An Adaptive CMMSE Receiver for Space–Time Block-Coded CDMA Systems in Frequency-Selective Fading Channels

Young-Ho Jung, *Member, IEEE*, Seung-Chul Hong, Seong Rag Kim, and Yong H. Lee, *Senior Member, IEEE*

***Abstract*—A technique that can suppress multiple-access interference (MAI) in space–time block-coded (STBC) multiple-input–multiple-output (MIMO) code-division multiple-access (CDMA) systems is developed. The proposed scheme, called a constrained minimum mean square error (CMMSE) receiver, is an extension of the CMMSE receiver for a single-input–single-output system to MIMO systems. It is shown that the complexity of the proposed CMMSE receiver is almost independent of the number of transmitter antennas. The advantage of the proposed receiver over the existing receivers for STBC CDMA systems is demonstrated by comparing the closed-form expressions of the signal-to-interference plus noise ratio and simulated bit error rates. The results indicate that the proposed CMMSE receiver can provide a significant performance improvement over the conventional receivers and that the gain achieved by suppressing the MAI can be larger than that from increasing the transmitter or receiver diversity.**

***Index Terms*—Adaptive interference mitigation, code division multiple access (CDMA), constrained minimum mean square error (CMMSE), multiple-input–multiple-output (MIMO), space–time block coding.**

I. Introduction

IT HAS BEEN recognized that the capacity of a communication system can be increased by employing a space–time block-coded (STBC) multiple-input–multiple-output (MIMO) system equipped with an maximum likelihood (ML) receiver that optimally combines received signals [1], [2]. However, in a code-division multiple-access (CDMA) environment, especially for frequency-selective fading channels, such a system suffers from multiple-access interference (MAI) and its performance rapidly degrades as the number of users increases. This is because the ML receiver treats MAI as additive white Gaussian noise (AWGN). As in the case of conventional CDMA systems, the suppression of MAI is also important in STBC CDMA systems.

Recently, several linear multiuser-detection (MUD) techniques have been proposed for reducing the effect of MAI in STBC CDMA communications over quasistatic channels. In [3] and [4], the authors introduce linear MUDs that jointly perform MAI suppression and space–time (ST) block decoding based on zero forcing (ZF), modified minimum mean square error (MMSE), and minimum variance criteria. These techniques require the knowledge of the spreading sequences for all users and simultaneously detect all users' symbols. Therefore, they are mainly useful for uplink communications. An ST-MUD algorithm that is applicable to downlink communications is proposed in [5]. This scheme consists of an MMSE-based linear filter for MAI suppression followed by an ST block decoder. However, it also requires excessive information including the spreading sequence of the desired user, a training sequence, and channel-state information (CSI) for evaluating the filter coefficients. In [6], a blind adaptive algorithm for joint MUD and ST block decoding is developed. The algorithm is based on subspace tracking and is useful for both up- and downlink communications. However, it needs P spreading sequences for each user, where P is the number of transmitter antennas, to distinguish the signals of a user transmitted from different antennas. As a result, the maximum number of total users in the CDMA system is reduced by the factor of P. In the current work, we develop an alternative algorithm that can avoid such difficulty by extending the adaptive constrained minimum mean square error (CMMSE) receiver for single-input–single-output (SISO) CDMA [7], [8] to STBC MIMO systems. The proposed receiver consists of adaptive CMMSE filters for MAI suppression followed by a combiner that jointly performs maximal ratio-type combining and ST block decoding. It is applicable to both up- and downlink communications. The information required by the adaptive technique is almost minimal in the sense that it only requires the spreading sequence of the desired user with the exception of periodically inserted pilot symbols, which are provided for tracking the channel parameters of the desired user. Due to the estimated channel parameters, the algorithm only needs one spreading sequence for each user, in contrast to [6]. Furthermore, the adaptive CMMSE receiver employs the MMSE criterion in [9] and can be applied to time-varying channel environments. A statistical analysis and computer simulation results indicate that the proposed receiver can provide a significant performance improvement over the existing receivers.

The organization of this paper is as follows. The system model is presented in Section II and the proposed CMMSE

Manuscript received September 15, 2002; revised July 14, 2003, and October 30, 2003. This work was supported in part by the University Information Technology Research Center Project. This paper was presented in part at the IEEE Vehicular Technology Conference, Vancouver, BC, Canada, September, 2002.

Y.-H. Jung is with the i-Networking Laboratory, Samsung Advanced Institute of Technology, Nongseori, Kiheungeup, Yonginsi, Kyungkido 449-712, Korea (e-mail: yh.jung@ieee.org).

S.-C. Hong is with Samsung Electronics, Maetandong, Youngtonggu, Suwonsi, Kyungkido 443-742, Korea (e-mail: seungchul.hong@samsung.com).

S. R. Kim is with the Mobile Communications Laboratory, Electronics and Telecommunications Research Institute (ETRI), Gajeongdong, Yuseonggu, Daejeon 305-350, Korea (e-mail: srkim@etri.re.kr).

Y. H. Lee is with the Department of Electrical Engineering and Computer Science, Korea Advanced Institute of Science and Technology, Guseongdong, Yuseonggu, Daejeon 305-701, Korea (e-mail: yohlee@ee.kaist.ac.kr).

Digital Object Identifier 10.1109/TVT.2004.825759

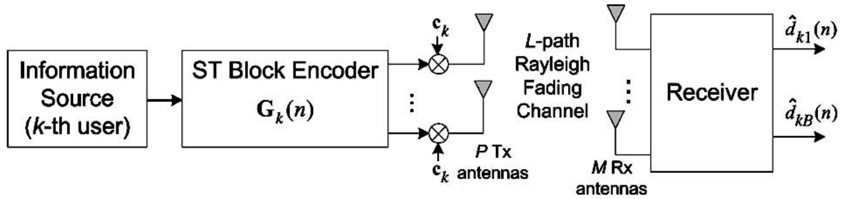


Fig. 1. STBC MIMO system with P transmitter (Tx) and M receiver (Rx) antennas in multipath fading channels.

TABLE I

VARIOUS ORTHOGONAL ST BLOCK CODE MATRICES WHEN INPUT BLOCK IS $[d_1, d_2, \ldots, d_B]$. HERE, THE NOTATIONS $\mathcal{G}_3$ AND $\mathcal{G}_4$ ARE INTRODUCED BECAUSE THEY ARE USEFUL FOR EXPRESSING THE COMPLEX CODE MATRICES

P	B	Real Orthogonal (rate1)	Complex Orthogonal (rate 1 or 1/2)
2	2	$\begin{bmatrix} d_1 & d_2 \\ d_2 & -d_1 \end{bmatrix}$	$\begin{bmatrix} d_1 & d_2^* \\ d_2 & -d_1^* \end{bmatrix}$ (rate 1)
3	4	$\mathcal{G}_3 = \begin{bmatrix} d_1 & -d_2 & -d_3 & -d_4 \\ d_2 & d_1 & d_4 & -d_3 \\ d_3 & -d_4 & d_1 & d_2 \end{bmatrix}$	$\begin{bmatrix} \mathcal{G}_3 & \mathcal{G}_3^* \end{bmatrix}$ (rate 1/2)
4	4	$\mathcal{G}_4 = \begin{bmatrix} d_1 & -d_2 & -d_3 & -d_4 \\ d_2 & d_1 & d_4 & -d_3 \\ d_3 & -d_4 & d_1 & d_2 \\ d_4 & d_3 & -d_2 & d_1 \end{bmatrix}$	$\begin{bmatrix} \mathcal{G}_4 & \mathcal{G}_4^* \end{bmatrix}$ (rate 1/2)

receiver for STBC CDMA systems is developed in Section III and is statistically analyzed in Section IV. Finally, Section V presents simulation results demonstrating the advantage of the proposed receiver over the existing receivers.

II. SYSTEM MODEL

The communication system considered in the current paper, as shown in Fig. 1, is a MIMO system with P transmitter and M receiver antennas operating in frequency-selective channels, which are assumed to be quasistatic so that the path gains are constant over an ST coded block and vary from one block to another. The information symbols for the kth user entering the ST block encoder are grouped into blocks of length B. Each block is then converted into a P-by-V ST coded block, denoted by $\mathbf{G}_k(n)$, where B/V is the code rate for the ST block code and n is the index of a coded block. For the sake of simplicity, the attention of the current paper is limited to rate 1 and 1/2 orthogonal ST block codes with $P \leq 8$ [2]. For such codes, rows of $\mathbf{G}_k(n)$ are mutually orthogonal and columns of $\mathbf{G}_k(n)$ consist of vectors whose entries are from either $\{\pm d_{k1}(n), \pm d_{k2}(n), \cdots, \pm d_{kB}(n)\}$ or $\{\pm d_{k1}^*(n), \pm d_{k2}^*(n), \cdots, \pm d_{kB}^*(n)\}$, where $[d_{k1}(n), d_{k2}(n), \cdots, d_{kB}(n)]$denotes the original data block entering the ST block encoder (for example, see Table I). For each time, P symbols in a column of $\mathbf{G}_k(n)$ are simultaneously spread and then transmitted through P antennas. Specifically, the symbols in the ith column of $\mathbf{G}_k(n)$, denoted by $\mathbf{g}_k^i(n)$, are spread at time $nV + i - 1$ by a normalized spreading code $\{c_k(j)\}$, where $c_k(j) \in \{-1/\sqrt{N}, 1/\sqrt{N}\}$ for $j = 0, 1, \cdots, N-1$ and $c_k(j) = 0$, otherwise. Here, N denotes the number of chips per symbol.

It is assumed that the propagation delay of the kth user's lth path, denoted by $\tau_{k,l}$, is a multiple of the chip duration T_c. The received signal is passed through a filter matched to the chip pulse shape and sampled at the chip rate. Suppose, without loss of generality, that the first user is the user of interest and that $\tau_{1,l} = (l-1)T_c$. Then, the sampled signal at the mth receiver antenna can be expressed as

$$y_m((nV+i-1)N+j) = \sum_{l=1}^{L} \mathbf{h}_{1,ml}(n)\mathbf{g}_k^i(n)c_k(j-l+1) + u_m((nV+i-1)N+j) \quad (1)$$

where $0 \leq j \leq N+L-2$, $\mathbf{h}_{1,ml}(n) = [h_{1,1ml}(n), \ldots, h_{1,Pml}(n)]$, and $\{h_{1,pml}(n) | \; l = 1, \ldots, L\}$ denotes the equivalent impulse response of the quasistatic channel, which is assumed to have L paths, between the pth transmitter and mth receiver antennas. The term $u_m(\cdot)$ denotes the sum of the intersymbol interference, multiple-access interference (MAI), and background noise. In vector form, $\mathbf{y}_{mi}(n) = [y_m((nV+i-1)N), y_m((nV+i-1)N+1), \ldots, y_m((nV+i)N+L-2)]^T$ is expressed as

$$\mathbf{y}_{mi}(n) = \sum_{l=1}^{L} \mathbf{h}_{1,ml}(n)\mathbf{g}_1^i(n)\mathbf{c}_{1l} + \mathbf{u}_{mi}(n). \quad (2)$$

