to revise their bid prices such that revised $BP_{VMtype} >= MeanBP_{VMtype}$ as calculated in eq^n 4 and rebid for the second half of the auction round i.e. from t=T/2 to t=T. As per the example in figure 3, user n1 is allowed to rebid for the second half of the current auction round i, with an increased bid price of each VM type. Finally, at time t=T, service provider repeats the winner determination procedure by calculating MeanTBP of all users who have submitted their bids between t=T/2 to t=T. These users include fresh bidders and also unselected bidders of the first half of auction round i. MeanTBP calculation is done as given in eq^n 3 for t=T/2 to t=T. Bidders whose TBP is equal or higher than the meanTBP are declared winners. These two winner-lists (first half and second half) are merged and arranged in non-increasing order of their TBPs. The purpose of dividing auction in two halves to declare two winner-lists is to give another chance to unselected users to rebid with an increased bid price. This will not only increase the profitability of the service provider but will also be fair to users. Hence, this auction strategy is termed as 'open' auction [7]. Note that the $MeanBP_{VMtype}$ calculated using eq^n 4 in the second half of auction will be set as SP_{VMtype} in the next $(i+1)^{th}$ round of resource allocation technique as stated in step 4.1.

4.3 Preference-driven Payment- Merged list of winners, arranged in decreasing order of TBPs, is forwarded to step 4.3 where payments are made by these winners. Winner's actual payment is calculated based on his chosen preference which the service provider offers as shown in table 2.

Table 2: Preferences Table

Task Deadline Option	Service Time Option	VMs Possession Option
D1: Fixed	S1: Immediate	P1: Full-time
D2: Flexible	S2: Flexible	P2: Partial

Every winner is given a flexibility to choose their task's deadline, service time and possession options. For example, if a winner's task deadline is flexible, but he needs immediate allocation of VMs and will hold all allocated VMs till his task is over, he will choose preference D2S1P1. A winner's preference will decide his actual payment (AP) criteria as outlined Table 3. Calculation of actual payment based on preference is also given in Table 3.

Table 3: Payment Table

Preferences	Actual Payment Criteria	Actual Payment (AP) Calculation by a winner i
D1S1P1	BP of the paying winner (i) + AC	$AP_i = \left[(RK1 * BPK1)_i + (RK2 * BPK2)_i + (RK3 * BPK3)_i \right] + AC$
D1S1P2	BP of the paying winner (i)	$AP_i = \left[(RK1 * BPK1)_i + (RK2 * BPK2)_i + (RK3 * BPK3)_i \right]$
D2S1P1	BP of the next winner (p) in the list	$AP_i = \left[(RK1_i * BPK1_p) + (RK2_i * BPK2_p) + (RK3_i * BPK3_p) \right]$
D2S1P2	BP of 2nd next winner (q) in the list	$AP_i = \left[(RK1_i * BPK1_q) + (RK2_i * BPK2_q) + (RK3_i * BPK3_q) \right]$
D2S2P1	BP of last winner (r) in the list	$AP_i = \left[(RK1_i * BPK1_r) + (RK2_i * BPK2_r) + (RK3_i * BPK3_r) \right]$
D2S2P1	MeanBP of each VM types	$AP_i = \left[(RK1_i * MeanBP_{K1}) + (RK2_i * MeanBP_{K2}) + (RK3_i * MeanBP_{K3}) \right]$

Actual payment made by a winner i is compared against his bid-price payment (BPP) which is calculated using equation 5.

$$BPP_i = \left[(RK1 * BPK1)_i + (RK2 * BPK2)_i + (RK3 * BPK3)_i \right] \quad \text{... (5)}$$

Further, utility of resource allocation for a winner is given by equation 6, as-

$$Utility_i = BPP_i - AP_i \quad \text{... (6)}$$

For example, consider a winner-list as shown below in figure 4. Offering the highest TBP among winners, user n2 gets the first chance to figure out his preferences. Suppose, n2 chooses preference D2S1P1 (i.e. flexible deadline but immediate VM allocation and full time possession of VMs), then n2 will make actual payment according to the bid prices offered by next user in the winner-list i.e. n3.

Winners' Ids	TBP
n2	36
n3	34
n11	25
n8	14

Figure 4: Example of winners list

User n3's bid prices are shown in figure 3 as BPK1 is 11, BPK2 is 10 and BPK3 is 13 and n2's VM requirements are RK1 is 3, RK2 is 6 and RK3 is 2, then using table 3, we get-

$$AP_{n2} = (3*11) + (6*10) + (2*13) = 119$$

If we calculate n2's best-bid price payment by considering his own bid-prices for K1, K2 and K2 type VMs which are 12, 14 and 10 respectively, then using eqn 3 we get-

$$BPP_{n2} = (3*12) + (6*14) + (2*10) = 140$$

As is evident from the calculations shown above, a cloud user will always pay less than what he best-bids for resources. However, in case of preference D1S1P2 actual payment is equal to a winner's best-bid payment. Also if a winner opts for preference D1S1P1, his actual payment will exceed his best-bid payment as in this case winner has chosen the highest priority preference.