Here, $\mathbf{c}_{1l} = [\mathbf{0}_{l-1}^T \; \mathbf{c}_1^T \; \mathbf{0}_{L-l}^T]^T$, $\mathbf{c}_1 = [c_1(0), \cdots, c_1(N-1)]^T$, $\mathbf{0}_l$ is an l-by-1 vector with all zero elements, and $\mathbf{u}_{mi}(n) = [u_m((nV+i-1)N), u_m((nV+i-1)N+1), \cdots, u_m((nV+i)N+L-2)]^T$. In matrix form, $\mathbf{Y}_m(n) = [\mathbf{y}_{m1}(n), \cdots, \mathbf{y}_{mV}(n)]$ is written as

$$\mathbf{Y}_m(n) = \mathbf{C}_1\mathbf{H}_{1,m}(n)\mathbf{G}_1(n) + \mathbf{U}_m(n) \quad (3)$$

where $\mathbf{C}_1 = [\mathbf{c}_{11} \cdots \mathbf{c}_{1L}]$, $\mathbf{U}_m(n) = [\mathbf{u}_{m1}(n) \cdots \mathbf{u}_{mV}(n)]$, and $\mathbf{H}_{1,m}(n)$ is an L-by-P matrix whose (l,p)th entry is given by $h_{1,pml}(n)$. Dropping the time index n and subscript indicating the desired user for notational simplicity, (2) and (3) can be rewritten as

$$\mathbf{y}_{mi} = \sum_{l=1}^{L} \mathbf{h}_{ml}\mathbf{g}^i\mathbf{c}_l + \mathbf{u}_{mi} \quad (4)$$

$$\mathbf{Y}_m = \mathbf{C}\mathbf{H}_m\mathbf{G} + \mathbf{U}_m. \quad (5)$$

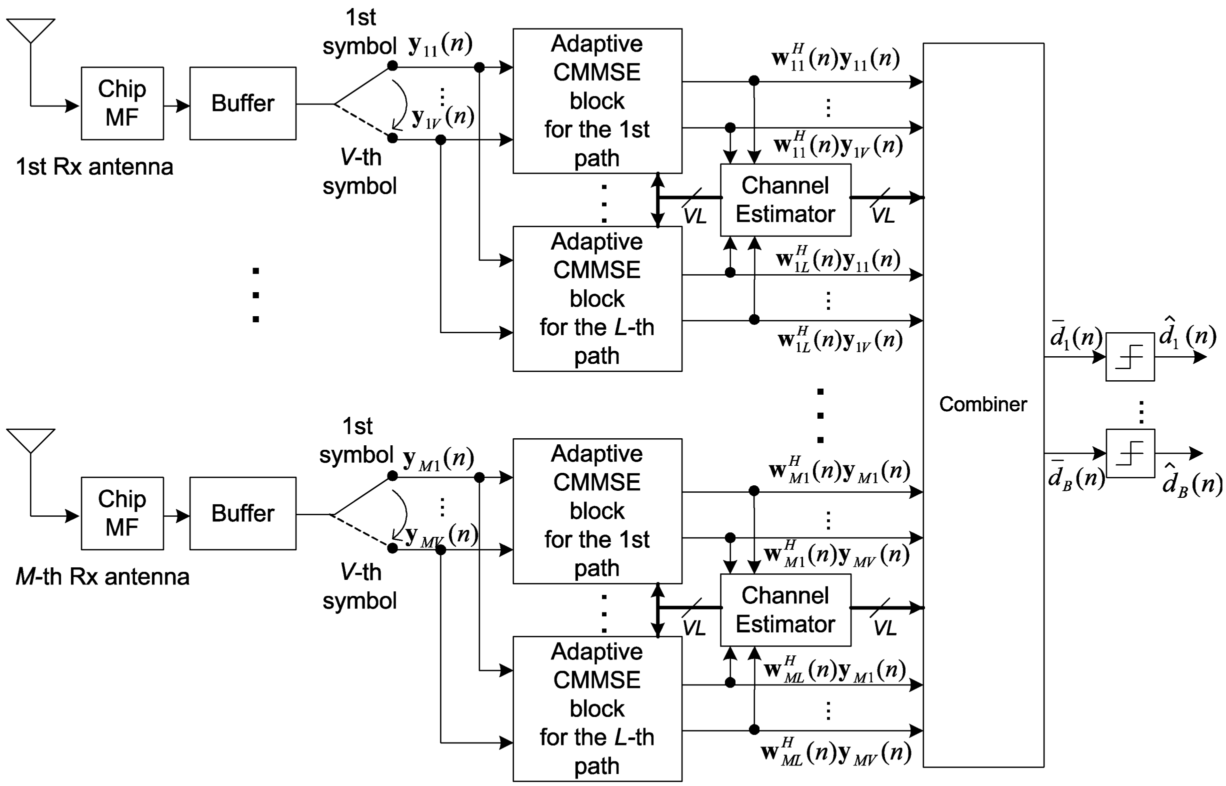


Fig. 2. Adaptive CMMSE receiver for STBC MIMO CDMA system with M receiver antennas.

The derivation of the CMMSE receiver requires the correlation matrices of $\{\mathbf{y}_{mi}\}$. These correlations are derived under the following assumption.

Assumption 1:

- Original data before ST coding are independent random variables with a zero mean and average power of one.
- Channel parameters $\{h_{k,pml}\}$ in (1) are independent for different values of p, m, and l and have a zero mean.
- Data and channel parameters are mutually independent.

Property 1: The correlation matrices of $\{\mathbf{y}_{mi}\}$, denoted by $\{\mathbf{R}_{\mathbf{y}_{mi}}\}$, are identical to each other.

Proof: Under Assumption 1 (presented above), the correlation matrix of $\mathbf{u}_{mi}$, $\mathbf{R}_{\mathbf{u}_{mi}} = E[\mathbf{u}_{mi}\mathbf{u}_{mi}^H]$ can be written as

$$\mathbf{R}_{\mathbf{u}_{mi}} = \sum_{k=2}^{K}\sum_{l=1}^{L} E\left[\sum_{p=1}^{P} |h_{k,pml}|^2 \mathbf{c}_{kl}\mathbf{c}_{kl}^H\right] + \sigma^2 \qquad (6)$$

where σ^2 is the variance of the background noise. Since the right-hand side (RHS) of (6) is not dependent upon i, $\mathbf{R}_{\mathbf{u}_{m1}} = \mathbf{R}_{\mathbf{u}_{m2}} =, \ldots, = \mathbf{R}_{\mathbf{u}_{mP}} \triangleq \mathbf{R}_{\mathbf{u}_m}$. Similarly, the correlation matrix of $\mathbf{y}_{mi}$, $\mathbf{R}_{\mathbf{y}_{mi}} = E[\mathbf{y}_{mi}\mathbf{y}_{mi}^H]$ can be expressed as

$$\mathbf{R}_{\mathbf{y}_{mi}} = \sum_{l=1}^{L} E\left[\sum_{p=1}^{P} |h_{1,pml}|^2 \mathbf{c}_{1l}\mathbf{c}_{1l}^H\right] + \mathbf{R}_{\mathbf{u}_m} \qquad (7)$$

and, thus, $\mathbf{R}_{\mathbf{y}_{m1}} = \mathbf{R}_{\mathbf{y}_{m2}} = \cdots = \mathbf{R}_{\mathbf{y}_{mP}} \triangleq \mathbf{R}_{\mathbf{y}_m}$.

III. Adaptive CMMSE Receiver for STBC CDMA Systems

The adaptive CMMSE receiver consisting of an adaptive filter, channel estimator, and detector is derived. The stability of the adaptive filter is analyzed and the receiver complexity is examined.

A. Filtering and Adaptation

At the receiver, due to the ST block coding, it is necessary to simultaneously determine $\mathbf{d} = [d_1, \ldots, d_B]$ after observing $\{\mathbf{y}_{mi} | 1 \le m \le M, 1 \le i \le V\}$, where $\mathbf{d}$ is the original data block before ST block coding. An efficient receiver structure suitable for this simultaneous detection is shown in Fig. 2. The signals entering each antenna are passed through L adaptive CMMSE blocks to filter the MAI for each transmission path. Then, the filtered signals from all antennas are combined for detecting elements of $\mathbf{d}$. In the CMMSE blocks, the MAI in $\{\mathbf{y}_{mi}\}$ is suppressed by linear filtering. To be specific, let $\mathbf{w}_{mli}$ be the filter weight for $\mathbf{y}_{mi}$ at the lth resolvable path. Then, the filter output is expressed as $\mathbf{w}_{mli}^H\mathbf{y}_{mi}$ and the optimal weights $\mathbf{w}_{mli}^o$ are obtained based on the following constrained optimization:

$$\mathbf{w}_{mli}^o = \arg\min_{\mathbf{w}_{mli}} E\left[\left|\mathbf{h}_{ml}\mathbf{g}^i - \mathbf{w}_{mli}^H\mathbf{y}_{mi}\right|^2\right] \quad \text{subject to } \mathbf{w}_{mli}^H\mathbf{c}_l = 1. \qquad (8)$$

This optimization problem is a direct extension of the SISO CMMSE optimization in [8] for MIMO systems. As in the case

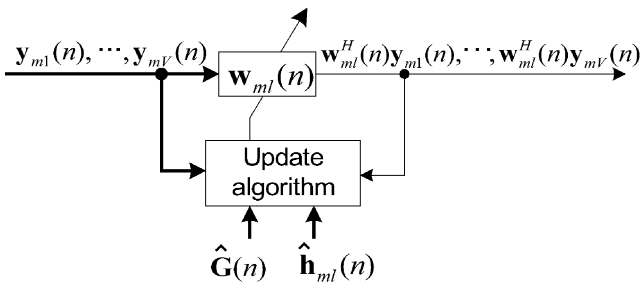


Fig. 3. Practical implementation of adaptive CMMSE block for the lth path of the mth antenna.

of SISO, the constraint in (8) prevents $\mathbf{w}_{mli}$ from being a zero vector and allows channel estimation from the outputs of the CMMSE filters. This receiver reduces to the conventional ML receiver, which is optimal for AWGN channels (henceforth, this receiver will be referred to as the ML-NO-MAI receiver) if $\mathbf{w}_{mli} = \mathbf{c}_l$ for all m and i. In this case, the CMMSE block becomes a despreader producing $\{\mathbf{c}_l^H \mathbf{y}_{mi}\}$. Each CMMSE block needs V filter weights $\{\mathbf{w}_{ml1}, \ldots, \mathbf{w}_{mlV}\}$ to suppress the MAI of $\{\mathbf{y}_{m1}, \ldots, \mathbf{y}_{mV}\}$ in the lth path of the mth antenna. The optimal weights $\{\mathbf{w}_{mli}^o\}$ can be obtained by using the method of Lagrange multipliers [10]. Specifically, it is possible to show that $\{\mathbf{w}_{mli}^o\}$ is represented as

$$\begin{aligned}\mathbf{w}_{mli}^o &= \mathbf{R}_{\mathbf{y}_m}^{-1}\mathbf{c}_l\left(\mathbf{c}_l^H \mathbf{R}_{\mathbf{y}_m}^{-1}\mathbf{c}_l\right)^{-1}\\ &\triangleq \mathbf{w}_{ml}^o \end{aligned} \tag{9}$$

where $\mathbf{R}_{\mathbf{y}_m}$ is given by (7). Note that $\mathbf{w}_{mli}^o$ is identical for all i. This observation and the fact that $\{\mathbf{y}_{mi} | i = 1, \ldots, V\}$ successively enter[1] the CMMSE block in a serial format considerably simplify the implementation. As shown in Fig. 3, the MAI of $\{\mathbf{y}_{mi} | i = 1, \ldots, V\}$ can be suppressed by a *single* filter, irrespective of the number of transmitter antennas P and ST block size V. To process $\{\mathbf{y}_{mi} | i = 1, \ldots, V\}$ using a single filter, it is necessary to modify the optimality criterion in (8). A natural choice for this purpose is the sum of the mean square errors (mses) $\sum_{i=1}^V E[|\mathbf{h}_{ml}\mathbf{g}^i - \mathbf{w}_{ml}^H \mathbf{y}_{mi}|^2]$. This criterion is indeed useful because it can also yield the optimal weight $\mathbf{w}_{ml}^o$ in (9). In particular, it can be shown that

$$\mathbf{w}_{ml}^o = \arg\min_{\mathbf{w}_{ml}} E\left[\sum_{i=1}^V \left|\mathbf{h}_{ml}\mathbf{g}^i - \mathbf{w}_{ml}^H \mathbf{y}_{mi}\right|^2\right] \quad \text{subject to } \mathbf{w}_{ml}^H \mathbf{c}_l = 1 \tag{10}$$

where $\mathbf{w}_{ml}^o$ is given in (9). The adaptive CMMSE block is designed based on the constrained optimization problem in (10). It employs a gradient-based adaptation rule that updates the tap weight on a *block-by-block* basis. Again referring to Fig. 3, for the nth coded block, all input vectors $\{\mathbf{y}_{mi}(n) | i = 1, \ldots, V\}$ are processed by a filter with weight $\mathbf{w}_{ml}(n)$ and the weight is updated after processing all $\{\mathbf{y}_{mi}(n) | i = 1, \ldots, V\}$. The adaptation rule can be derived by applying the method of Lagrange multipliers to (10). Let δ denote a Lagrange multiplier; then, the cost function $J_{ml}(n)$ that combines the two parts of the constrained optimization in (10) is written as

$$J_{ml}(n) = E\left[\sum_{i=1}^V |e_{mli}(n)|^2\right] + \mathrm{Re}\left[\left(\mathbf{w}_{ml}^H(n)\mathbf{c}_l - 1\right)\delta\right] \tag{11}$$

where $e_{mli}(n) = \mathbf{h}_{ml}(n)\mathbf{g}^i(n) - \mathbf{w}_{ml}^H(n)\mathbf{y}_{mi}(n)$ and $\mathrm{Re}[x]$ denotes the real part of x [10]. Since the gradient of $J_{ml}(n)$ is given by

$$\nabla J_{ml}(n) = -2E\left[\sum_{i=1}^V e_{mli}^*(n)\mathbf{y}_{mi}(n)\right] + \mathbf{c}_l\delta \tag{12}$$

the weight-update algorithm can be written as

$$\mathbf{w}_{ml}(n+1) = \mathbf{w}_{ml}(n) + \mu\left(\sum_{i=1}^V e_{mli}^*(n)\mathbf{y}_{mi}(n) - \frac{\mathbf{c}_l\delta}{2}\right) \tag{13}$$

where μ is the step size. The Lagrange multiplier δ that satisfies the constraint in (10) is given by

$$\delta = 2\sum_{i=1}^V e_{mli}^*(n)\mathbf{c}_l^H \mathbf{y}_{mi}(n). \tag{14}$$

Using (14) in (13) produces

$$\mathbf{w}_{ml}(n+1) = \mathbf{w}_{ml}(n) + \mu\mathbf{P}_{\mathbf{c}_l}^{\perp}\left(\sum_{i=1}^V e_{mli}^*(n)\mathbf{y}_{mi}(n)\right) \tag{15}$$

where $\mathbf{P}_{\mathbf{c}_l}^{\perp} = \mathbf{I} - \mathbf{c}_l\mathbf{c}_l^H$. This equation represents the proposed adaptation rule. For the CMMSE adaptation, it is natural to choose $\mathbf{c}_l$ as the initial weight vector. Such an initialization makes the CMMSE block initially act like the ML-NO-MAI receiver and generally guarantees convergence even without a training sequence when the symbol error probability of the ML-NO-MAI receiver is small (e.g., less than 10% [11, p. 95]). In other words, the rule in (15) can operate in a decision-directed mode, starting with $\mathbf{w}_{ml}(0) = \mathbf{c}_l$. This fact will be demonstrated in Section V through simulation.

B. Stability Analysis

This section analyzes the convergence characteristics of the rule in (15). For the analysis, it is convenient to define the weight-error vector $\boldsymbol{\epsilon}_{ml}(n)$, given by

$$\boldsymbol{\epsilon}_{ml}(n) = \mathbf{w}_{ml}(n) - \mathbf{w}_{ml}^o. \tag{16}$$

A property of $\boldsymbol{\epsilon}_{ml}(n)$ that is useful for the analysis is presented in

$$\boldsymbol{\epsilon}_{ml}(n) = \mathbf{P}_{\mathbf{c}_l}^{\perp}\boldsymbol{\epsilon}_{ml}(n). \tag{17}$$

which is true because $\mathbf{P}_{\mathbf{c}_l}^{\perp}\boldsymbol{\epsilon}_{ml}(n) = (\mathbf{I} - \mathbf{c}_l\mathbf{c}_l^H)\{\mathbf{w}_{ml}(n) - \mathbf{w}_{ml}^o\} = \boldsymbol{\epsilon}_{ml}(n) - \mathbf{c}_l\{\mathbf{c}_l^H\mathbf{w}_{ml}(n) - \mathbf{c}_l^H\mathbf{w}_{ml}^o\}$ and $\mathbf{c}_l^H\mathbf{w}_{ml}(n) = \mathbf{c}_l^H\mathbf{w}_{ml}^o = 1$ due to the constraint in (10).

[1]There is an overlap of $L-1$ chips between $\mathbf{y}_{mi}$ and $\mathbf{y}_{m(i+1)}$.

Subtracting the optimal tap weight $\mathbf{w}_{ml}^{o}$ from both sides of (15) produces

$$\begin{aligned}\boldsymbol{\epsilon}_{ml}(n+1) &= \left(\mathbf{I}-\mu\mathbf{P}_{\mathbf{c}_l}^{\perp}\sum_{i=1}^{V}\mathbf{y}_{mi}(n)\mathbf{y}_{mi}^{H}(n)\right)\boldsymbol{\epsilon}_{ml}(n)\\ &\quad+\mu\mathbf{P}_{\mathbf{c}_l}^{\perp}\sum_{i=1}^{V}\mathbf{y}_{mi}(n)e_{mli}^{o*}(n) \qquad (18)\\ &= \left(\mathbf{I}-\mu\sum_{i=1}^{V}\mathbf{P}_{\mathbf{c}_l}^{\perp}\mathbf{y}_{mi}(n)\mathbf{y}_{mi}^{H}(n)\mathbf{P}_{\mathbf{c}_l}^{\perp}\right)\boldsymbol{\epsilon}_{ml}(n)\\ &\quad+\mu\mathbf{P}_{\mathbf{c}_l}^{\perp}\sum_{i=1}^{V}\mathbf{y}_{mi}(n)e_{mli}^{o*}(n) \qquad (19)\end{aligned}$$

where $e_{mli}^{o}(n) = \mathbf{h}_{ml}(n)\mathbf{g}^{i}(n) - \mathbf{w}_{ml}^{oH}(n)\mathbf{y}_{m}(n)$ and the second equality comes from (17). Assuming that $E[\boldsymbol{\epsilon}_{ml}(n)] = 0$, $\boldsymbol{\epsilon}_{ml}(n)$, and $\mathbf{y}_{mi}(n)$ are uncorrelated, the correlation matrix for $\boldsymbol{\epsilon}_{ml}(n)$, defined as $\mathbf{K}_{ml}(n) = E[\boldsymbol{\epsilon}_{ml}(n)\boldsymbol{\epsilon}_{ml}^{H}(n)]$, can be computed as

$$\begin{aligned}\mathbf{K}_{ml}(n+1) &= \left(\mathbf{I}-\mu V\mathbf{P}_{\mathbf{c}_l}^{\perp}\mathbf{R}_{\mathbf{y}_m}\mathbf{P}_{\mathbf{c}_l}^{\perp}\right)\mathbf{K}_{ml}(n)\\ &\quad\times\left(\mathbf{I}-\mu V\mathbf{P}_{\mathbf{c}_l}^{\perp}\mathbf{R}_{\mathbf{y}_m}\mathbf{P}_{\mathbf{c}_l}^{\perp}\right)+\mu^{2}VJ_{\min}\mathbf{P}_{\mathbf{c}_l}^{\perp}\mathbf{R}_{\mathbf{y}_m}\mathbf{P}_{\mathbf{c}_l}^{\perp} \qquad (20)\end{aligned}$$

where $J_{\min} = E[|e_{mli}^{o}(n)|^{2}]$ denotes the mse corresponding to the optimal weight $\mathbf{w}_{ml}^{o}$. The estimation error can be expressed as

$$\begin{aligned}e_{mli}(n) &= \mathbf{h}_{ml}(n)\mathbf{g}^{i}(n)-\mathbf{w}_{ml}^{H}(n)\mathbf{y}_{mi}(n)\\ &= e_{mli}^{o}(n)+\boldsymbol{\epsilon}_{ml}^{H}(n)\mathbf{y}_{mi}(n)\\ &= e_{mli}^{o}(n)+\mathbf{P}_{\mathbf{c}_l}^{\perp}\boldsymbol{\epsilon}_{ml}^{H}(n)\mathbf{y}_{mi}(n). \qquad (21)\end{aligned}$$

Under the assumption that $e_{mli}(n)$ and $\mathbf{P}_{\mathbf{c}_l}^{\perp}\boldsymbol{\epsilon}_{ml}^{H}(n)\mathbf{y}_{mi}(n)$ are independent, the mse at iteration n, say $J_{mli}(n)$, can be represented as

$$\begin{aligned}J_{mli}(n) &= E\left[|e_{mli}(n)|^{2}\right]\\ &= J_{\min}+E\left[\mathbf{P}_{\mathbf{c}_l}^{\perp}\boldsymbol{\epsilon}_{ml}^{H}(n)\mathbf{y}_{mi}(n)\mathbf{y}_{mi}^{H}(n)\boldsymbol{\epsilon}_{ml}(n)\mathbf{P}_{\mathbf{c}_l}^{\perp}\right]\\ &= J_{\min}+\operatorname{tr}\left[\mathbf{P}_{\mathbf{c}_l}^{\perp}\mathbf{R}_{\mathbf{y}_m}\mathbf{P}_{\mathbf{c}_l}^{\perp}\mathbf{K}_{ml}(n)\right] \qquad (22)\end{aligned}$$

where $\operatorname{tr}[\cdot]$ means a trace operation. Following the approach in [10, p. 397], it can be shown that $J_{mli}(n)$ converges to a constant if and only if

$$0<\mu<\frac{2}{V\lambda_{ml,\max}} \qquad (23)$$

where $\lambda_{ml,\max}$ is the maximum eigenvalue of $\mathbf{P}_{\mathbf{c}_l}^{\perp}\mathbf{R}_{\mathbf{y}_m}\mathbf{P}_{\mathbf{c}_l}^{\perp}$. Furthermore, the average time constant can be shown to be

$$(\tau)_{\text{mse,av}} \simeq \frac{1}{2V\mu\lambda_{ml,\text{av}}} \qquad (24)$$

where $\lambda_{ml,\text{av}}$ is the average eigenvalue of $\mathbf{P}_{\mathbf{c}_l}^{\perp}\mathbf{R}_{\mathbf{y}_m}\mathbf{P}_{\mathbf{c}_l}^{\perp}$. Equation (24) indicates that the rate of convergence of $J_{mli}(n)$ is proportional to V. However, due to the blockwise adaptation that updates the weight after processing the V input vectors, increasing V does not imply faster adaptation. In fact, the speed of adaptation is not dependent upon V, but is dependent on μ and $\lambda_{ml,\text{av}}$. Since the adaptive CMMSE receiver for SISO CDMA can be thought of as a special case of the proposed receiver when $V=1$, the rate of convergence of the proposed receiver is essentially identical to its SISO counterpart that updates the weight at the symbol rate.

C. Channel Estimation

In practice, the error signal in (15) is evaluated using the channel estimates $\hat{\mathbf{h}}_{ml}(n)$. Suppose that a pilot block is inserted every Q-coded blocks for channel estimation. In the receiver, the outputs of the CMMSE filters are multiplied with pilot symbols to remove any data dependency. Specifically, at time jQ

$$s_{pml}(j) = \sum_{i=1}^{V} g_{(p,i)}^{*}(jQ)\mathbf{w}_{ml}^{H}(jQ)\mathbf{y}_{mi}(jQ) \qquad (25)$$

is evaluated, where $g_{(p,i)}(jQ)$ is the (p,i)th element of the STBC pilot matrix $\mathbf{G}(jQ)$. If $\mathbf{g}_p(jQ)$ is the pth row of $\mathbf{G}(jQ)$, then $s_{pml}(j)$ can be rewritten as

$$\begin{aligned}s_{pml}(j) &= \mathbf{w}_{ml}^{H}(jQ)\mathbf{Y}_{m}(jQ)\mathbf{g}_{p}^{H}(jQ)\\ &= \mathbf{w}_{ml}^{H}(jQ)\left\{\mathbf{C}\mathbf{H}_{m}(jQ)\mathbf{G}(jQ)+\mathbf{U}_{m}(jQ)\right\}\\ &\quad\times\mathbf{g}_{p}^{H}(jQ)\\ &= \mathbf{w}_{ml}^{H}(jQ)\left\{\mathbf{C}\mathbf{H}_{m}(jQ)\mathbf{f}_{p}+\mathbf{U}_{m}(jQ)\mathbf{g}_{p}^{H}(jQ)\right\}\\ &= \sum_{l'=1}^{L} h_{pml'}(jQ)\mathbf{w}_{ml}^{H}(jQ)\mathbf{c}_{l'}+\eta(jQ) \qquad (26)\end{aligned}$$

where $\mathbf{f}_p$ is a p-dimensional vector consisting of zero elements with the exception of the pth element, which is equal to one; $\eta(jQ) = \mathbf{U}_{m}(jQ)\mathbf{g}_{p}^{H}(jQ)$ denotes background noise plus the residual MAI; and the third equality comes from the orthogonality among the rows of $\mathbf{G}(jQ)$. Note that the terms in (26) are independent of the transmitted data. The channel parameters $\mathbf{h}_{m}^{p}(n) = [h_{pm1}(n),\ldots,h_{pmL}(n)]$ are estimated simultaneously from $\{s_{pml}(j)|l=1,\ldots,L, j=\lfloor n/Q\rfloor-J+1,\ldots,\lfloor n/Q\rfloor\}$, where J is a positive integer and $\lfloor n/Q\rfloor$ is the largest integer that does not exceed n/Q in an ML sense under Assumption 1 and the following additional assumptions.