5. Results and Discussion

The proposed resource allocation technique is experimented on CloudSim environment consisting 3 heterogeneous resources with multiple instances, a single service/resource provider and multiple users. In a single allocation round, four

winners were identified and their max and min actual payments as shown in figure 5. Figure 6 shows the variation of revenues earned by the service provider.
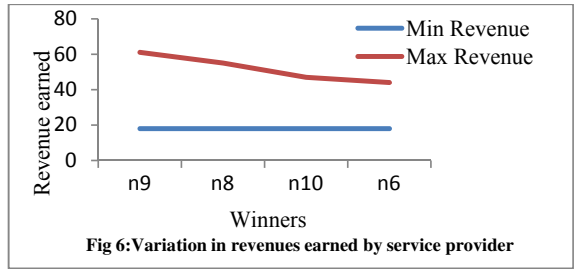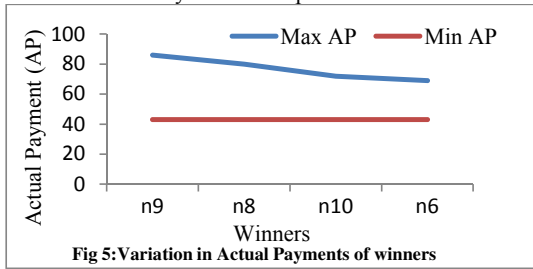

Fig 5:Variation in Actual Payments of winners

**Fig 6:Variation in revenues earned by service provider**

Figure 7 compares the minmum actual payment of a winner with his best-price payment. As shown in the figure, the proposed technique allows a winner to pay an amount which is considerably less than his bid price. This payment strategy works in favor of cloud users provided that they quote their 'best' bid price to win the auction. Figure 8 shows the variation in utilities earned by winners according to the actual payment they make based on their preferences.

Fig 7:Min AP Vs BPP of winners

Fig 8:Max and Min utilities of winners
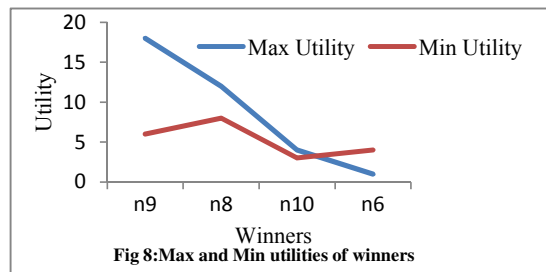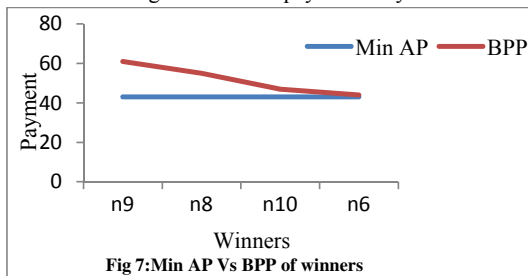
Our proposed allocation startegy is comapred with VCG auction mechanism and the results obtained are shown in figures 8 and 9. Figure 8 shows that the payment made by a winner in VCG is less as compared to that in the present work, which is beneficial to a service provider, however, at the same time, cloud users benefit by making a payment far less than their quoted price as shown in figure 7. Revenues earned by a service provider, in cases of VCG and the proposed work, are compared in figure 9 which clearly shows that the present auction technique out-performs the classical VCG auction technique.

Fig 8:VCG payment Vs Proposed Technique payment

Fig 9:Comparison of Revenues earned by service provider
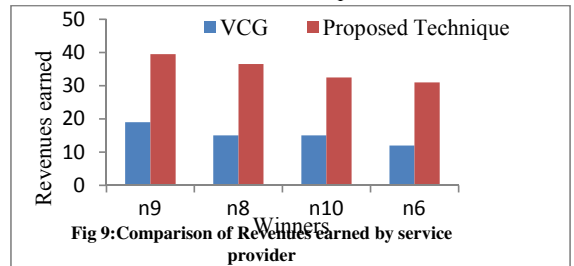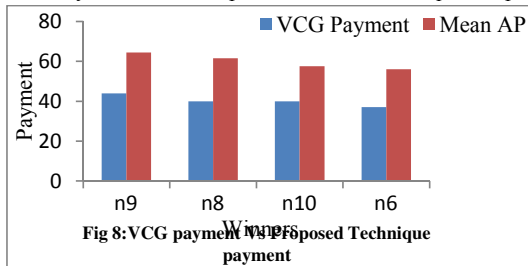
6. Conclusion

In the present work, a demand-driven preferential resource allocation technique is introduced involving a truthful auction process followed by a fair payment mechanism. This proposed technique involves bid revisions, re-biddings and multiple payment criteria for a cloud user whereas a service provider draws maximum advantage by following the current demands

of resources and setting their prices accordingly. A comparative study between the proposed technique and the well researched VCG mechanism is also presented which highlights the performance benefits of the present work.

Future work will extend the proposed allocation mechanism by including an energy efficient scheduling strategy to allot auction winners' tasks to suitable VMs such that minimum number of VMs is required to be activated at any time.

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