Assumption 2:

- $\{h_{pml}(n)\}$ are fixed during the JQ coding-block period ($h_{pml}(n) = h_{pml}$).
- $\{\mathbf{w}_{ml}(n)\}$ are given and fixed during the JQ coding-block period ($\mathbf{w}_{ml}(n) = \mathbf{w}_{ml}$).
- $\eta(jQ)$ is Gaussian.

Let $\mathbf{h}_{m}^{p} = [h_{pm1}\ldots h_{pmL}]^{T}, \mathbf{s}_{pm}(j) = [s_{pm1}(j)\ldots s_{pmL}(j)]^{T}$, and $\mathbf{W}_{m} = [\mathbf{w}_{m1}\ \mathbf{w}_{m2}\cdots\mathbf{w}_{mL}]$. Then, the joint probability density function (pdf) of $\mathbf{s}_{pm}(j)$, $j = \lfloor n/Q \rfloor - J + 1, \cdots, \lfloor n/Q \rfloor$, conditioned on $\mathbf{h}_m^p$, can be written as

$$f(\mathbf{s}_{pm}(j)|\mathbf{h}_m^p) = \prod_{j=\lfloor \frac{n}{Q} \rfloor - J+1}^{\lfloor \frac{n}{Q} \rfloor} \frac{e^{-(\mathbf{s}_{pm}(j)-\mathbf{m}_{pm})^H \mathbf{C}_{pm}^{-1} (\mathbf{s}_{pm}(j)-\mathbf{m}_{pm})}}{\pi^L \det(\mathbf{C}_{pm})} \tag{27}$$

where $\det(\cdot)$ denotes the determinant and $\mathbf{m}_{pm}$ and $\mathbf{C}_{pm}$ are the conditional mean and covariance matrix of $\mathbf{s}_{pm}$, respectively, which are given by $\mathbf{m}_{pm} = E[\mathbf{s}_{pm}(j)|\mathbf{h}_m^p] = \mathbf{W}_m^H \mathbf{C} \mathbf{h}_m^p$ and $\mathbf{C}_{pm} = E[(\mathbf{s}_{pm}(j) - \mathbf{m}_{pm})(\mathbf{s}_{pm}(j) - \mathbf{m}_{pm})^H |\mathbf{h}_m^p] = \mathbf{W}_m^H \mathbf{R}_\mathbf{u} \mathbf{W}_m$. The ML estimator that maximizes the pdf of (27) with respect to $\mathbf{h}_m^p$ can be obtained by minimizing the following log likelihood function:

$$\boldsymbol{\Lambda}(\mathbf{s}_{pm}(j)|\mathbf{h}_m^p) = \sum_{j=\lfloor \frac{n}{Q} \rfloor - J+1}^{\lfloor \frac{n}{Q} \rfloor} |\mathbf{s}_{pm}(j) - \mathbf{m}_{pm}|. \tag{28}$$

From (28), it is straightforward to show that the ML estimator is given by

$$\hat{\mathbf{h}}_m^p = \left(\mathbf{W}_m^H \mathbf{C}\right)^{-1} \frac{1}{J} \sum_{j=\lfloor \frac{n}{Q} \rfloor - J+1}^{\lfloor \frac{n}{Q} \rfloor} \mathbf{s}_{pm}(j). \tag{29}$$

This estimator is unbiased because $E[\hat{\mathbf{h}}_m^p] = (\mathbf{W}_m^H \mathbf{C})^{-1} \mathbf{m}_{pm} = \mathbf{h}_m^p$.

D. Data Detection

To derive the decision rule, a V-by-1 vector $\bar{\mathbf{r}}_{ml} = [\bar{r}_{ml1}, \cdots, \bar{r}_{mlV}]^T$ is defined, which collects all the CMMSE outputs for the lth path of the mth antenna. Let $\mathcal{D}$ and $\mathcal{D}^*$ be the sets of all p-dimensional vectors whose entries are from $\{\pm d_1, \cdots, \pm d_B\}$ and $\{\pm d_1^*, \cdots, \pm d_B^*\}$, respectively. Then, each column $\mathbf{g}^i$ of the code matrix $\mathbf{G}$ is either $\mathbf{g}^i \in \mathcal{D}$ or $\mathbf{g}^i \in \mathcal{D}^*$ (Table I). The elements of $\bar{\mathbf{r}}_{ml}$ are defined by

$$\bar{r}_{mli} = \begin{cases} \mathbf{w}_{ml}^H \mathbf{y}_{mi}, & \text{if } \mathbf{g}^i \in \mathcal{D} \\ \left(\mathbf{w}_{ml}^H \mathbf{y}_{mi}\right)^*, & \text{if } \mathbf{g}^i \in \mathcal{D}^*. \end{cases} \tag{30}$$

The vector $\bar{\mathbf{r}}_{ml}$ can be written as

$$\bar{\mathbf{r}}_{ml} = \widetilde{\mathbf{H}}_{ml} \mathbf{d} + \bar{\mathbf{v}}_{ml} + \bar{\mathbf{u}}_{ml} \tag{31}$$

where $\mathbf{d} = [d_1, \ldots, d_B]^T$ and $\widetilde{\mathbf{H}}_{ml}$ is a V-by-B matrix consisting of the channel coefficients, their complex conjugates, and zeros. $\widetilde{\mathbf{H}}_{ml}$ represents a combination of the ST code and channel. Specifically, the bth column of $\widetilde{\mathbf{H}}_{ml}$, denoted by $\widetilde{\mathbf{H}}_{ml}^b$, represents the effective channel vector between the bth original data d_b and the lth path of the mth receiver antenna. For a given ST-block code, $\widetilde{\mathbf{H}}_{ml}$ can be determined in a straightforward manner. The $\widetilde{\mathbf{H}}_{ml}$ matrices corresponding to the ST block codes in Table I are shown in Table II. The vectors $\bar{\mathbf{v}}_{ml}$ and $\bar{\mathbf{u}}_{ml}$ in (31) represent the interpath interference (IPI) and the residual MAI plus background noise, respectively. Their ith elements, $\bar{v}_{mli}$ and $\bar{u}_{mli}$, are given by

TABLE II

$\widetilde{\mathbf{H}}_{ml}$ FOR ST BLOCK CODE MATRICES IN TABLE I. NOTATIONS $\mathcal{H}_3$ AND $\mathcal{H}_4$ ARE INTRODUCED BECAUSE THEY ARE USEFUL FOR EXPRESSING $\widetilde{\mathbf{H}}_{ml}$ CORRESPONDING TO COMPLEX CODE MATRICES

P	Real Orthogonal	Complex Orthogonal
2	$\begin{bmatrix} h_{1ml} & h_{2ml} \\ h_{2ml} & -h_{1ml} \end{bmatrix}$	$\begin{bmatrix} h_{1ml} & h_{2ml} \\ h_{2ml}^* & -h_{1ml}^* \end{bmatrix}$
3	$\mathcal{H}_3 = \begin{bmatrix} h_{1ml} & h_{2ml} & h_{3ml} & 0 \\ h_{2ml} & -h_{1ml} & 0 & -h_{3ml} \\ h_{3ml} & 0 & -h_{1ml} & h_{2ml} \\ 0 & h_{3ml} & -h_{2ml} & -h_{1ml} \end{bmatrix}$	$\begin{bmatrix} \mathcal{H}_3 \\ \mathcal{H}_3^* \end{bmatrix}$
4	$\mathcal{H}_4 = \begin{bmatrix} h_{1ml} & h_{2ml} & h_{3ml} & h_{4ml} \\ h_{2ml} & -h_{1ml} & h_{4ml} & -h_{3ml} \\ h_{3ml} & -h_{4ml} & -h_{1ml} & h_{2ml} \\ h_{4ml} & h_{3ml} & -h_{2ml} & -h_{1ml} \end{bmatrix}$	$\begin{bmatrix} \mathcal{H}_4 \\ \mathcal{H}_4^* \end{bmatrix}$

$$\bar{v}_{mli} = \begin{cases} \sum_{l'=1, l' \neq l}^{L} (\widetilde{\mathbf{H}}_{ml'})_i \mathbf{d} \mathbf{w}_{ml}^H \mathbf{c}_{l'}, & \text{if } \mathbf{g}^i \in \mathcal{D} \\ \sum_{l'=1, l' \neq l}^{L} (\widetilde{\mathbf{H}}_{ml'})_i \mathbf{d} \left(\mathbf{w}_{ml}^H \mathbf{c}_{l'}\right)^*, & \text{if } \mathbf{g}^i \in \mathcal{D}^* \end{cases} \tag{32}$$

and

$$\bar{u}_{mli} = \begin{cases} \mathbf{w}_{ml}^H \mathbf{u}_{mi}, & \text{if } \mathbf{g}^i \in \mathcal{D} \\ \left(\mathbf{w}_{ml}^H \mathbf{u}_{mi}\right)^*, & \text{if } \mathbf{g}^i \in \mathcal{D}^* \end{cases} \tag{33}$$

where $(\widetilde{\mathbf{H}}_{ml'})_i$ is the ith row of $\widetilde{\mathbf{H}}_{ml'}$. Assuming that $(\bar{\mathbf{v}}_{ml} + \bar{\mathbf{u}}_{ml})$ is Gaussian, the following ML decision rule is obtained from (31):

$$\hat{\mathbf{d}}_{\mathrm{ML}} = \arg\min_{\hat{\mathbf{d}} \in \mathcal{D}} \sum_{m=1}^{M} \sum_{l=1}^{L} |\bar{\mathbf{r}}_{ml} - \widetilde{\mathbf{H}}_{ml} \hat{\mathbf{d}}|^2. \tag{34}$$

For orthogonal ST block codes in which the rows of $\mathbf{G}$ are mutually orthogonal, it turns out that the decision rule in (34) can be decomposed into the following B decision rules that separately determine $\{d_b\}$. For $b = 1, \ldots, B$

$$\hat{d}_{b,\mathrm{ML}} = \arg\min_{\hat{d}} |\bar{d}_b - \hat{d}| + \left(\sum_{m=1}^{M} \sum_{l=1}^{L} \left| \widetilde{\mathbf{H}}_{ml}^b \right|^2 - 1 \right) \cdot |\hat{d}|^2 \tag{35}$$

where $\bar{d}_b$ is the decision variable given by

$$\bar{d}_b = \sum_{m=1}^{M} \sum_{l=1}^{L} \left(\widetilde{\mathbf{H}}_{ml}^b \right)^H \bar{\mathbf{r}}_{ml}. \tag{36}$$

$\widetilde{\mathbf{H}}_{ml}^b$ is the bth column of $\widetilde{\mathbf{H}}_{ml}$ and $\hat{d}$ denotes a candidate symbol. The decision variable $\bar{d}_b$ is obtained by combining all the CMMSE outputs. As such, it is a maximal ratio combiner, because $\widetilde{\mathbf{H}}_{ml}^b$ represents an effective channel between d_b and the lth path of the mth receiver antenna. For constant envelop modulations such as phase-shift keying (PSK), $|\hat{d}|^2$ is constant

TABLE III
NUMBER OF MULTIPLICATIONS PER DATA SYMBOL FOR THE PROPOSED CMMSE RECEIVER

	Required number of multiplications
Filtering and adaptation	$\frac{1}{B(Q-1)} MNVL(3Q+1)$
Channel estimation	$\frac{M}{B(Q-1)}\{\frac{1}{3}L^3+\frac{5}{2}L^2+\frac{7}{6}L+P(NLJ+L^2+L)\}$
Data detection	MVL
Overall multiplications per data symbol	$\frac{M}{B(Q-1)}\{NVL(3Q+1)+\frac{1}{3}L^3+\frac{5}{2}L^2+\frac{7}{6}L+P(NLJ+L^2+L)\}$ $+MVL$

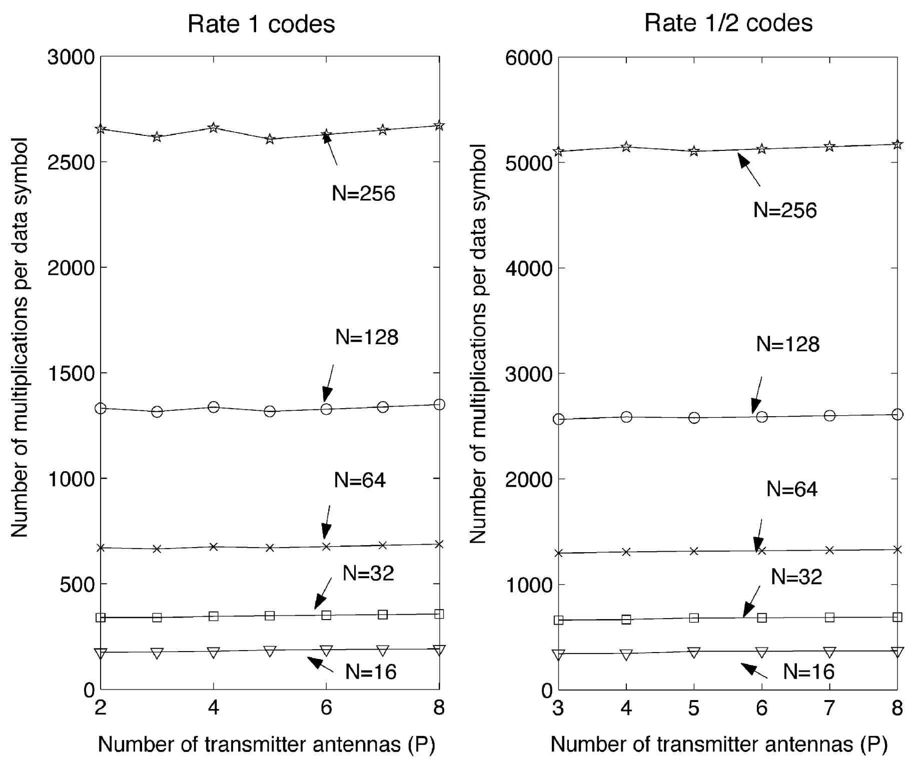


Fig. 4. Number of multiplications per data symbol for implementing a CMMSE receiver when $L = 3$, $Q = 10$, $M = 1$, and $J = 2$.

and, thus, the decision rule in (35) can be further simplified as

$$\hat{d}_{b,\mathrm{ML}} = \arg\min_{\hat{d}} |\bar{d}_b - \hat{d}|. \tag{37}$$

E. Computational Complexity

Table III and Fig. 4 show the number of multiplications involved in implementing the proposed CMMSE receiver, given a data frame consisting of one pilot block and $(Q-1)$ data blocks. The curves in Fig. 4 indicate that the computational complexity per data symbol is almost independent of the number of transmitter antennas P. This can also be seen from the expression of the overall multiplications in Table III. Note that the first term, consisting of the terms in the brace multiplied with $M/B(Q-1)$, is not proportional to P because $B \geq P$; $B = V$ for the rate 1 code, while $B = V/2$ for the rate 1/2 codes. The last term, MVL, is much lower than the other. The complexity of the proposed receiver increases with the number of receiver antennas M, yet remains almost constant irrespective of P.

IV. SINR ANALYSIS

The SINRs of the CMMSE and ML-NO-MAI receivers are derived. It is shown that the SINR of the former is always greater than or equal to that of the latter. The SIN's are derived under Assumptions 1, 2, and the following additional assumptions.

Assumption 3:

- Constant envelop modulation is used.
- Channel estimation is perfect.
- Adaptive filters are in a steady state, and the filter weight is identical to the optimal weight in (9).

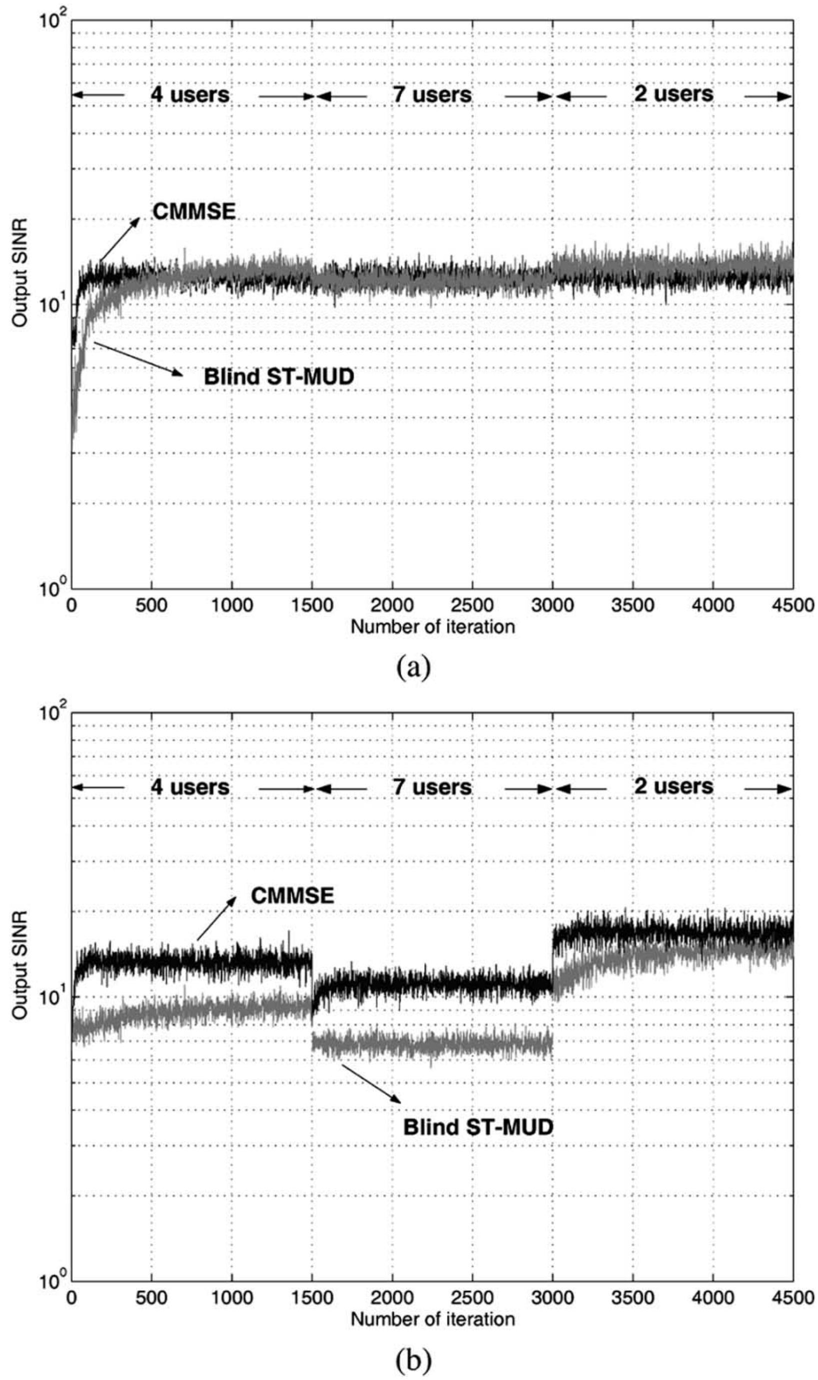


Fig. 5. Convergence characteristics in static channels ($P = 2$, $M = 2$, and $N = 15$). (a) Downlink ($L = 1$ and $E_b/N_0 = 5$ dB). (b) Uplink ($L = 3$ and $E_b/N_0 = 8$ dB).

Using (31) and $\mathbf{w}_{ml}^H \mathbf{c}_l = 1$ in (36), the decision variable of the CMMSE receiver is rewritten as

$$\bar{d}_b = \sum_{m=1}^{M} \sum_{l=1}^{L} \left| \widetilde{\mathbf{H}}_{ml}^b \right|^2 d_b + \sum_{m=1}^{M} \sum_{l=1}^{L} \left(\widetilde{\mathbf{H}}_{ml}^b \right)^H \bar{\mathbf{u}}_{ml} + \sum_{m=1}^{M} \sum_{l=1}^{L} \sum_{i=1}^{V} (\widetilde{\mathbf{H}}_{ml})_{(i,b)}^* \sum_{l'=1, l' \neq l}^{L} (\widetilde{\mathbf{H}}_{ml'})_i \mathbf{d} \mathbf{w}_{ml}^H \mathbf{c}_{l'} \quad (38)$$

where $(\widetilde{\mathbf{H}}_{ml})_{(i,b)}$ denotes the (i,b)th element of $\widetilde{\mathbf{H}}_{ml}$. The first term on the RHS of (38) represents the signal of interest, the second term denotes the sum of the MAI and the noise, and the third term denotes the IPI. The average signal power S_{CM} is given by

$$S_{\mathrm{CM}} = E \left[\left| \sum_{m=1}^{M} \sum_{l=1}^{L} \left| \widetilde{\mathbf{H}}_{ml}^b \right|^2 \right|^2 \right] \quad (39)$$

and the interferences and noise power, denoted by $I_{\rm CM}$, averaged over h_{pml}, $\bar{\mathbf{u}}_m$, and d_b, can be written as

$$\begin{aligned} I_{\rm CM} &= E\left[\left|\sum_{m=1}^{M}\sum_{l=1}^{L}\left(\widetilde{\mathbf{H}}_{ml}^{b}\right)^{H}\bar{\mathbf{u}}_{ml}\right.\right. \\ &\qquad + \sum_{m=1}^{M}\sum_{l=1}^{L}\sum_{i=1}^{V}(\widetilde{\mathbf{H}}_{ml})^{*}_{(i,b)} \\ &\qquad \left.\left.\times \sum_{l'=1,l'\neq l}^{L}(\widetilde{\mathbf{H}}_{ml'})_i \mathbf{d}\mathbf{w}_{ml}^{H}\mathbf{c}_{l'}\right|^{2}\right] \\ &= E\left[\left|\sum_{m=1}^{M}\sum_{l=1}^{L}\left(\widetilde{\mathbf{H}}_{ml}^{b}\right)^{H}\bar{\mathbf{u}}_{ml}\right|^{2}\right] \\ &\quad + E\left[\left|\sum_{m=1}^{M}\sum_{l=1}^{L}\sum_{i=1}^{V}(\widetilde{\mathbf{H}}_{ml})^{*}_{(i,b)}\right.\right. \\ &\qquad\qquad \left.\left.\times \sum_{l'=1,l'\neq l}^{L}(\widetilde{\mathbf{H}}_{ml'})_i \mathbf{d}\mathbf{w}_{ml}^{H}\mathbf{c}_{l'}\right|^{2}\right] \\ &= \sum_{m=1}^{M}\sum_{l=1}^{L}E\left[\left|\left|\widetilde{\mathbf{H}}_{ml}^{b}\right|^{2}\right|^{2}\right]\mathbf{w}_{ml}^{H}\mathbf{R}_{\mathbf{u}_m}\mathbf{w}_{ml} \\ &\quad + \sum_{m=1}^{M}\sum_{l=1}^{L}E\left[\left|\left|\widetilde{\mathbf{H}}_{ml}^{b}\right|^{2}\right|^{2}\right]\mathbf{w}_{ml}^{H} \\ &\quad \times\left(\sum_{l'=1,l'\neq l}^{L}\left|\widetilde{\mathbf{H}}_{ml}^{b}\right|^{2}\mathbf{c}_{l'}\mathbf{c}_{l'}^{H}\right)\mathbf{w}_{ml} \\ &= \sum_{m=1}^{M}\sum_{l=1}^{L}E\left[\left|\left|\widetilde{\mathbf{H}}_{ml}^{b}\right|^{2}\right|^{2}\right] \\ &\quad \times\left(\mathbf{w}_{ml}^{H}\mathbf{R}_{\mathbf{y}_m}\mathbf{w}_{ml} - E\left[\left|\widetilde{\mathbf{H}}_{ml}^{b}\right|^{2}\right]\right) \end{aligned} \tag{40}$$

where the second equality results from the independence assumptions. The SINR for the CMMSE receiver is given by $S_{\rm CM}/I_{\rm CM}$. As mentioned at the beginning of Section III, the ML-NO-MAI receiver is a special case of the CMMSE receiver, in which $\mathbf{w}_{ml}$ is fixed at $\mathbf{c}_l$. Therefore, the SINR of the ML-NO-MAI receiver is given by $S_{\rm ML}/I_{\rm ML}$, where $S_{\rm ML} = S_{\rm CM}$ in (39) and $I_{\rm ML}$ is obtained from $I_{\rm CM}$ in (40) by replacing $\mathbf{w}_{\rm ML}$ with $\mathbf{c}_l$. Specifically,

$$I_{\rm ML} = \sum_{m=1}^{M}\sum_{l=1}^{L}E\left[\left|\left|\widetilde{\mathbf{H}}_{ml}^{b}\right|^{2}\right|^{2}\right]\left(\mathbf{c}_l^{H}\mathbf{R}_{\mathbf{y}_m}\mathbf{c}_l - E\left[\left|\widetilde{\mathbf{H}}_{ml}^{b}\right|^{2}\right]\right). \tag{41}$$

The SINRs of the CMMSE and ML-NO-MAI receivers can be compared by examining $\mathbf{w}_{ml}^{H}\mathbf{R}_{\mathbf{y}_m}\mathbf{w}_{ml}$ in (40) and $\mathbf{c}_l^{H}\mathbf{R}_{\mathbf{y}_m}\mathbf{c}_l$ in (41): the one with the smaller value has the larger SINR. The comparison is made in the following property.

Property 3: $\mathbf{c}_l^{H}\mathbf{R}_{\mathbf{y}_m}\mathbf{c}_l \geq \mathbf{w}_{ml}^{H}\mathbf{R}_{\mathbf{y}_m}\mathbf{w}_{ml}$ for all m, l, and, thus, the average SINR of the proposed CMMSE receiver is greater than or equal to that of the ML-NO-MAI receiver.

Proof: See the Appendix.

The proposed receiver can outperform the ML-NO-MAI receiver at the expense of additional computation. The number of multiplications required by the latter is $({\rm MP}/(B(Q-1)))({\rm LJ}+L)+{\rm MVL}(1+(1/B))$, which is always considerably less than that of the former, shown in Table III.

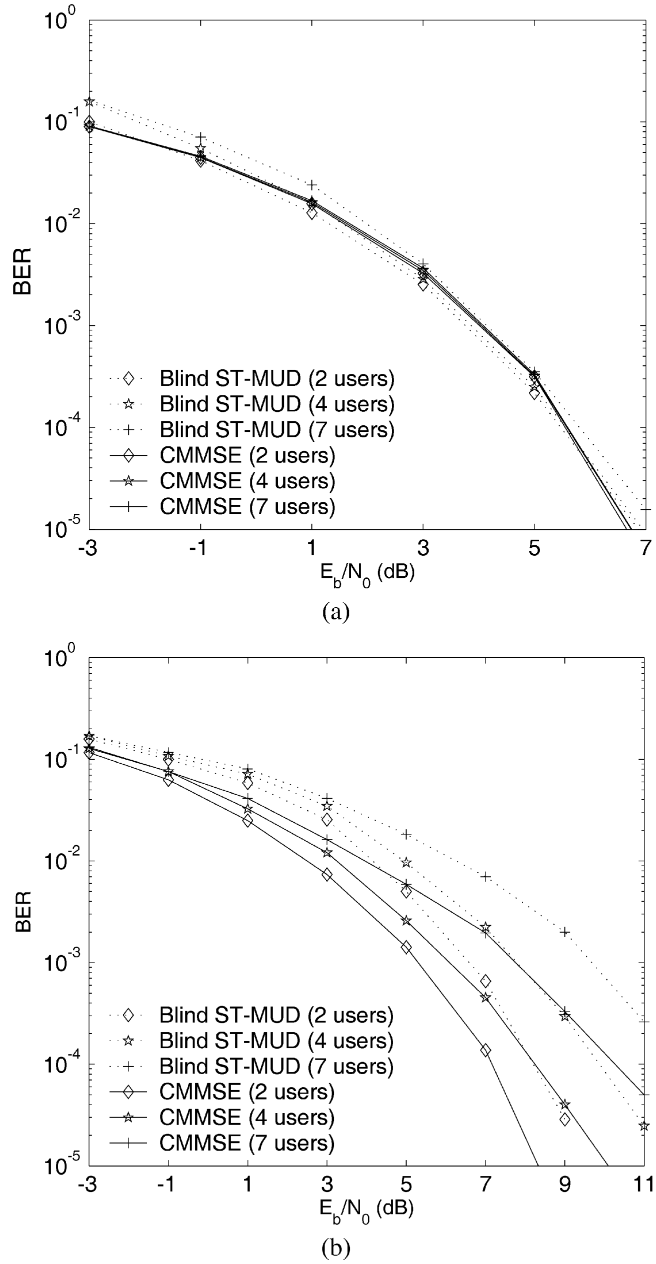


Fig. 6. BER performance comparison in statis channels ($P = 2, M = 2$, and $N = 15$). (a) Downlink ($L = 1$). (b) Uplink ($L = 3$).

V. Simulation Results

Computer simulations were conducted to examine the performance of the proposed receiver. In the first set of simulations, the CMMSE receiver was applied to a static channel and its performance was compared with that of the blind ST-MUD in [6]. Then, in the second set, the behavior of the CMMSE receiver was examined for a time-varying channel.

A. Performance for Fixed Channels

The simulation environment was identical to that of [6]. Modulation was binary PSK (BPSK); m-sequences of length

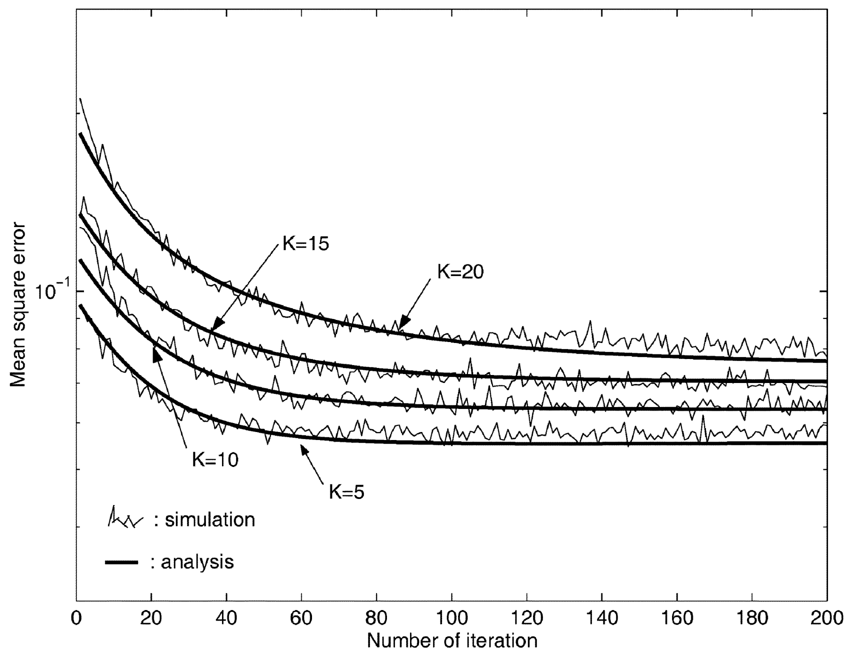


Fig. 7. Learning curves of CMMSE filter where K is number of users (no training sequence, downlink, $E_b/N_0 = 10$ dB, $P = 2$, $L = 3$, and $\mu = 0.02$).

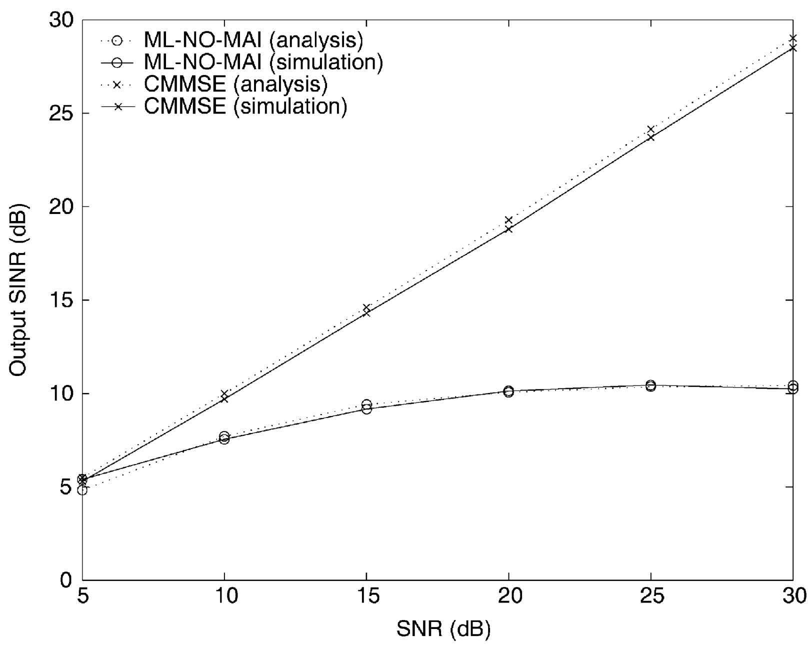


Fig. 8. Steady-state SINR comparison for circuit mode transmission (uplink, $P = 2$, $M = 1$, $L = 3$, and $K = 5$).

15 $(N = 15)$ and their shifted versions were used for spreading. The channel was complex Gaussian and the number of the transmitter and receiver antennas was two ($P = 2$, $M = 2$). Both synchronous flat fading $(L = 1)$, which represents downlink, and asynchronous multipath fading with three paths $(L = 3)$ having an identical average power, which represents uplink, were considered. For the downlink, all users experienced identical channels $(\mathbf{h}_{1,ml} = \mathbf{h}_{2,ml} =, \ldots, = \mathbf{h}_{K_m l})$ and propagation delays $(\tau_{1,l} = \tau_{2,l} = \cdots = \tau_{K,l})$, while for the uplink $\{\mathbf{h}_{k,ml}\}$ and $\{\tau_{k,l}\}$ were independent of each other for different values of k and $\tau_{k,l}$, $k \neq 1$, was randomly selected from $\{0, T_c, 2T_c, \ldots, (N-1)T_c\}$. The number of users K was varied from two to seven and the received power for all users was identical. The parameters for the CMMSE receiver were as follows. The step size μ for the adaptation was 0.02. To assist the channel estimation, one pilot block was inserted every ten code blocks $(Q = 10)$. The parameter J for the channel estimator was set at ten. The initial weights for the CMMSE filters were identical to those of the ML-NO-MAI receiver. In the simulation, SINR and BER values were evaluated under the scenario in [6], which was a circuit-mode CDMA communication.

Fig. 5 compares the output SINR. In this figure, the number of users for the first 1500 iterations was four and it became seven and two at iterations 1501 and 3001, respectively. With the help of pilot symbols, the CMMSE receiver could provide some performance gain over the blind ST-MUD. The former exhibited faster convergences and yielded higher steady-state SINR, espe-

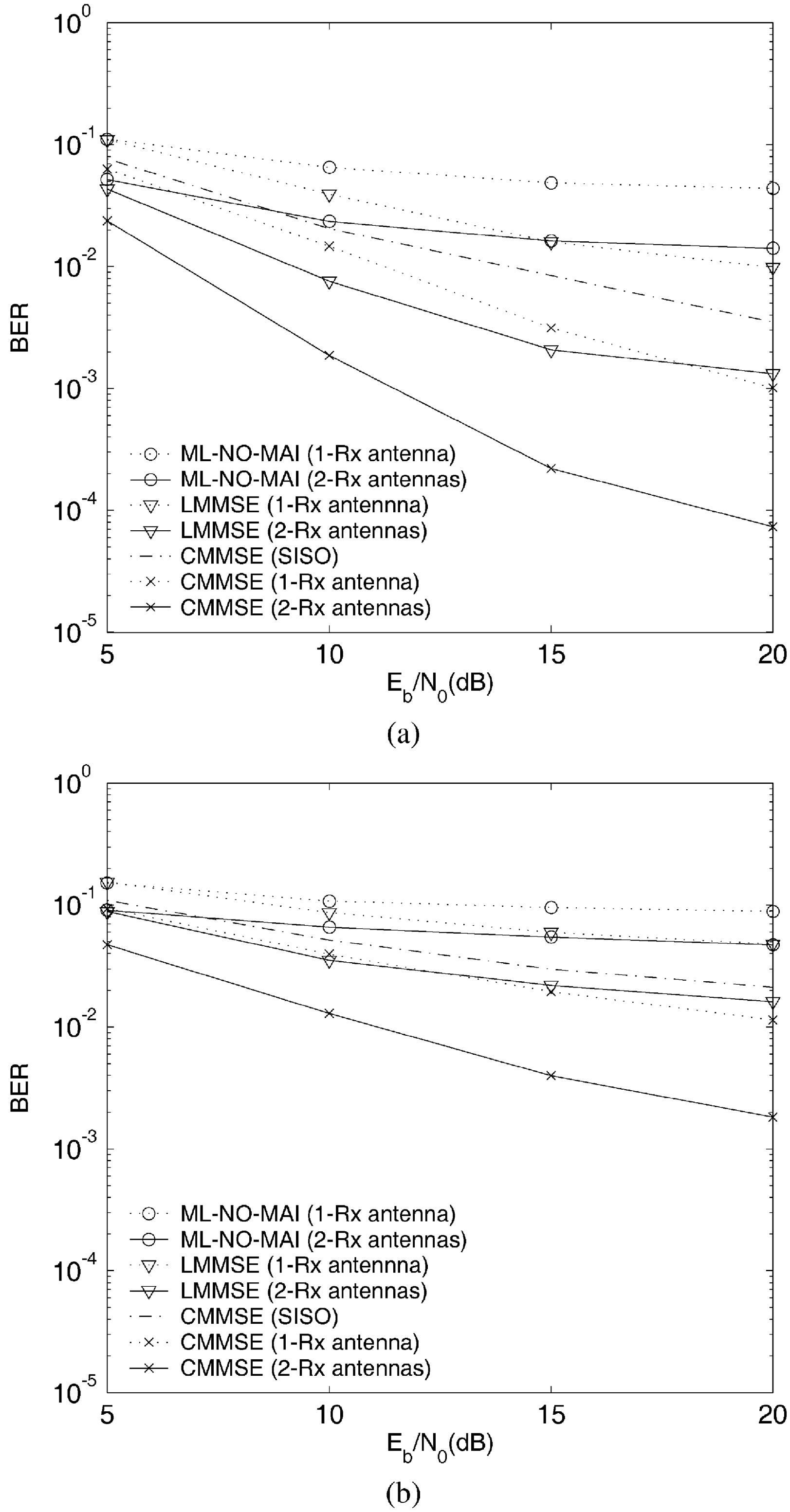


Fig. 9. BER versus E_b/N_0 in frequency-selective fading channel ($P = 2$, $L = 3$, and $K = 20$). (a) Downlink. (b) Uplink.

cially for the uplink. The steady-state performances of the two receivers in the downlink were comparable; this happened because the effect of MAI was not severe in the downlink.

Fig. 6 shows the steady-state BERs. As expected from the results in Fig. 5, the proposed receiver outperformed the blind ST-MUD in the uplink. The performance gap increased as the number of users increased, because new users caused additional MAI. For the downlink, in which the MAI was not severe, the two receivers exhibited comparable performances.

B. *Performance for Time-Varying Channels*

To consider a more practical situation, some parameters of the simulation have been modified as follows: quaternary PSK (QPSK) modulation, Gold code of length 63 ($N = 63$),

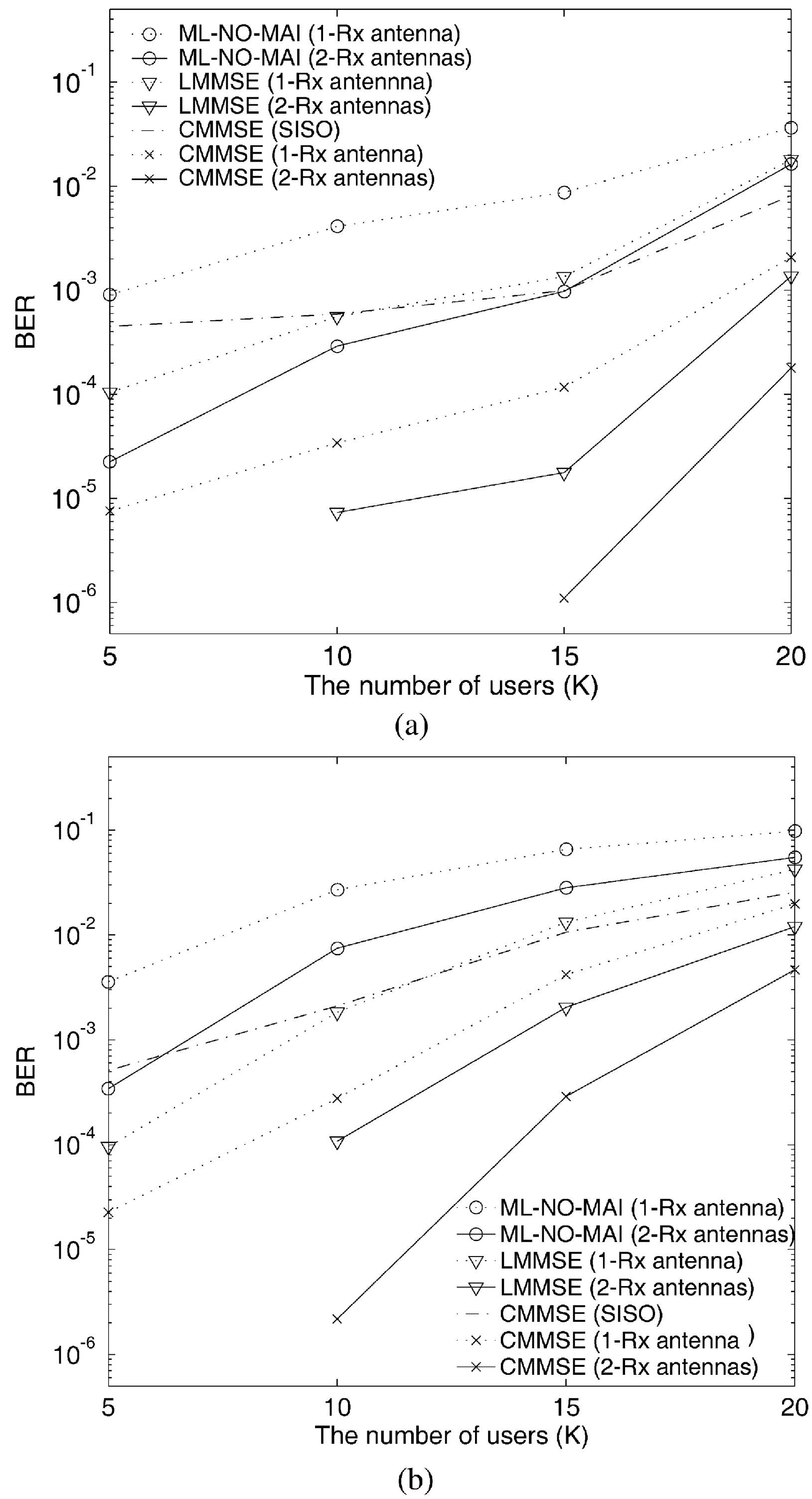


Fig. 10. BER versus number of users ($P = 2$, $L = 3$, and $E_b/N_0 = 15$ dB). (a) Downlink. (b) Uplink.

Rayleigh-fading channels with three paths ($L = 3$ for both up- and downlinks) having an identical average power, and $f_D T = 10^{-3}$ where f_D is the maximum Doppler frequency and T is the symbol duration. To facilitate channel tracking, the parameter J was reduced to two, while μ and Q remained the same. The number of users K was varied from five to 20.

As in the case of static channels, the CMMSE filters converged even without a training sequence. This is illustrated in Fig. 7, which shows various learning curves for the CMMSE adaptation obtained from the analysis and simulation. The results indicated that the CMMSE filter converged after about 100 iterations ($100 \times V$ symbol duration). In addition, there was a good agreement between the analysis and the simulation.

Both circuit and packet-mode communications were performed. In the circuit mode, all users in the system continuously transmitted data for a period of 5200 symbols, in which the initial 200 symbol period was considered as the training period. In this case, empirical SINR and BER values were obtained,

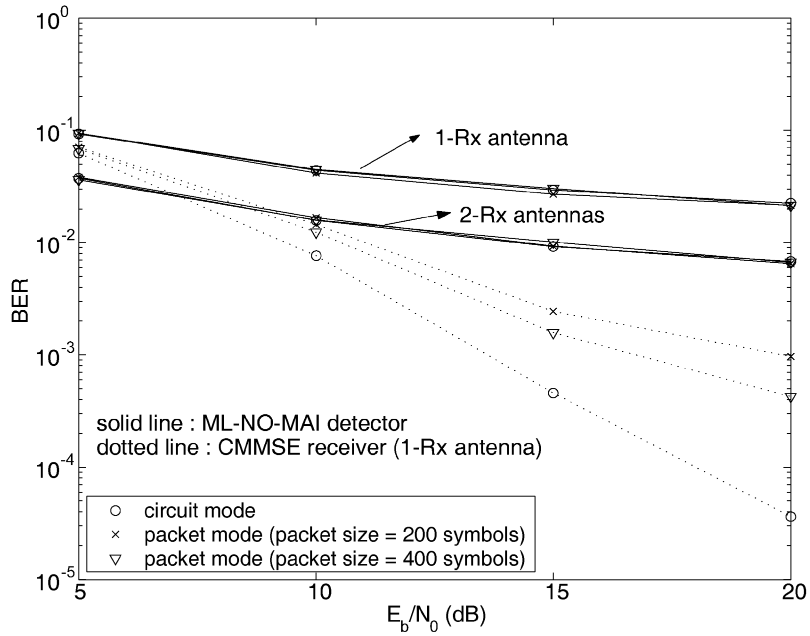


Fig. 11. BER performance degradation of CMMSE receiver with packet-mode communications (uplink, $P = 2$, $L = 3$, and $K = 10$).

while ignoring the training periods. In the packet-mode transmission, packets consisting of either 200 or 400 symbols were transmitted and 20% of the users were replaced with new ones at the end of each packet.[2] For the packet mode, there was no separate training period and the BER values were obtained considering all the symbols in the packets. The simulated SINR and BER values presented in this section are the averages of 10^7 symbols.

Fig. 8 shows the steady-state output SINRs obtained from the analysis and simulation. These values were evaluated for known channels. For the analytical SINR, optimal filter coefficients were used and the simulation was performed under a circuit mode. A remarkably good agreement was observed between the analysis and the simulation and, as expected from the SINR analysis, the CMMSE receiver performed better than the ML-NO-MAI receiver. The output SINR of the CMMSE receiver increased in proportion to the input SNR. However, the SINR curve for the ML-NO-MAI receiver exhibited saturation due to MAI.

Figs. 9 and 10 show the BERs for the circuit-mode communications when the number of transmitter antennas was two $(P = 2)$ and the number of receiver antennas was either one $(M = 1)$ or two $(M = 2)$. In Fig. 9, the number of users was 20 $(K = 20)$ and K varied from 5 to 20 in Fig. 10. For comparison, the BERs for CMMSE receivers in an SISO environment $(P = M = 1)$ and those for the linear MMSE (LMMSE) receiver in [9] are also shown. The LMMSE receiver for STBC CDMA systems is identical to the CMMSE receiver with the exception of weight adaptation and channel estimation. Its adaptation rule and channel estimate are given by

$$\mathbf{w}_{ml}(n+1) = \mathbf{w}_{ml}(n) + \mu \left(\sum_{i=1}^{V} e_{mli}^{*}(n)\mathbf{y}_{mi}(n) \right) \tag{42}$$

[2]This scenario is suitable for comparing circuit and packet modes in an uplink where channels and propagation delays vary whenever the users are replaced. Only an uplink was considered for the packet mode. For simplicity, it was assumed that each user could transmit their packets at times determined by the basestation.

and

$$\hat{h}_{pml}(n) = \frac{1}{J} \sum_{j=\left\lfloor \frac{n}{Q} \right\rfloor - J+1}^{\left\lfloor \frac{n}{Q} \right\rfloor} \mathbf{c}_l^H \mathbf{Y}_m(jQ)\mathbf{g}_p^H(jQ) \tag{43}$$

respectively. The BERs for the downlink were smaller than the corresponding BERs for the uplink because uplink communication causes more MAI than downlink communication. As expected, the CMMSE receiver for MIMO CDMA exhibited a significant performance improvement over the CMMSE receiver for SISO CDMA and outperformed the ML-NO-MAI and LMMSE receivers when the same number of receiver antennas was employed. Furthermore, the proposed CMMSE receiver with one receiver antenna $(M = 1)$ generally performed better than the ML-NO-MAI receiver with two receiver antennas $(M = 2)$, indicating that the gain achieved by increasing the transmitter or receiver diversity was less than that from suppressing the MAI.

Fig. 11 shows the BERs for the packet mode when $P = 2$, $M = 1$ or 2, $L = 3$, and $K = 10$. For comparison, the corresponding BERs for the circuit mode are also shown. Due to the adaptation within the packets, the BERs for the packet mode were somewhat larger than those for the circuit mode. However, as in the case of the circuit mode, the CMMSE receiver outperformed the ML-NO-MAI receiver and the former with $M = 1$ performed better than the latter with $M = 1$or 2.

Since the implementation of the CMMSE receiver with $M = 1$ may be simpler than that of the ML-NO-MAI receiver with $M = 2$ due to the additional antenna and RF/analog circuitry involved in the latter, the CMMSE receiver with $M = 1$ would appear to be a useful alternative to the ML-NO-MAI receiver with $M = 2$.

VI. Conclusion

An adaptive CMMSE receiver for STBC CDMA systems in frequency-selective fading channels was proposed. Analytical and simulation results demonstrated that the proposed receiver can provide a considerable performance improvement over the ML-NO-MAI receiver and existing adaptive MUD schemes for STBC CDMA. In particular, it was shown through simulation that the CMMSE receiver with one receiver antenna could perform better than the ML-NO-MAI receiver with two receiver antennas. The proposed adaptive CMMSE receiver can be applied to both up- and downlink communications operating in either a circuit or a packet mode. In addition, it only requires one spreading sequence for each user and periodically inserted pilot symbols for channel estimation. Accordingly, the CMMSE receiver would appear to be a useful alternative to the existing receivers for STBC CDMA systems. Further work in this area will include modifying the CMMSE adaptation for faster convergence.

APPENDIX
PROOF OF PROPERTY 3

From (8), $\mathbf{w}_{ml}^H \mathbf{R}_{\mathbf{y}_m} \mathbf{w}_{ml} = (\mathbf{c}_l^H \mathbf{R}_{\mathbf{y}_m}^{-1} \mathbf{c}_l)^{-1}$. By using the unitary similarity transformation, $\mathbf{R}_{\mathbf{y}_m} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H = \sum_{i=1}^{N+L-1} \lambda_i \mathbf{q}_i \mathbf{q}_i^H$ and $\mathbf{R}_{\mathbf{y}_m}^{-1} = \mathbf{Q}\mathbf{\Lambda}^{-1}\mathbf{Q}^H = \sum_{i=1}^{N+L-1} (1/\lambda_i) \mathbf{q}_i \mathbf{q}_i^H$, where $\mathbf{Q} = [\mathbf{q}_1 \cdots \mathbf{q}_{N+L-1}]$ is the $(N+L-1)$-by-$(N+L-1)$ matrix whose columns are an orthonormal set of eigenvectors and $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \cdots, \lambda_{N+L-1})$ is the eigenvalue matrix. Then, $\mathbf{w}_{ml}^H \mathbf{R}_{\mathbf{y}_m} \mathbf{w}_{ml} = (\mathbf{c}_l^H \mathbf{R}_{\mathbf{y}_m}^{-1} \mathbf{c}_l)^{-1} = (\sum_{i=1}^{N+L-1} \mathbf{c}_l^H \mathbf{q}_i \mathbf{q}_i^H \mathbf{c}_l \lambda_i^{-1})^{-1} = (\sum_{i=1}^{N+L-1} |\mathbf{c}_l^H \mathbf{q}_i|^2 \lambda_i^{-1})^{-1}$, and $\mathbf{c}_l^H \mathbf{R}_{\mathbf{y}_m} \mathbf{c}_l = \sum_{i=1}^{N+L-1} \mathbf{c}_l^H \mathbf{q}_i \mathbf{q}_i^H \mathbf{c}_l \lambda_i = \sum_{i=1}^{N+L-1} |\mathbf{c}_l^H \mathbf{q}_i|^2 \lambda_i$. Let $f(\lambda_i) = 1/\lambda_i$. Since $f(\lambda_i)$ is convex for $\lambda_i > 0$, from Jensen's inequality [12]

$$\sum_{i=1}^{N+L-1} \left|\mathbf{c}_l^H \mathbf{q}_i\right|^2 f(\lambda_i) \geq f\left(\sum_{i=1}^{N+L-1} \left|\mathbf{c}_l^H \mathbf{q}_i\right|^2 \lambda_i\right). \tag{44}$$

This is rewritten as

$$\sum_{i=1}^{N+L-1} \left|\mathbf{c}_l^H \mathbf{q}_i\right|^2 \lambda_i^{-1} \geq \frac{1}{\sum_{i=1}^{N+L-1} \left|\mathbf{c}_l^H \mathbf{q}_i\right|^2 \lambda_i}. \tag{45}$$

From the fact that $\mathbf{w}_{ml}^H \mathbf{R}_{\mathbf{y}_m} \mathbf{w}_{ml} = (\sum_{i=1}^{N+L-1} |\mathbf{c}_l^H \mathbf{q}_i|^2 \lambda_i^{-1})^{-1}$ and $\mathbf{c}_l^H \mathbf{R}_{\mathbf{y}_m} \mathbf{c}_l = \sum_{i=1}^{N+L-1} |\mathbf{c}_l^H \mathbf{q}_i|^2 \lambda_i$, (45) becomes

$$\frac{1}{\mathbf{w}_{ml}^H \mathbf{R}_{\mathbf{y}_m} \mathbf{w}_{ml}} \geq \frac{1}{\mathbf{c}_l^H \mathbf{R}_{\mathbf{y}_m} \mathbf{c}_l}. \tag{46}$$

Therefore, $\mathbf{c}_l^H \mathbf{R}_{\mathbf{y}_m} \mathbf{c}_l \geq \mathbf{w}_{ml}^H \mathbf{R}_{\mathbf{y}_m} \mathbf{w}_{ml}$. □

REFERENCES

[1] S. M. Alamouti, "A simple transmitter diversity scheme for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451–1458, Oct. 1998.

[2] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space–time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456–1467, July 1999.

[3] A. G. Dabak, T. Schmidl, and C. Sengupta, "Equalization and multi-user detection for space time block coding based transmit diversity (STTD) in frequency selective channels," in *Proc. IEEE Veh. Technol. Conf.*, vol. 2, Sept. 2000, pp. 506–513.

[4] X. Lu and H. Li, "Blind multiuser detection for space-time coded CDMA systems," in *Proc. Int. Conf. Acoustics, Speech, Signal Processing*, vol. 4, May 2001, pp. 2313–2316.

[5] G. Klang and A. F. Naguib, "Transmit diversity based on space–time block codes in frequency selective Rayleigh fading DS-CDMA systems," in *Proc. IEEE Veh. Technol. Conf.*, vol. 1, May 2000, pp. 264–268.

[6] D. Reynolds, X. Wang, and H. V. Poor, "Blind adaptive space–time multiuser detection with multiple transmitter and receiver antennas," *IEEE Trans. Signal Processing*, vol. 50, pp. 1261–1276, June 2002.

[7] S. R. Kim, Y. G. Jeong, and I. K. Choi, "A constrained MMSE receiver for DS/CDMA systems in fading channels," *IEEE Trans. Commun.*, vol. 48, pp. 1793–1796, Nov. 2000.

[8] S. C. Hong, J. H. Choi, S. R. Kim, and Y. H. Lee, "Constrained MMSE-RAKE receivers for DS/CDMA systems in multipath fading channels," in *Proc. IEEE Veh. Technol. Conf.*, vol. 2, May 2002, pp. 532–536.

[9] M. Latva-aho and M. J. Juntti, "LMMSE detection for DS-CDMA systems in fading channels," *IEEE Trans. Commun.*, vol. 48, pp. 194–199, Feb. 2000.

[10] S. Haykin, *Adaptive Filter Theory*, 3rd ed. Englewood Cliffs, NJ: Prentice-Hall, 1996.

[11] H. V. Poor and G. W. Wornell, *Wireless Communications Signal Processing Perspectives*. Englewood Cliffs, NJ: Prentice-Hall, 1998.

[12] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: Wiley, 1991.

Young-Ho Jung (S'98–M'04) received the B.S., M.S. and Ph.D. degrees in electrical engineering from Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, in 1998, 2000, and 2004, respectively.

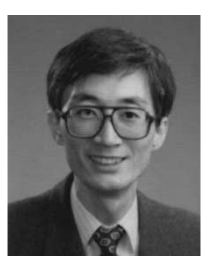
He has been with the Samsung Advanced Institute of Technology (SAIT), Kyungkido, Korea, since 2004. His primary research interests include multiple-input multiple output (MIMO), adaptive signal processing, adaptive modulation, and cellular orthogonal frequency-division multiplexing systems.

Seung-Chul Hong received the B.S., M.S., and Ph.D. degrees in electrical engineering from Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, in 1996, 1998, and 2002, respectively.

From September 2002 to November 2003, he was a Postdoctoral Associate at the Division of Engineering and Applied Sciences, Harvard University, Cambridge, MA. Since 2004, he has been with Samsung Electronics Company, Ltd., Kyungkido, Korea. His research interests include signal detection, channel estimation, multiuser diversity, smart antennas, and ultrawide-band communications.

Seong Rag Kim received the B.S. degree in electronics from the Kyungpook National University, Daegu, Korea, in 1981 and the M.S. degree in electrical engineering from the Korea Advanced Institute Science and Technology (KAIST), Seoul, Korea, in 1985. He received the Ph.D. degree in electrical engineering and computer science from the University of Illinois at Chicago, Chicago, in 1994.

In 1985, he joined the Electronics and Telecommunications Research Institute (ETRI), Taejon, Korea, where he currently is a Principal Member of Research Staff. His current research interests are mainly concentrated on multiple-input–multiple-output (MIMO), smart antenna, multiuser detection, and non-Gaussian signal processing.



Yong H. Lee (S'81–M'84–SM'98) received the B.S. and M.S. degrees in electrical engineering from Seoul National University, Seoul, Korea, in 1978 and 1980, respectively, and the Ph.D. degree in electrical engineering from the University of Pennsylvania, Philadelphia, in 1984.

From 1984 to 1988, he was an Assistant Professor with the Department of Electrical and Computer Engineering, State University of New York, Buffalo. Since 1989, he has been with the Department of Electrical Engineering and Computer Science, Korea Advanced Institute of Science and Technology, Daejeon, Korea, where he currently is a Professor. His research activities are in the area of communication signal processing, which includes synchronization, estimation, detection, interference cancellation, and efficient filter design for CDMA, TDMA, and orthogonal frequency-division multiplexing systems. He is also interested in designing and implementing transceivers.